

# Frege on Judgement and the Judging Agent

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## Introduction

Inspired by discussions with Wittgenstein, Philip E.B. Jourdain sent a letter to Frege in which he asked ‘whether you now regard assertion ( $\vdash$ ) as merely psychological’ (15.1.1914, *WB*, 126). Answering Jourdain, Frege writes in a draft letter that

judging (acknowledging as true) is certainly an inner mental process; but that something is true is independent of the knowing agent, is objective.<sup>1</sup>

However, a bit further on, Frege writes:

If one were to delete the judgement stroke from the premise sentences in the presentation of an inference in my ideography, something essential would be missing.<sup>2</sup>

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<sup>1</sup> (*WB*, 126). Translations are mine, unless indicated otherwise.

On the one hand, it seems that the act of judging is a mental act, depending on the judging agent, and thereby a psychological notion; on the other hand, the judgement stroke, the sign that a judgement has been made, is essential to Frege's logic. Is Frege able to defend the thesis that judgement is essential to logic, but that logic is independent of psychology nonetheless?

I defend the thesis that the distinction between an empirical and a logical notion of judgement is crucial to understanding the judgement stroke in Frege's logic. This claim holds for all periods we may distinguish in Frege's writings, since the judgement stroke is part of his logic throughout those writings, although there is a development in the way he elucidates the notion of a judgemental act. From around 1890, when the distinction between *Sinn* and *Bedeutung* is in place, Frege understands judging as the acknowledging of the truth of a Thought (*Gedanke*), and I take this elucidation to be central to his writings.<sup>3</sup>

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<sup>2</sup> 'Wenn man bei der Darstellung eines Schlusses in meiner Begriffsschrift die Urteilsstriche bei den Praemissensätzen wegliesse, fehlte etwas wesentliches.' (*WB*, 127). Translating 'Schluss' by 'inference' may be misleading as the term 'inference' is now often understood in a non-epistemic sense. See below.

<sup>3</sup> Before he made the distinction, when he was speaking of a judgeable content rather than a Thought, Frege already elucidated judging as acknowledging something as true. Compare (Frege 1882, 58) and (Frege 1879-1891, 7-8, written in the early 80s, cf. Hovens 1997). In a lecture of 1882, Frege says that if I want to assert a content as right, I put the judgement stroke at the left end of the content stroke (Frege 1883, 101). Frege's statement in the *Begriffsschrift* that all judgements are characterized by the predicate 'is a fact' should not be taken in a literal sense, as he immediately adds: 'We see that there cannot be any question here of subject and predicate in the ordinary sense' (*Bs*, §3). The judgement stroke is not a predicate in the *Begriffsschrift*: it has no conceptual content (*begrifflicher Inhalt*), and it has a unique syntax; for example, we are not allowed to put a negation sign in front of the judgement stroke.

I argue that the logical notion of judgement is not to be identified with the empirical notion of judgement (§1). Furthermore, one can raise doubts concerning a transcendental understanding of the notion (§2); and against an abstract, Platonic understanding, as well (§3). In the final section (§4), I explain that the logical notion of judgement is to be understood from a first-person perspective, whereas the empirical notion of judgement is to be understood from a third-person perspective. Readers of Frege have noted certain tensions in his writings. Armed with the distinction between first and third-person perspective, we may view these apparent tensions in a different light. Finally, I claim that the judging agent *can* be understood as a transcendental ego. This raises the question as to how one can explain that the judgements in Frege's ideography are fallible, which I discuss at the end of the paper.

A modern reader, immersed in a model-theoretic account of logic, may find it odd and old-fashioned that Frege gives a prominent place to the notion of judgement in his logic. Although modern logic finds its origin in Frege's logic, his idea of logic essentially differs from a model-theoretic account. The latter is shaped rather by Hilbert's view of logic and his notion of an axiom as a non-epistemic starting-point. For Hilbert, as soon as an axiom system is consistent, it specifies a class of models. Hilbert's view that formal systems are objects of study for meta-mathematical research, and his thesis that meanings can arbitrarily be given to formal systems and their axioms, have had a pervasive influence on the model-theoretic tradition. Because logic thus becomes a mere *calculus*, to which an endless variety of interpretations can be given, syntax takes priority over semantics. However, for Frege, the sentences of the *Begriffsschrift* have a meaning right from the start. It makes no sense to speak of a sentence without meaning. Whereas on the model-theoretic account truth comes in at the level of the meta-language, and has the role of a predicate, for Frege, the logical role of truth cannot be captured by a predicate, and certainly not by a meta-linguistic predicate. On Frege's

view of logic, we do not need a meta-theoretical perspective in which proofs of consistency of the logical system will be given. The fact that our axioms are true, guarantees that no inconsistency will arise. Or, to put it the other way round, if an inconsistency is found, one of the assertions in which a Thought is acknowledged as an axiom should be withdrawn, it being presupposed that nothing is wrong with the inference rules. Hilbert's view was reinforced by the success of logical positivism. Carnap's idea of logical tolerance in logic, in which axioms and inference rules can be chosen arbitrarily, and Schlick's critique of the traditional notion of an axiom as requiring the notion of self-evidence, seem to have given a definite blow to the idea of logic Frege stands for.

However, as important as a model-theoretic account of logic has been in the twentieth century, the philosophical assumptions on which it is based are not unproblematic. In order to function in an account of reasoning, a formal system needs to be acknowledged as correct by the reasoning agent. Premises and conclusions in reasoning are judgements rather than propositions. Where *proposition* is a non-epistemic notion, to be explained, for example, in terms of possible worlds, we cannot understand what judgement is without bringing in the notion of a judging agent. In our reasoning, the conclusion is drawn from the premises by an act of inference. Inference in this sense is agent-dependent, too. Logic as the study of inferences has to be epistemic right from the start. The act of inference needs to bring us to known truths, given that the premises are known. And it can only be knowledge-preserving in this sense, if it is itself an epistemic act.

