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## Integer and fractional quantum hall effects in lattice magnets

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# Integer and Fractional Quantum Hall effects in Lattice Magnets

J.W.F. Venderbos



# Integer and Fractional Quantum Hall effects in Lattice Magnets

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de graad van Doctor aan de universiteit Leiden,  
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volgens het besluit van het College voor Promoties  
te verdedigen op dinsdag 25 maart 2014  
te klokke 15:00 uur

door

Jörn Willem Friedrich Venderbos  
geboren te Kleve, Duitsland, in 1984

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