Integer and fractional quantum hall effects in lattice magnets

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CHAPTER 11

SUMMARY AND CONCLUSIONS

In this final section we give an account of the main results and conclusions that follow from this work. We summarize and discuss the key results of our work point by point.

(i) In this work we presented a detailed discussion of $2D$ particle-hole condensates from a symmetry perspective. For the square lattice and three representative hexagonal lattices we have decomposed all possible density waves for specified translational symmetry breaking based on lattice symmetries, yielding an arganization of these density waves in terms of basis functions of irreducible representations of the extended and bare points groups. Differentiating between site order (charge density waves), bond order (time-reversal preserving bond density waves), and flux order (imaginary bond density waves) has allowed for a gauge invariant classification of all distinct density waves just using a group theory toolkit.

(ii) The organization of density waves in terms lattice symmetries provided the framework to identify topological states of matter induced by interactions. In two dimensions there are two main classes of topological states: the QAH states which break time-reversal symmetry, and the QSH states which preserve time-reversal symmetry but must break spin rotation symmetry at least partially. When looking for candidate QAH states it is therefore sufficient to consider flux ordered states and spin density waves as these are time-reversal breaking states. Furthermore – and this is where the symmetry organization proves very powerful – only density wave formation breaking all reflection symmetries of the system can lead to QAH states,

meaning that only states transforming as A_2 allow for Chern insulators. Such states are straightforwardly identified within the group theoretical scheme producing a list of all possible flux ordered states of a given lattice and a given set of broken translations. We have demonstrated how known and well-studied flux ordered QAH states on for instance the honeycomb and kagome lattices (Haldane state [76] and chiral spin state [85]) follow directly from deriving and constructing flux states with proper symmetry.

In addition to these well-known QAH states, all of which do not break translational symmetry, we have shown the existance of a new class of QAH states of lattices with hexagonal symmetry, i.e. the flux ordered density waves. Guided purely by symmetry arguments we have identified flux density waves with M-point ordering vectors transforming as A_2 and leading to an insulating condensate ground state. Another class of M-point modulated QAH states on hexagonal lattices is given by the noncoplanar chiral spin density waves discussed in Section 10. The existence of such chiral states and their spontaneous QAH effects was shown for the triangular and honeycomb lattices in the context of local moment Kondo-Lattice [30, 40] and interacting single-band Hubbard models [160]. In the present work we show how these particular examples are part of a fundamental sequence of M-point hexagonal lattice spin density waves with a A_2 symmetry that do not break translational symmetry in spite of finite wave vector condensation. We refer to these states as part of a *fundamental sequence* since they all follow from the same underlying symmetry principle. Applying this principle to the kagome lattice, we identify a kagome lattice spin density wave with exactly the same properties.

In the context of interaction-driven topological insulating states the numerous possibilities arising from M-point order are particularly interesting as precisely the M-point vectors nest the Fermi surface of hexagonal lattices at the van Hove fillings where the density of states diverges. This inpsires hope that even infinitesimal interactions induce such states due to dominant instabilities towards such states. In fact, such an argument was put forward in case of triangular and honeycomb lattice spin density waves [160]. In contrast, interaction-induced topological states originating from low-energy Dirac fermions becoming massive, for instance within a mean field treatment of the honeycomb [84, 168], square [167] and kagome lattices [169, 170], require finite and large interaction strengths as a consequence of the linearly vanishing density of states at the Dirac points. This makes the scenario of spontaneously gapping out Dirac cones problematic, as recently shown in Refs. [211], because quantum fluctuations prevent the QAH state from fully developing.

The two sequences of hexagonal symmetry M-point ordered QAH states, i.e. flux order and spin density wave order, are of great significance for the second kind of topological states in $2D$, i.e. the QSH states. In this work we have explained how QSH states are trivially obtained from QAH states by constructing two copies of the latter for each of the two spin species with a relative sign difference. In that case the condensate function is proportional to $\vec{N} \cdot \vec{\sigma}$, with $\vec{N}$ the vector order parameter in spin space. This implies spin rotation symmetry around $\vec{N}$ is not broken, leading to a quantized spin Hall conductance. Full breaking of spin rotation symmetry, for instance by Rashba-type terms, does not immediately destroy the QSH state, but is generally harmful to its existence, as is signaled by the spin Hall conductance being no longer quantized [1]. In this work we propose a class of hexagonal spin triplet condensates which constitute QSH phases characterized by a matrix order parameter $\mathcal{R}$ instead of a vector $\vec{N}$ and fully break spin rotation symmmetry. They derive from a combination of spin and flux ordering with M-point ordering vectors.

