Effects of heavy fields on inflationary cosmology
Ortiz, P.

Author: Ortiz, Pablo
Title: Effects of heavy fields on inflationary cosmology
Issue Date: 2014-09-30

A

Conventions and useful formulae for Kähler manifolds

In this thesis we work in units of $c = \hbar = 1$. Thus, the Plank mass m_{pl} is simply given by

$$m_{pl}^{-2} \equiv \frac{G}{\hbar c} = G, \tag{A.0.1}$$

where G is the Newtons' constant. In general, we will also set the reduced Plank mass M_p to one:

$$M_p^{-2} = 8\pi G = 1, \tag{A.0.2}$$

except in the cases where it is more convenient to keep it explicitly for the sake of clarity. Unless specified, the following notation holds throughout the manuscript: the dot ($\dot{}$) denotes derivative with respect to cosmic time t. The prime ($'$) denotes derivative with respect to the argument. **Boldface** denotes a three-dimensional vector. The meaning of the main indices and symbols used along this thesis is summarised in the table A.1.

Here we compute some useful formulae related to the geometry of Kähler manifolds. First we give the expressions of the Christoffel symbols, Riemann tensor, and other useful quantities in terms of the Kähler function G and its derivatives. In A.1 we also give the explicit expressions in the case where there is a supersymmetric sector and a supersymmetry breaking sector, so we differentiate between the supersymmetric and the sGoldstino directions. In A.2 we focus on the case where the Kähler manifold is a direct product of these two sectors, so that $G_{\text{total}} = G_{\text{SUSY}} + G_{\cancel{\text{SUSY}}}$. In A.3 we rewrite these formulas in terms of the Kähler potential K and the superpotential W, which is especially useful in the case of a vanishing superpotential, since in that case the Kähler function G is ill defined. In

Symbol	Meaning	Chapters
$\mu = 0, 1, 2, 3$	Space-time indices, x^0 being the time coordinate	All
$i = 1, 2, 3$	Spatial indices	All
τ	Conformal time	1-3
ℓ	Multipole, angular scale	1-3
c_s	Speed of sound of the adiabatic mode	1-3
$s = \dot{c}_s / c_s H$	Rate of change of the speed of sound	1-3
X^i	Supersymmetry breaking fields, usually only X	4-5
z^i	Supersymmetry preserving fields	4
$A = (X, i)$	Index running over all scalar fields	4
$b = G^A G_A - 3$	Amount of supersymmetry breaking	4
$3\gamma = G^A G_A - 3$	Amount of supersymmetry breaking	5
$\lambda = 1, \ldots, \mathrm{N}$	Supersymmetric directions	5
m	Chiral fermion mass	5
μ	Eigenvalue of the Hessian of the potential (scalar mass)	5
m_h	Standard deviation of chiral fermion masses (largest mass in the typical spectrum)	5

Table A.1 – *Summary of indices and symbols.*

A.4 we focus on relevant quantities for inflation such as the scalar potential and its derivatives, and in A.5 we take the limit of vanishing superpotential, where we will see that the physical quantities are well defined in the limit $W \to 0$, as it should be. The generic expressions for the Christoffel symbols, the Riemann tensor, and other useful geometrical quantities derived from the Kähler function G are the following:

$$\Gamma^a_{bc} = G^{a\bar{d}}(\partial_b G_{c\bar{d}}) \ , \tag{A.0.3}$$

$$\partial_a G^{b\bar{c}} = -G^{b\bar{d}} G^{e\bar{c}} (\partial_a G_{e\bar{d}}) = -G^{e\bar{c}} \Gamma^b_{ae} \ , \tag{A.0.4}$$

$$\partial_a \Gamma^d_{bc} = G^{d\bar{e}} (\partial_a \partial_b G_{c\bar{e}}) - \Gamma^d_{ae} \Gamma^e_{bc} \ , \tag{A.0.5}$$

$$\nabla_{\bar{a}} \Gamma^d_{bc} = \partial_{\bar{a}} \Gamma^d_{bc} = G^{d\bar{e}} (\partial_{\bar{a}} \partial_b G_{c\bar{e}}) - \Gamma^e_{bc} G^{d\bar{f}} (\partial_{\bar{a}} G_{e\bar{f}}) \ , \tag{A.0.6}$$

$$R_{a\bar{b}c\bar{d}} = R_{c\bar{b}a\bar{d}} = R_{a\bar{d}c\bar{b}} = G_{e\bar{d}} (\partial_{\bar{b}} \Gamma^e_{ac}) = \partial_a \partial_{\bar{b}} G_{c\bar{d}} - \Gamma^e_{ac} (\partial_{\bar{b}} G_{e\bar{d}}) \ , \tag{A.0.7}$$

$$\begin{aligned} \nabla_a \nabla_b G_c = {} & \partial_a \partial_b G_c - G^{\bar{d}} (\partial_a \partial_b G_{c\bar{d}}) - \Gamma^d_{ab} (\partial_c G_d) - \Gamma^d_{bc} (\partial_a G_d) - \Gamma^d_{ac} (\partial_b G_d) \\ & + G_d (\Gamma^e_{ab} \Gamma^d_{ce} + \Gamma^e_{bc} \Gamma^d_{ae} + \Gamma^e_{ac} \Gamma^d_{be}) \ . \end{aligned} \tag{A.0.8}$$

A.1 Supersymmetric and sGoldstino directions

Here we specialise to situations where there is a supersymmetric sector embedded in a theory with supersymmetry breaking, which applies to chapters 4 and 5. Let us assume that the supersymmetry breaking direction is aligned in the X-field (sGoldstino) direction, as in sGoldstino inflation, while all the other fields (z_i) preserve supersymmetry. This translates into the conditions:

$$G_i(X,\bar{X},z_i^{(0)},\bar{z}_{\bar{i}}^{(0)}) = 0 \quad , \quad G_X(X,\bar{X},z_i^{(0)},\bar{z}_{\bar{i}}^{(0)}) \neq 0 \tag{A.1.1}$$

where $(z_i^{(0)},\bar{z}_{\bar{i}}^{(0)})$ is the so-called supersymmetric critical point. This means that any term containing one single derivative of G with respect to the z_i fields will vanish. We introduce the index notation (A,i), where A runs over all fields (X,z_i) and i runs only over the z_i fields. As in chapters 4 and 5, we will consider the case where supersymmetric sector is truncated. In this section we compute all the possible elements of the second and third covariant derivatives, Christoffel symbols and Riemann tensor for the particular case described above. The reader should keep in mind that all expressions are evaluated in the supersymmetric critical point.

- Second covariant derivatives:

$$\begin{aligned}
\nabla_X G_X &= \partial_X G_X - \Gamma^X_{XX} G_X \\
\nabla_X G_i &= \nabla_i G_X = 0 \\
\nabla_i G_j &= \partial_i G_j - \Gamma^X_{ij} G_X
\end{aligned} \tag{A.1.2}$$

- Christoffel symbols:

$$\begin{aligned}
\Gamma^X_{XX} &= G^{X\bar{A}}(\partial_X G_{X\bar{A}}) = G^{X\bar{X}}(\partial_X G_{X\bar{X}}) \\
\Gamma^i_{XX} &= G^{i\bar{A}}(\partial_X G_{X\bar{A}}) = 0 \\
\Gamma^X_{Xi} &= G^{X\bar{A}}(\partial_X G_{i\bar{A}}) = 0 \\
\Gamma^i_{Xj} &= G^{i\bar{A}}(\partial_X G_{j\bar{A}}) = G^{i\bar{k}}(\partial_X G_{j\bar{k}}) \\
\Gamma^X_{ij} &= G^{X\bar{A}}(\partial_i G_{j\bar{A}}) = G^{X\bar{X}}(\partial_i G_{j\bar{X}}) \\
\Gamma^k_{ij} &= G^{k\bar{A}}(\partial_i G_{j\bar{A}}) = G^{k\bar{l}}(\partial_i G_{j\bar{l}})
\end{aligned} \tag{A.1.3}$$

Note that the Christoffel symbols with a single spectator index vanish.

- Third covariant derivatives:

$$
\begin{aligned}
\nabla_X\nabla_X G_X &= \partial_X\partial_X G_X - G^{\bar{X}}(\partial_X\partial_X G_{X\bar{X}}) - 3\Gamma^X_{XX}(\partial_X G_X) + 3G_X(\Gamma^X_{XX})^2 \\
\nabla_X\nabla_X G_i &= \nabla_X\nabla_i G_X = \nabla_i\nabla_X G_X = 0 \\
\nabla_X\nabla_i G_j &= \partial_X\partial_i G_j - G^{\bar{X}}(\partial_X\partial_i G_{j\bar{X}}) - \Gamma^k_{Xi}(\partial_j G_k) - \Gamma^X_{ij}(\partial_X G_X) - \Gamma^k_{Xj}(\partial_i G_k) \\
&\quad + G_X(\Gamma^k_{Xi}\Gamma^X_{jk} + \Gamma^X_{ij}\Gamma^X_{XX} + \Gamma^k_{Xj}\Gamma^X_{ik}) \\
\nabla_i\nabla_j G_k &= \partial_i\partial_j G_k - G^{\bar{X}}(\partial_i\partial_j G_{k\bar{X}}) - \Gamma^l_{ij}(\partial_k G_l) - \Gamma^l_{jk}(\partial_i G_l) - \Gamma^l_{ik}(\partial_j G_l) \\
&\quad + G_X(\Gamma^l_{ij}\Gamma^X_{kl} + \Gamma^l_{jk}\Gamma^X_{il} + \Gamma^l_{ik}\Gamma^X_{jl})
\end{aligned}
\tag{A.1.4}
$$

- Riemann tensor:

$$
\begin{aligned}
R_{X\bar{X}X\bar{X}} &= \partial_X\partial_{\bar{X}} G_{X\bar{X}} - \Gamma^X_{XX}(\partial_{\bar{X}} G_{X\bar{X}}) \\
R_{X\bar{X}X\bar{i}} &= R_{X\bar{X}i\bar{X}} = R_{X\bar{i}X\bar{X}} = R_{i\bar{X}X\bar{X}} = 0 \\
R_{X\bar{X}i\bar{j}} &= R_{X\bar{j}i\bar{X}} = R_{i\bar{X}X\bar{j}} = R_{i\bar{j}X\bar{X}} = \partial_X\partial_{\bar{X}} G_{i\bar{j}} - \Gamma^k_{Xi}(\partial_{\bar{X}} G_{k\bar{j}}) \\
R_{X\bar{i}j\bar{k}} &= R_{j\bar{i}X\bar{k}} = R_{X\bar{k}j\bar{i}} = R_{j\bar{k}X\bar{i}} = \partial_X\partial_{\bar{i}} G_{j\bar{k}} - \Gamma^l_{Xj}(\partial_{\bar{i}} G_{l\bar{k}}) \\
R_{i\bar{X}j\bar{k}} &= R_{j\bar{X}i\bar{k}} = R_{i\bar{k}j\bar{X}} = R_{j\bar{k}i\bar{X}} = \partial_i\partial_{\bar{X}} G_{j\bar{k}} - \Gamma^l_{ij}(\partial_{\bar{X}} G_{l\bar{k}}) \\
R_{i\bar{j}k\bar{l}} &= R_{k\bar{j}i\bar{l}} = R_{i\bar{l}k\bar{j}} = R_{k\bar{l}i\bar{j}} = \partial_i\partial_{\bar{j}} G_{k\bar{l}} - \Gamma^m_{ik}(\partial_{\bar{j}} G_{m\bar{l}}) - \Gamma^X_{ik}(\partial_{\bar{j}} G_{X\bar{l}})
\end{aligned}
\tag{A.1.5}
$$

A.2 Separable Kähler function $G_{\textbf{total}} = G_{\textbf{SUSY}} + G_{\cancel{\textbf{SUSY}}}$

When the Kähler function is of the separable form

$$
G(X,\bar{X},z_i,\bar{z_{\bar{i}}}) = g(X,\bar{X}) + \tilde{g}(z_i,\bar{z_{\bar{i}}}) \ , \tag{A.2.1}
$$

is clear that all mixed derivatives vanish in every point. Furthermore, we impose the condition (A.1.1) at the supersymmetric critical point. This leads to a enormous simplification of our equations. We rewrite (A.1.2)-(A.1.5) for this simple case:

- Second covariant derivatives:

$$
\begin{aligned}
\nabla_X G_X &= \partial_X G_X - \Gamma^X_{XX} G_X \\
\nabla_X G_i &= \nabla_i G_X = 0 \\
\nabla_i G_j &= \partial_i G_j
\end{aligned}
\tag{A.2.2}
$$

• Christoffel symbols:

$$\begin{aligned}
\Gamma^X_{XX} &= G^{X\bar{X}}(\partial_X G_{X\bar{X}}) \\
\Gamma^i_{XX} &= \Gamma^X_{Xi} = \Gamma^i_{Xj} = \Gamma^X_{ij} = 0 \\
\Gamma^k_{ij} &= G^{k\bar{l}}(\partial_i G_{j\bar{l}})
\end{aligned} \tag{A.2.3}$$

