Strings and AdS/CFT at finite density

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Chapter 2

Fluctuations in finite density holographic quantum liquids

2.1 Introduction and summary

Perhaps the deepest open problem in condensed matter physics is the classification of compressible quantum liquids. This refers to stable states of zero temperature quantum matter that do not break any symmetry and support massless excitations. This question cannot be easily addressed within the confines of standard field theory. The issue arises when fermions are considered at finite density and the culprit is known as the "fermion sign" problem. Dealing with time-reversal symmetric finite density bosonic matter the methods of equilibrium statistical physics give a full control and invariably one finds that the ground states break symmetry. Dealing with incompressible quantum fluids like the fractional quantum Hall states the mass gap is quite instrumental to control the theory, revealing the profound non-classical phenomenon of topological order. The hardship is with the compressible quantum fluids: the only example which is fully understood is the Fermi-liquid.

The ease of the mathematical description of the Fermi-liquid as the adiabatic continuation of the Fermi-gas is in a way deceptive. Compared to classical fluids its low energy spectrum of non-charged excitations is amazingly rich. In addition to the zero sound, there is a continuum of volume conserving "shape fluctuations" of the Fermi-surface, corresponding with the particle-hole excitations (Lindhard continuum) of the conventional perturbative lore. Although serious doubts exist regarding the mathematical consistency and their relevance towards real physics, the "fractionalized (spin) liquids" that were constructed in condensed matter physics appear to be still controlled by the presence of a Fermi-surface while these are not Landau Fermi-liquids in the strict sense. This inspired Sachdev to put forward the interesting conjecture that the Fermi-surface might be ubiquitous for all compressible quantum liquids [1].

The gauge-gravity duality or AdS/CFT correspondence provides a unique framework to deal with these matters in a controlled way (see [1–6] for recent reviews). Although it addresses field theories that are at first sight very remote from the interacting electrons of condensed matter, there are reasons to believe that it reveals generic emergence phenomena associated with strongly interacting quantum systems. Field theories whose understanding is plagued by the "fermion sign" problem appear to be quite tractable in the dual gravitational description. With regard to unconventional Fermion physics, perhaps the most important achievement has been the discovery of the "AdS_2 metal" [7, 8], dual to the asymptotically AdS Reissner-Nordstrom black hole. On the field theory side this describes a local (purely temporal) quantum critical state that was not expected on basis of conventional field theoretic means. Although quite promising regarding the intermediate temperature physics (the "strange" normal states) in high Tc superconductors and so forth, this AdS_2 metal is probably not a stable state, given its zero temperature entropy. Much of recent activity has been devoted to the study of the instability of this metal towards bosonic symmetry breaking (holographic superconductivity [9], "stripe" instabilities [10]) and towards the stable Fermi liquid [11–13].

The top-down constructions might become quite instrumental in facilitating the search for truly new quantum liquids. An important category are the Dp/Dq brane intersections; the $p = 3$ case provides us with a set of especially tractable examples. The dynamics of the low energy degrees of freedom of the D3-Dp strings can be studied in the probe approximation where the back-reaction to the $AdS_5 \times S^5$ geometry can be neglected [14]. In this chapter we will consider D3 and Dp branes intersecting along 2+1 dimensions, where p=5 (p=7) corresponds to the (non)-supersymmetric system. As emphasized in [15] the nonsupersymmetric system can be viewed as a model of graphene: the brane intersection fermions are like the Dirac fermions moving on the 2+1D graphene backbone, (tunable to finite density by gating), interacting strongly through the gauge fields living in 3+1 dimensions. We will present a number of results for the longitudinal- and transversal *dynamical* charge susceptibilities (at finite frequency $\mathbf{w}$ and momenta $\mathbf{q}$)[1], in the absence and presence of a magnetic field, for both the supersymmetric and non-supersymmetric D3/Dp systems at finite density. We find very similar results in both the supersymmetric- and fermionic set ups, showing that these outcomes at strong 't Hooft coupling are not caused by the difference in the Lagrangians. We find suggestive indications for the presence of an entirely new form of quantum liquid, but we cannot be entirely conclusive. Our observations cannot entirely rule out the existence of a Fermi liquid with vanishing Fermi velocity.

In fact, the first study of these systems at finite density already produced evidence that some odd state is created. In ref. [16] it was observed that the density-dependent part of the heat capacity in the D3/Dp systems with $2+1$ dimensional intersection behaves like T^4. This is in contrast to the result for the Fermi-liquids which is set by the Sommerfeld law of the specific heat $C = \gamma T$, where the Sommerfeld coefficient γ is proportional to the quasiparticle mass. This behavior remains to be understood: for example, it is conceivable that the linear term in the heat capacity exists, but is parametrically suppressed in the holographic model. On a side, it is worth noting that in the context of pnictide superconductivity a rogue signal has been detected that refuses to disappear: this indicates that the electronic specific heat of the metal state $\sim T^3$ [17].

As mentioned above, besides the Lindhard continuum an interacting Fermi liquid will carry a single propagating mode called zero sound. Unlike the usual sound at finite temperature, translational invariance alone is not sufficient for establishing the existence of the zero sound mode. The discovery of zero sound associated with the brane intersection matter [16] is therefore significant. The fate of the holographic zero sound was further studied in [18–25] (see also [26, 27] for closely related work). At very low temperature the attenuation (damping) of this zero sound behaves like the ("collisionless") Fermi liquid zero sound, in the sense that it increases like the square of its momentum. In [24] it was found also that upon increasing temperature the zero sound velocity decreases while the attenuation increases, turning into a purely diffusive pole at high temperatures. This is different from the crossover from zero sound to ordinary sound as

[1] In this chapter we denote the values of frequency and momentum by bold letters. The usual letters, defined below, are reserved for dimensionless variables.

function of temperature in a single component Fermi-system like ^3He. In the brane intersection systems momentum is shared between the superconformal strongly coupled uncharged sector and the material system on the intersection, and the latter does not support hydrodynamical sound in isolation. Somehow, upon lowering temperature the momentum of the brane intersection matter becomes separately conserved, facilitating the emergence of the zero sound in the low temperature limit.

Given that zero sound is rather ubiquitous, one would like to obtain more direct information regarding the density fluctuations of the quantum liquid. These are expected to be contained in the fully dynamical, momentum and energy dependent charge susceptibility/density-density propagator associated with the conserved charge on the brane-intersection. One strategy is to look for the momentum dependence of the reactive response (real part) at zero frequency: one expects a singularity at twice the Fermi momentum, $2\mathbf{q}_F$ where the Luttinger's theorem implies that $\mathbf{q}_F$ is set by the bare chemical potential, $\mathbf{q}_F \sim \mu$. A number of papers has been devoted to the search of such singular behavior in the framework of AdS/CFT. In [19] the $\langle J^0 J^0 \rangle$ correlator has been computed in the holographic setup where the only charged degrees of freedom are four-dimensional fermions. The resulting function was completely smooth. In [28–31] the two-point function for global currents was computed for various systems and again the tree-level computation in the bulk did not show any nonanalytic behavior. Very recently it has been argued that a singularity can be observed in the systems where an exact result to all orders in α' is available [32].

Searching for the singularity at $2\mathbf{q}_F$ is in principle a tricky procedure because these "Friedel oscillation" singularities are strongly weakened by the self energy effects in the strongly interacting Fermi-liquid. Another way to probe for the signatures of the Fermi liquid is to compute the imaginary part of the dynamical density susceptibility in a large kinematical window because this spectral function shows directly the density excitations of the system. The result is well known in the weakly interacting Fermi liquid, see Fig. 1: besides the zero sound pole one finds the Lindhard continuum of particle hole excitations. It is worth noting that as the value of the Landau parameter F_0 increases, the spectral weight in the density response is increasingly concentrated in the zero sound poles, "hiding" the Lindhard continuum. In this regard the *transversal* density propagator is quite informative: since in this channel no collective modes are expected to form, this is the place to look for the incoherent Fermi-surface fluctuations. Unfortunately technical issues prevent us from accessing the regime of parametrically small Fermi velocity. Our holographic computations of the longitudinal and transversal dynamical charge susceptibilities are limited to a kinematical window where $\mathbf{w} \sim |\mathbf{q}|$.

Despite this caveat, the holographic density propagators that we compute reveal very interesting information. We find that the longitudinal density propagator is within our numerical resolution completely exhausted by the zero sound pole (Fig. 4). Regardless the precise nature of the underlying state this signals very strong density/density correlations in this liquid. The transversal charge propagator shows that sound is not the whole story. The "other stuff", albeit very unlike a Lindhard continuum, signals the presence of a sector of highly collective, deep IR density fluctuations: the imaginary part of the transversal propagator behaves like $\chi_t^{(i)}(\mathbf{q}, \mathbf{w}) \sim \mathbf{w}$. This response is surprisingly momentum independent and suggests local quantum criticality, which was instrumental in the "AdS_2 metal" setup. All of this seems to imply that we are indeed dealing with some entirely new quantum liquid.

To probe some of the features of this quantum liquid, we introduce an external magnetic field which is a valuable "experimental tool". This induces the gap in the spectrum that is visible in the holographic calculations. Dealing with a 2+1D Fermi-liquid one would expect the signatures of Landau levels also in the density response. In the strongly interacting system, the longitudinal response should reveal the "magneto-roton", the left over of zero sound in the system with a magnetic field which is well known from (fractional) quantum Hall systems [33] [2]. According to Kohn's theorem [35], the density spectrum should show a gap equal to the cyclotron frequency at zero momentum. Note that this theorem is very generic and only assumes that degrees of freedom, charged under the magnetic field, interact pairwise. Our holographic calculation reveals that: i) at small values of the magnetic field B the value of the gap[3] scales linearly with B, which is consistent with Kohn's theorem for the nonrelativistic fermions and ii) there are no signatures of Landau levels associated with incoherent particle-hole excitations (Fig. 2).

The remainder of this chapter is organized as follows. The next sec-

[2] See [34] for related work in the context of holography.

[3] This is also consistent with the observations made in [23, 36] where the same D3/D7 system, modified by the inclusion of flux through the internal cycles, is considered.

tion is devoted to the review of Landau Fermi liquid theory including the random phase approximation (RPA) for the dynamical response. In particular, we review the appearance of the zero sound mode in the RPA calculation of the density-density correlator. As the value of the interaction strength increases, the Lindhard continuum gets separated from the zero sound pole (Fig. 1) and gradually disappears. In the extreme limit of vanishing Fermi velocity, the spectral density is completely exhausted by the zero sound mode. We also review the RPA expectations for the 2+1 dimensional fermion system in the presence of magnetic field. There we expect Landau levels to contribute to the spectral density (Fig. 2).

In Section III we review the holographic description of the D3/Dp brane systems. The subject of our interest is the fermion matter, which is formed (at finite chemical potential for the fermion number) in the low energy theory living on intersection of the N_c $D3$ branes and N_f Dp branes. We consider the case of $N_c \gg N_f \sim 1$ and strong 't Hooft coupling λ, where the holographic description is applicable.

In Section IV we focus on the zero sound mode and show that it develops a gap in the presence of magnetic field. In the case of vanishing magnetic field, $B = 0$, we observe a zero sound mode whose speed is the same as that of the first sound. As long as the value of the magnetic field B is small compared to $\mathbf{w}^2, \mathbf{q}^2$ (in appropriate units), the sound mode peak in the spectral function is not significantly affected. On the other hand, the presence of the nonvanishing magnetic field leads to a gap in the dispersion relation for zero sound. (The effective action proposed by Nickel and Son [37] in the presence of the magnetic field gives vanishing sound velocity). In the regime of small magnetic field we derive the scaling behavior of the gap in the spectrum $\mathbf{w}_c$ as a function of magnetic field. The result, $\mathbf{w}_c \sim B$ is consistent with fermions acquiring an effective mass.

In Section V we investigate the current-current correlator at nonvanishing frequency $\mathbf{w}$ and momentum $\mathbf{q}$. We observe that in the longitudinal channel, the only nontrivial structure both in the real and in imaginary parts of the correlators is provided by the zero sound. There is no nontrivial structure in the transverse correlators when $B = 0$. We discuss our results in Section VI.

In Appendix we consider higher derivative corrections and show that when they are added to the DBI the correlators are not significantly modified.

2.2 Fermi liquid and the random phase approximation

In this section we review the application of the random phase approximation (RPA) for the computation of the density-density response function $\langle J_0(\mathbf{w},\mathbf{q})J_0(-\mathbf{w},-\mathbf{q})\rangle$ in Landau Fermi liquid theory. We consider the 2+1 dimensional theory for both cases of vanishing and non-vanishing magnetic field.