(iii) In this work we have demonstrated how interaction-induced semimetallic states are protected by lattice symmetries and the anti-unitary time-reversal symmetry. We have focused on two types of semimetallic states in $2D$, which are the Weyl semimetals (alternatively referred to as Dirac semimetal) and the QBC points. Both are characterized by topological winding numbers which rely on the presence of symmetries to make their definition and use meaningful. Our symmetry analysis of density waves provides a comprehensive and systematic framework to determine the symmetries which protect isolated degeneracies definining the semimetal. Both translations and global spin rotations can act as degeneracy protecting operations in combination point group elements.

In case of the square lattice the two density waves which are semimetals, $d_{x^2-y^2}$ (Weyl or Dirac semimetal) and d_{xy} (QBC), transform as $1D$ representations of the extended point group and one may therefore select symmetries thereof to prove the spectral degeneracy at high symmetry points of the Brillouin zone. For square lattice systems combinations of (bare) point group operations and translations can form good symmetries protecting degeracies. This is different in case of hexagonal lattice systems, where we found global spin rotations to have the potential to protect degeneracies for spinful M-point ordered condensates. In particular, we have seen how the chiral spin density waves are in fact translationally invariant states regardless of finite ordering vector condensation, which is a consequence of global spin rotations compensating the translation. Perhaps most remarkably, we have demonstrated how M-point modulated spin-flux density waves become symmetry protected semimetallic states and the protection crucially relies on global spin rotation equivalence.

In the presence of these spin-flux density waves, the nested hexagonal Fermi surface at van Hove fillings is gapped out except for the protected degeneracies located located at each of the inequivalent M' points of the reduced BZ (see Fig. 9.7). Degeneracies at the M' points are only protected if none of the lattice symmetries are broken. Indeed, we found in Section 9.4.1 that the degeneracy at M' is generally lifted by translational symmetry breaking. For the spin-flux density waves translations combined with (unitary) global spin rotations are symmetries and key in protecting the degeneracy. The spin-flux density waves only break spin rotation symmetry and preserve all lattice symmetries. The low-energy description around the degeneracy points at M' takes the form of a Dirac theory, but instead of the common double-node theory, it is a six-node theory, i.e. two nodes for each inequivalent M' point. As such the spin-flux density waves constitute a new $2D$ semimetallic state. Interestingly, it is possible to superimpose a spin rotation and time-reversal invariant density wave on this Dirac semital with the result of gappig out the Dirac nodes yielding an insulating QSH ground state. This is contrary to the canonical example of a spin-orbit coupling-induced Dirac mass in the spin-rotation invariant low-energy theory of graphene [1, 23]. In the latter case it is the *breaking* of spin rotation symmetry which induces a QSH mass gap.

In addition to these time-reversal invariant semimetallic states with six Dirac nodes, we have shown the emergence of another distinct semitallic state from time-reversal breaking spin density wave formation. Both the honeycomb and kagome lattices allow for translationally invariant M-point spin density waves with B_1 symmetry. The mean field spectrum of such a state is allowed to have Dirac nodes only at the K'_+ points of the reduced BZ, but not at the K'_-. We have found precisely this situation to occur for the B_1 M-point ordered spin density waves: two Dirac nodes at the equivalent K'_+ points (and none at K'_-) for commensurate electron fillings, which are however not equal to the van Hove fillings. As such, representing a time-reversal broken state, these density waves would appear to be similar to the square lattice $d_{x^2-y^2}$ state. It is however a truly distinct state, as it manifestly breaks time-reversal, instead of preserving a combination of time-reversal and translation. It therefore constitutes another new semimetallic state of hexagonal lattice systems.

We have seen examples of the second type of semimetals, the QBC points, in the context of both the square and hexagonal lattices. As noted, in case of the square lattice the d_{xy} state gives rise to a QBC point which is protected by a fourfold rotation and time-reversal symmetry. We have demonstrated that a QBC point can occur in hexagonal lattice systems for various cases of M-point ordering, as long as the system has C_{3v} symmetry. In particular, we have identified the occurance of QBC points as one of two possibilities in case of such M-ordering with C_{3v} symmetry within a low-energy description at the Γ point of the reduced BZ. Such a low-energy description is independent of the specific lattice. Generically QBC points can be destroyed by breaking the symmetries that protect it. Specifically, the opening of a gap is intrinsically connected to time-reversal symmetry breaking and it was shown recently [163] how the QBC of a uniaxial A_1 M-point ordered spin density wave is gapped out by developing a finite scalar spin chirality, reducing the symmetry to A_2. In the present work we embed this result in a general low-energy theory for M-point ordering in hexagonal lattices based on symmetry.