According to Frege in his mature writings, in every act of inference the premises should be known, thereby bringing us to a known conclusion: all acts of inference are part of an act of demonstration in his ideography. Frege does not make a distinction between demonstration and inference, though it is agreed that he should have done so. For we have to make allowance for inferences drawn from assumptions. An act is an act of demonstration

only if the premises are actually known. By contrast, an act of inference is knowledge-preserving, but even so it may not yield knowledge. Notwithstanding this identification of inference and demonstration, Frege's notion of inference is still of value. As Göran Sundholm has pointed out, Frege's notion of inference, holding among judgements, should not be understood in terms of logical consequence, which holds among non-epistemic propositions, but as standing in a tradition in which logic is primarily used in demonstrations (Sundholm 2012, 944, 945; cf. Sundholm 2009). In this tradition, the act of inference plays a central role. This idea of logic and inference is not only of historical importance. It plays, for example, an important role in Constructive Type Theory as it can be found in the writings of Per Martin-Löf.<sup>4</sup> Frege's thesis that judgement is essential to logic, far from being odd and old-fashioned, makes it possible to understand how logic is to be connected to the notion of judgement and the judging agent.

### **1. Judgement in logic is not an empirical notion**

When studying the act of judgement, one may distinguish two different points of view: one may study judgement from an empirical or from a logical point of view. From an empirical point of view, one understands judgement as an event in the world, to be represented by a predicate. Describing what John does, one may say 'John judges that snow is white.' Judging is here understood as a relation obtaining between John and the thought that snow is white. The sentence can thus be represented as  $Jab$ , where  $Jxy$  stands for the relation  $x$  judges  $y$ ,  $a$  stands for John and  $b$  stands for the thought *that snow is white*. Here, judging is expressed by

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<sup>4</sup> In (Schaar 2011) I explain in what sense the notion of judgemental act is of importance for Constructive Type Theory, and why the first-person perspective ought not to be neglected in logic.

a two-place predicate; it is conceived as a dual relation. The idea that judging is a dual relation has been very influential through theories of propositional attitude ascriptions. The term ‘propositional attitude’ was introduced by Russell in 1918, referring to a theory of judgement he himself had defended earlier.<sup>5</sup> On this account, judging is one of the propositional attitudes or acts, on a par with assuming and wishing. In his analysis of 1918, Russell seems to refer to the notion of psychological assertion present in *The Principles of Mathematics*.<sup>6</sup> According to that view, a proposition may be merely thought of, or it may be the object of judgement and assertion. In a study on Meinong dating from 1904, it becomes clear that on the account of judgement as a dual relation, propositions have only a contingent relation to truth: ‘some propositions are true and some false, just as some roses are red and some white’ (Russell 1904, 75).

The psychological, empirical notion of judgement is thus analysed as *Jab*, and it is this analysis that Russell criticises after 1905. By that time, he no longer considers the *that*-clause to be a semantic unity, for there is no propositional object for which the *that*-clause would stand, as Russell now claims. Instead, he considers judging to be a multiple relation between a judging agent, and the terms involved. Othello’s judgement that Desdemona loves Cassio is represented as *Jabcd*, where *a* stands for Othello, *b* for Desdemona, *c* for loving as term, and *d* for Cassio. As different as the two accounts may be, they agree in considering judging to be

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<sup>5</sup> (Russell 1918, 227). In fact, Russell uses ‘propositional verbs’ as a common name for ‘believe’, ‘wish’, etc., adding that ‘you might call them “attitudes”,’ where ‘them’ refers to the verbs. Strictly speaking, however, ‘attitudes’ does not refer to the verbs, but to what they stand for.

<sup>6</sup> (Russell 1903, §52 and §478). Besides this psychological notion of assertion, Russell acknowledges a logical notion of assertion. The latter is of interest, as it has an internal relation to *truth*. Russell identifies logical assertions with true propositions; cf. (idem, §§478-479). This position is problematic, as Russell himself realized (idem, §38), since the antecedents of hypothetical judgements are not asserted, although they may be true. This point will play a role in the section on Tyler Burge’s interpretation of Frege below.

a relation, an event in the world. We are analysing the judgement of John or Othello, and thus analyse these judgements from a third-person, external point of view. The fact that the judging subject may be denoted by the first-person pronoun 'I' is only an accidental case. The unique character of judgement that is to be captured by the judgement stroke, or the assertion sign in Russell's case, is missing in this explanation of judgement. If one attributes a judgement to Othello by means of the predicate letter  $J$ , one is still in need of a special sign of judgemental force showing that the predicate  $Jxbcd$  is actually attributed to Othello. If one understands the act of judging as a relation, one treats it as an event in the world to be expressed by a predicate, that is, by a function expression. There seems to be no place for a normative notion of truth on this account; judging is just one of the events in the world, being an object of study for psychology and philosophy of mind. The empirical notion of judgement is not essential to logic, at least on the non-psychological account of logic Frege is aiming at. In order to understand the role judgement plays in Frege's logic we need an analysis of judgement from a logical point of view, an analysis in which the relation between judgement and truth is elucidated. We need to analyse the act of judgement itself, not attributions of judgement to others. As David Bell has put it, 'it is to direct discourse, and not to oratio obliqua, that one must look for the key to the nature of human judgement' (Bell 1979, 7).

Recently, Mark Textor has pointed out that judgement is, for Frege, not one of the propositional attitudes. As Wolfgang Kühne puts it, judgement is not an act directed to a proposition (Kühne 2003, 260). We do not judge a propositional object; we judge a proposition *to be true*. According to Textor, Frege's notion of judgement is not that of a dual relation; it is rather a *three-place relation* between a Thought, a thinker and a truth-value (Textor 2010, 647). Textor's main thesis is that for Frege judging is a 'special case of acknowledging the reference of an expression, namely the reference of an assertoric

sentence.’ (Textor 2010, 629). This interpretation is confirmed by Frege’s repeated elucidation of judging as acknowledging the truth of a Thought (*SB*, 34, note; Frege 1918, 62). According to Textor, this means that judgement is a *species* of ontic acknowledgement (Textor 2010, 641). Acknowledging in its ontic use is a *non-propositional* attitude; one accepts an object or a kind of object. Just as one may acknowledge non-real numbers in one’s ontology, one may acknowledge the truth of a Thought. The act of acknowledging is, on Textor’s reading, *factive*: one can only acknowledge what is there. One can acknowledge the truth of a Thought only if the Thought is true.