• Third covariant derivatives:

$$\begin{aligned}
\nabla_X\nabla_X G_X &= \partial_X\partial_X G_X - G^{\bar{X}}(\partial_X\partial_X G_{X\bar{X}}) - 3\Gamma^X_{XX}(\partial_X G_X) + 3G_X(\Gamma^X_{XX})^2 \\
\nabla_X\nabla_X G_i &= \nabla_X\nabla_i G_j = 0 \\
\nabla_i\nabla_j G_k &= \partial_i\partial_j G_k - \Gamma^l_{ij}(\partial_k G_l) - \Gamma^l_{jk}(\partial_i G_l) - \Gamma^l_{ik}(\partial_j G_l)
\end{aligned} \tag{A.2.4}$$

• Riemann tensor:

$$\begin{aligned}
R_{X\bar{X}X\bar{X}} &= \partial_X\partial_{\bar{X}} G_{X\bar{X}} - \Gamma^X_{XX}(\partial_{\bar{X}} G_{X\bar{X}}) \\
R_{X\bar{X}X\bar{\imath}} &= R_{X\bar{X}i\bar{X}} = R_{X\bar{\imath}X\bar{X}} = R_{i\bar{X}X\bar{X}} = 0 \\
R_{X\bar{X}i\bar{\jmath}} &= R_{X\bar{\jmath}i\bar{X}} = R_{i\bar{X}X\bar{\jmath}} = R_{i\bar{\jmath}X\bar{X}} = 0 \\
R_{X\bar{\imath}j\bar{k}} &= R_{j\bar{\imath}X\bar{k}} = R_{X\bar{k}j\bar{\imath}} = R_{j\bar{k}X\bar{\imath}} = 0 \\
R_{i\bar{X}j\bar{k}} &= R_{j\bar{X}i\bar{k}} = R_{i\bar{k}j\bar{X}} = R_{j\bar{k}i\bar{X}} = 0 \\
R_{i\bar{\jmath}k\bar{l}} &= R_{k\bar{\jmath}i\bar{l}} = R_{i\bar{l}k\bar{\jmath}} = R_{k\bar{l}i\bar{\jmath}} = \partial_i\partial_{\bar{\jmath}} G_{k\bar{l}} - \Gamma^m_{ik}(\partial_{\bar{\jmath}} G_{m\bar{l}})
\end{aligned} \tag{A.2.5}$$

A.3 Geometric quantities in terms of K and W

It can be useful to have the expressions obtained along this thesis in terms of the Kähler potential K and the superpotential W, especially if we want to analyse the case where $W = 0$. Is a critical case in the sense that many quantities diverge, but not the physical ones, as we will see. Moreover, some models only work in this case (see [86]), so it convenient to have the expressions displayed above in terms of K and W, since the Kähler function G is not well defined in that case.

We introduce the Kähler covariant derivative $D_a W \equiv W_a + K_a W$. Notice that we are not specifying any sectors, so lower case letters run over all possible values. When we distinguish between supersymmetric and non-supersymmetric sectors, as in section A.1, we will reintroduce the notation with capital letters $A = (X, i)$. The Kähler function, first derivatives, and metric are given by:

$$G = K + \ln |W|^2 \ , \ e^G = e^K |W|^2 \tag{A.3.1}$$

$$G_a = K_a + \frac{1}{W} W_a = \frac{1}{W} D_a W \ , \ G_{\bar{b}} = K_{\bar{b}} + \frac{1}{\bar{W}} \bar{W}_{\bar{b}} = \frac{1}{\bar{W}} D_{\bar{b}} \bar{W} \tag{A.3.2}$$

$$G_{a\bar{b}} = K_{a\bar{b}} \ , \ G^{a\bar{b}} = K^{a\bar{b}} \tag{A.3.3}$$

$$G^{\bar{b}} = G^{a\bar{b}} G_a = K^{a\bar{b}} \left(K_a + \frac{1}{W} W_a \right) \tag{A.3.4}$$

Using the previous equations, we can already rewrite the scalar potential:

$$\begin{aligned} V &= e^K \left(K^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} - 3|W|^2 \right) \\ &= e^K \left(K^{a\bar{b}} \left(W_a + K_a W \right) \left(\bar{W}_{\bar{b}} + K_{\bar{b}} \bar{W} \right) - 3|W|^2 \right) \end{aligned} \tag{A.3.5}$$

The Christoffel symbols and higher derivatives of the Kähler function read:

$$\Gamma^a_{bc} = K^{a\bar{d}} K_{bc\bar{d}} \tag{A.3.6}$$

$$\partial_a G_b = G_{ab} = K_{ab} + \frac{1}{W} W_{ab} - \frac{1}{W^2} W_a W_b \tag{A.3.7}$$

$$\begin{aligned} G_{abc} = K_{abc} &+ \frac{1}{W} W_{abc} - \frac{1}{W^2} (W_{ab} W_c + W_{ac} W_b + W_{bc} W_a) \\ &+ \frac{2}{W^3} W_a W_b W_c \end{aligned} \tag{A.3.8}$$

Using the above expressions we can compute the second and third covariant derivatives, given by:

$$\nabla_a G_b = K_{ab} + \frac{1}{W} W_{ab} - \frac{1}{W^2} W_a W_b - K^{c\bar{d}} K_{ab\bar{d}} \left(K_c + \frac{1}{W} W_c \right) \tag{A.3.9}$$

$$\begin{aligned} \nabla_a \nabla_b G_c = K_{abc} &+ \frac{1}{W} W_{abc} - \frac{1}{W^2} (W_{ab} W_c + W_{ac} W_b + W_{bc} W_a) + \frac{2}{W^3} W_a W_b W_c \\ &- K^{e\bar{d}} K_{abc\bar{d}} \left(K_e + \frac{1}{W} W_e \right) - K^{d\bar{e}} K_{ab\bar{e}} \left(K_{cd} + \frac{1}{W} W_{cd} - \frac{1}{W^2} W_c W_d \right) \\ &- K^{d\bar{e}} K_{bc\bar{e}} \left(K_{ad} + \frac{1}{W} W_{ad} - \frac{1}{W^2} W_a W_d \right) \\ &- K^{d\bar{e}} K_{ca\bar{e}} \left(K_{bd} + \frac{1}{W} W_{bd} - \frac{1}{W^2} W_b W_d \right) \\ &+ \left(K_d + \frac{1}{W} W_d \right) K^{e\bar{f}} K^{d\bar{g}} \left(K_{ab\bar{f}} K_{ce\bar{g}} + K_{bc\bar{f}} K_{ae\bar{g}} + K_{ca\bar{f}} K_{be\bar{g}} \right) \end{aligned} \tag{A.3.10}$$

Last, the Riemann tensor is simply written as:

$$R_{a\bar{b}c\bar{d}} = K_{a\bar{b}c\bar{d}} - K^{e\bar{f}} K_{ac\bar{f}} K_{\bar{b}\bar{d}e} \ . \tag{A.3.11}$$

A.4 Physical quantities relevant for inflation

In this section we compute several quantities that are relevant for the inflationary dynamics, such as the scalar potential and the elements of the mass matrix. In this section we will use the index notation $A=(X,i)$, where X represents the sGoldstino and i runs over the spectator fields z_i. First we will write explicitly the scalar potential and its first and second derivatives, and then we evaluate the scalar potential and its first derivative at the supersymmetric critical point, where the spectator sector satisfies $G_i(X,z_0^i)=0$. These expressions can be used to calculate, for instance, the 'potential' slow-roll parameters, and get an estimate of the viability and amount of inflation for a given model. One can also study the stability of a given model as we did in chapters 4 and 5 by using the elements of the mass matrix. In any case, it is convenient to have the explicit expressions for cases where the superpotential vanishes at one or more points of the inflationary trajectory. The generic lengthy expressions for the scalar potential and its derivatives are the following:

$$V=e^K\left(K^{A\bar{B}}\left(W_A+K_AW\right)\left(\bar{W}_{\bar{B}}+K_{\bar{B}}\bar{W}\right)-3|W|^2\right)\ , \tag{A.4.1}$$

$$\begin{aligned}V_A=&\,e^K\left(K_A+\frac{1}{W}W_A\right)\left(K^{B\bar{C}}\left(W_B+K_BW\right)\left(\bar{W}_{\bar{C}}+K_{\bar{C}}\bar{W}\right)-2|W|^2\right)\\&+e^KK^{B\bar{C}}\left(K_{\bar{C}}\bar{W}+\bar{W}_{\bar{C}}\right)\Big(WK_{AB}+W_{AB}\\&-\frac{1}{W}W_AW_B-K^{D\bar{E}}K_{AB\bar{E}}\left(K_DW+W_D\right)\Big)\ ,\end{aligned} \tag{A.4.2}$$

$$\begin{aligned}\nabla_A\nabla_{\bar{B}}V=&\left[K_{A\bar{B}}-\left(K_A+\frac{1}{W}W_A\right)\left(K_{\bar{B}}+\frac{1}{\bar{W}}\bar{W}_{\bar{B}}\right)\right]V\\&+\left(K_A+\frac{1}{W}W_A\right)V_{\bar{B}}+\left(K_{\bar{B}}+\frac{1}{\bar{W}}\bar{W}_{\bar{B}}\right)V_A+e^K|W|^2K^{C\bar{D}}\\&\times\left[K_{AC}+\frac{1}{W}W_{AC}-\frac{1}{W^2}W_AW_C-K^{E\bar{F}}K_{AC\bar{F}}\left(K_E+\frac{1}{W}W_E\right)\right]\\&\times\left[K_{\bar{B}\bar{D}}+\frac{1}{\bar{W}}\bar{W}_{\bar{B}\bar{D}}-\frac{1}{\bar{W}^2}\bar{W}_{\bar{B}}\bar{W}_{\bar{D}}-K^{E\bar{F}}K_{\bar{B}\bar{D}E}\left(K_{\bar{F}}+\frac{1}{\bar{W}}\bar{W}_{\bar{F}}\right)\right]\\&+e^K|W|^2\Bigg[\left(-K_{A\bar{B}C\bar{D}}+K^{E\bar{F}}K_{AC\bar{F}}K_{\bar{B}\bar{D}E}\right)K^{C\bar{H}}K^{I\bar{D}}\\&\times\left(K_{\bar{H}}+\frac{1}{\bar{W}}\bar{W}_{\bar{H}}\right)\left(K_I+\frac{1}{W}W_I\right)+K_{A\bar{B}}\Bigg]\ ,\end{aligned}$$

$$
\begin{aligned}
\nabla_A \nabla_B V = &\left[K_{AB} + \frac{1}{W}W_{AB} - \frac{1}{W^2}W_A W_B - K^{E\bar{F}}K_{AB\bar{F}}\left(K_E + \frac{1}{W}W_E\right)\right.\\
&\left. - \left(K_A + \frac{1}{W}W_A\right)\left(K_B + \frac{1}{W}W_B\right)\right]V\\
&+ \left(K_A + \frac{1}{W}W_A\right)V_B + \left(K_B + \frac{1}{W}W_B\right)V_A + 2e^K|W|^2\\
&\times \left[K_{AB} + \frac{1}{W}W_{AB} - \frac{1}{W^2}W_A W_B - K^{E\bar{F}}K_{AB\bar{F}}\left(K_E + \frac{1}{W}W_E\right)\right]\\
&+ e^K|W|^2K^{C\bar{D}}\left(K_{\bar{D}} + \frac{1}{\bar{W}}\bar{W}_{\bar{D}}\right)\\
&\times \left[K_{ABC} + \frac{1}{W}W_{ABC} - \frac{1}{W^2}(W_{AB}W_C + W_{AC}W_B + W_{BC}W_A)\right.\\
&+ \frac{2}{W^3}W_A W_B W_C - K^{E\bar{D}}K_{ABC\bar{D}}\left(K_E + \frac{1}{W}W_E\right)\\
&- K^{D\bar{E}}K_{AB\bar{E}}\left(K_{CD} + \frac{1}{W}W_{CD} - \frac{1}{W^2}W_C W_D\right)\\
&- K^{D\bar{E}}K_{BC\bar{E}}\left(K_{AD} + \frac{1}{W}W_{AD} - \frac{1}{W^2}W_A W_D\right)\\
&- K^{D\bar{E}}K_{CA\bar{E}}\left(K_{BD} + \frac{1}{W}W_{BD} - \frac{1}{W^2}W_B W_D\right) + \left(K_D + \frac{1}{W}W_D\right)\\
&\left.\times K^{E\bar{F}}K^{D\bar{G}}\left(K_{AB\bar{F}}K_{CE\bar{G}} + K_{BC\bar{F}}K_{AE\bar{G}} + K_{CA\bar{F}}K_{BE\bar{G}}\right)\right]
\end{aligned}
$$

When there is a spectator sector sitting on a supersymmetric critical point $z_i^{(0)}$, this configuration is also a critical point of the scalar potential, $V_i\big|_0 = 0$, as it has been extensively reviewed along this thesis. However, this is not the case for the supersymmetry breaking sector. As we have done previously, considering the supersymmetry breaking aligned with the sGoldstino direction X, the scalar potential and its first derivative in the sGoldstino direction will be:

$$
V\big|_0 = e^K\left(K^{X\bar{X}}\left(W_X + K_X W\right)\left(\bar{W}_{\bar{X}} + K_{\bar{X}}\bar{W}\right) - 3|W|^2\right) \tag{A.4.3}
$$

$$
\begin{aligned}
V_X\big|_0 = &\, e^K\left(K_X + \frac{1}{W}W_X\right)\left(K^{X\bar{X}}\left(W_X + K_X W\right)\left(\bar{W}_{\bar{X}} + K_{\bar{X}}\bar{W}\right) - 2|W|^2\right)\ ,\\
&+ e^K K^{X\bar{X}}\left(K_{\bar{X}}\bar{W} + \bar{W}_{\bar{X}}\right)\left(WK_{XX} + W_{XX} - \frac{1}{W}W_X W_X\right.\\
&\left. - K^{X\bar{X}}K_{XX\bar{X}}\left(K_X W + W_X\right)\right)\ .
\end{aligned} \tag{A.4.4}
$$

A.5 Vanishing superpotential

Now we will take the limit $W \to 0$. This does not make sense in sGoldstino inflation, since in that limit we would also have $W_X = 0$, which implies that the field X preserves supersymmetry. Hence, we cannot have sGoldstino inflation when $W = 0$. But this limit might be very interesting to analyse models for which the superpotential vanishes during inflation. Notice that once we fix the superpotential, we are fixing (some of) the fields. If the superpotentials of different sectors are combined by multiplication, the dynamical sector can be used to stabilise the inflationary trajectory, and this is precisely the functional freedom claimed in [86]. We assume that supersymmetry is broken in at least one of the sectors, which is necessary in order to achieve inflation or stable dS vacua.