Due to the interaction of quasiparticles, the variation of quasiparticle energy due to small perturbation of the distribution function, is given by (see, e.g, [38])

$$\delta\varepsilon(\mathbf{q}) = \int d\mathbf{q}' f(\mathbf{q},\mathbf{q}')\delta n(\mathbf{q}') \tag{2.1}$$

Because the small changes of quasiparticle density occur in the vicinity of a Fermi surface, one considers the function $f(\mathbf{q},\mathbf{q}')$ to be dependent on the momenta on the Fermi surface, and therefore it boils down to a function of the angle between $\mathbf{q}$ and $\mathbf{q}'$:

$$\frac{m^*}{\pi} f(\theta) = 2F(\theta)\,. \tag{2.2}$$

where, as usual, the effective mass at the Fermi surface is defined via

$$m^* = \frac{\mathbf{q}_F}{v_F}, \qquad v_F = \frac{\partial\epsilon(\mathbf{q})}{\partial\mathbf{q}}|_{\mathbf{q}=\mathbf{q}_F} \tag{2.3}$$

Landau parameters F_l are the coefficients of the expansion of $F(\theta)$ in Legendre polynomials:

$$F(\theta) = \sum_l (2l+1)F_l P_l(\cos\theta) \tag{2.4}$$

The Fermi liquid has a collective excitation at vanishing temperature called zero sound. In the case of $F_l = 0,\ l > 0$, the speed of zero sound u_0 can be determined from

$$\frac{s}{2}\log\frac{s+1}{s-1} - 1 = \frac{1}{F_0}, \qquad s = \frac{u_0}{v_F} \tag{2.5}$$

which, in the limit $F_0 \gg 1$ gives $s \sim \sqrt{F_0}$.

To compute the dynamical collective responses of a Fermi liquid, one evaluates the time dependent mean field (random phase approximation) obtained by summing up the quasiparticle “bubble” diagrams. Assuming for simplicity only the presence of a contact interaction, with effective coupling constant $V \simeq F_0$, the nth diagram is equal to $V^{n-1}(\chi_0(\mathbf{q},\mathbf{w}))^n$. The susceptibility in the RPA is then given by the sum of a geometric progression:

$$\chi(\mathbf{q},\mathbf{w}) = \frac{\chi_0(\mathbf{q},\mathbf{w})}{1 - V\chi_0(\mathbf{q},\mathbf{w})}\,, \tag{2.6}$$

Express $\chi = \chi^{(r)} + i\chi^{(i)}$, hence

$$\chi^{(i)}(\mathbf{q},\mathbf{w}) = \frac{\chi_0^{(i)}(\mathbf{q},\mathbf{w})}{(1 - V\chi_0^{(r)}(\mathbf{q},\mathbf{w}))^2 + (\chi_0^{(i)}(\mathbf{q},\mathbf{w}))^2}\,. \tag{2.7}$$

Then we study density of excitations by plotting $\chi^{(i)}(\mathbf{q},\mathbf{w})$. The result for vanishing magnetic field is presented in Fig. 3.1, where we plot the susceptibility (for $q_F = 0.2$) at strong and weak coupling V. In the case

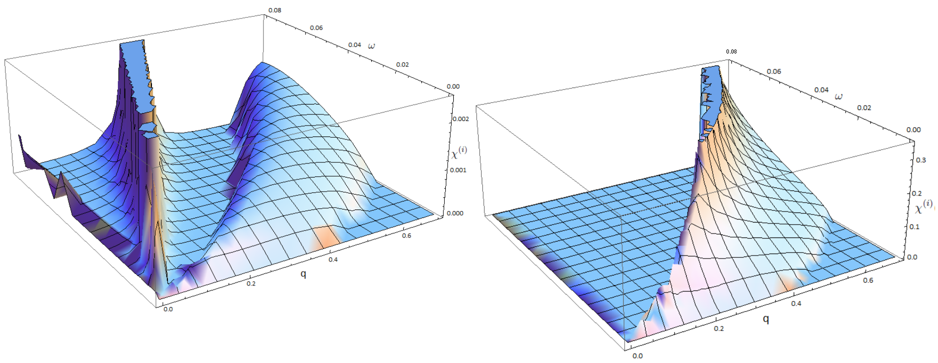


Figure 2.1. Spectral density $\chi^{(i)}(\mathbf{q},\mathbf{w})$ at strong coupling ($V = 50$, left graph) and weak coupling ($V = 3$, right graph) in the random phase approximation, at vanishing magnetic field. Fermi momentum is put to $q_F = 0.2$. Note that at strong coupling zero sound is well separated from the particle-hole continuum, while at weak coupling zero sound merges with the left edge of the particle-hole continuum. At small frequencies particle-hole continuum sharply ends at $\mathbf{q} = 2\mathbf{q}_F$.

of strong coupling there is a finite gap, separating the zero sound collective mode, and the band of the particle-hole excitations. For given small frequency $\mathbf{w}$ the width of the gap is given by $\delta\mathbf{q} \simeq \frac{\mathbf{w}}{u_0}(s-1)$. Note the non-analytic step behavior at $\mathbf{q} = 2\mathbf{q}_F$, originating from the free response function $\chi_0^{(i)}(\mathbf{q},\mathbf{w})$. In the case of weak coupling the zero sound mode merges with the left edge of particle-hole band.

The location of zero sound pole is determined as a solution to equations $\chi_0^{(i)}(\mathbf{q},\mathbf{w}) = 0$, $\chi_0^{(r)}(\mathbf{q},\mathbf{w}) = 1/V$. The real part $\chi_0^{(r)}(\mathbf{q},\mathbf{w})$ of Lindhard function for 2D Fermi gas is given by (see, e.g., [39]):

$$\chi_0^{(r)}(\mathbf{q},\mathbf{w}) = -\left(1 + \frac{\mathbf{q}_F}{\mathbf{q}}\left[\mathrm{sign}(\nu_-)\theta(|\nu_-|-1)\sqrt{\nu_-^2-1}\right.\right.$$
$$\left.\left. - \ \mathrm{sign}(\nu_+)\theta(|\nu_+|-1)\sqrt{\nu_+^2-1}\right]\right), \tag{2.8}$$

where $\nu_\pm = \frac{\mathbf{w}\pm\varepsilon_{\mathbf{q}}}{\mathbf{q}v_F}$. For large $\frac{\mathbf{w}}{\mathbf{q}v_F} = s \gg 1$ one may expand

$$\chi_0^{(r)}(\mathbf{q},\mathbf{w}) \simeq \frac{\mathbf{q}^2 v_F^2}{2\mathbf{w}^2}. \tag{2.9}$$

Therefore, for the speed of zero sound one obtains $s = \sqrt{V/2}$, exactly as it follows at large F_0 from the equation (2.5).

Suppose now that besides F_0 there is also non-vanishing "mass" Landau parameter F_1. In the relativistic case, the value of m^* is related to the value of the chemical potential [40],

$$m^* = \mu\left(1 + \frac{F_1}{3}\right) \tag{2.10}$$

The speed of zero sound u_0 then satisfies equation

$$\frac{s}{2}\log\frac{s+1}{s-1} - 1 = \frac{1 + F_1/3}{F_0 + F_0F_1/3 + F_1 s^2}, \qquad s = \frac{u_0}{v_F}. \tag{2.11}$$

For free fermions in a magnetic field B, the Lindhard function is equal to (see, e.g., [39])

$$\chi_0(\mathbf{q},\mathbf{w}) = \frac{1}{2\pi\ell^2}\sum_{n,n'}\frac{f(\varepsilon_n) - f(\varepsilon_{n'})}{\mathbf{w} + (n-n')\mathbf{w}_c + i\eta}|F_{n',n}(\mathbf{q})|^2, \tag{2.12}$$

where

$$F_{n',n}(\mathbf{q}) = \sqrt{\frac{n!}{n'!}}\left(\frac{(\mathbf{q}_y - i\mathbf{q}_x)\ell}{\sqrt{2}}\right)^{n'-n} e^{-\mathbf{q}^2\ell^2/4} L_n^{n'-n}\left(\frac{\mathbf{q}^2\ell^2}{2}\right), \tag{2.13}$$

for $n' \geq n$. Here we have introduced the cyclotron frequency $\mathbf{w}_c = B/m^\star$ and the magnetic length $\ell = \frac{1}{\sqrt{B}}$. The functions $L_n^{n'-n}$ are Laguerre polynomials, and $f(\varepsilon_n)$ is an occupation number for the nth Landau level.

We would like to compute the effect of the magnetic field on the density-density response function of the interacting fermions. Let us write the quasiparticle interaction Hamiltonian

$$H_{int} = \sum_{\mathbf{q}} V_{\mathbf{q}} n_{\mathbf{q}} n_{-\mathbf{q}} \tag{2.14}$$

in the basis of Landau levels wavefunctions. The corresponding matrix elements of the density fluctuation operator $n_{\mathbf{q}} = \sum_{\mathbf{k}} c_{\mathbf{k}} c^\dagger_{\mathbf{k}+\mathbf{q}}$ are given by

$$\langle n'\mathbf{k}'_y | n_{\mathbf{q}} | n\mathbf{k}_y \rangle = \exp\left(-i \frac{\mathbf{q}_x(\mathbf{k}_y + \mathbf{k}'_y)\ell^2}{2} \right) F_{n'n}(\mathbf{q}) \delta_{\mathbf{k}_y - \mathbf{k}'_y, \mathbf{q}_y} \,. \tag{2.15}$$

The density fluctuation operator in the basis of Landau level wavefunctions is then given by

$$n_{\mathbf{q}} = \sum_{n, \mathbf{k}_y,\, n', \mathbf{k}'_y} \langle n'\mathbf{k}'_y | n_{\mathbf{q}} | n\mathbf{k}_y \rangle c_{n\mathbf{k}_y} c^\dagger_{n'\mathbf{k}'_y} \,. \tag{2.16}$$

Note that

$$\Big(\langle n\mathbf{k}_y | n_{\mathbf{q}} | n'\mathbf{k}'_y \rangle \Big)^\star = \langle n'\mathbf{k}'_y | n_{-\mathbf{q}} | n\mathbf{k}_y \rangle \tag{2.17}$$

implies $(n_{\mathbf{q}})^\dagger = n_{-\mathbf{q}}$. Substituting (2.16) into the interaction Hamiltonian (2.14), assuming again only a contact interaction of plane waves $V_{\mathbf{q}} \equiv V \simeq F_0$, and considering all quasiparticles in the same Landau level n, one obtains

$$\begin{aligned} H_{int} = V &\sum_{\mathbf{q}, \mathbf{k}_y, \mathbf{k}'_y} c_{n\mathbf{k}_y} c^\dagger_{n\mathbf{k}_y - \mathbf{q}_y} c_{n\mathbf{k}'_y} c^\dagger_{n\mathbf{k}'_y + \mathbf{q}_y} \\ &\exp\left(-i\ell^2 \mathbf{q}_x(\mathbf{k}_y - \mathbf{k}'_y - \mathbf{q}_y) - \frac{\mathbf{q}^2\ell^2}{2} \right) [L_n^0(\mathbf{q}^2\ell^2/2)]^2 \,. \end{aligned} \tag{2.18}$$

Let us choose the momentum to be in y-direction, then

$$H_{int} = \sum_{\mathbf{q}_y, \mathbf{k}_y, \mathbf{k}'_y} V_{\mathbf{q}_y} c_{n\mathbf{k}_y} c^\dagger_{n\mathbf{k}_y - \mathbf{q}_y} c_{n\mathbf{k}'_y} c^\dagger_{n\mathbf{k}'_y + \mathbf{q}_y} \,, \tag{2.19}$$

where $V_{\mathbf{q}_y} = [L_n^0(\mathbf{q}^2\ell^2/2)]^2 \exp\left(-\frac{\mathbf{q}^2\ell^2}{2} \right) V$.

We can explicitly demonstrate that the zero sound mode is gapped in the magnetic field, with the gap being equal to $\mathbf{w}_c$, in agreement with the Kohn's theorem [35]. For this aim we are to solve equation $\chi_0^{(r)}(\mathbf{q},\mathbf{w}) = 1/V_{\mathbf{q}}$ again. From (2.12), (2.13) one may obtain the following expression for $\chi_0^{(r)}$:

$$\chi_0^{(r)}(\mathbf{q},\mathbf{w}) = \frac{e^{-\mathbf{q}^2\ell^2/2}}{2\pi\hbar\ell^2}\sum_{k=1}^{\infty}\sum_j{}' \frac{j!}{(j+k)!}\left(\frac{\mathbf{q}^2\ell^2}{2}\right)^k \left[L_j^k\left(\frac{\mathbf{q}^2\ell^2}{2}\right)\right]^2 \frac{2k\mathbf{w}_c}{\mathbf{w}^2-(k\mathbf{w}_c)^2}\,, \tag{2.20}$$

where the prime denotes summation in the range $\max(0,\nu-k)\le j\le\nu$, and ν is the number of occupied Landau levels. Following [39], we consider this equation for small $\mathbf{q}$ and $\mathbf{w}\simeq\mathbf{w}_c$. Then the main contribution in the sum over k comes from the term with $k=1$, and we obtain equation:

$$\mathrm{const}\frac{\mathbf{q}^2}{\mathbf{w}^2-\mathbf{w}_c^2}\simeq\frac{1}{V}\,, \tag{2.21}$$

and therefore the zero sound dispersion relation is given by

$$\mathbf{w}=\sqrt{\mathbf{w}_c^2+c\mathbf{q}^2}\,, \tag{2.22}$$

where $c\sim V\mathbf{w}_c$ is a constant. Similarly, for any integer M, there is a mode with dispersion relation

$$\mathbf{w}=\sqrt{(M\mathbf{w}_c)^2+c'\mathbf{q}^{2M}}\,. \tag{2.23}$$

We plot RPA computations of two-point function, for $\omega_c = 0.25$, restricting to the first two first branches, in Fig. 2.2.