Textor’s reading of Frege is to be credited insofar as he understands that for Frege judging is not a dual relation. Judging is not simply a relation between a judging subject and a propositional object. Furthermore, Textor understands that, for Frege, the notion of judgement has a special relation to *truth*. However, Textor’s interpretation contains some problems. According to Textor, judging is a ‘mental *relation* between a thinker, a thought, and an object, namely a truth-value’ (idem, 615, abstract; italics mine). If a particular judging, say, that snow is white, would be a relation between a subject, a Thought, and a truth-value, as Textor claims, it should be represented as *Jabc*, where *Jxyz* stands for the relation *x* judges *y* to be *z*, *a* stands for John, *b* stands for the Thought *that snow is white*, and *c* stands for *the True*. However, the notion of judgement that is essential to Frege’s logic is not a relation, for, on Frege’s account, relations are to be represented by a predicate, whereas the act of judgement is not to be represented this way. Predicates are used to describe what happens in the world, but the act of judging that is essential to logic is not an event in the world among other events.

Frege thus needs a logical notion of judgement, to be distinguished from judgement as empirical phenomenon. If, with Textor, one understands judgement as a *mental relation*, one misses an understanding of the role of the judgement stroke. The judgement stroke is a sign that differs essentially from words that are used to describe or express a content. It is used to



*show*, rather than describe that a judgement has been made; it is a *pragmatic* sign of judgemental or assertive *force* (cf. Bell 1979, 97-98). By using the judgement stroke in his logic, Frege goes beyond semantics. Although natural language does not have a special sign for assertive force and the assertive force is often taken to be represented by the predicate, in logic the assertive force has to be separated from the predicate, as Frege puts it in ‘What may I regard as the result of my work?’ (*NS*, 200; *PW*, 184). If one were to express the judgemental force by a predicate, one would miss the essence of Frege’s logic. The judgement stroke is a sign of its own special kind (*GG*, §26, p. 44). In its primordial, logical sense, judgement is an act that cannot be represented in any ordinary sense of that term. Since we do make assertions in our logical system, we need a special sign of assertive force (cf. Frege 1896, 232, orig. 377).

Secondly, on Textor’s interpretation of Frege, an incorrect judgement is impossible: it is impossible to acknowledge a false proposition as true, for one can only acknowledge what is there. Textor takes this consequence of his interpretation to be an argument for his position, given that Frege, on Textor’s reading, does not allow for incorrect judgements. In case the content is false, there may at most be an *attempt* to judge the content to be true. Because ontic acknowledging is factive, we cannot make an incorrect judgement. It is true that, with respect to inference, Frege claims that one can infer only from propositions that are rightly acknowledged to be true. If the premises are not true, we can only speak of a pseudo-inference (*Pseudoschluss*, letter to Dingler, 13.1.1917, *WB*, 30; cf. *WB*, 127). Frege does not seem to be that strict with respect to judgement; he does not speak of a pseudo-judgement if one judges a false proposition to be true.<sup>7</sup> I will come back to the problem of incorrect judgement in my last section.

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<sup>7</sup> And when he speaks of a pseudo-assertion (*Scheinbehauptung*) he has something else in mind (cf. Frege 1918, 63).

As a third objection against Textor's interpretation, one may say that, for Frege, judgement is *sui generis*, a kind of its own. In 'Über Sinn und Bedeutung', Frege says that the act of judgement is one of a kind and incomparable.<sup>8</sup> The elucidations he gives of judgement are not meant as definitions, for there is not a *genus* to which the logical judgement belongs. It is precisely for this reason that judgement cannot be compared to other mental acts. Although Textor acknowledges that judgement is for Frege a logically primitive activity, nevertheless, by taking judgement to be a *species* of the higher *genus* ontic acknowledgement, he is committed to the thesis that Frege's notion of judgement is to be defined in terms of *genus* and specific difference. By contrast, ontic acknowledgement is rather to be understood as a special kind of judgement; acknowledging the existence of witches is judging that the concept of witch is not an empty concept; at least, on a Fregean view. However, sometimes it seems to be correct to describe judging as a special kind of mental act. In such cases, we are considering judgement as one of the propositional attitudes, and thus as a species belonging to a genus. Here, however, our aim is to analyse the empirical notion of judgement; we are not elucidating the primitive notion of judgement that is essential to Frege's conception of logic. If one focuses on questions within the philosophy of mind, judging may be understood as one mental act among others, but the importance of judgement for logic cannot be captured this way. From a *logical* point of view, judgement is a primitive, indefinable notion.

## 2. Judgement as a transcendental notion

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<sup>8</sup> 'Das Urteilen ist eben etwas ganz Eigenartiges und Unvergleichliches' (*SB*, 35; cf. *NS*, 16). Although the translation of 'ganz Eigenartig' as 'quite peculiar' (Frege 1997, 159) is not wrong, 'Eigenartig' primarily means *being one of a kind*, which makes more sense here.

If Frege's notion of judgement is not an empirical notion, may a transcendental interpretation of Frege's writings help us here? Is the logical notion of judgement involved in the question as to what belongs to the condition of the possibility of knowledge? Or, is the judging agent perhaps to be understood as a transcendental ego? Gottfried Gabriel has rightly pointed to the Neo-Kantian background of Frege's logic. According to him, Frege argues for the acceptance of basic logical laws in a transcendental way: 'the acceptance of basic logical laws forms the condition of the possibility of our judging.' (Gabriel 2013, 289). Gabriel supports this thesis by means of a passage from the preface of the *Grundgesetze* (idem, 288), of which only sentence (1) is quoted by him:

(1) [W]e must recognize it [a basic law such as the law of identity] if we are not to throw our thought into confusion and in the end renounce judgement altogether. (2) I do not wish to either dispute or endorse this view and only remark that what we have here is not a logical implication (*Folgerung*). (3) What is given is not a ground of *being true*, but of our *holding as true*. (4a) And furthermore, this impossibility of our rejecting the law does not prevent us from supposing that there are beings who do reject it; (4b) but it does prevent us from supposing that these beings are right in doing so; (4c) it also prevents us from doubting whether we or they are right. At least this goes for me. (5) If others dare to recognize and doubt a law in the same breath, then it seems to me like trying to jump out of one's own skin (*aus der eignen Haut zu fahren*), against which I can only urgently warn. (Frege *GG*, xvii; Frege 1997, 204)

In sentence (2), one can read that Frege does not endorse the view expressed by sentence (1). As he argues in sentence (3), the point gives a ground merely for our holding something to be true, not for a law of truth; it gives nothing but a psychological justification for the logical

laws. In sentence (1) Frege is presenting not his own view, but an existing opinion (view, *Meinung*), most likely the position of certain Neo-Kantians. A further argument against a transcendental reading of the logical laws is presented in sentence (4a): as it is possible to assume that there are beings rejecting these laws, the basic laws cannot form a condition for the possibility of any judgement. The point Frege is making is rather that our acknowledgement of a logical law prevents us from acknowledging that those who reject the law do so rightly (4b). Sentence (5) does seem to support a transcendental reading. Nevertheless, I will explain in my last section below that sentence (5) is to be understood by means of the idea of a first-person perspective.