Then, in the special case of $W \to 0$ the divergent terms of the expressions in section A.4 cancel with each other. The expressions for the scalar potential and its first derivative in this case read:

$$V \xrightarrow{W \to 0} e^K K^{A\bar{B}} W_A \bar{W}_{\bar{B}} \ , \tag{A.5.1}$$

$$V_A \xrightarrow{W \to 0} e^K K^{B\bar{C}} \bar{W}_{\bar{C}} \left[W_A K_B + K_A W_B + W_{AB} - K^{D\bar{E}} K_{AB\bar{E}} W_D \right] \ . \tag{A.5.2}$$

$\mathcal{B}$

Small spectral index for inflection point inflation

In this appendix we derive the spectral index and power spectrum for inflection point inflation, following the work of Refs. [197, 198]. To a very good approximation the inflationary observables only depend on the η-parameter at the extremum and on the number of efolds.

Expanding the potential around the inflection point gives:

$$V = V_0(1 + 1/2\eta_0\phi^2 + C_3\phi^3 + C_4\phi^4 + ...), \tag{B.0.1}$$

with $\eta, C_3 < 0$ so that the field rolls towards the minimum at positive ϕ values. Inflation ends when the C_3 term becomes important, and $\epsilon \approx 1$, which occurs for field values $\phi_f^2 \sim \sqrt{2}/(3|C_3|)$. We can calculate the number of efolds

$$N \approx \int_{\phi_f}^{\phi_N} \frac{V}{V'} = \frac{1}{\eta} \log\left[\frac{\phi}{3C_3\phi + \eta}\right]_{\phi_f}^{\phi_N}, \tag{B.0.2}$$

where we used $V \approx V_0$ above. The above expression can be inverted to obtain the value of the inflaton field N efolds before the end of inflation ϕ_N:

$$\phi_N = \frac{\mathrm{e}^{N\eta_0}\eta_0/C3}{-3(\mathrm{e}^{N\eta_0} - 1) - \eta_0/(\phi_f C_3)} \approx \frac{\mathrm{e}^{N\eta_0}\eta_0}{-3C_3(\mathrm{e}^{N\eta_0} - 1)}, \tag{B.0.3}$$

where in the second step we used $\eta_0/(\phi_f|C_3|) \ll 1$. This is a good approximation as $\eta_0 \ll 1$ is fine-tuned, whereas C_3, and thus ϕ_f, is naturally of order one[1]. Note

[1] To be precise, $C_3 = \mathcal{O}(1)$ for $\phi_0 \sim 1$. For minima at smaller field values generically C_3 increases, as a sharper turnover of the potential is needed. We do not find valid solutions for minima for $\phi_0 \gg 1$ much larger, as then other local minima at smaller field values appear.

that in this limit, the number of efolds is independent of the end of inflation, as ϕ_f has dropped out of the equation. As a result the inflationary observables are insensitive to the precise coefficients of the higher order terms in (B.0.1). The spectral index is

$$n_s \approx 1 + 2\eta \approx 1 + 2\eta_0 + 12C_3\phi_N \approx 1 - 2\eta_0 \frac{(e^{\eta_0 N} + 1)}{(e^{\eta_0 N} - 1)}, \tag{B.0.4}$$

where we used that $\epsilon \ll \eta$. For $N < 50 - 60$ one finds $n_s < 0.92 - 0.93$ for the whole range of $|\eta_0| \lesssim 10^{-2}$. The power spectrum is

$$P_\zeta = \frac{V}{150\pi^2\epsilon} = \frac{3C_3^2 e^{-4N\eta_0}(e^{N\eta_0} - 1)^4 V_0}{25\pi^2\eta_0^4} \tag{B.0.5}$$

with $P_\zeta = 4 \times 10^{-10}$ measured by WMAP.

For the first example (4.3.12) in the text $\eta_0 = 0$ and $C_3 = -2.39$. For $\eta_0 = 0$, the expressions simplify to

$$n_s - 1 = -\frac{4}{N}, \quad P_\zeta = \frac{3C_3^2 N^4 V_0}{25\pi^2}, \quad (\text{for } \eta_0 = 0). \tag{B.0.6}$$

Choosing $N = 50$ this gives $n_s = 0.92$ and $V_0 = 9 \times 10^{-16}$. The second example (4.3.13) has $C_3 = -3.69$, and gives the same spectral index and similar $V_0 = 4 \times 10^{-16}$. The gravitino mass today is related to the inflationary scale via $m_{3/2} = e^{K/2}W|_{\min} \sim 10^2\sqrt{V_0} \sim 10^{-7}$, far above the electroweak scale.

C

Mass spectrum for quasi-separable Kähler functions

In this appendix we derive in full detail the result in (5.3.8), which refers to the eigenvalues of the mass matrix for Kähler functions with small coupling between the heavy and light sectors. On our way, we will also derive the result (5.3.15) for separable Kähler functions. We will briefly review eigenvalue perturbation theory and afterwards we will use this to calculate the perturbed eigenvalues for a Kähler function with a small mixing between sectors.

Perturbation theory

Consider a $n \times n$ square matrix $\mathcal{H} = \mathcal{H}_0 + \delta\mathcal{H}$, where the elements of $\delta\mathcal{H}$ are much smaller than those of $\mathcal{H}_0$. Let us denote by $\lambda_{0,i}$ the eigenvalues of $\mathcal{H}_0$, where $i = 1, \ldots, n$. The eigenvectors corresponding to those eigenvalues form a orthonormal basis with which one can build the matrix A that diagonalises $\mathcal{H}_0$, that is, $A^\dagger \mathcal{H}_0 A = \mathrm{diag}(\lambda_{0,1}, \ldots, \lambda_{0,n})$.

Then, to first order in perturbation theory, the eigenvalues of the full matrix $\mathcal{H}$ will be given by

$$\lambda_i = \left(A^\dagger \mathcal{H} A\right)_{ii} = \lambda_{0,i} + \left(A^\dagger \delta\mathcal{H} A\right)_{ii} \ , \quad i = 1, \ldots, n \ . \tag{C.0.1}$$

In other words, the perturbation over the 'bare' eigenvalues is given by the diagonal elements of the matrix perturbation in the basis that diagonalises the 'bare' matrix.

Perturbed eigenvalues

Let us consider a Kähler function with a small interaction term:

$$G(H, \bar{H}, L, \bar{L}) = A(H, \bar{H}) + B(L, \bar{L}) + \epsilon\, G_{\mathrm{int}}(H, \bar{H}, L, \bar{L}) \ , \tag{C.0.2}$$

where the heavy fields H^α are consistently truncated at the supersymmetric critical point such that[1]

$$A_\alpha|_{H_0} = G_{\mathrm{int},\alpha}|_{H_0} = 0 \ . \tag{C.0.3}$$

We take the Hessian matrix in section 5.6.3, given by

$$\mathcal{H} = \begin{pmatrix} \nabla_\alpha \nabla_{\bar\beta} V & \nabla_{\bar\alpha} \nabla_{\bar\beta} V \\ \nabla_\alpha \nabla_\beta V & \nabla_{\bar\alpha} \nabla_\beta V \end{pmatrix} , \tag{C.0.4}$$

where the elements are

$$\nabla_\alpha \nabla_\beta V|_{H_0,L_0} = \mathrm{e}^G \left[(3\gamma+2)\nabla_\alpha G_\beta + G^i\, \nabla_i(\nabla_\alpha G_\beta)\right] \ , \tag{C.0.5}$$
$$\nabla_\alpha \nabla_{\bar\beta} V|_{H_0,L_0} = \mathrm{e}^G \left[\delta_{\alpha\bar\beta}\,(3\gamma+1) + \delta^{\gamma\bar\sigma}\,(\nabla_\gamma G_\alpha)\left(\nabla_{\bar\sigma} G_{\bar\beta}\right) - R_{i\bar{j}\alpha\bar\beta}\, G^i G^{\bar{j}}\right] \ .$$

We define the following quantities

$$M_{\alpha\beta} \equiv \nabla_\alpha G_\beta = M_{\beta\alpha} \ , \tag{C.0.6}$$
$$Y_{\alpha\beta} \equiv G^i\, \nabla_i\,(\nabla_\alpha G_\beta) = Y_{\beta\alpha} \ , \tag{C.0.7}$$
$$\Omega_{\alpha\beta} \equiv -R_{i\bar{j}\alpha\bar\beta}\, G^i G^{\bar{j}} = \Omega^*_{\beta\alpha} \ . \tag{C.0.8}$$

One can recast the mass matrix $\mathcal{H}$ in terms of the quantities above as follows:

$$\mathcal{H} = \mathcal{H}_0 + \delta\mathcal{H} \tag{C.0.9}$$

$$= \begin{pmatrix} \mathrm{e}^G[(3\gamma+1)\mathbb{I}+MM^*] & \mathrm{e}^G(3\gamma+2)M^* \\ \mathrm{e}^G(3\gamma+2)M & \mathrm{e}^G[(3\gamma+1)\mathbb{I}+M^*M] \end{pmatrix} + \begin{pmatrix} \mathrm{e}^G\Omega & \mathrm{e}^G Y^* \\ \mathrm{e}^G Y & \mathrm{e}^G \Omega^T \end{pmatrix} ,$$

where the terms in $\delta\mathcal{H}$ are at least $\mathcal{O}(\epsilon)$. It is possible to perform a transformation of the fields such that

$$M \to \tilde{M} = UMU^t \ , \tag{C.0.10}$$

where U is a unitary matrix. Thanks to the symmetry properties of M, we can easily see that MM^* is hermitian and hence it can be diagonalised by a unitary transformation. In fact, given the transformation of M, it follows that

$$\tilde{M}\tilde{M}^* = U(MM^*)U^\dagger = \mathrm{diag}\left(|\lambda_1|^2\, \mathbb{I}_{n_1}, \ldots, |\lambda_p|^2\, \mathbb{I}_{n_p}\right) = \tilde{M}^*\tilde{M} \ , \tag{C.0.11}$$

where n_p is the degeneracy of the p^{th} eigenvalue. A direct consequence of the above is that $[\tilde{M}, \tilde{M}\tilde{M}^*] = 0$, which means that $\tilde{M}$ is block diagonal in the subspaces of dimension n_p. We will denote each of those matrices by $\tilde{M}_p$, satisfying

$$\tilde{M}_p\tilde{M}_p^* = |\lambda_p|^2\, \mathbb{I}_{n_p} \ . \tag{C.0.12}$$

[1]Since we want to impose this condition for any value of ϵ, both functions A and G_{int} must satisfy this requirement.

Now, since $\tilde{M}_p$ is complex and symmetric, we can always rewrite it using Takagi's factorisation, i.e. $\tilde{M}_p = V_p D_p V_p^t$, where V_p is unitary and D_p is diagonal and contains the non-negative square roots of the eigenvalues of $\tilde{M}_p \tilde{M}_p^\dagger$. Therefore, we may write $\tilde{M}_p = V_p V_p^t |\lambda_p|$.