2.3 Dp brane in $AdS_5\times S^5$ background

We study strongly interacting massless fermions at zero temperature and finite density. A good field theoretical model of such a system is $\mathcal{N}=4$ SYM theory with gauge group $SU(N_c)$, coupled to matter in the fundamental representation. A convenient way to study strongly coupled theories is provided by holography where one considers a dual gravitational theory, taking the limit of large 't Hooft coupling $\lambda = g_{YM}^2 N_c$, and

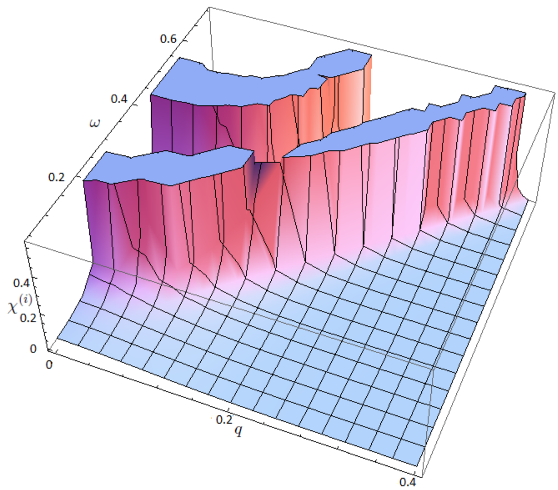


Figure 2.2. Spectral density in the random phase approximation of the 2 + 1 dimensional Fermi liquid in the plane of the magnetic field, with cyclotron frequency $\omega_c = 0.25$. First two of infinitely many collective excitation branches are shown. Each branch starts at ($q = 0$, $\omega = M\omega_c$), where M is an integer.

the limit of large N_c. The dual gravitational background is created by $N_c \gg 1$ $D3$ branes, and has an $AdS_5 \times S^5$ geometry. The coupling to fundamental matter is realized by considering an embedding of a probe Dp brane in the $AdS_5 \times S^5$ background [14]. We will consider $D3/Dp$ configurations with $d = 2 + 1$ dimensional intersections.

Let us now provide a more detailed description of the bulk gravitational theory set-up. Consider $AdS_5 \times S^5$ geometry, with the metric

$$ds^2 = L^2 \left(r^2(-dt^2 + dx_\alpha dx^\alpha) + \frac{dr^2}{r^2} + d\Omega_5^2 \right) . \tag{2.24}$$

Here L is the radius of S^5 and scale of curvature of AdS_5. We will study the probe Dp brane, embedded in the geometry described by (2.24). We represent the metric on S^5 as

$$d\Omega_5^2 = d\Omega_n^2 + \sin^2\tilde{\theta} d\Omega_{5-n}^2 = d\theta^2 + \sin^2\theta d\Omega_{n-1}^2 + \cos^2\theta d\Omega_{5-n}^2 \,,$$

where $n = p + 1 - d$. Then we define coordinates $\rho\,, f$ via the relation

$$\rho = r\sin\theta\,, \qquad f = r\cos\theta\,, \quad r^2 = \rho^2 + f^2\,, \tag{2.25}$$

and write

$$d\theta^2 = \frac{(f - \rho\,\partial_\rho f)^2}{r^4} d\rho^2\,, \quad dr^2 = \frac{(\rho + f\,\partial_\rho f)^2}{r^2} d\rho^2\,, \tag{2.26}$$

which gives the following induced Dp brane world-volume metric

$$ds^2_{Dp} = L^2 \left(r^2(-dt^2 + dx_i dx^i) + \frac{1}{r^2}\left(1 + (\partial_\rho f)^2\right) d\rho^2 + \frac{\rho^2}{r^2} d\Omega^2_{n-1} \right)\,. \tag{2.27}$$

The coordinate $f(\rho)$ defines an embedding of the Dp brane in the AdS background (2.24). In the case of the trivial embedding $f(\rho) \equiv 0$, which is what we are going to deal with in this chapter, Dp brane crosses the Poincaré horizon of the AdS space. In the case of $d = 3$ $p = 7$ such a configuration becomes stable only for sufficiently large values of chemical potential $\bar{\mu}_{ch}$ in the dual field theory [41]. (See also [42] for the phase structure of the similar model in the presence of the magnetic field.) Note that holographically computed correlators do not depend on the dimensionality of the probe brane; in particular our results apply in the case of stable supersymmetric D3/D5 defect theory.

Subsequently we add a gauge field A_μ on the world-volume of the probe $D7$ brane. In general we are interested in non-vanishing magnetic field B. So we consider the following components of the field strength:

$$F_{12} = B\,, \qquad F_{0\rho} = -\partial_\rho A_0(\rho)\,. \tag{2.28}$$

Consequently the DBI action for the Dp brane is given by [4]

$$S_{DBI} \simeq \frac{N_c}{L^4} \int d^{p+1}x \sqrt{-\det(G+F)} = \int d\Omega_{n-1} \int d^d x\, S\,, \tag{2.29}$$

where we have denoted

$$S \simeq N_c L^{p-5} \int d\rho \rho^{d-3} \sqrt{(L^4\rho^4 + B^2)(1 - (\partial_\rho A_0)^2 L^{-4})}\,. \tag{2.30}$$

Now rescale gauge field on the world-volume as

$$\bar{A}_\mu = \frac{A_\mu}{L^2}\,, \tag{2.31}$$

[4] We adopt the convention $2\pi\alpha' = 1$. For our purposes we are ignoring the total numerical coefficient, which leaves us with an overall normalization of the action proportional to $\frac{1}{g_s} \sim \frac{N_c}{\lambda} \sim \frac{N_c}{L^4}$.

which yields the DBI action in the form,

$$S \simeq N_c L^{p-3} \int d\rho \rho^{d-3} \sqrt{(\rho^4 + \bar{B}^2)(1 - (\partial_\rho \bar{A}_0)^2)}\,, \qquad (2.32)$$

where $\bar{B} = B/L^2$.

In the case of a non-vanishing magnetic field there is also a Chern-Simons term in the total action for the Dp brane. It can be shown that this term vanishes in the case of $f \equiv 0$ embedding.

The boundary value of $\bar{A}_0$ is equal to the chemical potential of the dual field theory: $\bar{A}_0(\rho = \infty) = \bar{\mu}_{ch}$. Due to $f(\rho = 0) = 0$ and the initial condition $\bar{A}_0(r = 0) = 0$ (imposed to ensure that chemical potential vanishes when the charge density is zero) we obtain $\bar{A}_0(\rho = 0) = 0$, and therefore the chemical potential may be expressed as

$$\bar{\mu}_{ch} = \int_0^\infty d\rho\, \partial_\rho \bar{A}_0\,. \qquad (2.33)$$

Introducing a constant of integration $\hat{d}$, the solution of the equation of motion for $\partial_\rho \bar{A}_0$ field strength becomes,

$$\partial_\rho \bar{A}_0 = \frac{\hat{d}^2}{\sqrt{\hat{d}^4 + \rho^4 + \bar{B}^2}}\,. \qquad (2.34)$$

Using this expression and eq. (2.33), we obtain the value of the chemical potential

$$\bar{\mu}_{ch} = \int_0^\infty d\rho\, \partial_\rho \bar{A}_0 = \frac{4\Gamma(5/4)^2}{\sqrt{\pi}} \frac{\hat{d}^2}{(\hat{d}^4 + \bar{B}^2)^{1/4}}\,. \qquad (2.35)$$

2.4 Holographic zero sound

In this and the next sections we study $D3/Dp$ system with $d = 2 + 1$ dimensional intersection, described by trivial $f(\rho) \equiv 0$ embedding of the probe Dp brane in the $AdS_5 \times S^5$ background. We consider the gauge field on the Dp brane world-volume, solve its classical equations of motion and use AdS/CFT to find the two-point functions of the $U(1)$ current in the dual field theory. In this section we show the existence of holographic zero sound in the $D3/Dp$ configuration, to observe that it develops a gap as the magnetic field is turned on. In the next section we will study the current-current correlation function numerically.

2.4.1 Zero sound in the $D3/Dp$ system with $d = 2+1$ dimensional intersection

Equation (2.34) is the expression for the background field strength $\partial_\rho \bar{A}_0$. Let us turn on small fluctuations $\bar{a}_0$, $\bar{a}_1$, $\bar{a}_2$, dependent on coordinates x^0, x^2, ρ. In addition let us fix the gauge $\bar{a}_\rho = 0$. The longitudinal response is described holographically by the $\bar{a}_0$ and $\bar{a}_2$ components of the gauge field, and the transverse response is described by the $\bar{a}_1$ component. The DBI action, expanded up to the second order in fluctuations, then takes the form [5]

$$S= \int d\rho \left(\sqrt{\frac{\rho^4+\bar{B}^2}{1-(\partial_\rho \bar{A}_0)^2}} \left(-\frac{(\partial_\rho \bar{a}_0)^2}{1-(\partial_\rho \bar{A}_0)^2} + \frac{\rho^4(\partial_\rho \bar{a}_2)^2-(\partial_0 \bar{a}_2-\partial_2 \bar{a}_0)^2}{\rho^4+\bar{B}^2} \right) + \right.$$
$$+ \sqrt{\frac{1-(\partial_\rho \bar{A}_0)^2}{\rho^4+\bar{B}^2}} \left(\frac{\rho^4(\partial_2 \bar{a}_1)^2}{\rho^4+\bar{B}^2} + \frac{\rho^4(\partial_\rho \bar{a}_1)^2-(\partial_0 \bar{a}_1)^2}{1-(\partial_\rho \bar{A}_0)^2} \right) + \tag{2.36}$$
$$\left. + \frac{2\bar{B}\partial_\rho \bar{A}_0}{\sqrt{(\rho^4+\bar{B}^2)(1-(\partial_\rho \bar{A}_0)^2)}} (\partial_2 \bar{a}_1 \partial_\rho \bar{a}_0 - \partial_0 \bar{a}_1 \partial_\rho \bar{a}_2 + (\partial_0 \bar{a}_2 - \partial_2 \bar{a}_0)\partial_\rho \bar{a}_1) \right)$$

Note that the last line in (2.36) describes a coupling of the transverse and longitudinal gauge potential components. Bellow we will consider Fourier transform of the gauge field

$$\bar{a}_\mu(\rho, x^0, x^2) = \int \frac{d\mathbf{w}d\mathbf{q}}{(2\pi)^2} e^{-i\mathbf{w}x_0+i\mathbf{q}x_2} \tilde{a}_\mu(\rho, \mathbf{w}, \mathbf{q}) \tag{2.37}$$

Now we substitute eq. (2.34) into the action (2.36), define $b^2 = \bar{B}^2/\hat{d}^4$, and introduce a new variable $z = \frac{\hat{d}}{\rho}$, so that $z = 0$ is a boundary and $z = \infty$ is a Poincaré horizon of AdS_5. In addition, we make the quantities $\mathbf{w}$, $\mathbf{q}$ dimensionless, by measuring these in units of $\hat{d}$: $\mathbf{w} = \omega\hat{d}$, $\mathbf{q} = q\hat{d}$. We also denote for shortness of notation

$$\zeta = 1 + (1+b^2)z^4 \tag{2.38}$$

Then the action (2.36) becomes written as

$$S= \int \frac{dz}{1+b^2z^4} \left(-\zeta^{3/2}a_0'^2 + \zeta^{1/2}a_2'^2 - \zeta^{1/2}(\partial_0 a_2 - \partial_2 a_0)^2 + \zeta^{-1/2}(\partial_2 a_1)^2 \right.$$
$$\left. - \zeta^{1/2}(\partial_0 a_1)^2 + \zeta^{1/2}a_1'^2 - 2bz^4(\partial_2 a_1 a_0' - \partial_0 a_1 a_2' + (\partial_0 a_2 - \partial_2 a_0)a_1') \right), \tag{2.39}$$

[5] We thank J. Shock for comments on this action.