The main argument, though, against a transcendental reading of these laws is that the basic laws, and logical laws in general, are, for Frege, descriptive laws. A basic law is a most general scientific truth, and such a law is not in need of a transcendental argument, because one is, on Frege's view, entitled to acknowledge it to be true as soon as one understands the basic notions involved; identity is such a basic, primitive notion.

However, Frege's ideography goes beyond a mere description of logical laws. The judgement stroke escapes a purely descriptive view on logic. It is a sign of assertive force, and there is no assertive force without an agent. If one understands the judgement in logic as a non-empirical notion, the judging agent may be understood as a non-empirical ego. Are we entitled to call the judging agent a transcendental ego? If we use the term in a strict Kantian sense, in which the transcendental ego is to account for the unity of consciousness, and for the unity of the judgemental content, as well, it seems that the transcendental ego plays no role in Frege's logic. However, we can take the *transcendental ego* in a more general sense as the ego that cannot be described by any predicate, but is needed as a presupposition for the possibility of any ideography. Can we call the agent of judgement in a logical sense a transcendental ego in this sense? If we do so, how can we explain the fact that a mistake is

made in Frege's ideography as presented in the *Grundgesetze*? The Basic Law (V) is preceded by a judgement stroke, but, as Frege came to realize, it implies a contradiction. If the judgement stroke is a sign of judgement made by a transcendental ego, no mistake seems to be possible. I come back to this problem at the end of the final section.

This interpretation of Frege, in which the primitive notions of judgement and truth cannot be captured in empirical terms, seems to be confirmed by Wayne Martin's reading of Frege, when he compares the role that *truth* plays in Frege's writings with the role it plays in Heidegger's thought. Martin brings out the notion of truth as a unique presupposition for logic: 'logic presupposes and cannot explicate a pre-logical understanding of truth.' (Martin 2006, 100). This Heideggerian thesis Martin also finds in Frege's writings: 'Frege insists ... that the most basic logical notion is neither concept nor judgment but truth ... Here, Frege effectively approaches the central claim of Heidegger's mature philosophical logic.' (idem, 102). It is true that in early and later writings Frege has claimed that the aim of logic is to know the laws of truth (Frege 1918, 58); the logical laws are a development of the content of the word 'true' (Frege 1879-1891, 3). However, judgement seems to play an equally important role, as Frege characterizes the laws of logic both as the laws of truth and as the laws of judgement. The point of logic is equally to give the laws of correct inferences (idem, 3), as to give the laws of judgement (Frege 1897, 157). Essential to logic is the truth claimed in the act of judgement, truth as we use it in our practice of judgements and inferences. As Frege explicitly claims in 'My Basic Logical Insights [1915]', the essence of logic cannot be found in the word 'true', but lies in the assertive force (*NS*, 272; *PW*, 252). When Frege calls the logical laws *laws of truth*, he is speaking about truth as it is claimed in the act of judgement. It is for this reason that the logical laws can be understood both as *laws of truth* and as *laws of judgement*.

A comparison between Frege and the earlier phenomenological tradition seems to be more relevant, at least for the interpretation of Frege proposed in the last section below, in which the first-person perspective plays a central role. Like Frege, Franz Brentano understands the act of judgement to be logically primitive: it cannot be explained as a special kind of presentation. Because Brentano's account of judgement is a non-propositional one, a meaningful comparison is to be given on a more general level.<sup>9</sup>

According to Brentano, a conceptual understanding of judgement is to be obtained by descriptive psychology, later to be called 'phenomenology'. Descriptive psychology is distinguished from genetic psychology, which studies the causal relations between mental events. Descriptive psychology gives us *a priori* truths; for example, every judgement is dependent upon an act of presentation. The logician needs to take into account these truths, and descriptive psychology, or phenomenology, is in this sense foundational to logic.

Frege's elucidations of primitive terms differ in an important way from the *a priori* truths given in the phenomenological tradition. Whereas for Brentano and Husserl descriptive psychology or phenomenology is a science that precedes logic, for Frege logic is the foundational science. Primitive notions, such as judgement and truth can only be understood by relating them to each other in *elucidations*. Frege's claim that judging is acknowledging the truth of a *Gedanke* is such an elucidation. What precedes logic is propaedeutic, consisting

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<sup>9</sup> A more extensive comparison between Frege and the phenomenological tradition has to wait for another occasion. As an example, in 1884, one of Brentano's students, Anton Marty, gives an evaluation of the first sections of Frege's *Begriffsschrift*, and criticises the idea that every judgement presupposes a connection of presentations (*Vorstellungsverbindung*). According to Marty, the 'is' and 'is not' ('+' and '-') signs of Brentano's logic are on a par with Frege's judgement stroke (cf. Marty 1918, 56 ff), a point that is in need of further investigation.

of elucidations, sharply to be distinguished from *a priori* truths, and from definitions as well, which do have a role within logic as science. For Frege, the foundation of logic consists in axioms and definitions, not in elucidations given in natural language. The aim of elucidations is a practical one, and good will and some guess work is needed from those who try to understand them (Frege 1906, 288, orig. 301; Frege 1914, *NS*, 224; cf. Weiner 2010, 58-61). Elucidations are to be taken with a pinch of salt.