Given this, let us transform the fields once more, in such a way that the resulting transformation of $\tilde{M}_p$ is the following:

$$\tilde{M}_p \to M'_p = V_p^\dagger \tilde{M}_p V_p^* = |\lambda_p| \, \mathbb{I}_{n_p} \ . \qquad \text{(C.0.13)}$$

After this, the unperturbed mass matrix $\mathcal{H}_0$ has been rewritten in a new basis as $\mathcal{H}'_0$ and it has four blocks of size $n_1 + \cdots + n_p$ each, which are diagonal:

$$\mathcal{H}'_0 = \mathrm{e}^G \left(\begin{array}{cc|cc} \left[(3\gamma+1)+|\lambda_1|^2\right]\mathbb{I}_{n_1} & \mathbf{0} & (3\gamma+2)|\lambda_1|\mathbb{I}_{n_1} & \mathbf{0} \\ & \ddots & & \ddots \\ \mathbf{0} & \left[(3\gamma+1)+|\lambda_p|^2\right]\mathbb{I}_{n_p} & \mathbf{0} & (3\gamma+2)|\lambda_p|\mathbb{I}_{n_p} \\ \hline (3\gamma+2)|\lambda_1|\mathbb{I}_{n_1} & \mathbf{0} & \left[(3\gamma+1)+|\lambda_1|^2\right]\mathbb{I}_{n_1} & \mathbf{0} \\ & \ddots & & \ddots \\ \mathbf{0} & (3\gamma+2)|\lambda_p|\mathbb{I}_{n_p} & \mathbf{0} & \left[(3\gamma+1)+|\lambda_p|^2\right]\mathbb{I}_{n_p} \end{array} \right) . \qquad \text{(C.0.14)}$$

We can always solve the eigenvalue problem by rearranging rows and columns to make the mass matrix block diagonal, with blocks of dimension $2n_1, \ldots, 2n_p$ given by the matrices

$$\mathcal{H}'^{(p)}_0 = \mathrm{e}^G \begin{pmatrix} \left(|\lambda_p|^2+3\gamma+1\right)\mathbb{I}_{n_p} & (3\gamma+2)|\lambda_p|\,\mathbb{I}_{n_p} \\ (3\gamma+2)|\lambda_p|\,\mathbb{I}_{n_p} & \left(|\lambda_p|^2+3\gamma+1\right)\mathbb{I}_{n_p} \end{pmatrix} \ ; \ \mathcal{H}'_0 = \begin{pmatrix} \mathcal{H}'^{(1)}_0 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathcal{H}'^{(p)}_0 \end{pmatrix} . \qquad \text{(C.0.15)}$$

Although the previous step is not strictly necessary, it makes the eigenvalue problem more visual to solve it for each subspace. We easily find the eigenvalues, which have degeneracy n_p each, and are given by:

$$m^2_{p\pm} = \mathrm{e}^G \left[|\lambda_p|^2 + (3\gamma+1) \pm |\lambda_p|(3\gamma+2)\right] = \mathrm{e}^G \left[\left(|\lambda_p| \pm \tfrac{1}{2}(3\gamma+2)\right)^2 - \tfrac{9}{4}\gamma^2\right] . \qquad \text{(C.0.16)}$$

This is the result displayed in (5.3.15). The eigenvectors are easily found through the equation:

$$\begin{pmatrix} \mp(3\gamma+2)|\lambda_p|\,\mathbb{I}_{n_p} & (3\gamma+2)|\lambda_p|\,\mathbb{I}_{n_p} \\ (3\gamma+2)|\lambda_p|\,\mathbb{I}_{n_p} & \mp(3\gamma+2)|\lambda_p|\,\mathbb{I}_{n_p} \end{pmatrix} \begin{pmatrix} \vec{a} \\ \vec{b} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \vec{u}_\pm = \frac{1}{\sqrt{2}|\vec{a}|} \begin{pmatrix} \vec{a} \\ \pm\vec{a} \end{pmatrix} . \qquad \text{(C.0.17)}$$

Since they are n_p times degenerate, we can choose n_p linearly independent vectors with a 1 in the p^{th} position and 0 in all the others. The matrix of change of basis is then:

$$A_p = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbb{I}_{n_p} & \mathbb{I}_{n_p} \\ \mathbb{I}_{n_p} & -\mathbb{I}_{n_p} \end{pmatrix} \Rightarrow A_p^\dagger \mathcal{H}'^{(p)}_0 A_p = \mathrm{diag}\left(m^2_{p+}\mathbb{I}_{n_p}, m^2_{p-}\mathbb{I}_{n_p}\right) \ . \qquad \text{(C.0.18)}$$

We can repeat the process for every block, which leads to our final result for the unperturbed mass matrix:

$$A = \begin{pmatrix} A_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & A_p \end{pmatrix} \quad \Rightarrow \quad A^\dagger \mathcal{H}'_0 A = \begin{pmatrix} m_{1+}^2 \mathbb{I}_{n_1} & & & & \mathbf{0} \\ & m_{1-}^2 \mathbb{I}_{n_1} & & & \\ & & \ddots & & \\ & & & m_{p+}^2 \mathbb{I}_{n_p} & \\ \mathbf{0} & & & & m_{p-}^2 \mathbb{I}_{n_p} \end{pmatrix} . \tag{C.0.19}$$

We retrieve the results of [3, 83, 84]. Now we just have to express the perturbed matrix $\delta\mathcal{H}'$ in the basis that diagonalises $\mathcal{H}'_0$. In order to do that, we first have to undo the rearranging of rows and columns we did to get to (C.0.15) (which was only done to facilitate the discussion). Instead of rearranging, it is easy to realise that the matrix that diagonalises $\mathcal{H}'_0$ (C.0.14) is

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbb{I}_{n_1+\cdots+n_p} & \mathbb{I}_{n_1+\cdots+n_p} \\ \mathbb{I}_{n_1+\cdots+n_p} & -\mathbb{I}_{n_1+\cdots+n_p} \end{pmatrix} \Rightarrow \tag{C.0.20}$$

$$\Rightarrow A^\dagger \mathcal{H}'_0 A = \mathrm{diag}\left(m_{1+}^2 \mathbb{I}_{n_1}, \ldots, m_{p+}^2 \mathbb{I}_{n_p}, m_{1-}^2 \mathbb{I}_{n_1}, \ldots, m_{p-}^2 \mathbb{I}_{n_p}\right) \tag{C.0.21}$$

Therefore, the leading order correction to the eigenvalues (C.0.16) due to $\delta\mathcal{H}$ in (C.0.9) is given by the diagonal elements of the matrix

$$A^\dagger \delta\mathcal{H}\, A = \frac{1}{2} \begin{pmatrix} \mathrm{e}^G \left[(\Omega + \Omega^T) + (Y + Y^*)\right] & \mathrm{e}^G \left[(\Omega - \Omega^T) + (Y - Y^*)\right] \\ \mathrm{e}^G \left[(\Omega - \Omega^T) - (Y - Y^*)\right] & \mathrm{e}^G \left[(\Omega + \Omega^T) - (Y + Y^*)\right] \end{pmatrix} , \tag{C.0.22}$$

where Ω and Y have been already transformed according to (C.0.10) and (C.0.13). To first order in perturbation theory, the eigenvalues then read

$$m_{p\pm}^2 + \delta m_{p\pm}^2 = \mathrm{e}^G \left\{ |\lambda_p|^2 + (3\gamma + 1) + \omega_p \pm \left[|\lambda_p|(3\gamma + 2) + y_p\right] \right\} , \tag{C.0.23}$$

where $\omega_p \equiv \Omega_{pp} = -R_{i\bar{j}p\bar{p}} G^i G^{\bar{j}}$ and $y_p \equiv \mathrm{Re}(Y_{pp}) = \mathrm{Re}(G^i \nabla_i (\nabla_p G_p))$. This result was derived in [3] for the simplest case of one light field and one heavy field[2]. We emphasise that the quantities y_p and ω_p are $\mathcal{O}(\epsilon)$ plus subleading corrections.

The matrix Y is proportional to the derivative of the fermion mass matrix along the sGoldstino direction. Thus, in the basis that diagonalises M, it is possible to show that $\tilde{Y} \equiv V^\dagger Y V^*$ has the following form:

$$\tilde{Y} = V (G^i \partial_i X) V^t = G^i \partial_i D + G^i \left(V^\dagger \partial_i V D - D\, V^t \partial_i V^*\right) , \tag{C.0.24}$$

where the unitary matrix V and the diagonal matrix D are the ones appearing in the Takagi's factorisation of $M = V D V^t$. Due to the unitarity of V the

[2] The notation in [3] is slightly different, it corresponds to $\gamma \to b/3$.

matrix $V^\dagger\partial_i V$ is anti-hermitian, and therefore its diagonal elements have to be purely imaginary $(V^\dagger\partial_i V)_{pp} = \mathrm{i}\theta_p$, with $\theta_p \in \mathbb{R}$. Then, in this basis, the diagonal elements of Y read

$$\tilde{Y}_{pp} = G^i\left(\partial_i|\lambda_p| + \mathrm{i}2|\lambda_p|\theta_p\right), \qquad \Longrightarrow \qquad y_p = G^i\partial_i|\lambda_p|, \tag{C.0.25}$$

implying that the perturbation y_p is just proportional to the derivative of the eigenvalues of the matrix M along the sGoldstino direction. In order to reduce the dependence on $|\lambda_p|$ of the perturbation parameters appearing in the Hessian, it is convenient to write it in terms of $\tilde{y}_p \equiv y_p/|\lambda_p|$, which gives

$$\tilde{y}_p = G^i\partial_i \log(|\lambda_p|). \tag{C.0.26}$$

$\mathcal{D}$

Random matrix theory: atypical minima and fluctuated spectra

In this appendix we review the expressions for the probability of occurrence of atypical fluctuations of the fermionic mass spectra, and in particular we will discuss the probability distribution of the lightest and largest fermion masses. As we have discussed in the main text, the CI-ensemble describes the statistical properties of the fermion mass matrix $\mathcal{M}_h$ for a generic supersymmetric sector. The CI-ensemble is closely related to the set of Wishart ensemble [212] for which there are many results in the literature regarding fluctuated spectra. For this reason we will first discuss known results for the Wishart ensemble, and then we will translate them into properties of the fermion mass spectrum in a generic supersymmetric sector.

D.1 Typical spectral density in the Wishart and CI-ensembles

The Wishart ensemble is composed of matrices of the form $\mathcal{W} = AA^\dagger$, where A is an $\mathrm{N} \times \mathrm{M}$ real or complex matrix, (with $\mathrm{M} \geq \mathrm{N}$), whose entries are independent and identically distributed (i.i.d.) random variables drawn from a statistical distribution with zero mean and variance σ^2: $A_{IJ} \in \Omega(0, \sigma)$. When $\Omega = N(0, \sigma)$ is a normal distribution, the joint probability distribution for the ordered eigenvalues $\lambda_1 \leq \lambda_2, \ldots, \leq \lambda_{\mathrm{N}}$ is [237]:

$$f(\lambda_1, \ldots, \lambda_{\mathrm{N}}) = \mathcal{C}\ \exp\left(-\frac{\beta}{2}\left(\frac{1}{\sigma^2}\sum_{a=1}^{\mathrm{N}}\lambda_a\ - 2\sum_{a<b}^{\mathrm{N}}\ln|\lambda_b - \lambda_a| - \xi\ \sum_{a=1}^{\mathrm{N}}\ln\lambda_a\right)\right)\ , \tag{D.1.1}$$

where $\xi = \text{M} - \text{N} + 1 - 2/\beta$, and $\beta = 1, 2$ for real and complex matrices, respectively. The eigenvalue density function for the eigenvalues of $\mathcal{W}$ is given by the Marčenko-Pastur law [231],

$$\rho_{\text{MP}}(\lambda)\, d\lambda = \frac{1}{2\pi\sigma^2\lambda}\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}\, d\lambda\,, \tag{D.1.2}$$

with support $\lambda \in [\lambda_-, \lambda_+]$, where

$$\lambda_\pm = \text{N}\sigma^2(1 \pm \sqrt{\eta})^2\,, \qquad \text{and} \qquad \eta = \text{M}/\text{N} \geq 1. \tag{D.1.3}$$

The joint probability distribution for the eigenvalues of a matrix from the CI-ensemble was given in eq. (5.4.8). As was pointed out in [212], the p.d.f. of the eigenvalues of a Wishart matrix (D.1.1) reduces to (5.4.8) for $\beta = 1$ and $\text{M} = \text{N} + 1$ after doing the identification $\lambda_a \leftrightarrow m_\lambda^2$. Moreover, as we are interested in results to leading leading order in $1/\text{N}$, it will be sufficient to discuss square Wishart matrices $\text{N} \approx \text{M}$. Thus, since the fermion mass matrix of a generic supergravity theory can be identified with an element of the CI ensemble, the typical spectral density of the fermion masses m_λ is also given by the Marčenko-Pastur law (D.1.2) with $\lambda = m^2$. Defining $m_h \equiv 2\sqrt{\text{N}}\sigma$, we have that to leading order in $1/\text{N}$ the fermion mass density function reads:

$$\rho_{\text{MP}}(m^2)dm^2 = \frac{2\text{N}}{\pi m_h^2 m}\sqrt{m_h^2 - m^2}\; dm^2, \tag{D.1.4}$$

which has support in $m^2 \in \left[0, m_h^2\right]$. In the limit $\text{N} \to \infty$ the bounds of the support coincide with the expectation value of the smallest and largest fermionic masses squared, m_1^2 and m_{N}^2 respectively [238]:

$$\mathbb{E}[m_1^2] = 0, \qquad\qquad \mathbb{E}[m_{\text{N}}^2] \approx m_h^2. \tag{D.1.5}$$