(iv) We have established a robust connection between the *symmetry* of density

waves and their *low-energy interpretation* within the framework of a Dirac theory. The back bone of this connection is the rule which assigns density waves transforming as $1D$ representations the meaning of generalized masses, either gapping out the spectrum or making the two valleys inequivalent. In turn, density waves transforming as $2D$ representations have the interpretation of gauge field components in the low-energy Dirac theory, shifting the Dirac nodes in momentum space. We have shown in case of the kagome lattice, where we apply this rule, how decomposing site, bond and flux order using group theory is sufficient to determine the nature of their electronic properties.

In particular, the symmetry organization of density waves provides a straightforward way to find density waves states which enter as gauge field components in the Dirac theory. Such states may arise as a consequence of electronic interactions, but could alternatively be induced by application of external fields such as strain or modulated substrate potentials [180]. Knowledge of which states correspond to gauge fields allows to assess in which systems such gauge fields may be generated by either one of those mechanisms. This opens up the possibility to address and study the physics of non-Abelian $SU(2)$ fields in a condensed matter setting in a general way.

(v) The organization of density waves as basis functions of irreducible representations of extended point groups reveals their degeneracies. Density waves which are partners belonging to a larger dimensional extended point group representation will be energetically degenerate. If there is a dominant electronic instability towards the formation of such a state, then it applies to all partners in the representation. An illustrative example of such kind of partnership is given by the two independent hexagonal lattice K-point density waves corresponding Kekule masses (see Sections 9.4.1 and 9.4.2). Both for the honeycomb and kagome lattices these transform as E_1', a $2D$ representation of C_{6v}''. This is reflected in the low-energy theory as they correspond to compatible Dirac mass gaps, making them energetically equivalent.

For hexagonal lattice M-point ordering the irreducible representations of the extended point group C_{6v}''' are all three-dimensional, a consequence of the three inequivalent M-points. The three partners of such a representation are energetically equivalent from an electronic instability point of view. If the system gains condensation energy by the formation of one of such states in the triplet, then it will gain energy linearly and independently by forming the other partners as well. This is why our convention of decomposing these $3D$ representations into sums of representations of the bare group is particularly useful and relevant. Each of those triplets is decomposed as the sum of a $1D$ and a $2D$ representation and the $1D$ representation corresponds to a state which is the most symmetric superposition of the partners which transform into each other under rotations (in other words behave as the $\vec{Q}_\mu$ vectors). We therefore expect these states, i.e. states transforming as $1D$ bare point group representations, to develop from electronic instabilities favoring the given triplet representation. Hence, the symmetry organization gives insight into condensation energetics.

(vi) A large part of this work is devoted to hexagonal lattice density waves. We have discussed three prominent examples of hexagonal lattices and have found remarkable similarities. Again focusing on the case of M-point ordering, we have shown explicitly that density waves of the same symmetry have the same physical properties, independent of the lattice considered. To give an example, the uniaxial spin density waves of A_1 symmetry, which exist for all three lattices considered, are topological semimetals with a QBC point in either the spin-up or spin-down sector. The chiral spin density waves with A_2 symmetry, which are closely related to the uniaxial A_1 spin density waves, are all gapped for appropriate filling and correspond to a Chern insulating state. In addition, for both the honeycomb and kagome lattices there is a noncoplanar spin density wave with B_1 symmetry which induces Dirac points at the K'_+ (or K'_-) points for commensurate fillings ($n = 1/8$ for honeycomb, $n = 1/12$ for kagome). All of these key electronic characteristics are connected to the symmetry of the (spin) density wave and transcend the lattice specific setting. The same is true for the time-reversal invariant spin-flux density waves. We have explicitly shown the equivalent electronic properties for such a state on the triangular and honeycomb lattices. We found these particular density waves to break no other symmetry than spin-rotation symmetry, leading to a new kind of $2D$ semimetallic state not specific to a lattice structure.

The importance of symmetry rather than lattice structure also manifests itself in the context of K-point ordering. For instance, both for the honeycomb and kagome lattices, we found that any E'_1 doublet corresponds to two independent yet compatible Dirac masses. In case of the honeycomb lattice there only exists such a bond order doublet, while for the kagome lattice there is a site and bond order doublet. In the same way any E'_2 doublet leads to a Dirac valley inequivalence, separating the Dirac nodes in energy but not in momentum.

These examples mentioned here highlight the general conclusion that density waves with the same symmetry affect the electronic properties in the same way if the low-energy description of the electronic degrees of freedom is equivalent. Indeed, all the hexagonal lattices have an M-point nested Fermi surface at van Hove fillings and both the kagome and honeycomb spectra exhibit Dirac nodes. It is the symmetry of the density waves which is decides what happens to the electrons close to the Fermi surface or Fermi points.