Strings and AdS/CFT at finite density

Goykhman, M.

Author: Goykhman, Mikhail
Title: Strings and AdS/CFT at finite density
Issue Date: 2014-06-24

Chapter 5

Discussion

In this concluding chapter we are going to look at the problems solved in this thesis in the general context of the AdS/CFT correspondence and strongly coupled quantum field theories. We are going to discuss strongly coupled IR phases of a non-abelian gauge quantum field theory with the fundamental matter. Consider $SU(N_c)$ gauge theory coupled to N_f flavors of quarks and N_f flavors of anti-quarks. Let us start in the UV where the gauge interaction is weak and one can rely on perturbation theory to compute the beta function for the gauge coupling. If $N_f/N_c < 11/2$ then the gauge coupling grows as one moves towards the IR, *i.e.*, the theory is asymptotically free. If $N_f/N_c = 11/2 - \epsilon$ and $\epsilon \ll 1$, then the theory flows to a perturbative IR fixed point. It is called a Banks-Zaks fixed point.

The question one can ask is whether there is a finite range of values of $x = N_f/N_c$ for which the IR theory is in a non-abelian Coulomb phase. In real-world QCD we have $N_c = 3$ and $N_f = 6$, so that $x = 2$. The IR phase of this QCD is confining. It has been suggested in the literature that for $x_* < x < 11/2$ QCD flows to the IR fixed point, with $x_* \simeq 4$. It is a well-known non-perturbative problem to find the value of x_*, and the exact solution to this problem of a conformal window has not yet been found.

However the solution is known for the supersymmetric QCD. Let us promote the gauge boson to the $\mathcal{N} = 1$ gauge supermultiplet, and promote the quarks to the chiral superfields. The superpartner of the gluon is gluino, which is a Weyl fermion, and the superpartners of the quarks are squarks, each squark is a complex-valued scalar. Therefore we have a vector superfield, N_f chiral superfields and N_f chiral anti-superfields. The resulting theory is asymptotically free when $x = N_f/N_c < 3$. This value differs from the value $11/2$ for a non-supersymmetric QCD because now we have more fields with the gauge group charge.

First observation to make is that in the $\mathcal{N} = 1$ supersymmetric gauge theory the conformal symmetry group $SO(2,4)$ is enhanced to the superconformal symmetry group $SU(2,2|1)$. This allows one to formulate the requirement of unitarity of a conformal field theory in terms of the bound on the R-charge, which is the charge with respect to the $U(1)$ subgroup of the $SU(2,2|1)$ group. The R-charge is expressed in terms of N_f and N_c. Using these facts one derives that a unitary superconformal fixed point in the IR exists when $3/2 < x < 3$, *i.e.*, the lower edge of the conformal window is located at $x_* = 3/2$. In fact in the $\mathcal{N} = 1$ supersymmetric field theory when $N_f < 3N_c/2$ but $N_f > N_c + 1$, a unitary superconformal fixed point exists, but to describe it one needs to switch to a Seiberg-dual magnetic theory, which is IR-free in this region.

Perhaps new methods for solving the problem of a conformal window in a non-supersymmetric QCD are needed. The AdS/CFT correspondence can be suitable for this purpose. In fact in the literature there exist holographic solutions to the problem of a conformal window for QCD in a Veneziano limit, $N_f, N_c \gg 1$, $N_f/N_c =$ finite. The idea of the solution is that if one starts at $x = 11/2$ and decreases the value of x, then at some point $x = x_*$ conformal phase transition takes place, which can be seen in the dual gravity theory in AdS. It has been suggested that as the field theory passes through the point of conformal phase transition, the scalar field in AdS (playing the role of an order parameter for the phase transition) develops a non-trivial profile. So the problem of a conformal window is reformulated holographically as the question of when a non-trivial configuration is a preferred state for this scalar field. Unfortunately the known holographic solutions are far from being rigorous, and are probably good only on a qualitative level. One of the principal obstacles to solving the problem of a conformal window holographically is that one needs to consider string theory in AdS space instead of its supergravity approximation.

Let us fix N_f and N_c such that the IR theory is confining, like real-world QCD. Up till now we have been considering QCD at vanishing temperature T and vanishing chemical potential μ. One can ask what happens as the values of T and μ are increased. This is the problem of the (T, μ) phase diagram of QCD. Although the precise form of the phase diagram of QCD is not known it is known that at large values of temperature or chemical potential conformal symmetry is restored, and the system of quarks goes into either color superconducting phase or quark-gluon plasma phase. We studied a similar problem in chapter 3, describing the quark bi-linear operator by the dual tachyon field in AdS, with the dynamics defined by the tachyon Dirac-Born-Infeld action. When the values of the temperature or the chemical potential are large enough the preferred tachyon state in AdS is trivial: the tachyon identically vanishes. However we have found that as one moves towards the origin of the (T, μ) plane the tachyon prefers to be in the state with a non-trivial profile. For the dual field theory it means that the bi-linear quark operator develops expectation value. It is a second-order phase transition at small values of temperature and a first-order phase transition at small values of chemical potential.