D.2 Probability distributions of the limiting eigenvalues

Let us first discuss the probability distribution of the largest eigenvalue λ_{N} of a real, almost square Wishart matrix, $\beta = 1$, $\text{M} \approx \text{N}$. The probability distribution of large $\mathcal{O}(\sigma^2\text{N})$ fluctuations of λ_{N} far to the right and left of its mean value λ_+ was calculated in [239] and [240], respectively, and are given by:

$$\begin{aligned} t > \lambda_+ : \quad & \lim_{\text{N}\to\infty} \mathbb{P}(\lambda_{\text{N}} \leq t) \;\approx\; 1 - \left(\sqrt{x+1} + \sqrt{x}\right)^{2\text{N}} \mathrm{e}^{-2\text{N}\sqrt{x(x+1)}}, \\ t < \lambda_+ : \quad & \lim_{\text{N}\to\infty} \mathbb{P}(\lambda_{\text{N}} \leq t) \;\approx\; \left(\frac{x+1}{\sqrt{\mathrm{e}}}\right)^{\frac{\text{N}^2}{2}} \mathrm{e}^{-\frac{\text{N}^2}{4}(1-x)^2}\;, \end{aligned} \tag{D.2.1}$$

where $x \equiv (t - \lambda_+)/\lambda_+$. For large but finite values of N, the maximum value of a Wishart matrix, λ_{N}, *typically* fluctuates over a region of size $\mathcal{O}(\sigma^2\, \text{N}^{-1/3})$, and the corresponding probability distribution for these small fluctuations can be approximated by the Tracy-Widom distribution $F_1(x)$ [232, 238, 239]:

$$\mathbb{P}(\lambda_{\mathrm{N}} \leq t) \approx F_1\left(\frac{\eta^{\frac{1}{12}}\,\mathrm{N}^{\frac{1}{3}}\,(t-\lambda_+)}{\sigma^{\frac{2}{3}}\lambda_+^{\frac{2}{3}}}\right) \approx F_1\left(2^{\frac{2}{3}}\,\mathrm{N}^{\frac{2}{3}}\,\frac{(t-\lambda_+)}{\lambda_+}\right)\,, \tag{D.2.2}$$

where we have used the leading order approximation $\eta = 1$ and $\lambda_+ \approx 4\mathrm{N}\sigma^2$ for large N in the last step. For the asymptotic values of the probability (D.2.2), see [241] and references therein. In particular, to leading order in $1/\mathrm{N}$, the cumulative probability distribution for the largest eigenvalue λ_{N} is:

$$\begin{aligned} t > \lambda_+ : \quad & \lim_{\mathrm{N}\to\infty} \mathbb{P}_{\mathrm{N}}(\lambda_{\mathrm{N}} \leq t) &\approx& \; 1 - \frac{\mathrm{e}^{-\frac{4}{3}\mathrm{N}\,x^{\frac{3}{2}}}}{8\sqrt{\pi}\,\mathrm{N}\,x^{\frac{3}{2}}} - \frac{\mathrm{e}^{-\frac{8}{3}\mathrm{N}\,x^{\frac{3}{2}}}}{64\pi\mathrm{N}\,x^{\frac{3}{2}}}\,, \\ t < \lambda_+ : \quad & \lim_{\mathrm{N}\to\infty} \mathbb{P}_{\mathrm{N}}(\lambda_{\mathrm{N}} \leq t) &\approx& \; \tau_1\,\frac{\mathrm{e}^{-\frac{1}{6}|x|^3\mathrm{N}^2}}{2^{\frac{1}{24}}\,\mathrm{N}^{\frac{1}{24}}\,|x|^{\frac{1}{16}}}\,, \end{aligned} \tag{D.2.3}$$

where $\tau_1 \equiv 2^{-11/48}\,\mathrm{e}^{\frac{1}{2}\zeta'(-1)}$, and $\zeta'(-1) = -0.16542\ldots$ is the derivative of the Riemann zeta function evaluated at -1. It is easy to check that, to leading order in $\mathcal{O}(1/\mathrm{N})$, the probability distributions in (D.2.1) match the tail behaviour of the Tracy-Widom distributions in the limit $t \to \lambda_+$, which describes small fluctuations.

The probability distribution of the smallest eigenvalue λ_1 of a real square Wishart matrix was derived in [233]. To leading order in $1/\mathrm{N}$ it is given by:

$$\lim_{\mathrm{N}\to\infty} \mathbb{P}(\lambda_1 \geq t) \approx \frac{\lambda_+}{4\mathrm{N}^2}\,\mathrm{e}^{-\frac{2\mathrm{N}^2}{\lambda_+}t}. \tag{D.2.4}$$

D.3 Probability of atypical field configurations

In chapter 5, where we study the stability of a consistently truncated supersymmetric sector in models with a separable Kähler function, we estimated the probability of occurrence of critical points with light scalar fields, i.e. with a mass $\mu^2|_{min} \leq \alpha^2$, in the regime $m_h < 1-\alpha$. We argued that, due to the relation between scalar and fermion masses, this would require the largest fermion mass fermion to be above its expectation value $m_{\mathrm{N}} \geq 1-\alpha > m_h$. Using the first equation in (D.2.1), and taking into account the relation between the Wishart and CI-ensembles, we find

$$1-\alpha > m_h : \quad \lim_{\mathrm{N}\to\infty} \mathbb{P}(m_{\mathrm{N}} \geq 1-\alpha) \approx \left(\sqrt{x+1}+\sqrt{x}\right)^{2\mathrm{N}}\,\mathrm{e}^{-2\mathrm{N}\sqrt{x(x+1)}}, \tag{D.3.1}$$

with $x = (1-\alpha)^2/m_h^2 - 1$. The Tracy-Widom distribution gives an accurate description in the limit $m_h \to (1-\alpha)^-$, where the deviations of m_{N} from its expectation value are small.

Now we turn to the stability of the truncated sector, when the sector surviving the truncation is driving a period of inflation, and the Kähler function is also separable in the two sectors. In general, in the regime where the mass scale m_h is larger than the gravitino mass ($m_h > 1$), the typical spectrum contains tachyons (see right plot in Fig. 5.5). However, as illustrated by (5.6.7), there is an exponentially suppressed probability that the fermionic spectrum fluctuates in such a way that the scalar spectrum is free of tachyons. There are two possible types of configurations which are non-tachyonic: when the fermion masses are confined to $m_\lambda < 1$, or to $m_\lambda > 3\gamma + 1$, for all λ. It is interesting to check, for a configuration with a Hubble parameter given in terms of γ, in what regime of parameters one of these types of critical points becomes more abundant than the other. The probability that the fermion masses are bounded below as $m_\lambda \geq 3\gamma + 1$, can be calculated from (D.2.4):

$$\lim_{\text{N}\to\infty} \mathbb{P}\Big(m_1 \geq 3\gamma + 1\Big) \approx \frac{m_h^2}{4\text{N}^2}\, \mathrm{e}^{-\frac{2\text{N}^2}{m_h^2}(3\gamma+1)^2} \,. \tag{D.3.2}$$

On the other hand, the probability that the fermion masses are bounded above by $m_{\text{N}} = 1$, also to leading order in $1/\text{N}$, can be derived from the second equation in (D.2.1):

$$\lim_{\text{N}\to\infty} \mathbb{P}\Big(m_{\text{N}} \leq 1\Big) \approx \frac{\mathrm{e}^{-\text{N}^2/4}}{m_h^{\text{N}^2}}\, \mathrm{e}^{-\frac{\text{N}^2}{4}\left(2-\frac{1}{m_h^2}\right)^2} \,. \tag{D.3.3}$$

From the above expressions it can be seen that a fluctuation to a minimum of the Kähler function, where all $m_\lambda < 1$ (D.3.2), becomes more likely than a fluctuation to large fermionic masses (D.3.3) when the following condition is satisfied:

$$(2+\text{N}^2)\log m_h - 2\log(2\text{N}) - \frac{2\text{N}^2}{m_h^2}(1+3\gamma)^2 + \frac{\text{N}^2}{4} + \frac{\text{N}^2}{4}\left(2 - \frac{1}{m_h^2}\right)^2 < 0 \,. \tag{D.3.4}$$

In the regime $\text{N} \gg m_h \gg 1$ and with γ comparable to m_h the expression above simplifies to

$$\gamma^2 \geq \frac{m_h^2}{18}\,(\log m_h + \tfrac{5}{4}) \,, \tag{D.3.5}$$

which can be rewritten in terms of the Hubble parameter as follows:

$$H^2 \gtrsim \frac{m_h m_{3/2}}{3\sqrt{2}}\sqrt{\log m_h + \frac{5}{4}} \,. \tag{D.3.6}$$

We can see that regardless of the value of the mass scale of the supersymmetric sector m_h, there is always a value of the Hubble parameter (D.3.6) above which the largest fraction of stable critical points corresponds to a minimum of the Kähler function. This is particularly important for cosmological models which involve a large inflationary scale H and a low supersymmetry breaking scale. This result depends strongly on the value of the mass scale of the supersymmetric sector m_h which, as we discussed in section 5.4.2, should also be regarded as a random variable depending on the value of the gravitino mass $m_{3/2}$, and its statistical properties should be derived from a realistic characterisation of the String Theory Landscape.

Bibliography

[1] A. Achucarro, V. Atal, P. Ortiz, and J. Torrado, *Localized correlated features in the CMB power spectrum and primordial bispectrum from a transient reduction in the speed of sound*, *Phys.Rev.* **D89** (2014) 103006, [`arXiv:1311.2552`].

[2] A. Achucarro, V. Atal, B. Hu, P. Ortiz, and J. Torrado, *Inflation with moderately sharp features in the speed of sound: GSR and in-in formalism for power spectrum and bispectrum*, *Phys.Rev.* **D90** (2014) 023511, [`arXiv:1404.7522`].

[3] A. Achucarro, S. Mooij, P. Ortiz, and M. Postma, *Sgoldstino inflation*, *JCAP* **1208** (2012) 013, [`arXiv:1203.1907`].

[4] K. Sousa and P. Ortiz, *Perturbative Stability along the Supersymmetric Directions of the Landscape*, `arXiv:1408.6521`.

[5] A. H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, *Phys.Rev.* **D23** (1981) 347–356.

[6] A. A. Starobinsky, *Relict Gravitation Radiation Spectrum and Initial State of the Universe. (In Russian)*, *JETP Lett.* **30** (1979) 682–685.

[7] A. A. Starobinsky, *Dynamics of Phase Transition in the New Inflationary Universe Scenario and Generation of Perturbations*, *Phys.Lett.* **B117** (1982) 175–178.

[8] K. Sato, *First Order Phase Transition of a Vacuum and Expansion of the Universe*, *Mon.Not.Roy.Astron.Soc.* **195** (1981) 467–479.

[9] A. D. Linde, *A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems*, *Phys.Lett.* **B108** (1982) 389–393.

[10] A. Albrecht and P. J. Steinhardt, *Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking*, *Phys.Rev.Lett.* **48** (1982) 1220–1223.

[11] A. A. Penzias and R. W. Wilson, *A Measurement of excess antenna temperature at 4080-Mc/s*, *Astrophys.J.* **142** (1965) 419–421.

[12] W. H. Kinney, *TASI Lectures on Inflation*, `arXiv:0902.1529`.

[13] D. Baumann, *TASI Lectures on Inflation*, `arXiv:0907.5424`.

[14] D. Langlois, *Lectures on inflation and cosmological perturbations*, *Lect.Notes Phys.* **800** (2010) 1–57, [`arXiv:1001.5259`].

[15] V. Mukhanov, *Physical foundations of cosmology.* Cambridge Univ. Pr., Cambridge, UK, 2005.

[16] D. H. Lyth and A. R. Liddle, *The primordial density perturbation: Cosmology, inflation and the origin of structure.* Cambridge Univ. Pr., Cambridge, UK, 2009.

[17] A. Friedmann, *On the Possibility of a world with constant negative curvature of space*, *Z.Phys.* **21** (1924) 326–332.

[18] G. Lemaître, *Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques*, *Annales de la Societe Scietifique de Bruxelles* **47** (1927) 49–59.

[19] H. P. Robertson, *Kinematics and World-Structure*, *Ap. J.* **82** (Nov., 1935) 284.

[20] H. P. Robertson, *Kinematics and World-Structure II.*, *Ap. J.* **83** (Apr., 1936) 187.

[21] H. P. Robertson, *Kinematics and World-Structure III.*, *Ap. J.* **83** (May, 1936) 257.

[22] A. G. Walker, *On milne's theory of world-structure*, *Proceedings of the London Mathematical Society* **s2-42** (1937), no. 1 90–127, [`http://plms.oxfordjournals.org/content/s2-42/1/90.full.pdf+html`].

[23] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, *Spontaneous Creation of Almost Scale - Free Density Perturbations in an Inflationary Universe*, *Phys.Rev.* **D28** (1983) 679.

[24] V. F. Mukhanov, *Gravitational Instability of the Universe Filled with a Scalar Field*, *JETP Lett.* **41** (1985) 493–496.

[25] M. Sasaki, *Large Scale Quantum Fluctuations in the Inflationary Universe*, *Prog.Theor.Phys.* **76** (1986) 1036.

[26] T. Bunch and P. Davies, *Quantum Field Theory in de Sitter Space: Renormalization by Point Splitting*, *Proc.Roy.Soc.Lond.* **A360** (1978) 117–134.

[27] U. H. Danielsson, *A Note on inflation and transPlanckian physics*, *Phys.Rev.* **D66** (2002) 023511, [hep-th/0203198].

[28] B. R. Greene, K. Schalm, G. Shiu, and J. P. van der Schaar, *Decoupling in an expanding universe: Backreaction barely constrains short distance effects in the CMB*, *JCAP* **0502** (2005) 001, [hep-th/0411217].

[29] P. D. Meerburg, J. P. van der Schaar, and P. S. Corasaniti, *Signatures of Initial State Modifications on Bispectrum Statistics*, *JCAP* **0905** (2009) 018, [arXiv:0901.4044].