where we have omitted bars for simplicity of notation, and prime denotes differentiation w.r.t. z. In momentum representation

$$\begin{aligned}S = \int \frac{dz}{1+b^2 z^4} \Big(&-\zeta^{3/2} a_0'(\omega,\, q) a_0'(-\omega,\, -q) + \zeta^{1/2} a_2'(\omega,\, q) a_2'(-\omega,\, -q)]\\
&+ \zeta^{1/2} E(\omega,\, q) E(-\omega,\, -q) + \zeta^{-1/2} q^2 a_1(\omega,\, q) a_1(-\omega,\, -q)\\
&- \zeta^{1/2}\omega^2 a_1(\omega,\, q) a_1(-\omega,\, -q) + \zeta^{1/2} a_1'(\omega,\, q) a_1'(-\omega,\, -q)\\
&+ 2ibz^4 (q a_1(-\omega,\, -q) a_0'(\omega,\, q) + \omega a_1(-\omega,\, -q) a_2'(\omega,\, q)\\
&+ E(\omega,\, q) a_1'(-\omega,\, -q))\Big)\ ,\end{aligned} \tag{2.40}$$

where we have omitted tildes for simplicity of notation and introduced the gauge-invariant electric field strength [43],

$$E(\omega,\, q) = \omega a_2(\omega,\, q) + q a_0(\omega,\, q)\,. \tag{2.41}$$

In addition we have Gauss's law [6]

$$\omega \zeta^{3/2} a_0'(\omega,\, q) + q \zeta^{1/2} a_2'(\omega,\, q) = 0 \tag{2.42}$$

Together with

$$E'(\omega,\, q) = \omega a_2'(\omega,\, q) + q a_0'(\omega,\, q)\,, \tag{2.43}$$

eq. (2.42) gives

$$a_0'(\omega,\, q) = \frac{q}{q^2 - \zeta\omega^2} E'\,, \tag{2.44}$$

$$a_2'(\omega,\, q) = \frac{\omega\zeta}{\omega^2\zeta - q^2} E'\,. \tag{2.45}$$

Plugging these expressions into the action (2.40), we obtain

$$\begin{aligned}S = \int \frac{dz}{1+b^2 z^4} \Bigg(&\frac{q^2 - \zeta\omega^2}{\zeta^{1/2}} a_1^2 - \zeta^{3/2} \frac{E'^2}{\zeta\omega^2 - q^2} + \zeta^{1/2} E^2 + \zeta^{1/2} a_1'^2\\
&+ 2ibz^4 (E a_1)'\Bigg)\ .\end{aligned} \tag{2.46}$$

[6] This is an equation of motion for a_z. To derive it replace

$$a_2' \to a_2' - \partial_2 a_z\,, \quad a_0' \to a_0' - \partial_0 a_z$$

in the Lagrangian (2.39) and leave only terms linear in derivatives of a_z, because only these will survive when we consider the equation of motion for a_z in the $a_z = 0$ gauge. Then use the Fourier transform (2.37).

Corresponding fluctuation equations are

$$E''+\left(\omega^2-\frac{q^2}{1+(1+b^2)z^4}\right)E-\frac{4ibz^3(\omega^2(1+(1+b^2)z^4)-q^2)a_1}{(1+b^2z^4)(1+(1+b^2)z^4)^{3/2}} \tag{2.47}$$
$$+\frac{2}{z}\left(\frac{1}{1+((1+b^2)z^4)^{-1}}+2\left(\frac{1}{1+b^2z^4}-\frac{1-(q/\omega)^2(1+(1+b^2)z^4)^{-2}}{1-(q/\omega)^2(1+(1+b^2)z^4)^{-1}}\right)\right)E'=0\,,$$

$$a_1''+2z^3\left(\frac{1+b^2}{1+(1+b^2)z^4}-\frac{2b^2}{1+b^2z^4}\right)a_1'+\left(\omega^2-\frac{q^2}{1+(1+b^2)z^4}\right)a_1$$
$$+\frac{4ibz^3E}{(1+b^2z^4)(1+(1+b^2)z^4)^{1/2}}=0\,. \tag{2.48}$$

Vanishing magnetic field

In this subsection we set the magnetic field to zero. Fluctuations of E and a_1 fields then decouple, and we can consider separately transverse and longitudinal responses,

$$E''+\frac{2}{z}\left(\frac{1}{1+z^{-4}}+2\left(1-\frac{1-(q/\omega)^2(1+z^4)^{-2}}{1-(q/\omega)^2(1+z^4)^{-1}}\right)\right)E'$$
$$+(\omega^2-q^2(1+z^4)^{-1})E=0\,, \tag{2.49}$$

$$a_1''+\frac{2z^3}{1+z^4}a_1'+\left(\omega^2-\frac{q^2}{1+z^4}\right)a_1=0\,. \tag{2.50}$$

Let us first study the longitudinal response. In the near-horizon $z\gg 1$ region eq. (2.49) becomes:

$$E''+\frac{2}{z}E'+\omega^2E=0\,, \tag{2.51}$$

The general solution of (2.51) is a linear combination of $e^{\pm i\omega z}/z$. We choose the solution with the incoming near-horizon behavior, since it corresponds to retarded propagator in the dual field theory [44]:

$$E=C\frac{e^{i\omega z}}{z}\,. \tag{2.52}$$

The constant C is undetermined, because the fluctuation equation is linear. When $\omega z\ll 1$, we obtain

$$E=C\left(\frac{1}{z}+i\omega\right)\,. \tag{2.53}$$

Condition (2.52) together with the boundary condition $E(0) = 0$ (imposed to get normalizable solutions) defines an eigenvalue problem for the fluctuation equation (2.49). In the limit $\omega z, qz \ll 1$, (2.49) reduces to,

$$E'' + \frac{2}{z}\left(\frac{1}{1+z^{-4}} + 2\left(1 - \frac{1-(q/\omega)^2(1+z^4)^{-2}}{1-(q/\omega)^2(1+z^4)^{-1}}\right)\right)E' = 0\,, \qquad (2.54)$$

having as general solution,

$$E(z) = C_1 + C_2(q^2 - 2\omega^2)\sqrt{i}F\left(i\sinh^{-1}(\sqrt{i}z)|-1\right) - \frac{C_2 q^2 z}{\sqrt{1+z^4}}\,, \quad (2.55)$$

where $F(z)$ is an elliptic integral of the first kind. In the limit $z \to \infty$ it has an expansion

$$\sqrt{i}F\left(i\sinh^{-1}(\sqrt{i}z)|-1\right) \to -K(1/2) + \frac{1}{z} + O\left(\frac{1}{z^5}\right)\,, \qquad (2.56)$$

where $K(z)$ is the complete elliptic integral of the first kind. The solution (2.55) becomes in this limit

$$E(z) = C_1 - C_2 K(1/2)(q^2 - 2\omega^2) - \frac{2C_2}{z}\omega^2. \qquad (2.57)$$

Now we compare (2.53) and (2.57), and obtain as result

$$C_1 = \left(i\omega - \frac{(q^2-2\omega^2)K(1/2)}{2\omega^2}\right)C\,, \qquad C_2 = -\frac{C}{2\omega^2} \qquad (2.58)$$

Recalling the boundary condition $E(0) = 0$, we deduce from (2.55) that $C_1 = 0$, and consequently

$$\left(1 + \frac{i\omega}{K(1/2)}\right)^{-1} = \frac{2\omega^2}{q^2}\,, \qquad (2.59)$$

which in the limit of small q, ω is solved by the considering leading orders in momentum q,

$$\omega = \pm\frac{q}{\sqrt{2}} - \frac{iq^2}{4K(1/2)}\,. \qquad (2.60)$$

This excitation has been identified before, and is called [16] holographic zero sound. In the $d = 2+1$ dimensional system this mode has been observed in [23]. Note that the speed of sound does not depend on dimensionality p of a probe brane and for any value of p is equal to the speed of the usual sound in the hydrodynamic regime [45]. In Section IV we will study current-current two-point functions, and the peak in the spectral function, corresponding to zero sound mode, will also be observed in the numerics.

Now, let us consider the fluctuation equation (2.50) for the transverse gauge field component, in the limit ω, $q \ll 1$. Then eq. (2.50) becomes

$$a_1'' + \frac{2z^3}{1+z^4} a_1' = 0\,, \tag{2.61}$$

with an exact solution being

$$a_1(z) = C_1 + C_2\sqrt{i} F\left(i \sinh^{-1}(\sqrt{i}z)| - 1\right)\,. \tag{2.62}$$

In the near-horizon $z \to \infty$ limit it is expanded as

$$a_1(z) \simeq C_1 - C_2 K(1/2) + C_2/z\,. \tag{2.63}$$

Comparing it with the incoming-wave solution (2.114), one obtains

$$C_1 = (i\omega + K(1/2))\, C\,, \qquad C_2 = C\,. \tag{2.64}$$

Then, near-boundary $z \ll 1$ expansion of (2.62) is given by

$$a_1(z) \simeq A + Bz\,, \tag{2.65}$$

where

$$A = (i\omega + K(1/2))\, C\,, \qquad B = -C\,. \tag{2.66}$$

Therefore one may find the current two-point function $\langle J^1 J^1 \rangle = \frac{B}{A}$. In particular, its imaginary part is given by

$$\mathrm{Im}\langle J^1 J^1 \rangle \simeq \frac{\omega}{[K(1/2)]^2}\,. \tag{2.67}$$

We provide numerical results for the transverse fluctuations in Section IV.

Non-vanishing magnetic field

In this subsection we are going to study the case of small magnetic field, $b \ll 1$, which will allow us to achieve some simplifications. Let us rewrite the action (2.46) as

$$S = \int dz \left(\mathcal{G}_E E'^2 + \mathcal{U}_E E^2 + \mathcal{G}_a a_1'^2 + \mathcal{U}_a a_1^2 + \mathcal{C}^{(1)}(E a_1)' \right) , \tag{2.68}$$

where we have denoted

$$\mathcal{G}_E = - \frac{(1+(1+b^2)z^4)^{1/2}}{(1+b^2 z^4)\left(\omega^2 - \frac{q^2}{1+(1+b^2)z^4}\right)} , \quad \mathcal{U}_E = \frac{(1+(1+b^2)z^4)^{1/2}}{1+b^2 z^4} , \tag{2.69}$$

$$\mathcal{C}^{(1)} = \frac{2ibz^4}{1+b^2 z^4} , \quad \mathcal{U}_a = - \frac{(1+(1+b^2)z^4)^{1/2}\left(\omega^2 - \frac{q^2}{1+(1+b^2)z^4}\right)}{1+b^2 z^4} , \tag{2.70}$$

$$\mathcal{G}_a = \frac{(1+(1+b^2)z^4)^{1/2}}{1+b^2 z^4} . \tag{2.71}$$

In the case of $b \ll 1$, we can approximate

$$\mathcal{G}_E = - \frac{(1+z^4)^{1/2}}{(1+b^2 z^4)\left(\omega^2 - \frac{q^2}{1+z^4}\right)} , \quad \mathcal{U}_E = \frac{(1+z^4)^{1/2}}{1+b^2 z^4} , \tag{2.72}$$

$$\mathcal{G}_a = \frac{(1+z^4)^{1/2}}{1+b^2 z^4} , \quad \mathcal{U}_a = - \frac{(1+z^4)^{1/2}\left(\omega^2 - \frac{q^2}{1+z^4}\right)}{1+b^2 z^4} , \tag{2.73}$$

$$\mathcal{C}^{(1)} = \frac{2ibz^4}{1+b^2 z^4} . \tag{2.74}$$

In the near-horizon limit, for $\omega > 0$, integrating the $\mathcal{C}^{(1)}$ term by parts, we arrive at

$$S = \int \frac{dz\, z^2}{1+b^2 z^4} \left(- \frac{E'^2}{\omega^2} + E^2 + a_1'^2 - (\omega^2 - b^2 q^2) a_1^2 - 8ibE a_1 \frac{z}{1+b^2 z^4} \right) . \tag{2.75}$$

Moreover, for $z \gg 1/\sqrt{b}$ and $z^3 \gg 1/(b(\omega^2 - b^2 q^2)^{1/2})$, we actually obtain decoupled system of equations

$$E'' - \frac{2}{z} E' + \omega^2 E = 0 \quad \Rightarrow \quad E = \tilde{C}_1 (1 - i\omega z) e^{i\omega z} , \tag{2.76}$$

$$a_1''-\frac{2}{z}a_1'+(\omega^2-b^2q^2)a_1=0 \;\Rightarrow\; a_1=\tilde{C}_2(1-i\sqrt{\omega^2-b^2q^2}z)e^{i\sqrt{\omega^2-b^2q^2}z}\,. \tag{2.77}$$

Now, assume that $\omega^2 \gg b^2q^2$, and perform the linear transformation in (2.75)

$$E = i\omega(\chi_1\tilde{E}+\chi_2\tilde{a}_1)\,, \qquad a_1 = \chi_1\tilde{E}-\chi_2\tilde{a}_1\,, \tag{2.78}$$

with arbitrary constant coefficients χ_1, χ_2, which brings the action to the form

$$S = \int \frac{2dz\,z^2}{1+b^2z^4}\left(\chi_1^2\left(\tilde{E}'^2-\omega^2\left(1-\frac{4bz}{\omega(1+b^2z^4)}\right)\tilde{E}^2\right)\right.$$
$$\left. +\,\chi_2^2\left(\tilde{a}_1'^2-\omega^2\left(1+\frac{4bz}{\omega(1+b^2z^4)}\right)\tilde{a}_1^2\right)\right)\,. \tag{2.79}$$

Corresponding equations of motion are

$$\tilde{E}''+\frac{2}{z}\frac{1-b^2z^4}{1+b^2z^4}\tilde{E}'+\omega^2\left(1-\frac{4bz}{\omega(1+b^2z^4)}\right)\tilde{E}=0\,, \tag{2.80}$$

$$\tilde{a}_1''+\frac{2}{z}\frac{1-b^2z^4}{1+b^2z^4}\tilde{a}_1'+\omega^2\left(1+\frac{4bz}{\omega(1+b^2z^4)}\right)\tilde{a}_1=0\,. \tag{2.81}$$

The solutions are

$$\tilde{E} = \frac{e^{\pm i\omega z}}{z}+\frac{b}{\omega}(1\mp i\omega z)e^{\pm i\omega z}\,, \tag{2.82}$$

$$\tilde{a}_1 = \frac{e^{\pm i\omega z}}{z}-\frac{b}{\omega}(1\mp i\omega z)e^{\pm i\omega z}\,. \tag{2.83}$$

We impose the incoming-wave behavior,

$$E = i\omega\left(\frac{\chi_1+\chi_2}{z}+(\chi_1-\chi_2)\frac{b}{\omega}(1-i\omega z)\right)e^{i\omega z}\,, \tag{2.84}$$

$$a_1 = \left(\frac{\chi_1-\chi_2}{z}+(\chi_1+\chi_2)\frac{b}{\omega}(1-i\omega z)\right)e^{i\omega z}\,, \tag{2.85}$$

which leaves us with two constant of integration $\chi_1 \pm \chi_2$.