### 3. Judgement as ideal notion

Tyler Burge claims that Frege's use of *reason* (*Vernunft*) is not to be understood in a transcendental way. On Burge's interpretation an ideally rational mind plays a central role in Frege's account of objectivity. The task of logic and mathematics is an investigation of *the* Mind, not of minds ('die Erforschung des Geistes ..., nicht der Geister', Frege 1918, 74; Frege 1997, 342). In the *Grundlagen*, Frege makes a distinction between what is actual ('wirklich') and what is objective (*GLA*, §26; cf. *GG*, II, §74). Whereas the earth is actual, because it may stand in causal relations, the equator is objective, but not actual.<sup>10</sup> Frege explains the objectivity of what is not actual as an objectivity that is independent of sensations and presentations, but not independent of reason (*Vernunft*). A certain number (*Anzahl*) is dependent on the (objective) concepts we use to carve the world, but this makes, for example, the number of lime trees not less objective. The result is not created in thought, but acknowledged by thought. Reason as it is used here by Frege is not to be identified with the

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<sup>10</sup> 'The world of actuality is a world in which this acts (litt.: works) on that' ('Die Welt des Wirklichen ist eine welt in der Dieses auf Jenes wirkt', Frege 1918, 76; Frege 1997, 343). Alternative interpretations of Frege's concept of *wirklich* are discussed in (Künne 2010, 374 ff, and 536-541).

rationality of individual acts, for the number of lime trees at a certain moment is not dependent upon any individual judger, however rational he or she may be. The realm of Thoughts belongs to the realm of reason, too, as Thoughts are objective, but not actual. Thoughts are independent of individual minds, but not independent of reason.

Can one extend this role of reason in Frege's writings to his view on judgements? According to Burge, Frege's logic makes use of a notion of judgement as an idealized abstraction, as not being acts of individuals: 'Individuals can instantiate these judgements through their acts of judgement, but the abstract judgements themselves seem to be independent of individual mental acts.' (Burge 2000, 357). Burge's reading of Frege is to be applauded insofar as he recognizes that a non-empirical notion of judgement plays a role in Frege's logic. However, on Burge's reading, Frege seems to allow for a Platonic realm of judgements, besides the realm of Thoughts he already acknowledges. What are these judgements, on Burge's reading? Are they abstract, possible acts of judgement associated with ideal logical thinking (Burge 1998, 319, note 2)? Or are they nothing but true propositions? The latter interpretation cannot be right, as Burge himself acknowledges, since we are in need of a notion of non-judged truths to account, for example, for true, but non-judged antecedents in hypothetical judgements.<sup>11</sup> And there is no textual evidence for the idea that judgements in the ideography are abstract, possible acts of judgements, as we will see below. The judgements in the *Begriffsschrift* seem rather to be made by its author, as Frege says: 'With this judgement stroke I close a sentence, ... and the content of the sentence thus closed I assert as being true by the same sign' (Frege 1896, 232, orig.: 377; cf. Frege 1891, orig., 22). It is true that in the *Grundgesetze* Frege formulates the point without mentioning himself as asserter: 'in " $\vdash 2^2 = 4$ " it is asserted that the square of 2 is 4.' (Frege 1893, § 5; cf.

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<sup>11</sup> (Burge 2000, 357). Cf. Russell's point in footnote 6.



Frege 1891, orig.: 22, note 7). But as will be discussed in the next section, it is essential to Frege's logic that the asserter is not mentioned in the use of the sign for assertive force.

Burge takes Frege's 'judgements of pure thought' (*Urtheile des reinen Denkens*) in §13 of the *Begriffsschrift* to be idealized acts of judgement (Burge 1998, 319, note 2), but I can find no support for this reading. When Frege speaks of 'judgements of pure thought', he has non-empirical judgements in mind, judgments of logic and arithmetic; being non-empirical relates to the *content* of these judgements. Therefore, the fact that Frege speaks in this context of 'judgements of pure thought' does not give evidence for the thesis that Frege is speaking here of idealized acts of judgement independent of any individual judger.

Burge claims that Frege was interested in judgement as norm-yielding form, not in judgement as human activity (Burge 1992, 311). Judgement itself is not norm-yielding, though; it is the truth aimed at in judgement that yields the norm. It is true that Frege is not interested in judgement as a mental process, but this does not imply that he is speaking of an altogether different kind of judgement, of judgements made by an ideally rational mind. What is unique is the perspective from which these judgements are studied: they are studied from a logical point of view. There are not two kinds of judgements, as Burge assumes: human judgements that happen at a certain time and place, and abstract idealized judgements outside space and time. There is only one kind of judgement, namely, the human, fallible judgement, but it may be looked at from two different perspectives: as a phenomenon in the empirical world, to be studied by psychology, and as a logical phenomenon, standing under objective logical norms constituted by the norm of truth and the laws of truth. Apart from the final judgement that awaits us all on Judgement Day, judgement is a fallible affair. The judgement stroke in Frege's ideography is a sign of judgemental force, and, as said in § 2 above, there is no judgemental force without a judging agent. By using the judgement stroke in front of the logical axioms and theorems in his ideography Frege claims that they are true, and known by

him. As soon as Frege acknowledged that there are doubts about Basic Law (*V*), the truth claim had to be withdrawn, as we can see from the presentation of the law in the afterword of the *Grundgesetze* from October 1902: the ‘law’ is no longer preceded by a judgement stroke. What is left is nothing but the empirical fact that Frege once judged the content to be true. The judgement stroke may still be present in front of the sentence expressing ‘Basic Law’ (*V*) in one’s copy of the *Grundgesetze*, but it has lost its proper function. Wittgenstein’s remark in the *Tractatus* (4.442) that the judgement stroke in Frege’s ideography only shows that Frege holds the relevant proposition to be true, applies to the situation in which the judgement stroke has lost its proper function. Only then do we take it as a sign of an empirical fact: Frege once held the proposition to be true.

#### **4. Judgement from a first-person point of view**

In the previous sections, Frege’s notion of judgement has been explained along a *via negativa*: the logical judgement is not to be studied from a third-person, external point of view, as an event in the world, nor is it to be understood as an idealized abstraction. Furthermore, doubts have been raised as to whether the logical judgement can be understood in transcendental terms. In this section, I suggest that Frege’s logical notion of judgement is best seen as a first-person notion. The relation between the first-person point of view and assertion can be explained by means of Moore’s paradox. This explanation also shows that the first-person point of view is the point of view of the first-person *present*. Although Moore’s sentence ‘It’s raining, but I don’t believe that it is.’ may be true, the assertion of the sentence creates a paradox. Furthermore, the paradox does not arise if the grammatical first-person present is not in place. No paradox arises when one asserts ‘It rains, but John doesn’t believe it’, or ‘It was