[30] M. G. Jackson and K. Schalm, *Model Independent Signatures of New Physics in the Inflationary Power Spectrum*, *Phys.Rev.Lett.* **108** (2012) 111301, [arXiv:1007.0185].

[31] **WMAP** Collaboration, C. Bennett *et. al.*, *Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results*, *Astrophys.J.Suppl.* **208** (2013) 20, [arXiv:1212.5225].

[32] **Planck Collaboration** Collaboration, P. Ade *et. al.*, *Planck 2013 results. I. Overview of products and scientific results*, arXiv:1303.5062.

[33] V. F. Mukhanov and G. V. Chibisov, *Quantum Fluctuation and Nonsingular Universe. (In Russian)*, *JETP Lett.* **33** (1981) 532–535.

[34] **Planck collaboration** Collaboration, P. Ade *et. al.*, *Planck 2013 results. XV. CMB power spectra and likelihood*, arXiv:1303.5075.

[35] T. Allen, B. Grinstein, and M. B. Wise, *Nongaussian Density Perturbations in Inflationary Cosmologies*, *Phys.Lett.* **B197** (1987) 66.

[36] T. Falk, R. Rangarajan, and M. Srednicki, *The Angular dependence of the three point correlation function of the cosmic microwave background radiation as predicted by inflationary cosmologies*, *Astrophys.J.* **403** (1993) L1, [astro-ph/9208001].

[37] A. Gangui, F. Lucchin, S. Matarrese, and S. Mollerach, *The Three point correlation function of the cosmic microwave background in inflationary models*, *Astrophys.J.* **430** (1994) 447–457, [astro-ph/9312033].

[38] V. Acquaviva, N. Bartolo, S. Matarrese, and A. Riotto, *Second order cosmological perturbations from inflation*, *Nucl.Phys.* **B667** (2003) 119–148, [astro-ph/0209156].

[39] J. M. Maldacena, *Non-Gaussian features of primordial fluctuations in single field inflationary models*, *JHEP* **0305** (2003) 013, [astro-ph/0210603].

[40] L. Keldysh, *Diagram technique for nonequilibrium processes*, *Zh.Eksp.Teor.Fiz.* **47** (1964) 1515–1527.

[41] S. Weinberg, *Quantum contributions to cosmological correlations*, *Phys.Rev.* **D72** (2005) 043514, [hep-th/0506236].

[42] V. F. Mukhanov, *CMB-slow, or how to estimate cosmological parameters by hand*, *Int.J.Theor.Phys.* **43** (2004) 623–668, [astro-ph/0303072].

[43] U. Seljak and M. Zaldarriaga, *A Line of sight integration approach to cosmic microwave background anisotropies*, *Astrophys.J.* **469** (1996) 437–444, [astro-ph/9603033].

[44] A. Lewis, A. Challinor, and A. Lasenby, *Efficient computation of CMB anisotropies in closed FRW models*, *Astrophys.J.* **538** (2000) 473–476, [astro-ph/9911177].

[45] J. Lesgourgues, *The Cosmic Linear Anisotropy Solving System (CLASS) I: Overview*, arXiv:1104.2932.

[46] D. Blas, J. Lesgourgues, and T. Tram, *The Cosmic Linear Anisotropy Solving System (CLASS) II: Approximation schemes*, *JCAP* **1107** (2011) 034, [arXiv:1104.2933].

[47] **Planck Collaboration** Collaboration, P. Ade *et. al.*, *Planck 2013 results. XVI. Cosmological parameters*, arXiv:1303.5076.

[48] **Planck Collaboration** Collaboration, P. Ade *et. al.*, *Planck 2013 Results. XXIV. Constraints on primordial non-Gaussianity*, arXiv:1303.5084.

[49] D. Spergel, R. Flauger, and R. Hlozek, *Planck Data Reconsidered*, arXiv:1312.3313.

[50] E. Komatsu, D. N. Spergel, and B. D. Wandelt, *Measuring primordial non-Gaussianity in the cosmic microwave background*, *Astrophys.J.* **634** (2005) 14–19, [astro-ph/0305189].

[51] M. Bucher, B. Van Tent, and C. S. Carvalho, *Detecting Bispectral Acoustic Oscillations from Inflation Using a New Flexible Estimator*, `arXiv:0911.1642`.

[52] J. Fergusson, M. Liguori, and E. Shellard, *The CMB Bispectrum*, *JCAP* **1212** (2012) 032, [`arXiv:1006.1642`].

[53] D. Babich, P. Creminelli, and M. Zaldarriaga, *The Shape of non-Gaussianities*, *JCAP* **0408** (2004) 009, [`astro-ph/0405356`].

[54] L. Senatore, K. M. Smith, and M. Zaldarriaga, *Non-Gaussianities in Single Field Inflation and their Optimal Limits from the WMAP 5-year Data*, *JCAP* **1001** (2010) 028, [`arXiv:0905.3746`].

[55] **BICEP2 Collaboration** Collaboration, P. Ade *et. al.*, *BICEP2 I: Detection Of B-mode Polarization at Degree Angular Scales*, `arXiv:1403.3985`.

[56] D. H. Lyth, *What would we learn by detecting a gravitational wave signal in the cosmic microwave background anisotropy?*, *Phys.Rev.Lett.* **78** (1997) 1861–1863, [`hep-ph/9606387`].

[57] M. J. Mortonson and U. Seljak, *A joint analysis of Planck and BICEP2 B modes including dust polarization uncertainty*, `arXiv:1405.5857`.

[58] R. Flauger, J. C. Hill, and D. N. Spergel, *Toward an Understanding of Foreground Emission in the BICEP2 Region*, `arXiv:1405.7351`.

[59] A. Zee, *A Broken Symmetric Theory of Gravity*, *Phys.Rev.Lett.* **42** (1979) 417.

[60] L. Smolin, *Towards a Theory of Space-Time Structure at Very Short Distances*, *Nucl.Phys.* **B160** (1979) 253.

[61] B. Spokoiny, *Inflation and generation of perturbations in broken symmetric theory of gravity*, *Phys.Lett.* **B147** (1984) 39–43.

[62] D. Salopek, J. Bond, and J. M. Bardeen, *Designing Density Fluctuation Spectra in Inflation*, *Phys.Rev.* **D40** (1989) 1753.

[63] F. L. Bezrukov and M. Shaposhnikov, *The Standard Model Higgs boson as the inflaton*, *Phys.Lett.* **B659** (2008) 703–706, [`arXiv:0710.3755`].

[64] J. Garcia-Bellido, D. G. Figueroa, and J. Rubio, *Preheating in the Standard Model with the Higgs-Inflaton coupled to gravity*, *Phys.Rev.* **D79** (2009) 063531, [`arXiv:0812.4624`].

[65] F. Bezrukov, D. Gorbunov, and M. Shaposhnikov, *On initial conditions for the Hot Big Bang*, *JCAP* **0906** (2009) 029, [`arXiv:0812.3622`].

[66] J. Barbon and J. Espinosa, *On the Naturalness of Higgs Inflation*, *Phys.Rev.* **D79** (2009) 081302, [arXiv:0903.0355].

[67] D. P. George, S. Mooij, and M. Postma, *Quantum corrections in Higgs inflation: the real scalar case*, *JCAP* **1402** (2014) 024, [arXiv:1310.2157].

[68] F. Quevedo, *Lectures on string/brane cosmology*, *Class.Quant.Grav.* **19** (2002) 5721–5779, [hep-th/0210292].

[69] R. Kallosh, *On inflation in string theory*, *Lect.Notes Phys.* **738** (2008) 119–156, [hep-th/0702059].

[70] L. McAllister and E. Silverstein, *String Cosmology: A Review*, *Gen.Rel.Grav.* **40** (2008) 565–605, [arXiv:0710.2951].

[71] D. Baumann and L. McAllister, *Advances in Inflation in String Theory*, *Ann.Rev.Nucl.Part.Sci.* **59** (2009) 67–94, [arXiv:0901.0265].

[72] D. Baumann and L. McAllister, *Inflation and String Theory*, arXiv:1404.2601.

[73] K. A. Olive, *Inflation*, *Phys.Rept.* **190** (1990) 307–403.

[74] D. H. Lyth and A. Riotto, *Particle physics models of inflation and the cosmological density perturbation*, *Phys.Rept.* **314** (1999) 1–146, [hep-ph/9807278].

[75] M. Yamaguchi, *Supergravity based inflation models: a review*, *Class.Quant.Grav.* **28** (2011) 103001, [arXiv:1101.2488].

[76] M. Nakahara, *Geometry, topology and physics.* Taylor & Francis, New York, USA, 1990.

[77] D. Freedman and A. Van Proeyen, *Supergravity.* Cambridge University Press, 2012.

[78] E. D. Stewart, *Inflation, supergravity and superstrings*, *Phys.Rev.* **D51** (1995) 6847–6853, [hep-ph/9405389].

[79] M. K. Gaillard, H. Murayama, and K. A. Olive, *Preserving flat directions during inflation*, *Phys.Lett.* **B355** (1995) 71–77, [hep-ph/9504307].

[80] M. Kawasaki, M. Yamaguchi, and T. Yanagida, *Natural chaotic inflation in supergravity*, *Phys.Rev.Lett.* **85** (2000) 3572–3575, [hep-ph/0004243].

[81] S. Antusch, M. Bastero-Gil, K. Dutta, S. F. King, and P. M. Kostka, *Chaotic Inflation in Supergravity with Heisenberg Symmetry*, *Phys.Lett.* **B679** (2009) 428–432, [arXiv:0905.0905].

[82] P. Binetruy, G. Dvali, R. Kallosh, and A. Van Proeyen, *Fayet-Iliopoulos terms in supergravity and cosmology*, *Class.Quant.Grav.* **21** (2004) 3137–3170, [hep-th/0402046].

[83] A. Achucarro and K. Sousa, *F-term uplifting and moduli stabilization consistent with Kahler invariance*, *JHEP* **0803** (2008) 002, [arXiv:0712.3460].

[84] A. Achucarro, S. Hardeman, and K. Sousa, *F-term uplifting and the supersymmetric integration of heavy moduli*, *JHEP* **0811** (2008) 003, [arXiv:0809.1441].

[85] A. Achucarro, S. Hardeman, and K. Sousa, *Consistent Decoupling of Heavy Scalars and Moduli in N=1 Supergravity*, *Phys.Rev.* **D78** (2008) 101901, [arXiv:0806.4364].

[86] R. Kallosh, A. Linde, and T. Rube, *General inflaton potentials in supergravity*, *Phys.Rev.* **D83** (2011) 043507, [arXiv:1011.5945].

[87] J. Garcia-Bellido and D. Wands, *Metric perturbations in two field inflation*, *Phys.Rev.* **D53** (1996) 5437–5445, [astro-ph/9511029].

[88] C. Gordon, D. Wands, B. A. Bassett, and R. Maartens, *Adiabatic and entropy perturbations from inflation*, *Phys.Rev.* **D63** (2001) 023506, [astro-ph/0009131].

[89] S. Groot Nibbelink and B. van Tent, *Density perturbations arising from multiple field slow roll inflation*, hep-ph/0011325.

[90] S. Groot Nibbelink and B. van Tent, *Scalar perturbations during multiple field slow-roll inflation*, *Class.Quant.Grav.* **19** (2002) 613–640, [hep-ph/0107272].

[91] D. Wands, N. Bartolo, S. Matarrese, and A. Riotto, *An Observational test of two-field inflation*, *Phys.Rev.* **D66** (2002) 043520, [astro-ph/0205253].

[92] C. M. Peterson and M. Tegmark, *Testing Two-Field Inflation*, *Phys.Rev.* **D83** (2011) 023522, [arXiv:1005.4056].

[93] S. Cremonini, Z. Lalak, and K. Turzynski, *Strongly Coupled Perturbations in Two-Field Inflationary Models*, *JCAP* **1103** (2011) 016, [arXiv:1010.3021].

[94] C. M. Peterson and M. Tegmark, *Testing multifield inflation: A geometric approach*, *Phys.Rev.* **D87** (2013), no. 10 103507, [arXiv:1111.0927].

[95] A. Avgoustidis, S. Cremonini, A.-C. Davis, R. H. Ribeiro, K. Turzynski, *et. al.*, *The Importance of Slow-roll Corrections During Multi-field Inflation*, *JCAP* **1202** (2012) 038, [arXiv:1110.4081].

[96] K. A. Malik and D. Wands, *Cosmological perturbations*, *Phys.Rept.* **475** (2009) 1–51, [arXiv:0809.4944].

[97] D. Wands, K. A. Malik, D. H. Lyth, and A. R. Liddle, *A New approach to the evolution of cosmological perturbations on large scales*, *Phys.Rev.* **D62** (2000) 043527, [astro-ph/0003278].

[98] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, and L. Senatore, *The Effective Field Theory of Inflation*, *JHEP* **0803** (2008) 014, [arXiv:0709.0293].

[99] C. Burgess, J. M. Cline, and R. Holman, *Effective field theories and inflation*, *JCAP* **0310** (2003) 004, [hep-th/0306079].

[100] S. Weinberg, *Effective Field Theory for Inflation*, *Phys.Rev.* **D77** (2008) 123541, [arXiv:0804.4291].

[101] A. J. Tolley and M. Wyman, *The Gelaton Scenario: Equilateral non-Gaussianity from multi-field dynamics*, *Phys.Rev.* **D81** (2010) 043502, [arXiv:0910.1853].