When $\omega \sim q \ll 1$, we can consider fluctuation equations (2.49), (2.50), as for the case of vanishing magnetic field. Then we perform computations along the lines of the previous subsection, using now near-horizon boundary conditions (2.84) and (2.85).

First, we match (2.84) in $\omega z \ll 1$ limit,

$$E = i(b(\chi_1 - \chi_2) + i\omega^2(\chi_1 + \chi_2)) + i\omega\frac{\chi_1 + \chi_2}{z} \tag{2.86}$$

with eq. (2.57). Requiring that $C_1 = 0$, we arrive at

$$q^2 - 2\omega^2 - \frac{2}{K(1/2)}\left(\omega b\frac{\chi_1 - \chi_2}{\chi_1 + \chi_2} + i\omega^3\right) = 0\,. \tag{2.87}$$

Then, we match (2.85) in $\omega z \ll 1$ limit,

$$a_1 = i\omega(\chi_1 - \chi_2) + (\chi_1 + \chi_2)\frac{b}{\omega} + \frac{\chi_1 - \chi_2}{z} \tag{2.88}$$

with eq. (2.63). Again, imposing normalizability condition $C_1 = 0$, we obtain

$$\omega + \frac{1}{K(1/2)}\left(b\frac{\chi_1 + \chi_2}{\chi_1 - \chi_2} + i\omega^2\right) = 0\,. \tag{2.89}$$

Solving (2.87) together with (2.89), we get [7]

$$q^2 - 2\omega^2 + \frac{2b^2}{[K(1/2)]^2} + \frac{i\omega}{K(1/2)}(q^2 - 4\omega^2) = 0\,. \tag{2.90}$$

We see that in the presence of a magnetic field b zero sound mode develops a gap ω_c in the spectrum,

$$\omega_c = \frac{b}{K(1/2)}\,. \tag{2.91}$$

2.4.2 Effective theory for the sound mode

Zero sound may also be studied in the framework of Ref. [37]. First, one introduces a hypersurface $z = z_\Lambda$ in the bulk, integrating out degrees of freedom in the UV region $0 \le z \le z_\Lambda$. The UV physics is then effectively encoded in the action by,

$$S = \frac{1}{2}\int d^3x\big(f_0^2(\partial_0\phi - W_0 + w_0)^2 - f_2^2(\partial_2\phi - W_2 + w_2)^2\big)\,, \tag{2.92}$$

[7] Equivalently, we can obtain this result requiring that (2.87) and (2.89) have a non-trivial solution for $\chi_1 \pm \chi_2$.

where $W_\mu = a_\mu(z=0)$, $w_\mu = a_\mu(z=z_\Lambda)$, and the "Godstone boson" ϕ corresponds to breaking of the $U(1)$ symmetry with a gauge field $W_\mu - w_\mu$. The zero sound mode may be interpreted in such a framework as a mode coming from an excitation of the field ϕ, and therefore the speed of zero sound is given by the expression $v = f_2/f_0$. Let us now compare the effective field theory action for the UV degrees of freedom with the bulk DBI action. To render the relation between bulk and boundary to be precise, we specify the zero boundary condition $W_\mu = 0$, putting the Goldstone boson ϕ to zero:

$$S = \frac{1}{2}\int d^3x(f_0^2 w_0^2 - f_2^2 w_2^2) \tag{2.93}$$

Let us consider all fields to be only z-dependent, in which case transverse fluctuations decouple, and we can put these to zero. Then we can rewrite the bulk theory action (2.39) in a form

$$S \simeq \frac{1}{2}\int d^3x \frac{dz}{1+b^2z^4}(h^3(z)\tilde{a}_0'^2 - h(z)\tilde{a}_2'^2)\,, \tag{2.94}$$

where we have defined $h(z) = \sqrt{1+(1+b^2)z^4}$. The solutions of the equations of motion on $\tilde{a}_0$, $\tilde{a}_2$, satisfying zero boundary condition at the AdS boundary, while being defined on the hypersurface $z = z_\Lambda$, are now given by:

$$w_0 = C_0\int_0^{z_\Lambda}\frac{dz(1+b^2z^4)}{h^3(z)}\,, \qquad w_2 = C_2\int_0^{z_\Lambda}\frac{dz(1+b^2z^4)}{h(z)}\,. \tag{2.95}$$

To match the bulk action and the boundary theory (2.93), we evaluate the action (2.94) on the solution of the EOM, which leaves us with the boundary terms at $z = z_\Lambda$ only

$$S \simeq \frac{1}{2}\int d^3x(C_0 w_0 - C_2 w_2)\,, \tag{2.96}$$

which in turn with the help of (2.95), may be rewritten as (2.93) with

$$f_0^{-2} = \int_0^{z_\Lambda}\frac{dz(1+b^2z^4)}{h^3(z)}\,, \qquad f_2^{-2} = \int_0^{z_\Lambda}\frac{dz(1+b^2z^4)}{h(z)}\,. \tag{2.97}$$

Therefore the speed of zero sound is given by

$$u_0^2 = \int_0^{z_\Lambda}\frac{dz(1+b^2z^4)}{h^3(z)}\left(\int_0^{z_\Lambda}\frac{dz(1+b^2z^4)}{h(z)}\right)^{-1}\,. \tag{2.98}$$

When $b \ll 1$, one obtains

$$u_0^2 \simeq \frac{1}{2 + \frac{8\pi^{1/2}}{3\Gamma[1/4]^2} b^2 z_\Lambda^3} , \tag{2.99}$$

and therefore for $b^2 z_\Lambda^3 \ll 1$ one recovers the value of the speed of zero sound in vanishing b-field, $u_0 = 1/\sqrt{2}$, while for $b^2 z_\Lambda^3 \gg 1$ the speed of zero sound approaches zero. In this regime the description of the low energy physics by the effective action (2.92) presumably breaks down; it would be interesting to write the low energy description that would account for the gap in the spectrum.

2.4.3 Thermodynamic properties of trivial embeddings

We will study the thermodynamics of the trivial Dp brane embedding, to obtain as a result the value of the speed of the usual first (hydrodynamic) sound. We consider here the $D3/Dp$ system with a $2+1$ dimensional intersection, and in the Appendix we will study the supersymmetric $D3/D7$ system with a $3+1$ dimensional intersection, in the presence of a non-vanishing magnetic field.

The total prefactor of the action is irrelevant for the computation of the speed of first sound. The grand canonical potential is given by the equation

$$\Xi = -S = \int d\rho (\rho^4 + \bar{B}^2)(\rho^4 + \bar{B}^2 + \hat{d}^4)^{-1/2} = a \frac{2\bar{B}^2 - \hat{d}^4}{(\bar{B}^2 + \hat{d}^4)^{1/4}} , \tag{2.100}$$

where $a = \Gamma(1/4)^2/(12\sqrt{\pi})$. Using (2.35) one may calculate the charge density as,

$$\hat{\rho} = -\frac{\partial \Xi}{\partial \bar{\mu}_{ch}} , \tag{2.101}$$

to find the energy density, being at zero temperature equal to the free energy,

$$\epsilon = \Xi + \bar{\mu}_{ch}\hat{\rho} = 2a(\bar{B}^2 + \hat{d}^4)^{3/4} . \tag{2.102}$$

Consequently, the speed of sound is given by

$$u^2 = \frac{\partial P}{\partial \epsilon} = -\frac{\partial \Xi}{\partial \epsilon} = \frac{1}{2} \frac{1 + 2b^2}{1 + b^2} . \tag{2.103}$$

Notice that this result is independent of p, which agrees with [45]. Observe that when the magnetic field vanishes we retrieve the value $u^2 = 1/2$, which we observed before in the dispersion relation (2.60).

Notice also that all the steps performed in the above may be combined into one expression (use $\partial S/\partial\bar{\mu}_{ch} = \hat{d}^2$):

$$u^2 = \frac{\partial S/\partial\bar{\mu}_{ch}}{\bar{\mu}_{ch}\partial^2 S/\partial\bar{\mu}_{ch}^2} = \frac{1}{2}\frac{\partial\log\bar{\mu}_{ch}}{\partial\log\hat{d}}\,. \tag{2.104}$$

2.5 Holographic current-current correlators at finite frequency and momentum

In the previous section we have shown that a propagating mode (zero sound) develops a gap in the presence of the magnetic field. In this section we compute numerically the two-point function of the $U(1)$ currents. First we set magnetic field to zero. We identify the holographic zero sound as a peak in the spectral function. We start by computing the density-density correlator $\langle J^0 J^0\rangle$ using the linearized DBI action. We then proceed to computing the transverse correlator $\langle J^1 J^1\rangle$. After that we proceed to the case of non-vanishing magnetic field and show that the gap in the zero sound spectrum shows itself on the numeric graphs.

2.5.1 Fluctuations of electric field strength E

Consider the fluctuation equation (2.47), near the boundary $z = 0$ for any value of magnetic field:

$$E'' - (q^2 - \omega^2)E = 0\,. \tag{2.105}$$

Its general solution is of the form,

$$E = \mathcal{A}_E F_I + \mathcal{B}_E F_{II}\,, \tag{2.106}$$

where we have denoted the two independent solutions as

$$F_I = 1 + \frac{q^2-\omega^2}{2}z^2 + \frac{(q^2-\omega^2)^2}{24}z^4 + \cdots\,, \tag{2.107}$$

$$F_{II} = z + \frac{q^2-\omega^2}{6}z^3 + \cdots\,. \tag{2.108}$$

The on-shell action is therefore given by

$$S_{on-shell} \simeq \lim_{\varepsilon\to 0}\int d\omega dq \mathcal{A}_E(\omega,q)\mathcal{A}_E(-\omega,-q)\frac{1}{q^2-\omega^2}\frac{\mathcal{B}_E(\omega,q)}{\mathcal{A}_E(\omega,q)}|_{z=\varepsilon}\,. \tag{2.109}$$

Non-vanishing Green functions are

$$\begin{aligned}
\langle J^0(\omega,q)J^0(-\omega,-q)\rangle &= \lim_{\varepsilon\to 0}\frac{\delta^2 S_{on-shell}}{\delta a_0(z=\varepsilon,\omega,q)\delta a_0(z=\varepsilon,-\omega,-q)} \\
&= \lim_{\varepsilon\to 0}\frac{\delta^2 S_{on-shell}}{\delta E(z=\varepsilon,\omega,q)\delta E(z=\varepsilon,-\omega,-q)} \\
&\times\frac{\delta E(\omega,q,z)}{\delta a_0(\omega,q,z)}\frac{\delta E(-\omega,-q,z)}{\delta a_0(-\omega,-q,z)} = \qquad (2.110)\\
&= -\frac{q^2}{q^2-\omega^2}\frac{\mathcal{B}_E(\omega,q)}{\mathcal{A}_E(\omega,q)}\,,\\
\langle J^2(\omega,q)J^2(-\omega,-q)\rangle &= \lim_{\varepsilon\to 0}\frac{\delta^2 S_{on-shell}}{\delta a_2(z=\varepsilon,\omega,q)\delta a_2(z=\varepsilon,-\omega,-q)} = \\
&= \lim_{\varepsilon\to 0}\frac{\delta^2 S_{on-shell}}{\delta E(z=\varepsilon,\omega,q)\delta E(z=\varepsilon,-\omega,-q)} \\
&\times\frac{\delta E(\omega,q,z)}{\delta a_2(\omega,q,z)}\frac{\delta E(-\omega,-q,z)}{\delta a_2(-\omega,-q,z)} = \qquad (2.111)\\
&= -\frac{\omega^2}{q^2-\omega^2}\frac{\mathcal{B}_E(\omega,q)}{\mathcal{A}_E(\omega,q)}\,,\\
\langle J^0(\omega,q)J^2(-\omega,-q)\rangle &= \lim_{\varepsilon\to 0}\frac{\delta^2 S_{on-shell}}{\delta a_2(z=\varepsilon,\omega,q)\delta a_0(z=\varepsilon,-\omega,-q)} = \\
&= \lim_{\varepsilon\to 0}\frac{\delta^2 S_{on-shell}}{\delta E(z=\varepsilon,\omega,q)\delta E(z=\varepsilon,-\omega,-q)} \\
&\times\frac{\delta E(\omega,q,z)}{\delta a_2(\omega,q,z)}\frac{\delta E(-\omega,-q,z)}{\delta a_0(-\omega,-q,z)} = \qquad (2.112)\\
&= -\frac{\omega q}{q^2-\omega^2}\frac{\mathcal{B}_E(\omega,q)}{\mathcal{A}_E(\omega,q)}\,.
\end{aligned}$$