In this thesis we have also studied conformal phase transition which takes place in a strongly coupled quantum field theory conformal in its single-trace sector. This means that the coupling constants of the single-trace gauge-invariant operators (including the gauge coupling) do not run and reside at their fixed points, while the coupling constants of the double-trace, and more generally the multi-trace operators, run along the RG flow. In chapter 3 we studied such a quantum field theory in a Wilsonian holographic renormalization framework. In a Wilsonian holographic renormalization one integrates out a part of the AdS space between the boundary of AdS and the surface located at the fixed radial coordinate $z = b$ of AdS. Integrated out geometry is encoded in the effective boundary action defined at $z = b$. In the model considered in chapter 3 we introduced the tachyon field in AdS, with the dynamics determined by the tachyon-DBI action, dual to a single trace operator in the boundary field theory. We applied holographic renormalization to the tachyon DBI action and derived the effective holographic Wilsonian boundary action. This action satisfies the equation similar to the Callan-Symanzik equation, from which one can determine the running of the multi-trace coupling constants. The question we asked in chapter 3 was whether there exists a fixed point for the beta function of the double-trace coupling constant and when does this fixed point disappear and conformal phase transition takes place. We have found that conformal phase transition occurs when the mass of the tachyon crosses the Breitenlohner-Freedman bound. It is an infinite-order BKT type of phase transition, with an order parameter of the phase transition being a dynamically generated mass scale.

We have reviewed three kinds of phase transitions in a non-abelian gauge theories: the lower edge of a conformal window, the finite temperature/chemical potential and the running of the multi-trace coupling constants; all occurring between symmetric and massive phases. The AdS/CFT correspondence makes it possible to study these phase transitions, with various degrees of rigor and various assumptions and simplifications being made. It would be interesting to construct more rigorous holographic models of conformal phase transition at the lower edge of a conformal window, in particular for the $\mathcal{N} = 1$ supersymmetric gauge theories. Models like that exist in the literature, but the precise theory still remains to be found.

In the holographic examples described above the fermionic bi-linear operator on the boundary field theory was described by the scalar (tachyon) field in the AdS. In chapter 3 we used the tachyon-DBI action to describe dynamics of the tachyon field. Tachyon-DBI action appears in string theory as the low-energy action describing the embedding profile of a probe D-brane. In chapter 3 we considered various potentials for the tachyon field, some of which have been taken from the Dp-brane DBI action ('hard-wall' potential). We have also studied the tachyon DBI action with the generic 'soft-wall' tachyon potential. For a gravitational theory to be physically consistent it must be embeddable into string theory. In chapter 2 we considered the explicit string theory holographic realization of the matter degrees of freedom. Matter fields of a quantum field theory are massless modes of an open strings which stretch between the $D3$-branes, creating the $AdS_5 \times S^5$ background, and a probe branes in this background. Matter is described holographically by the dynamics of the probe branes. Such models, both supersymmetric and non-supersymmetric, have been extensively studied in the literature. The main focus of chapter 2 was the study of the probe brane matter in the background of a constant magnetic field.

Majority of quantum fields theories which have been studied by the methods of AdS/CFT correspondence are taken at an infinite number of degrees of freedom. This is a direct consequence of the fact that $SU(N)$ gauge theory is dual to a classical (supergravity) theory in AdS only when N is large. Any finite N means that quantum corrections in the bulk should be taken into account. At the same time it is known that interaction between a gauge-invariant IR operators is subleading in the $1/N$ expansion, this phenomenon is known as large-N factorization. In chapter 4 we studied holographically a quantum field theory dual to a two-dimensional charged black hole. It can be considered as Little String Theory at a finite density, the latter is defined as the low-energy limit of the $SU(N)$ gauge theory on the world-volume of an $NS5$-branes at a vanishing string coupling. Such a theory can be described holographically. When the N is large, the supergravity approximation of the bulk dynamics is valid (compare with the AdS/CFT correspondence, in which case the supergravity approximation is valid for the large 't Hooft coupling). String theory in the two-dimensional charged black hole background is exactly solvable, and therefore we do not have to resort to the supergravity approximation, which means we can describe holographically a finite-density matter at a finite N.

Results of chapter 4 give rise to the following problem. In chapter 4 we considered type-II superstring theory in a two-dimensional charged black hole background. We have found that the dual field theory behaves as a sum of two non-interacting fluids. Each fluid supports a gapless excitation, and we have verified that at low frequencies and momenta the dispersion relations of these excitations coincide with the dispersion relations obtained in the supergravity approximation. One might wonder whether there exists a theory in a two-dimensional charged black hole background which describes a hydrodynamics of a field theory at a finite density and a finite N. It is known that one can consider classical heterotic string theory in a coset realization of the two-dimensional charged black hole. In chapter 4 we studied heterotic gravity in the two-dimensional charged black hole and found out that the dual field theory is described by hydrodynamics, by deriving a dispersion relation of the diffusion mode in the shear channel. It would be interesting to see whether this dispersion relation is corrected in heterotic string theory, and whether the viscosity over entropy ratio gets stringy corrections. However as one tries to answer these questions the problem comes up. One needs to perform a first quantization and construct a spectrum of heterotic string in a two-dimensional charged black hole. This geometry is represented as a coset over $U(1)$ group. As one tries to construct quantum spectrum of this theory chiral anomaly appears because in heterotic string theory only the right-handed fermions couple to the $U(1)$ gauge field. Usually in string theory on coset space, described by a gauged WZW model, a chiral anomaly of fermions is compensated by a classical anomaly of bosons (which appears due to the asymmetric gauging). It would be interesting to find how this problem is resolved for heterotic string in the two-dimensional charged black hole.

There are numerous applications of the AdS/CFT correspondence to strongly coupled systems which we did not have a chance to discuss in this thesis. Since the AdS/CFT correspondence sometimes turns out to be the only available analytical tool for the study of the low-energy strongly coupled systems we advocate that a lot of effort should be invested into it.