[102] A. Achucarro, J.-O. Gong, S. Hardeman, G. A. Palma, and S. P. Patil, *Mass hierarchies and non-decoupling in multi-scalar field dynamics*, *Phys.Rev.* **D84** (2011) 043502, [arXiv:1005.3848].

[103] A. Achucarro, J.-O. Gong, S. Hardeman, G. A. Palma, and S. P. Patil, *Features of heavy physics in the CMB power spectrum*, *JCAP* **1101** (2011) 030, [arXiv:1010.3693].

[104] G. Shiu and J. Xu, *Effective Field Theory and Decoupling in Multi-field Inflation: An Illustrative Case Study*, *Phys.Rev.* **D84** (2011) 103509, [arXiv:1108.0981].

[105] S. R. Behbahani, A. Dymarsky, M. Mirbabayi, and L. Senatore, *(Small) Resonant non-Gaussianities: Signatures of a Discrete Shift Symmetry in the Effective Field Theory of Inflation*, *JCAP* **1212** (2012) 036, [arXiv:1111.3373].

[106] S. Cespedes, V. Atal, and G. A. Palma, *On the importance of heavy fields during inflation*, *JCAP* **1205** (2012) 008, [arXiv:1201.4848].

[107] A. Achucarro, J.-O. Gong, S. Hardeman, G. A. Palma, and S. P. Patil, *Effective theories of single field inflation when heavy fields matter*, arXiv:1201.6342.

[108] X. Gao, D. Langlois, and S. Mizuno, *Influence of heavy modes on perturbations in multiple field inflation*, *JCAP* **1210** (2012) 040, [arXiv:1205.5275].

[109] A. Achucarro, V. Atal, S. Cespedes, J.-O. Gong, G. A. Palma, *et. al.*, *Heavy fields, reduced speeds of sound and decoupling during inflation*, *Phys.Rev.* **D86** (2012) 121301, [arXiv:1205.0710].

[110] E. Castillo, B. Koch, and G. Palma, *On the integration of fields and quanta in time dependent backgrounds*, arXiv:1312.3338.

[111] A. Achucarro, J.-O. Gong, G. A. Palma, and S. P. Patil, *Correlating features in the primordial spectra*, *Phys.Rev.* **D87** (2013) 121301, [arXiv:1211.5619].

[112] J.-O. Gong, K. Schalm, and G. Shiu, *Correlating correlation functions of primordial perturbations*, arXiv:1401.4402.

[113] **Planck Collaboration** Collaboration, P. Ade *et. al.*, *Planck 2013 results. XXII. Constraints on inflation*, arXiv:1303.5082.

[114] A. A. Starobinsky, *Spectrum of adiabatic perturbations in the universe when there are singularities in the inflation potential*, *JETP Lett.* **55** (1992) 489–494.

[115] L.-M. Wang and M. Kamionkowski, *The Cosmic microwave background bispectrum and inflation*, *Phys.Rev.* **D61** (2000) 063504, [astro-ph/9907431].

[116] J. A. Adams, B. Cresswell, and R. Easther, *Inflationary perturbations from a potential with a step*, *Phys.Rev.* **D64** (2001) 123514, [astro-ph/0102236].

[117] J.-O. Gong, *Breaking scale invariance from a singular inflaton potential*, *JCAP* **0507** (2005) 015, [astro-ph/0504383].

[118] X. Chen, R. Easther, and E. A. Lim, *Generation and Characterization of Large Non-Gaussianities in Single Field Inflation*, *JCAP* **0804** (2008) 010, [arXiv:0801.3295].

[119] F. Arroja, A. E. Romano, and M. Sasaki, *Large and strong scale dependent bispectrum in single field inflation from a sharp feature in the mass*, *Phys.Rev.* **D84** (2011) 123503, [arXiv:1106.5384].

[120] J. Martin and L. Sriramkumar, *The scalar bi-spectrum in the Starobinsky model: The equilateral case*, *JCAP* **1201** (2012) 008, [arXiv:1109.5838].

[121] P. Adshead, C. Dvorkin, W. Hu, and E. A. Lim, *Non-Gaussianity from Step Features in the Inflationary Potential*, *Phys.Rev.* **D85** (2012) 023531, [arXiv:1110.3050].

[122] F. Arroja and M. Sasaki, *Strong scale dependent bispectrum in the Starobinsky model of inflation*, *JCAP* **1208** (2012) 012, [arXiv:1204.6489].

[123] Y.-i. Takamizu, S. Mukohyama, M. Sasaki, and Y. Tanaka, *Non-Gaussianity of superhorizon curvature perturbations beyond δ N formalism*, *JCAP* **1006** (2010) 019, [arXiv:1004.1870].

[124] N. Bartolo, D. Cannone, and S. Matarrese, *The Effective Field Theory of Inflation Models with Sharp Features*, *JCAP* **1310** (2013) 038, [arXiv:1307.3483].

[125] E. D. Stewart, *The Spectrum of density perturbations produced during inflation to leading order in a general slow roll approximation*, *Phys.Rev.* **D65** (2002) 103508, [astro-ph/0110322].

[126] J. Choe, J.-O. Gong, and E. D. Stewart, *Second order general slow-roll power spectrum*, *JCAP* **0407** (2004) 012, [hep-ph/0405155].

[127] C. Dvorkin and W. Hu, *Generalized Slow Roll for Large Power Spectrum Features*, *Phys.Rev.* **D81** (2010) 023518, [arXiv:0910.2237].

[128] P. Adshead, W. Hu, C. Dvorkin, and H. V. Peiris, *Fast Computation of Bispectrum Features with Generalized Slow Roll*, *Phys.Rev.* **D84** (2011) 043519, [arXiv:1102.3435].

[129] V. Miranda, W. Hu, and P. Adshead, *Warp Features in DBI Inflation*, *Phys.Rev.* **D86** (2012) 063529, [arXiv:1207.2186].

[130] P. Adshead, W. Hu, and V. Miranda, *Bispectrum in Single-Field Inflation Beyond Slow-Roll*, *Phys.Rev.* **D88** (2013) 023507, [arXiv:1303.7004].

[131] W. Hu, *Generalized Slow Roll for Non-Canonical Kinetic Terms*, *Phys.Rev.* **D84** (2011) 027303, [arXiv:1104.4500].

[132] M. Park and L. Sorbo, *Sudden variations in the speed of sound during inflation: features in the power spectrum and bispectrum*, *Phys.Rev.* **D85** (2012) 083520, [arXiv:1201.2903].

[133] M. Nakashima, R. Saito, Y.-i. Takamizu, and J. Yokoyama, *The effect of varying sound velocity on primordial curvature perturbations*, *Prog.Theor.Phys.* **125** (2011) 1035–1052, [arXiv:1009.4394].

[134] R. Bean, X. Chen, G. Hailu, S.-H. H. Tye, and J. Xu, *Duality Cascade in Brane Inflation*, *JCAP* **0803** (2008) 026, [arXiv:0802.0491].

[135] R. H. Ribeiro, *Inflationary signatures of single-field models beyond slow-roll*, *JCAP* **1205** (2012) 037, [arXiv:1202.4453].

[136] X. Gao, D. Langlois, and S. Mizuno, *Oscillatory features in the curvature power spectrum after a sudden turn of the inflationary trajectory*, *JCAP* **10** (2013) 23, [arXiv:1306.5680].

[137] R. Saito and Y.-i. Takamizu, *Localized Features in Non-Gaussianity from Heavy Physics*, *JCAP* **1306** (2013) 031, [arXiv:1303.3839].

[138] T. Noumi and M. Yamaguchi, *Primordial spectra from sudden turning trajectory*, arXiv:1307.7110.

[139] L. Covi, J. Hamann, A. Melchiorri, A. Slosar, and I. Sorbera, *Inflation and WMAP three year data: Features have a Future!*, *Phys.Rev.* **D74** (2006) 083509, [astro-ph/0606452].

[140] M. Benetti, M. Lattanzi, E. Calabrese, and A. Melchiorri, *Features in the primordial spectrum: new constraints from WMAP7+ACT data and prospects for Planck*, *Phys.Rev.* **D84** (2011) 063509, [arXiv:1107.4992].

[141] P. Adshead and W. Hu, *Fast Computation of First-Order Feature-Bispectrum Corrections*, *Phys.Rev.* **D85** (2012) 103531, [arXiv:1203.0012].

[142] M. Benetti, *Updating constraints on inflationary features in the primordial power spectrum with the Planck data*, *Phys.Rev.* **D88** (Oct., 2013) 087302, [arXiv:1308.6406].

[143] J. Hamann, L. Covi, A. Melchiorri, and A. Slosar, *New Constraints on Oscillations in the Primordial Spectrum of Inflationary Perturbations*, *Phys.Rev.* **D76** (2007) 023503, [astro-ph/0701380].

[144] M. Benetti, S. Pandolfi, M. Lattanzi, M. Martinelli, and A. Melchiorri, *Featuring the primordial power spectrum: new constraints on interrupted slow-roll from CMB and LRG data*, *Phys.Rev.* **D87** (2013) 023519, [arXiv:1210.3562].

[145] J. Martin and C. Ringeval, *Superimposed oscillations in the WMAP data?*, *Phys.Rev.* **D69** (2004) 083515, [astro-ph/0310382].

[146] R. Flauger, L. McAllister, E. Pajer, A. Westphal, and G. Xu, *Oscillations in the CMB from Axion Monodromy Inflation*, *JCAP* **1006** (2010) 009, [arXiv:0907.2916].

[147] M. Aich, D. K. Hazra, L. Sriramkumar, and T. Souradeep, *Oscillations in the inflaton potential: Complete numerical treatment and comparison with the recent and forthcoming CMB datasets*, *Phys.Rev.* **D87** (2013) 083526, [arXiv:1106.2798].

[148] P. D. Meerburg, R. A. M. J. Wijers, and J. P. van der Schaar, *WMAP7 constraints on oscillations in the primordial power spectrum*, *MNRS* **421** (Mar., 2012) 369–380, [`arXiv:1109.5264`].

[149] H. Peiris, R. Easther, and R. Flauger, *Constraining Monodromy Inflation*, *JCAP* **1309** (2013) 018, [`arXiv:1303.2616`].

[150] P. D. Meerburg, D. N. Spergel, and B. D. Wandelt, *Searching for Oscillations in the Primordial Power Spectrum: Perturbative Approach (Paper I)*, `arXiv:1308.3704`.

[151] P. D. Meerburg and D. N. Spergel, *Searching for Oscillations in the Primordial Power Spectrum: Constraints from Planck (Paper II)*, `arXiv:1308.3705`.

[152] B. Audren, J. Lesgourgues, K. Benabed, and S. Prunet, *Conservative Constraints on Early Cosmology: an illustration of the Monte Python cosmological parameter inference code*, *JCAP* **1302** (2013) 001, [`arXiv:1210.7183`].

[153] L. Verde, *Statistical methods in cosmology*, *Lect.Notes Phys.* **800** (2010) 147–177, [`arXiv:0911.3105`].

[154] X. Chen, R. Easther, and E. A. Lim, *Large Non-Gaussianities in Single Field Inflation*, *JCAP* **0706** (2007) 023, [`astro-ph/0611645`].

[155] K. M. Smith, C. Dvorkin, L. Boyle, N. Turok, M. Halpern, *et. al.*, *On quantifying and resolving the BICEP2/Planck tension over gravitational waves*, `arXiv:1404.0373`.

[156] V. Miranda, W. Hu, and P. Adshead, *Steps to Reconcile Inflationary Tensor and Scalar Spectra*, `arXiv:1403.5231`.

[157] R. Bousso, D. Harlow, and L. Senatore, *Inflation After False Vacuum Decay: New Evidence from BICEP2*, `arXiv:1404.2278`.

[158] D. K. Hazra, A. Shafieloo, G. F. Smoot, and A. A. Starobinsky, *Ruling out the power-law form of the scalar primordial spectrum*, `arXiv:1403.7786`.

[159] A. Ashoorioon, K. Dimopoulos, M. Sheikh-Jabbari, and G. Shiu, *Non-Bunch-Davis Initial State Reconciles Chaotic Models with BICEP and Planck*, `arXiv:1403.6099`.

[160] J.-Q. Xia, Y.-F. Cai, H. Li, and X. Zhang, *Evidence for bouncing evolution before inflation after BICEP2*, `arXiv:1403.7623`.

[161] C. Burgess, M. Horbatsch, and S. Patil, *Inflating in a Trough: Single-Field Effective Theory from Multiple-Field Curved Valleys*, *JHEP* **1301** (2013) 133, [`arXiv:1209.5701`].

[162] D. Baumann and D. Green, *Equilateral Non-Gaussianity and New Physics on the Horizon*, *JCAP* **1109** (2011) 014, [arXiv:1102.5343].

[163] S. Pi and M. Sasaki, *Curvature Perturbation Spectrum in Two-field Inflation with a Turning Trajectory*, *JCAP* **1210** (2012) 051, [arXiv:1205.0161].

[164] X. Chen and Y. Wang, *Quasi-Single Field Inflation with Large Mass*, *JCAP* **1209** (2012) 021, [arXiv:1205.0160].

[165] Y.-F. Cai, W. Zhao, and Y. Zhang, *CMB Power Asymmetry from Primordial Sound Speed Parameter*, *Phys.Rev.* **D89** (2014) 023005, [arXiv:1307.4090].