raining, but I didn't believe it'. Even when I assert 'It rains, but Maria van der Schaar doesn't believe it', the paradox does not arise, because I may not realize that I am the bearer of that name. The paradox arises only when the asserter refers to himself by the first-person indexical, while using the present tense. An interesting account of the paradox is given by Jonathan Adler and Bradley Armour-Garb. In their argument they make use of what I will call *the AA-G principle*: 'For you fully to believe *that p* is for it to be true *that p* (from your first-personal point of view)' (Adler and Armour-Garb 2007, 153, 154). In a sincere assertion *that p*, they claim, one expresses one's belief *that p*. In one's assertion of the Moore sentence, one expresses: *I believe [that it rains, and that I do not believe that it rains]*. From this it follows: *I believe that it rains*, and, *I believe that I do not believe that it rains*. Because of the AA-G principle, it follows from the latter that *I do not believe that it rains*, which is in explicit contradiction with *I believe that it rains* (Adler and Armour-Garb 2007, 147-149). Without proposing an analysis of Moore's paradox here, the analysis given above is of interest to our topic, for it makes use of the idea of a first-person point of view. It uses the idea of a first person without bringing in the idea of first-person access to one's beliefs. In contrast to other explanations of Moore's paradox, the truth of the principle 'If I believe that *S*, then I believe that I believe that *S*' is not presupposed. In general, the idea of a first-person perspective is not to be identified with the idea of (privileged) first-person access to one's own thoughts and feelings.

The central point of the idea that Frege understands judgement not only in a logical sense, but also from a first-person point of view, is that the ideography is essentially a first-person engagement: each of us can make use of the ideography as a calculus only if we have made

the axioms and inference rules evident to ourselves.<sup>12</sup> In this sense, the calculus is essentially embedded in the ideography as universal language, as a science of logic. By putting the judgement stroke in front of an axiom, the agent claims not only that the Thought is true, but that anyone who understands the Thought, thereby acknowledges it as true, and is thus entitled to use it as an axiom. By putting the judgement stroke in front of a theorem, the agent claims that anyone who knows the axioms, and has made the relevant inference rules evident to himself is entitled to use the theorem as a logical law. These judgements are thus made from a first-person perspective, but they are non-personal at the same time.

This thesis can be extended to assertive force in general, and some modifications of assertive force may count as first-person and non-personal, too. When one says in a discussion ‘I doubt that what you say is true’, one is generally not thereby expressing one’s subjective feelings of doubt. It is rather claimed that there are reasons to doubt that the content is true. Such a doubt is not personal, although it is first-person insofar as the reasons for doubt are judgements made from a first-person perspective. In general, though, the judgement stroke differs in this respect from other signs of force. When one uses a sign intimating a certain wish, one does not claim that anyone who understands the content will have the same wish. In contrast to judging, wishing is personal.

The distinction between a first-person and a third-person perspective, and, in general, the distinction between a logical and an empirical notion of judgement, may help us to explain some tensions in Frege’s writings. On the one hand, Frege claims in ‘Die Verneinung’ that a

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<sup>12</sup> One should make the mode(s) of inference evident to oneself by means of the explanation given in *Bs*, §6 or *GG*, §14 ff., which is possible as soon as one has understood the basic notions involved (*Bs*, §5). The act of inference in which the mode of inference is applied will thereby be an epistemic act. The question how the idea of a first-person perspective may clarify Frege’s notion of evidence and justification deserves another paper.

judgement is an act of judging, as a jump is a deed of jumping, and, one cannot fully understand the act without knowing the agent:

If a judgement is an act, it happens at a certain time and thereafter belongs to the past.

With an act there also belongs an agent, and we do not know the act completely if we do not know the agent. (Frege 1919, 151, note 4; Frege 1997, 354)

On the other hand, in the draft letter to Jourdain, Frege writes:

If I assert something as true, I do not want to speak about myself, about a process in my mind. And in order to understand it, one need not know who asserted it. (*WB*, 126, 127)

Frege says both that one cannot understand the judgemental act without knowing who the agent is, and that one *can* understand an assertion without knowing who made the assertion. Equipped with the distinction between a logical and an empirical notion of judgement, we may say that when judgement is compared to jumping, happening at a certain time, and then belonging to the past, Frege thinks of judgement as an empirical phenomenon, to be represented by the predicate  $Jxy$ , in which case we have to add a name for the judging agent in order to fully understand this particular event of judging. In contrast, in the passage in the draft letter, Frege speaks of judgement from a logical point of view, to be represented by the judgement stroke. In order to understand the assertion, we do not need to know who made it, although the agent is relevant insofar as he is responsible for the assertion made. The sign of judgemental force is not to be accompanied by a name of the judging agent. When the descriptive part of the assertion contains the first-person indexical, the agent knowingly

speaks about himself; only in such special cases do we have to know who made the assertion to fully understand it.

Another tension in Frege's writings is created by the fact that Frege speaks of the Thought sometimes as the object of judgement, sometimes as the content of judgement. As in Russell's analysis of propositional attitudes, in the second half of Frege's 'Über Sinn und Bedeutung', thinking and believing are understood as relations between a subject and a Thought.<sup>13</sup> In the sentence 'Copernicus believed that the movements of the planets are circles', the *that*-clause refers to the Thought that the movements of the planets are circles, and the relation of believing is to be expressed by a two-place predicate. Thoughts may thus be understood as objects. Belief, opinion, being convinced, judging and understanding are treated here as having a Thought as their *object*, because these propositional attitudes are understood from a third-person point of view.

In Frege's elucidations of the logical notion of judgement, the Thought is understood to be the *content* of the judgement, not something to which the judgement is directed, but what directs the judgement to the truth-value of the Thought. The judgement made manifest by the assertion 'John is a football player' is not about the Thought that John is a football player; it is about John and the concept of being a football player precisely insofar as they contribute to the truth-value of that Thought. From a first-person perspective, the Thought is the *content* of one's judgement.