Note that these expression agree with the Ward identity for the $U(1)$ conserved current J^μ,

$$\omega\langle J^0(\omega,q)J^0(-\omega,-q)\rangle - q\langle J^0(\omega,q)J^2(-\omega,-q)\rangle = 0\,. \tag{2.113}$$

We evaluate numerically the ratio $\mathcal{B}_E/\mathcal{A}_E$ on the solution of equation (2.47) with incoming-wave near horizon behavior (2.52). In Fig. 2.3 we present numerical results for the real and imaginary parts of the $\mathcal{B}_E/\mathcal{A}_E$ for different values of ω, q in the case of $b = 0$. The holographic zero sound corresponds to the peak in the spectral density.

2.5.2 Fluctuations of the transverse component of the gauge field

In this subsection we will compute numerically the holographic two-point function for the transverse current $\langle J^1(x)J^1(y)\rangle$. Let us put $b = 0$.

In the near horizon regime $z \to \infty$ the bulk solution, corresponding to the retarded current-current propagator in the dual field theory, takes the incoming-wave form

$$a_1 = C\frac{e^{i\omega z}}{z}\,, \tag{2.114}$$

and in the vicinity of the boundary, the equation of motion becomes

$$a_1'' - (q^2 - \omega^2)a_1 = 0\,, \tag{2.115}$$

with a general solution being a combination of F_I and F_{II} (2.107), (2.108),

$$a_1 = \mathcal{A}_a F_I + \mathcal{B}_a F_{II}\,. \tag{2.116}$$

The results of numerical evaluations of the holographic two-point function $\langle J^1(q)J^1(-q)\rangle = \frac{\mathcal{B}_a}{\mathcal{A}_a}$ are presented in Fig. 2.4. We see that it does not reveal any structure.

2.5.3 Non-vanishing magnetic field

In the case of $b \neq 0$ fluctuations of the longitudinal $E(x^0, x^2, z)$ and transverse $a_1(x^0, x^2, z)$ components of the gauge field are no longer decoupled[8]. They are described by the action (2.68), which can be written as

$$\begin{aligned} S = \int dz\, \Bigg(& \left(-(\mathcal{G}_E E')' + \mathcal{U}_E E - \frac{1}{2}(\mathcal{C}^{(1)})' a_1\right) E + \\ & + \left(-(\mathcal{G}_a a_1')' + \mathcal{U}_a a_1 - \frac{1}{2}(\mathcal{C}^{(1)})' E\right) a_1 \Bigg) + \\ & + \left[\mathcal{G}_E E E' + \mathcal{G}_a a_1 a_1' + \mathcal{C}^{(1)} E a_1\right]_{z=0}^{z=\infty}\,. \end{aligned} \tag{2.117}$$

[8]We thank R. Davison for pointing this out to us.

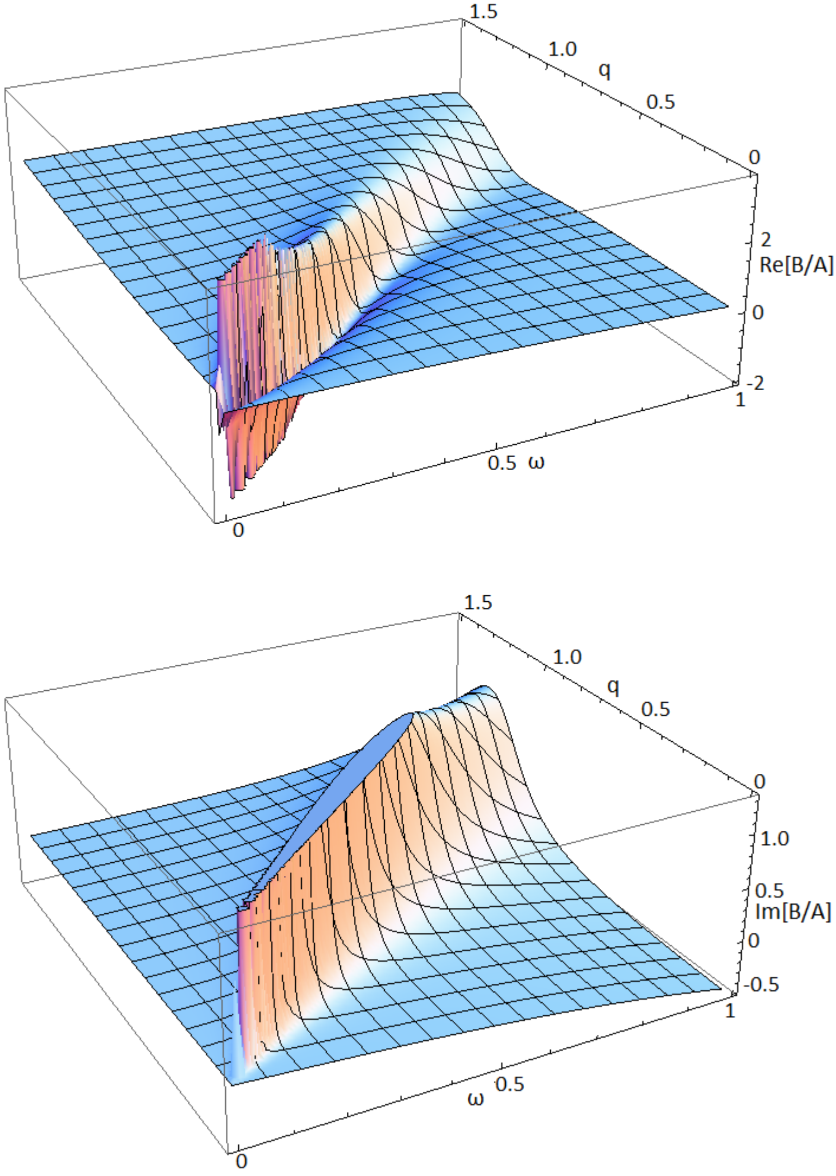
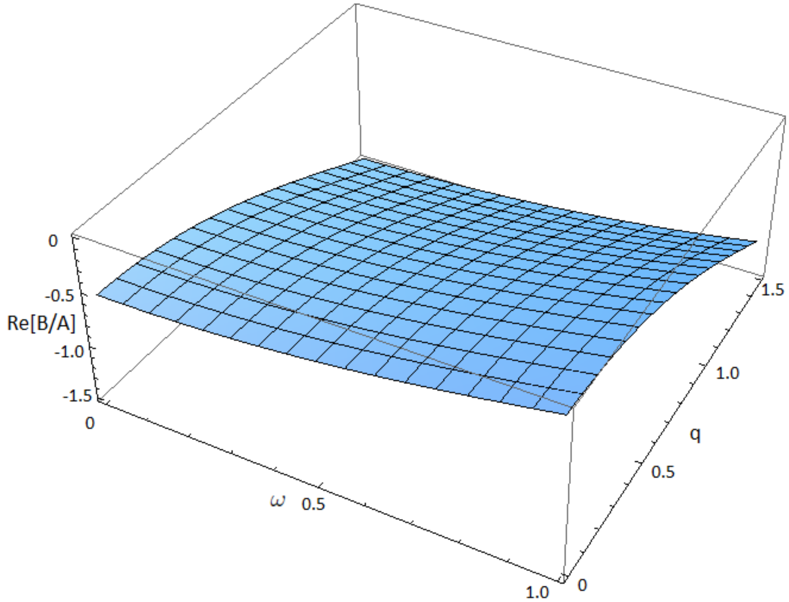


Figure 2.3. Real and imaginary parts of the $\mathcal{B}_E/\mathcal{A}_E$ in the $D3/Dp$ system with $d = 2+1$ dimensional intersection. The spectrum of excitations is exhausted by the holographic zero sound mode with the speed of sound $u_0 = \frac{1}{\sqrt{2}}$, and the attenuation $\Gamma_q \simeq q^2$.



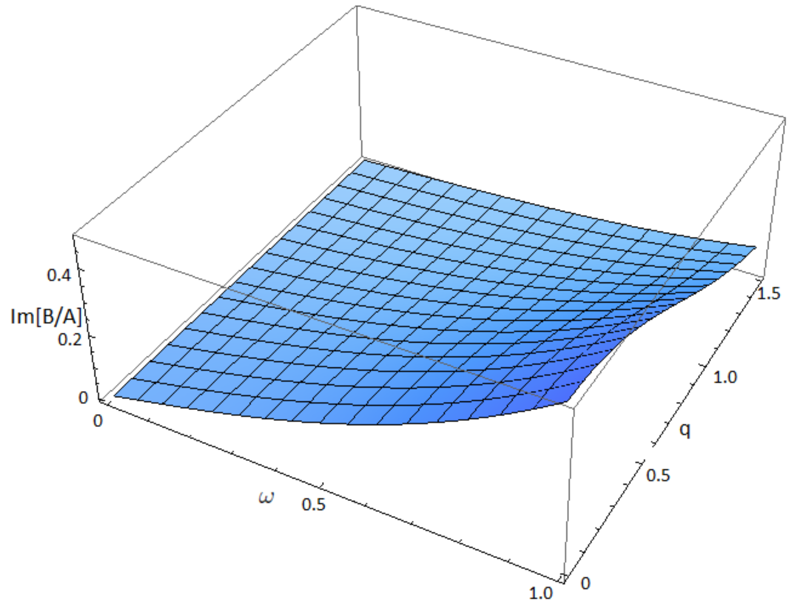


Figure 2.4. Real and imaginary parts of the correlation function $\langle J^1(-q)J^1(q)\rangle$ in the $D3/D7$ system with $d = 2+1$ dimensional intersection. No non-trivial collective excitation modes are observed. For small frequencies and momenta $\omega, q \ll 1$, the imaginary part of the correlation function behaves as $\mathrm{Im}[\langle J^1(-q)J^1(q)\rangle] \sim \omega$, independently of a particular value of q.

The first two lines vanish on shell. In the last line the cross term does not contribute to the variation of the on-shell action by the boundary $z = 0$ values of the fields E and a_1, because

$$\mathcal{C}^{(1)}|_{z=0} = 0\,. \tag{2.118}$$

The on-shell action is then given by the boundary term

$$S_{on-shell} \simeq \lim_{\varepsilon\to 0} \int d\omega dq \left(\frac{1}{q^2-\omega^2} E E' + a_1 a_1' \right)_{z=\varepsilon}\,. \tag{2.119}$$

Near the boundary the solutions to equations of motion are given by

$$E = \mathcal{A}_E F_I + \mathcal{B}_E F_{II}\,, \qquad a_1 = \mathcal{A}_a F_I + \mathcal{B}_a F_{II}\,, \tag{2.120}$$

where $F_{I,\,II}$ are defined by (2.107), (2.108).