[166] M. Konieczka, R. H. Ribeiro, and K. Turzynski, *The effects of a fast-turning trajectory in multiple-field inflation*, arXiv:1401.6163.

[167] V. Miranda and W. Hu, *Inflationary Steps in the Planck Data*, arXiv:1312.0946.

[168] P. Adshead and W. Hu, *Bounds on non-adiabatic evolution in single-field inflation*, arXiv:1402.1677.

[169] D. Cannone, N. Bartolo, and S. Matarrese, *Perturbative Unitarity of Inflationary Models with Features*, arXiv:1402.2258.

[170] J.-O. Gong and E. D. Stewart, *The Density perturbation power spectrum to second order corrections in the slow roll expansion*, *Phys.Lett.* **B510** (2001) 1–9, [astro-ph/0101225].

[171] P. Creminelli and M. Zaldarriaga, *Single field consistency relation for the 3-point function*, *JCAP* **0410** (2004) 006, [astro-ph/0407059].

[172] N. Kaloper and A. Lawrence, *Natural Chaotic Inflation and UV Sensitivity*, arXiv:1404.2912.

[173] X. Chen and M. H. Namjoo, *Standard Clock in Primordial Density Perturbations and Cosmic Microwave Background*, arXiv:1404.1536.

[174] A. Lewis and S. Bridle, *Cosmological parameters from CMB and other data: a Monte- Carlo approach*, *Phys. Rev.* **D66** (2002) 103511, [astro-ph/0205436].

[175] A. Achucarro, V. Atal, B. Hu, P. Ortiz, and J. Torrado , in preparation.

[176] J. Garriga and V. F. Mukhanov, *Perturbations in k-inflation*, *Phys.Lett.* **B458** (1999) 219–225, [hep-th/9904176].

[177] K. Sousa, *Consistent supersymmetric decoupling in cosmology*, *PhD Thesis (2012), ISBN 978-90-8593-127-0.*

[178] X. Chen and Y. Wang, *Quasi-Single Field Inflation and Non-Gaussianities*, *JCAP* **1004** (2010) 027, [arXiv:0911.3380].

[179] M. Dine, L. Randall, and S. D. Thomas, *Baryogenesis from flat directions of the supersymmetric standard model*, *Nucl.Phys.* **B458** (1996) 291–326, [hep-ph/9507453].

[180] M. Dine, L. Randall, and S. D. Thomas, *Supersymmetry breaking in the early universe*, *Phys.Rev.Lett.* **75** (1995) 398–401, [hep-ph/9503303].

[181] L. Alvarez-Gaume, C. Gomez, and R. Jimenez, *Minimal Inflation*, *Phys.Lett.* **B690** (2010) 68–72, [arXiv:1001.0010].

[182] L. Alvarez-Gaume, C. Gomez, and R. Jimenez, *A Minimal Inflation Scenario*, *JCAP* **1103** (2011) 027, [arXiv:1101.4948].

[183] L. Alvarez-Gaume, C. Gomez, and R. Jimenez, *Phenomenology of the minimal inflation scenario: inflationary trajectories and particle production*, *JCAP* **1203** (2012) 017, [arXiv:1110.3984].

[184] S. de Alwis, *Effective potentials for light moduli*, *Phys.Lett.* **B626** (2005) 223–229, [hep-th/0506266].

[185] S. de Alwis, *On integrating out heavy fields in SUSY theories*, *Phys.Lett.* **B628** (2005) 183–187, [hep-th/0506267].

[186] D. Gallego and M. Serone, *An Effective Description of the Landscape. I.*, *JHEP* **0901** (2009) 056, [arXiv:0812.0369].

[187] D. Gallego and M. Serone, *An Effective Description of the Landscape - II*, *JHEP* **0906** (2009) 057, [arXiv:0904.2537].

[188] D. Gallego, *On the Effective Description of Large Volume Compactifications*, *JHEP* **1106** (2011) 087, [arXiv:1103.5469].

[189] L. Brizi, M. Gomez-Reino, and C. A. Scrucca, *Globally and locally supersymmetric effective theories for light fields*, *Nucl.Phys.* **B820** (2009) 193–212, [arXiv:0904.0370].

[190] S. Hardeman, J. M. Oberreuter, G. A. Palma, K. Schalm, and T. van der Aalst, *The everpresent eta-problem: knowledge of all hidden sectors required*, *JHEP* **1104** (2011) 009, [arXiv:1012.5966].

[191] A. Achucarro, S. Hardeman, J. M. Oberreuter, K. Schalm, and T. van der Aalst, *Decoupling limits in multi-sector supergravities*, *JCAP* **1303** (2013) 038, [arXiv:1108.2278].

[192] P. Binetruy and M. K. Gaillard, *Temperature Corrections, Supersymmetric Effective Potentials and Inflation*, *Nucl.Phys.* **B254** (1985) 388.

[193] S. C. Davis and M. Postma, *Successfully combining SUGRA hybrid inflation and moduli stabilisation*, *JCAP* **0804** (2008) 022, [arXiv:0801.2116].

[194] L. Covi, M. Gomez-Reino, C. Gross, J. Louis, G. A. Palma, *et. al.*, *de Sitter vacua in no-scale supergravities and Calabi-Yau string models*, *JHEP* **0806** (2008) 057, [arXiv:0804.1073].

[195] L. Covi, M. Gomez-Reino, C. Gross, J. Louis, G. A. Palma, *et. al.*, *Constraints on modular inflation in supergravity and string theory*, *JHEP* **0808** (2008) 055, [arXiv:0805.3290].

[196] I. Ben-Dayan, R. Brustein, and S. P. de Alwis, *Models of Modular Inflation and Their Phenomenological Consequences*, *JCAP* **0807** (2008) 011, [arXiv:0802.3160].

[197] P. Brax, S. C. Davis, and M. Postma, *The Robustness of n(s) 0.95 in racetrack inflation*, *JCAP* **0802** (2008) 020, [arXiv:0712.0535].

[198] A. D. Linde and A. Westphal, *Accidental Inflation in String Theory*, *JCAP* **0803** (2008) 005, [arXiv:0712.1610].

[199] M. Dine and L. Pack, *Studies in Small Field Inflation*, *JCAP* **1206** (2012) 033, [arXiv:1109.2079].

[200] R. Barbieri, E. Cremmer, and S. Ferrara, *Flat and Positive Potentials in $N = 1$ Supergravity*, *Phys.Lett.* **B163** (1985) 143.

[201] D. H. Lyth and T. Moroi, *The Masses of weakly coupled scalar fields in the early universe*, *JHEP* **0405** (2004) 004, [hep-ph/0402174].

[202] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart, and D. Wands, *False vacuum inflation with Einstein gravity*, *Phys.Rev.* **D49** (1994) 6410–6433, [astro-ph/9401011].

[203] A. D. Linde, *Chaotic Inflation*, *Phys.Lett.* **B129** (1983) 177–181.

[204] A. D. Linde, *Hybrid inflation*, *Phys.Rev.* **D49** (1994) 748–754, [astro-ph/9307002].

[205] G. Dvali, Q. Shafi, and R. K. Schaefer, *Large scale structure and supersymmetric inflation without fine tuning*, *Phys.Rev.Lett.* **73** (1994) 1886–1889, [hep-ph/9406319].

[206] A. D. Linde and A. Riotto, *Hybrid inflation in supergravity*, *Phys.Rev.* **D56** (1997) 1841–1844, [hep-ph/9703209].

[207] S. Ashok and M. R. Douglas, *Counting flux vacua*, *JHEP* **0401** (2004) 060, [hep-th/0307049].

[208] M. R. Douglas, *Statistics of string vacua*, hep-ph/0401004.

[209] F. Denef and M. R. Douglas, *Distributions of nonsupersymmetric flux vacua*, *JHEP* **0503** (2005) 061, [hep-th/0411183].

[210] F. Denef and M. R. Douglas, *Distributions of flux vacua*, *JHEP* **0405** (2004) 072, [hep-th/0404116].

[211] D. Marsh, L. McAllister, and T. Wrase, *The Wasteland of Random Supergravities*, *JHEP* **1203** (2012) 102, [arXiv:1112.3034].

[212] T. C. Bachlechner, D. Marsh, L. McAllister, and T. Wrase, *Supersymmetric Vacua in Random Supergravity*, *JHEP* **1301** (2013) 136, [arXiv:1207.2763].

[213] Y. Sumitomo and S.-H. H. Tye, *A Stringy Mechanism for A Small Cosmological Constant*, *JCAP* **1208** (2012) 032, [arXiv:1204.5177].

[214] Y. Sumitomo and S. H. Tye, *A Stringy Mechanism for A Small Cosmological Constant - Multi-Moduli Cases -*, *JCAP* **1302** (2013) 006, [arXiv:1209.5086].

[215] Y. Sumitomo and S.-H. H. Tye, *Preference for a Vanishingly Small Cosmological Constant in Supersymmetric Vacua in a Type IIB String Theory Model*, *Phys.Lett.* **B723** (2013) 406–410, [arXiv:1211.6858].

[216] K. Metallinos, *Numerical exploration of the string theory landscape*, *ProQuest Dissertations and Theses* (2013) 114.

[217] M. D. Marsh, L. McAllister, E. Pajer, and T. Wrase, *Charting an Inflationary Landscape with Random Matrix Theory*, *JCAP* **1311** (2013) 040, [arXiv:1307.3559].

[218] F. G. Pedro and A. Westphal, *The Scale of Inflation in the Landscape*, arXiv:1303.3224.

[219] S. B. Giddings, S. Kachru, and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys.Rev.* **D66** (2002) 106006, [hep-th/0105097].

[220] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, *De Sitter vacua in string theory*, *Phys.Rev.* **D68** (2003) 046005, [hep-th/0301240].

[221] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, *JHEP* **0503** (2005) 007, [hep-th/0502058].

[222] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski, and S. Pokorski, *Stability of flux compactifications and the pattern of supersymmetry breaking*, *JHEP* **0411** (2004) 076, [hep-th/0411066].

[223] M. Rummel and A. Westphal, *A sufficient condition for de Sitter vacua in type IIB string theory*, *JHEP* **1201** (2012) 020, [arXiv:1107.2115].

[224] M. Gomez-Reino and C. A. Scrucca, *Locally stable non-supersymmetric Minkowski vacua in supergravity*, *JHEP* **0605** (2006) 015, [hep-th/0602246].

[225] M. Gomez-Reino and C. A. Scrucca, *Constraints for the existence of flat and stable non-supersymmetric vacua in supergravity*, *JHEP* **0609** (2006) 008, [hep-th/0606273].

[226] M. Gomez-Reino and C. A. Scrucca, *Metastable supergravity vacua with F and D supersymmetry breaking*, *JHEP* **0708** (2007) 091, [arXiv:0706.2785].

[227] L. Covi, M. Gomez-Reino, C. Gross, G. A. Palma, and C. A. Scrucca, *Constructing de Sitter vacua in no-scale string models without uplifting*, *JHEP* **0903** (2009) 146, [arXiv:0812.3864].

[228] P. Breitenlohner and D. Z. Freedman, *Positive Energy in anti-De Sitter Backgrounds and Gauged Extended Supergravity*, *Phys.Lett.* **B115** (1982) 197.

[229] D. Gallego, *Light field integration in SUGRA theories*, arXiv:1301.6177.

[230] L. Mehta, *Random Matrices.* Academic Press, 1991.

[231] V. A. Marčenko and L. Pastur, *Distribution of the eigenvalues in certain sets of random matrices*, *Math USSR-Sb* **1** (1967) 457–483.

[232] C. A. Tracy and H. Widom, *Level spacing distributions and the Airy kernel*, *Commun.Math.Phys.* **159** (1994) 151–174, [hep-th/9211141].

[233] A. Edelman, *Eigenvalues and condition numbers of random matrices*, *SIAM Journal on Matrix Anal. and Applic.* **9** (1988) 543–560.

[234] A. Borghese, D. Roest, and I. Zavala, *A Geometric bound on F-term inflation*, *JHEP* **1209** (2012) 021, [arXiv:1203.2909].

[235] R. Kallosh and A. D. Linde, *Landscape, the scale of SUSY breaking, and inflation*, *JHEP* **0412** (2004) 004, [hep-th/0411011].

[236] J. J. Blanco-Pillado, R. Kallosh, and A. D. Linde, *Supersymmetry and stability of flux vacua*, *JHEP* **0605** (2006) 053, [hep-th/0511042].

[237] J. Wishart, *The generalised product moment distribution in samples from a normal multivariate population*, *Biometrika* **20A** (1928) no. 1, 32–52.

[238] M. Johnstone, *On the distribution of the largest eigenvalue in principal components analysis*, *Ann. Statist* **29** (2001) 295–327.

[239] K. Johansson, *Shape Fluctuations and Random Matrices*, *Communications in Mathematical Physics* **209** (2000) 437–476, [math/9903134].

[240] P. Vivo, S. N. Majumdar, and O. Bohigas, *Large deviations of the maximum eigenvalue in Wishart random matrices*, *Journal of Physics A Mathematical General* **40** (2007) 4317–4337, [cond-mat/0701371].

[241] C. Tracy and H. Widom, *The distributions of random matrix theory and their applications*, in *New Trends in Mathematical Physics* (V. Sidoravi?ius, ed.), pp. 753–765. Springer Netherlands, 2009.