The distinction between the logical judgement and the empirical judgement can also be used to clarify the question whether the act of judging is, for Frege, an act of knowing, resulting in a piece of knowledge. On the one hand, Wolfgang Kühne and Michael Kremer

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<sup>13</sup> Frege differs from Russell in his analysis of orders and wishes. For Frege, in 1892, the *that*-clause in the sentence 'Napoleon ordered that the troops should retreat' is not referring to a Thought, but to an order, differing from a Thought in not being a bearer of truth and falsity. The imperative sentence itself, uttered by Napoleon: 'Retreat the troops!' has as its *Sinn* the order to retreat the troops, while not having a *Bedeutung* (SB, 38, 39).

rightly claim that Frege allows for the fact that someone may judge a falsehood to be true: ‘What is true, is true independent of our acknowledgement. We can make mistakes.’ (Frege 1879-1891, 2). The translation of ‘anerkennen’ as ‘recognizing’ has perhaps played a misleading role, because ‘recognizing’ has a clear epistemic meaning, whereas ‘anerkennen’ is probably not to be understood in an epistemic sense (Künne 2010, 430). On the other hand, Mark Textor, Wolfgang Carl, and Tom Ricketts have claimed that Frege uses in his logic a strict notion of judgement: we cannot judge a falsehood. Ricketts does not attribute to Frege the view that we are infallible in our judgements. The point is rather that Frege would not use the term ‘judgement’ in case the content is false (Ricketts 1996, 131). One of Ricketts’ arguments is that Frege uses ‘judgement’ (*Urteil*) in contexts where the Thought is true, while using ‘holding true’ (*Fürwahrhalten*) in other contexts.

Ricketts is right to hold that there is some system in Frege’s terminology. In the first place, although this is not explicitly noted by Frege or Ricketts themselves, judging (*Urteilen*) is an act, while holding true (believing, *Fürwahrhalten*) is a state or disposition. Early Frege explains inference as an act of judgement justified by judgements already made. Inference is rightly explained as a special case of the *act* of judgement, and is not understood in terms of belief as a mental state. Secondly, Frege distinguishes in the ‘Logik’ (1897, *NS*, 157; *PW*, 145) laws of judgement from laws of holding true or believing. The normative laws of judgement are the laws of truth, the logical laws. The laws of holding true are empirical laws about how people think. There is no logic of belief on Frege’s account, because he understands the laws of belief to be empirical laws about how people think. In the *Preface* to the *Grundgesetze*, and in ‘Der Gedanke’, Frege no longer uses the term ‘laws of judgement’ when he speaks of the logical laws, afraid that this might lead to confusion. He still uses, though, the term ‘laws of holding true’ for the empirical laws, in contrast to the logical laws of truth: ‘I understand by logical laws not psychological laws of holding true, but laws of

truth' (*GG*, xvi). Although Frege's terminology is not fixed, in most of the important passages the term 'judgement' is used by Frege for the logical notion, whereas he uses the dispositional term 'holding true' in a psychological, empirical sense. We thus see that the terminological distinction between 'judging' and 'holding true' reflects a distinction between the logical and the empirical point of view.

The thesis that we cannot judge a falsehood is supported by those passages in Frege's writings in which no conceptual distinction between judgement and knowledge is acknowledged where this would be relevant. In 'Über Sinn und Bedeutung', Frege writes that we obtain a piece of knowledge only when the Thought is combined with its truth-value: judging is an advancing from the Thought to *its* truth-value (*SB*, 35). And at the end of his life, he writes: 'A piece of knowledge (*Eine Erkenntnis*) arises, because a Thought is acknowledged as being true.' (*NS*, 286). Furthermore, within Frege's logic, each judgement should be known. The judgement sign is a sign that the content is acknowledged to be true. At the same time, within the ideography, one is *entitled* to make the judgement only if one *knows* that the content is true. For the asserted content is either an axiom, which is known as soon as one understands the content, or a theorem, which is known because one has justified it by means of known premises in an epistemic act of inference.

The distinction between first and third-person perspective may be used to clarify the lack of conceptual distinction in these places between judgement and knowledge. When we look at judgement from a third-person point of view, there is a conceptual difference between, on the one hand, judgement or belief, and, on the other hand, knowledge, but from a first-person point of view one takes one's judgement to be true and justified, that is, as knowledge.<sup>14</sup> The central role of the first-person perspective in the notion of judgement in

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<sup>14</sup> Markus Stepanians has already noted that a judgement represents itself phenomenologically as a knowing from the point of view of the first-person present (Stepanians 1998, 105).



Frege's ideography may explain that in crucial passages no conceptual space between judgement and knowledge is acknowledged. It is only from a third-person perspective that Frege can say: once I judged the fifth basic law to be true, but I now understand that the act cannot have resulted in a piece of knowledge. There is no reason to think that Frege would deny that he had made a judgement. From a third-person perspective, he does allow for a conceptual distinction between judgement and knowledge.

The distinction between the first and the third-person perspective may also help to elucidate two passages in Ricketts' paper that may puzzle the reader when put together. On the one hand, Ricketts claims that Frege does not endorse Moore's and Russell's theory of judgement, in which judgement is understood as a binary relation between minds and propositions (Ricketts 1996, 130). On the other hand, he says: 'Construing the clauses of indirect discourse to be proper names, he [Frege] takes judging to be a relation that holds between cognizers and thoughts.' (*idem*, 139). Whereas in the first passage Ricketts speaks of Frege's logical notion of judgement, the context of the latter claim makes it clear that the judgement as a propositional attitude is at stake, that is, judgement from a third-person, external point of view. Frege endorses the thesis that judgement is a dual relation if it is understood as an event in the world, to be represented by a predicate, but he does not do so if judgement is understood in a logical sense.

Finally, the distinction between first-person and third-person perspective may be used to clarify a difficult passage in the draft letter to Jourdain mentioned earlier:

Whoever understands a sentence uttered with assertoric force adds to it his acknowledgement of the truth. If a sentence uttered with assertoric force expresses a false Thought, then it is logically useless and strictly speaking, incomprehensible.<sup>15</sup>

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<sup>15</sup> (WB, 127). I owe this reference to Michael Kremer.

Frege is not denying that we can understand a sentence  $S$  expressing a false Thought, which is uttered by someone with assertive force. Viewing judgement from a third-person point of view, we may say ‘He judges that  $S$ ’, and this makes sense when the Thought expressed by  $S$  is false. What Frege *is* denying is that we can understand the assertion from a first-person perspective. From a first-person perspective, that is, from a logical point of view, understanding a declarative sentence uttered with assertive force is grasping the Thought expressed by the sentence, *and* acknowledging it to be true. Both the wider context of the passage and the phrase ‘logically useless’ show that assertion is to be understood in a logical sense. Understanding assertion in this sense means that one is able to make use of it for oneself as a premise: ‘Only after a Thought has been acknowledged by me as true, can it be a premise for me.’ (*WB*, 118, remarks for Jourdain, probably from 1910). If logic is essentially done from a first-person perspective, one can also understand why Frege repeatedly says that one’s premises have to be true. Applying to judgement the AA-G principle mentioned at the beginning of this section, one gets: for one to judge that  $S$  is for it to be true that  $S$  (from one’s first-person point of view).<sup>16</sup>

