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Quantum computation with Majorana zero modes in superconducting circuits

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Quantum computation with Majorana modes in superconducting circuits

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geboren te Rome, Italië in 1986

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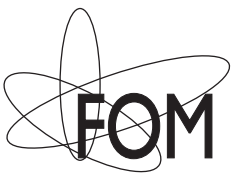

On the cover: schematic illustration of a superconducting circuit where Majorana modes can be braided (front) and where the braiding result can be measured (back), see chapter 4.

Alla mia famiglia

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