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Signatures of Majorana zero-modes in nanowires, quantum spin Hall edges, and quantum dots

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**Signatures of Majorana
zero-modes
in nanowires,
quantum spin Hall edges,
and quantum dots**

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Cover: Evolution of the differential conductance of a superconducting quantum dot upon variation of an external parameter (such as the magnetic field). The circles indicate the poles of the scattering matrix in the complex energy plane. The accumulation of poles on the imaginary axis gives rise to a Y-shaped conductance resonance profile, which can be mistaken for a Majorana zero-mode. (See chapter 5.)

To my mother and grandmother

献给我的母亲和姥姥

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