Michael Kremer has asked what precisely is taken to be incomprehensible in the passage quoted above. Is it ‘(a) the sentence  $p$ , which happens to express a false Thought, and to be uttered by speaker  $S$  with assertive force’, or is it ‘(b) the sentence- $p$ -uttered-with-assertive-force-by- $S$ ’? (Kremer 2000, 567-568). He rightly answers that it is (b) that is intended: understanding (b) requires ‘a sharing not only in thoughts grasped, but also in judgements’ (*idem*, 568). I subscribe to Kremer’s interpretation so far. In order to explain that, on Frege’s account,  $p$  needs to be true, Kremer makes use of the idea that the purpose of

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<sup>16</sup> This leaves untouched the criticism mentioned in the introduction that Frege’s view of logic does not allow for drawing inferences from assumptions.

assertion is, for Frege, ‘to give the hearer H a new starting point for inferences, which H can use to acquire further knowledge.’ (idem). And this purpose can only be fulfilled if the Thought expressed is *true*. Kremer concludes that assertion ‘is [for Frege] an act that has its place in the joint venture of science, of adding to and building up a structure of knowledge’ (idem). However, I do not think that this conclusion is supported by the passage, or by Frege’s writings in general. As I read Frege, his notion of knowledge and science is primarily first-person. For Frege, an assertion is primarily the manifestation of an act of judgement. A mathematician can never use a conclusion proven by someone else without having himself acknowledged each of the premises and each mode of inference used in the process of demonstration. The social aspect of science is grounded rather in the fact that judgements made by different people may have the same Thought as content. A Thought may be the common property of many (*SB*, 29), but it can only become a common property by being actualised in particular acts of thinking, questioning and judging. Our aim in science is the truth or falsity of the Thought, not the Thought as such (idem, 33, 35). Judgements thus play a crucial role in science. Frege’s point in the passage is that I cannot use a Thought as a premise for making further inferences unless I myself have acknowledged it as true. ‘The acknowledgement of the truth of the premises is necessary.’ (Letter to Dingler, 31.1.1917, *WB*, 30). As I understand him, Frege is claiming in the passage above that if one sees that others are drawing conclusions from judgements containing false Thoughts, one cannot logically make sense of this. As soon as one understands that a mistake has been made, the judgement no longer has any logical role to play. Whereas Kremer takes Frege’s project of logic as science to be a ‘joint venture’, I take a first-person notion of understanding to be central to Frege’s idea of logic as science.

In what sense would it be possible to understand Frege’s logic as a joint pursuit? In a dialogical context, the judgement stroke is understood as a sign of assertive force. The fact

that for Frege, judgement and assertion are counterpart notions, makes it possible to introduce ideas from speech act theory to elucidate his notion of judgement. In a dialogical context, the judging agent is held responsible for the assertion made. We have seen that the judgement stroke does not describe a situation in the world. It is a sign of judgemental or assertive force: a pragmatic, not a semantic sign. Frege's theory of judgement is thus naturally embedded in a pragmatic account of assertion and inference, but his logic is primarily first-person nonetheless. Whereas we would now explain the inner notion of judgement in terms of the outer notion of assertion, this order of explanation is not endorsed by Frege. Notions of speech act theory for Frege are secondary in the order of explanation. The task of the logician is to find the laws of logic, the basic laws of truth, and to present the logical truths that can be derived from them in the right order, so that one can understand why they are true. The creative work of the logician consists in finding and elucidating new concepts, through which a new axiom may be acknowledged, and new inferences become possible. Here the logician can only give some elucidations in the hope that others will grasp the primitive concepts for themselves, and thus be able to understand why the relevant Thought is a basic law and can therefore be used as an axiom. In this way Frege hoped to show that all truths of arithmetic can be demonstrated by logical means alone.

We have seen that the presence of the judgement stroke does not make Frege's logic psychologistic, since the logical notion of judgement is to be distinguished from the empirical notion. But now we are confronted with a related problem because of the introduction of the idea of a first-person perspective: Does not the idea that Frege's logical notion of judgement is to be understood from a first-person perspective imply a form of relativism? If there is no distinction between being judged to be true and being true, from a first-person point of view, there is indeed a threat of relativism. However, Frege says in the *Grundgesetze* that being true

is not the same as being held to be true (*Fürwahrgehaltenwerden*) and that it is not to be reduced to the latter (*GG*, xv). Given the fact that error is possible, one cannot but acknowledge that our holding something to be true is not the same as being true. Even from a first-person perspective, one may acknowledge the general fact that error is possible. The fact, though, that we have actually made a mistake can only be determined by invoking a third-person perspective. The fact that Frege uses the term ‘being held to be true’ shows that he is speaking here of judgement from a third-person point of view. Presented with a conflict between our actual judgement, which is made from a first-person perspective, and our former judgement, which is considered from a third-person perspective, we may conclude that our former judgement is based on a mistake. In the presentation of his ideography as a science, there is, for Frege, no conceptual distinction between a Thought being acknowledged as true and the Thought being known. However, after having read Russell’s letter, Frege understands that a contradiction is implied by Basic Law (*V*). Now, he considers the ideography as presented in the *Grundgesetze* from a third-person point of view, and decides that he can no longer use the judgement sign in front of ‘Basic Law’ (*V*).

We are now able to answer the question put forward in section 2. How can we understand the judging agent in Frege’s ideography as a transcendental ego, if these judgements are fallible, that is, if mistakes are possible? From a first-person perspective, the judging agent may indeed be understood as a transcendental ego. From this perspective, being acknowledged as true conceptually involves being true; the question of fallibility does not arise. However, Frege does not define truth in terms of judgement; he is a realist with respect to truth. When we look at our past judgements, that is, when we see them from a third-person point of view, we understand that error is possible; *being held to be true* does not involve *being true*. It is thus that we acknowledge the fallibility of our judgements, and of the ideography as presented. In section 2, I pointed out that in the preface to the *Grundgesetze*

Frege does not give a transcendental argument for the logical laws. As we have seen, Frege did not deny that there may be others rejecting the logical laws we have acknowledged. But he did deny, in sentence (5) in the quotation given there, the possibility that one can acknowledge and doubt the same logical law at once, that is, from a first-person perspective.<sup>17</sup>

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