To compute current-current two-point function numerically, we follow [46], where general system of coupled equations in the bulk is studied. For arbitrary two independent solutions $\Phi_{(1)}$, $\Phi_{(2)}$ of the coupled system of fluctuation equations (2.47), (2.48), we define the matrix $H = \big(\Phi_{(1)},\ \Phi_{(2)}\big)$. Near the boundary it is expanded as

$$H = \mathcal{A} F_I + \mathcal{B} F_{II}\,. \tag{2.121}$$

On-shell action (2.119) may be rewritten as

$$S_{on-shell} \simeq \int d\omega dq\, \Phi^T M\, \Phi'\,, \tag{2.122}$$

where

$$M = \begin{pmatrix} \frac{1}{q^2-\omega^2} & 0 \\ 0 & 1 \end{pmatrix}\,. \tag{2.123}$$

The matrix of correlation functions is then given by (see eq. (2.34) in [46])

$$G \simeq M \mathcal{B} \mathcal{A}^{-1}\,. \tag{2.124}$$

In such a form the current-current correlation matrix G is explicitly independent of a linear change of fields

$$\Phi_{(1)} \to r_1 \Phi_{(1)} + r_2 \Phi_{(2)}\,,\ \Phi_{(2)} \to r_3 \Phi_{(1)} + r_4 \Phi_{(2)} \ \Rightarrow\ H \to H R\,, \tag{2.125}$$

where $R = \begin{pmatrix} r_1 & r_3 \\ r_2 & r_4 \end{pmatrix}$ is some arbitrary non-degenerate matrix. If $\Phi_{(1),(2)} = \begin{pmatrix} E^{(1),(2)} \\ a_1^{(1),(2)} \end{pmatrix}$ are some arbitrary independent solutions, then due to (2.121) we get

$$\mathcal{A} = \begin{pmatrix} \mathcal{A}_E^{(1)} & \mathcal{A}_E^{(2)} \\ \mathcal{A}_a^{(1)} & \mathcal{A}_a^{(2)} \end{pmatrix}, \qquad \mathcal{B} = \begin{pmatrix} \mathcal{B}_E^{(1)} & \mathcal{B}_E^{(2)} \\ \mathcal{B}_a^{(1)} & \mathcal{B}_a^{(2)} \end{pmatrix}, \tag{2.126}$$

and therefore using (2.124) we obtain

$$G \simeq \frac{1}{\mathcal{A}_E^{(1)}\mathcal{A}_a^{(2)} - \mathcal{A}_a^{(1)}\mathcal{A}_E^{(2)}} \begin{pmatrix} \frac{\mathcal{B}_E^{(1)}\mathcal{A}_a^{(2)} - \mathcal{B}_E^{(2)}\mathcal{A}_a^{(1)}}{q^2-\omega^2} & \frac{\mathcal{B}_E^{(2)}\mathcal{A}_E^{(1)} - \mathcal{B}_E^{(1)}\mathcal{A}_E^{(2)}}{q^2-\omega^2} \\ \mathcal{B}_a^{(1)}\mathcal{A}_a^{(2)} - \mathcal{B}_a^{(2)}\mathcal{A}_a^{(1)} & \mathcal{A}_E^{(1)}\mathcal{B}_a^{(2)} - \mathcal{B}_a^{(1)}\mathcal{A}_E^{(2)} \end{pmatrix}, \tag{2.127}$$

Near-horizon solutions are given by (2.76), (2.77), which we can write as a linear combination of two independent solutions

$$\tilde{\Phi}_{(1)} = \begin{pmatrix} (1 - i\omega z)e^{i\omega z} \\ (1 - i\sqrt{\omega^2 - b^2q^2}z)e^{i\sqrt{\omega^2-b^2q^2}z} \end{pmatrix}, \tag{2.128}$$

$$\tilde{\Phi}_{(2)} = \begin{pmatrix} (1 - i\omega z)e^{i\omega z} \\ -(1 - i\sqrt{\omega^2 - b^2q^2}z)e^{i\sqrt{\omega^2-b^2q^2}z} \end{pmatrix} \tag{2.129}$$

Arbitrary near-horizon behavior, with the most general form (up to simultaneous rescaling of all fields by the same factor) may therefore be written as a linear combination of these two solutions,

$$\Phi = \begin{pmatrix} (1 - i\omega z)e^{i\omega z} \\ c(1 - i\sqrt{\omega^2 - b^2q^2}z)e^{i\sqrt{\omega^2-b^2q^2}z} \end{pmatrix} = \frac{1+c}{2}\tilde{\Phi}_{(1)} + \frac{1-c}{2}\tilde{\Phi}_{(2)}\,. \tag{2.130}$$

On the other hand, fluctuation equations may be rewritten as

$$\Omega_1\Phi'' + \Omega_2\Phi' + \Omega\Phi = 0\,, \tag{2.131}$$

with matrices $\Omega_{1,2,3}$, being determined from (2.47), (2.48). Therefore linear combination of near-horizon solutions (2.129) results in the same linear combination of the solutions near the boundary. Recall that the matrix correlation function (2.127) is the same for any such a non-degenerate linear combination.

We therefore fix two arbitrary near-horizon conditions, say (2.129), determine corresponding coefficients $\mathcal{A}_E^{(1),(2)}$, $\mathcal{A}_a^{(1),(2)}$ and $\mathcal{B}_E^{(1),(2)}$, $\mathcal{B}_a^{(1),(2)}$

by integrating numerically fluctuation equations (2.47), (2.48) up to the boundary and matching corresponding solutions with (2.120), and compute the correlation matrix (2.127). Each of the four components of the correlation matrix shows a gapped zero sound mode.

In figure 2.5 we plot the real and imaginary parts of the G_{22} component, for $b = 0.001$. We see the gapped zero sound mode, with the gap which scales as $\omega_c \sim b$, in agreement with analytic result $\omega_c = \frac{b}{K(1/2)}$ of the previous section.

2.6 Discussion

In this chapter we have studied current-current two-point functions at strong coupling. We have considered the current-current correlators at finite momenta, but did not observe any nontrivial structure in the spectral function, other than the zero sound[9].

It is instructive to compare the holographic density-density correlator with the form expected from the random phase approximation and reviewed in Section II. Within RPA the zero sound mode presents itself as a smeared delta-function like peak in Fig. 1, the Lindhard particle-hole continuum starts at $\mathbf{q} \simeq \mathbf{w}/v_F$ and sharply ends at $\mathbf{q} \simeq 2\mathbf{q}_F$. The absence of the Lindhard continuum in the holographic computations can be explained by parametrically large values of the Landau parameters. The key point is eq. (2.11) which implies that since the zero sound velocity that we observe is $\mathcal{O}(1)$, the value of Fermi velocity scales like $v_F \sim 1/\sqrt{F_0 F_1}$. The regime of validity of our calculations is limited to $\mathbf{w} \sim \mathbf{q}$, and therefore the Lindhard continuum cannot be observed for parametrically large values of the Landau parameters. In the following we offer some speculations on how such a scenario can play out.

We can argue that the Fermi velocity is parametrically small. Recall that $q \simeq \mathbf{q}\sqrt{\lambda}/\mu$. Hence, eq. (2.60) implies that the zero sound attenuation is $\alpha \sim \mathbf{w}^2\sqrt{\lambda}/\mu$. According to [47] this can be expressed in terms of the quasi-particle lifetime as

$$\alpha \simeq \frac{1}{\tau}\frac{m^*}{\mu}v_F^2 F_2^2 \sim \frac{\mathbf{w}^2}{\mu}F_0^2 F_2^2 \tag{2.132}$$

[9] Note that our models are different from those studied in [7, 8], where poles at finite momenta were observed in the holographic two-point functions of operators with nonvanishing charge under global $U(1)$.

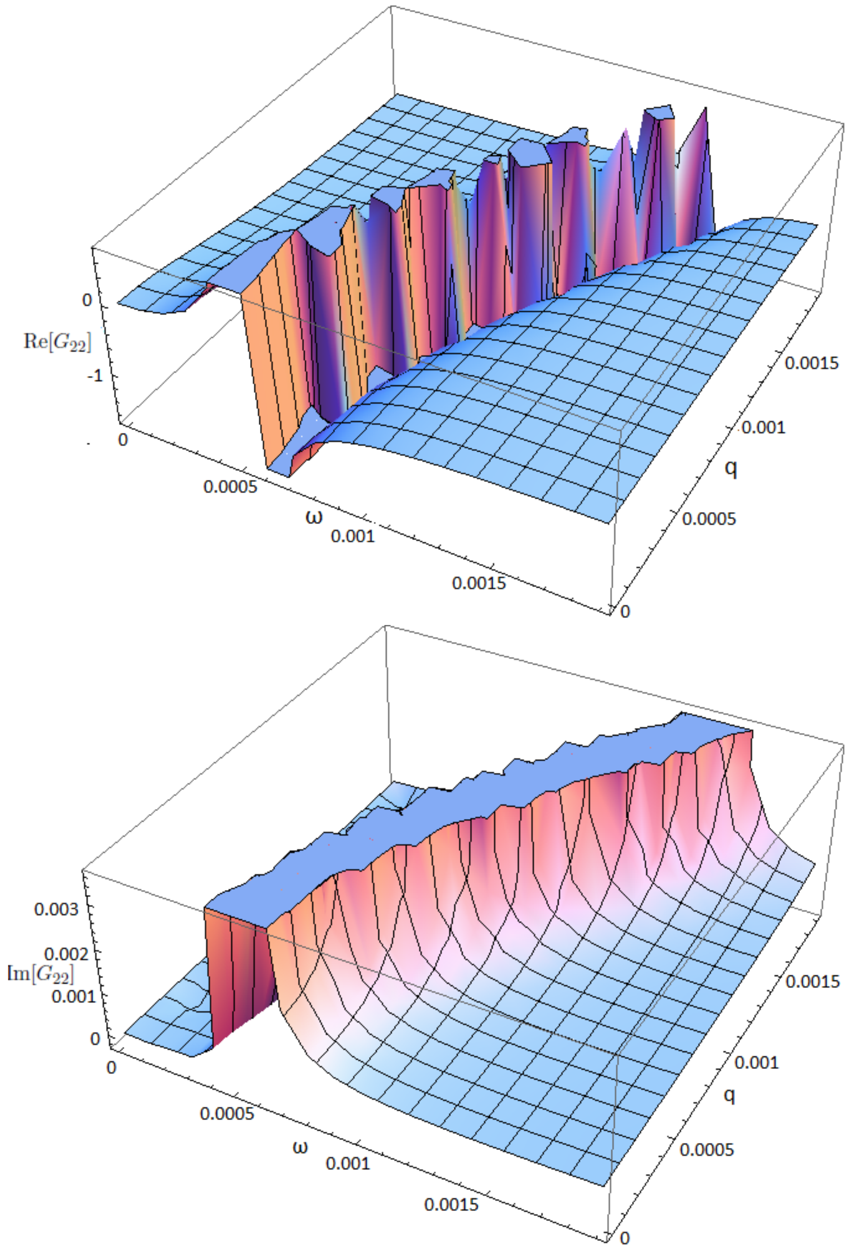


Figure 2.5. Real and imaginary parts of the component G_{22} of the correlation matrix (2.124) in the $D3/D7$ system with $d = 2 + 1$ dimensional intersection, for the magnetic field $b = 0.001$. The spectrum of excitations is exhausted by a gapped zero sound mode, with the value of the gap $\omega_c \sim b$.

To derive the second approximate equality we used the Fermi liquid estimate $1/\tau \sim \mathbf{w}^2 m^* {F_0}^2/\mathbf{q}_F^2 \sim \mathbf{w}^2 F_0^2/E_F^*$. Eq. (2.132) implies that the Landau parameters are indeed parametrically large, $F_0^2 F_2^2 \sim \sqrt{\lambda}$.

We have analyzed the system in the presence of magnetic field and observed a gap in the excitation spectrum $\mathbf{w}_c$. We derived a scaling relation $\mathbf{w}_c \simeq B/\mu$. Note that the gap in the spectrum of non relativistic fermions scales linearly with B, while the relativistic fermions obey $\sqrt{B}$ scaling; Kohn's theorem implies that the gap in the spectrum of excitations is not changed when the pairwise interaction is turned on. In our setup charged fermions interact and can exchange momentum with $\mathcal{N}=4$ SYM degrees of freedom; the linear scaling of the gap with the magnetic field is consistent with the assumption that the effective degrees of freedom have an effective mass $m^* \simeq \mu$. [According to eq. (2.10) this implies that $F_1 = \mathcal{O}(1)$; a scenario consistent with the discussion above may involve a parametrically large $F_0 \sim \lambda^{1/4}$ but finite F_n, $n > 0$.] We do not quite understand the mechanism of dynamical mass generation at finite density – it is clearly very different from dynamical mass generation in a strongly interacting fermion system at zero density[10].

We already emphasized that a priori the very existence of zero sound is nontrivial, given the interaction of the charged matter with the uncharged superconformal degrees of freedom. It would be interesting to make this picture more precise and to see whether there is any relation to the recent studies of fermions in magnetic fields in the context of holography [48–50, 52, 51]. It would also be interesting to compare our results with the correlators computed in the charged magnetic brane background [53].

At this point it is worth recalling the relation between the charge density and the value of the chemical potential, given by (2.35). As usual, the value of the charge density is proportional to $\hat{d}$, $\rho \simeq N_c \lambda^{(p-5)/4} \hat{d}$ and the proportionality coefficient strongly depends on the dimensionality of the probe brane. The incompressibility $\partial \hat{d}/\partial\mu$ is a smooth non-vanishing function of μ, b. This implies that we cannot rule out the existence of gapless modes in our system[11]. Indeed, the analysis that led to the existence of the zero sound implicitly assumed $\omega \sim q \sim b$, and can be shown to break down for $|\omega| < bq$. We leave the search for gapless quasi normal modes for future work. Let us also note that a smooth compressibility is not

[10] The holographic dual of the latter involves repulsion of the probe brane from the bulk of of the AdS space; see e.g. [41] for a recent discussion.

[11] We thank D. Son for pointing this out to us.

compatible with the existence of Landau levels for the effective fermions.

Appendix is devoted to the subject of higher derivative corrections to the DBI action for the probe Dp brane. The possibility of breakdown of the DBI description in the extreme infrared (very close to the horizon) was pointed out in [54]. Two possible causes were identified in the presence of the electric flux on the brane: strong back-reaction and vanishing of the effective string tension. The strength of back-reaction from the flavor branes is governed by the ratio N_f/N_c. We did not investigate $1/N_c$ corrections in this chapter, although it is a very interesting problem. Instead, we explored the effects of the breakdown of the DBI description due to the vanishing of the effective string tension near the horizon. The strength of this effect is controlled by an inverse power of 't Hooft coupling. In principle, such effects can be described by going to higher orders in the α' expansion of the effective action for open strings, which corresponds to adding higher derivative terms to the DBI Lagrangian. Unfortunately we are not aware of the precise structure of higher derivative corrections to DBI in the presence of the worldvolume electric field. However we were able to model this situation by writing generic higher derivative terms which become important near the horizon and completely change the effective metric for fluctuations there.

The effect of such terms is confined to a very small region (which scales as an inverse power of 't Hooft coupling in suitable units); outside of this region the second order differential equations derived from the DBI are applicable. In principle, one can solve the higher order fluctuation equation outwards from the horizon, and then feed the resulting solution into the second order equation. From the point of view of the latter, this amounts to modifying the boundary conditions: an outgoing wave (with a small coefficient) is added to the incoming wave near the horizon. We verify that this does not introduce any qualitative new features in the two-point functions.

2.7 Appendix: Higher-derivative corrections to $L_{DBI}(a_1)$

The DBI description might break down in the near-horizon region [54], and therefore higher derivative corrections become essential in that region. Consider higher derivative correction to the DBI Lagrangian of the form

[55, 56]
$$\frac{\epsilon}{2}\sqrt{-g}g^{\mu\lambda}g^{\nu\rho}g^{\alpha\beta}(\nabla_\alpha F_{\lambda\rho})(\nabla_\beta F_{\mu\nu})\,, \tag{2.133}$$
Alternatively, using the Bianchi identity, one may rewrite it as
$$\tilde{\epsilon}\sqrt{-g}g_{\mu\nu}\nabla_\lambda F^{\lambda\mu}\nabla_\sigma F^{\sigma\nu}\,. \tag{2.134}$$
Here $\epsilon \sim \ell_s^2 \sim \frac{1}{\sqrt{\lambda}}$.

In this section we put $L = 1$. The induced $AdS_4 \times S^4$ metric on the trivially embedded Dp brane world-volume then takes the form
$$ds^2 = \rho^2(-(dx^0)^2 + (dx^1)^2 + (dx^2)^2) + \frac{d\rho^2}{\rho^2} + d\Omega_4^2\,, \tag{2.135}$$
This corresponds to non-vanishing Christoffel symbols in the AdS subspace,
$$\Gamma^\rho_{\rho\rho} = -\frac{1}{\rho}\,, \quad \Gamma^\rho_{ij} = -\rho^3\eta_{ij}\,, \quad \Gamma^i_{\rho j} = \frac{1}{\rho}\delta^i_j\,, \tag{2.136}$$
where $\eta_{00} = -1$, $\eta_{11} = \eta_{22} = 1$. We fix the background value of $A_0'(\rho)$ (2.34) (with $\bar{B} = 0$) and study the dynamics of the fluctuation field $a_1(\rho, x^0, x^2)$. Consequently, the non-vanishing components of the field strength tensor covariant derivatives
$$\nabla_\alpha F_{\mu\nu} = \partial_\alpha F_{\mu\nu} - \Gamma^\tau_{\alpha\mu}F_{\tau\nu} - \Gamma^\tau_{\alpha\nu}F_{\mu\tau} \tag{2.137}$$
are given by
$$\nabla_1 F_{0\rho} = -\frac{1}{\rho}F_{01}\,, \quad \nabla_\rho F_{0\rho} = \partial_\rho F_{0\rho}\,, \quad \nabla_\rho F_{01} = \partial_\rho F_{01} - \frac{2}{\rho}F_{01}\,, \tag{2.138}$$
$$\nabla_1 F_{01} = \rho^3 F_{0\rho}\,, \quad \nabla_2 F_{01} = \partial_2 F_{01}\,, \quad \nabla_0 F_{01} = \partial_0 F_{01} - \rho^3 F_{\rho 1}\,, \tag{2.139}$$
$$\nabla_2 F_{12} = \partial_2 F_{12} + \rho^3 F_{1\rho}\,, \quad \nabla_0 F_{12} = \partial_0 F_{12}\,, \quad \nabla_\rho F_{12} = \partial_\rho F_{12} - \frac{2}{\rho}F_{12}\,, \tag{2.140}$$
$$\nabla_2 F_{\rho 1} = \partial_2 F_{\rho 1} - \frac{1}{\rho}F_{21}\,, \quad \nabla_0 F_{\rho 1} = \partial_0 F_{\rho 1} - \frac{1}{\rho}F_{01}\,, \quad \nabla_\rho F_{\rho 1} = \partial_\rho F_{\rho 1}\,. \tag{2.141}$$

Let us now substitute the quantities (2.138)-(2.141) into the Lagrangian (2.133), which becomes in momentum representation,
$$\Delta L = \epsilon[-\rho^2(\partial_\rho A_0)^2 - \rho^4(\partial_\rho^2 A_0)^2 + \frac{1}{\rho^2}\left(\frac{(q^2-\omega^2)^2}{\rho^2} + 5q^2 - 6\omega^2\right)a_1^2$$
$$+ 2(\rho^2 + q^2 - \omega^2)(\partial_\rho a_1)^2 + \rho^4(\partial_\rho^2 a_1)^2 + \frac{4(\omega^2 - q^2)}{\rho}a_1\,\partial_\rho a_1]\,. \tag{2.142}$$

To obtain the corrected equation of motion of the background field $\partial_\rho A_0$, we put the a_1 fluctuations to zero and write the total, DBI + corrections, Lagrangian as

$$L = \rho^2\sqrt{1-(\partial_\rho A_0)^2} - \epsilon[\rho^2(\partial_\rho A_0)^2 + \rho^2(\partial_\rho^2 A_0)^2]\,. \qquad (2.143)$$

The corresponding equation of motion

$$\frac{\rho^2\partial_\rho A_0}{\sqrt{1-(\partial_\rho A_0)^2}} + 2\epsilon[\rho^2\,\partial_\rho A_0 - \partial_\rho\left(\rho^4\,\partial_\rho^2 A_0\right)] = \hat{d}^2 \qquad (2.144)$$

is solved to first order in ϵ by

$$\partial_\rho A_0 = \frac{\hat{d}^2}{\sqrt{\rho^4+\hat{d}^4}} + \delta\,\partial_\rho A_0\,, \qquad (2.145)$$

where we have denoted the correction to the background as

$$\delta\,\partial_\rho A_0 = -\frac{2\hat{d}^2\epsilon\rho^6(\hat{d}^8+16\hat{d}^4\rho^4+3\rho^8)}{(\rho^4+\hat{d}^4)^4}\,. \qquad (2.146)$$

Note that (2.146) approaches zero as $\mathcal{O}(\rho^6)$, near the horizon $\rho = 0$. Therefore the correction to the behavior of the background potential $\partial_\rho A_0$ does not substantially affect the near-horizon physics. Using the $z = 1/\rho$ radial coordinate, and considering the near horizon limit $\omega z \gg 1$, we obtain from (2.142) the correction to the near-horizon DBI Lagrangian

$$\Delta L = \epsilon\left((q^2-\omega^2)z^2(2a_1'^2 + (q^2-\omega^2)a_1^2) + (2a_1' + z a_1'')^2\right)\,. \qquad (2.147)$$

This is to be added to the quadratic DBI near-horizon Lagrangian,

$$L_{DBI} = z^2(a_1'^2 - \omega^2 a_1^2)\,. \qquad (2.148)$$

As a result we obtain the following near-horizon Lagrangian:

$$L = \left(\left(1+2\epsilon(q^2-\omega^2)\right)z^2 + 2\epsilon\right)a_1'^2 + \left(-\omega^2+\epsilon(q^2-\omega^2)^2\right)z^2 a_1^2 \\ + 2\epsilon(z a_1'^2)' + \epsilon z^2(a_1'')^2\,. \qquad (2.149)$$

Up to a total derivative term[12] and $\mathcal{O}(\epsilon)$ modification of the DBI behavior, this Lagrangian therefore may be rewritten as

$$L = z^2(a_1'^2 - \omega^2 a_1^2) + \epsilon z^2(a_1'')^2\,, \qquad (2.150)$$

[12] Corresponding boundary terms $2\epsilon z a_1'^2$, evaluated on non-perturbed solution $e^{i\omega z}/z$, vanish when $z \gg 1$.

with associated equation of motion

$$a_1'' + \frac{2}{z}a_1' + \omega^2 a - \epsilon\left(a_1'''' + \frac{4}{z}a_1'''\right) = 0\,. \tag{2.151}$$

To estimate the relative significance of the correction and DBI terms, let us compare terms a_1'' and $\epsilon\left(a_1'''' + \frac{4}{z}a_1'''\right)$, when evaluated on the non-perturbed near-horizon solution $e^{i\omega z}/z$:

$$a_1'' \simeq \frac{1+(\omega z)^2}{z^3}\,, \qquad \epsilon\left(a_1'''' + \frac{4}{z}a_1'''\right) \simeq \epsilon\frac{\omega^4}{z}\,. \tag{2.152}$$

We observe that this correction is negligible.

Unfortunately we are not aware of the exact form of the higher derivative corrections to the DBI action in the presence of the electric field on the world-volume of the probe Dp brane. In the following we will simply assume a particular expression for the higher derivative corrections to the Lagrangian for the transverse fluctuations:

$$L = z^2(a_1'^2 - \omega^2 a_1^2) + \epsilon z^{2+\nu}(a_1'')^2\,, \tag{2.153}$$

with $\nu > 0$. To estimate the significance of the correction term we need to compare contributions to the equation of motion from the terms $\mathcal{O}(1)$

$$a_1'' \simeq \frac{1+(\omega z)^2}{z^3} \tag{2.154}$$

and $\mathcal{O}(\epsilon)$

$$\epsilon\left(z^\nu a_1'''' + 2(\nu+2)z^{\nu-1}a_1''' + (\nu+1)(\nu+2)z^{\nu-2}a_1''\right) \simeq \epsilon z^{\nu-5}(1+(\omega z)^4)\,. \tag{2.155}$$

Therefore, if $0 < \nu \le 2$, the correction becomes significant when $z \gg \frac{1}{(\epsilon\omega^2)^{1/\nu}}$ (see the hierarchy of scales in Fig. 2.6). If $\nu > 2$, considering modes with sufficiently low frequency $\omega < \epsilon^{1/(\nu-2)}$ the correction becomes significant when $z \gg \epsilon^{1/(2-\nu)}$ (see Fig. 2.7). Finally, if $\nu > 2$ and $\omega > \epsilon^{1/(\nu-2)}$, the Fig. 2.6 is applicable, and the correction is significant when $z \gg \frac{1}{(\epsilon\omega^2)^{1/\nu}}$. Hence, in the region $z \ll \frac{1}{(\epsilon\omega^2)^{1/\nu}}$ the DBI description is valid, provided that $0 < \nu \le 2$ or $\nu > 2$, $\omega > \epsilon^{1/(\nu-2)}$. The DBI description is valid in the region $z \ll \epsilon^{1/(2-\nu)}$ for $\nu > 2$, $\omega < \epsilon^{1/(\nu-2)}$.

The behavior of a_1 in the limit $z \gg 1$ where the DBI description is valid, is different from the incoming-wave (2.114): it has a qualitative form

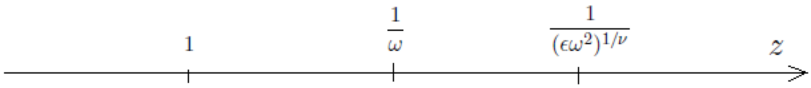


Figure 2.6. Hierarchy of scales in the near-horizon region for $0 < \nu \leq 2$.

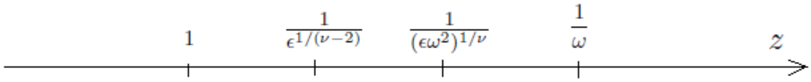


Figure 2.7. Hierarchy of scales in the near-horizon region for $\nu > 2$ and $\omega < \epsilon^{1/(\nu-2)}$.

of "incoming wave" + $\mathcal{O}(\epsilon)$ "outgoing wave". It is worth noting that the effect of higher derivative corrections on the current-current correlation function is essentially the same as an effect of non-zero b-field. We verified that such a modification does not lead to any nontrivial structure in the spectral density.

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