Effects of heavy fields on inflationary cosmology
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A

Conventions and useful formulae for Kähler manifolds

In this thesis we work in units of $c = \hbar = 1$. Thus, the Plank mass m_{pl} is simply given by

$$m_{pl}^{-2} \equiv \frac{G}{\hbar c} = G, \qquad \text{(A.0.1)}$$

where G is the Newtons' constant. In general, we will also set the reduced Plank mass M_p to one:

$$M_p^{-2} = 8\pi G = 1, \qquad \text{(A.0.2)}$$

except in the cases where it is more convenient to keep it explicitly for the sake of clarity. Unless specified, the following notation holds throughout the manuscript: the dot ($\dot{}$) denotes derivative with respect to cosmic time t. The prime ($'$) denotes derivative with respect to the argument. **Boldface** denotes a three-dimensional vector. The meaning of the main indices and symbols used along this thesis is summarised in the table A.1.

Here we compute some useful formulae related to the geometry of Kähler manifolds. First we give the expressions of the Christoffel symbols, Riemann tensor, and other useful quantities in terms of the Kähler function G and its derivatives. In A.1 we also give the explicit expressions in the case where there is a supersymmetric sector and a supersymmetry breaking sector, so we differentiate between the supersymmetric and the sGoldstino directions. In A.2 we focus on the case where the Kähler manifold is a direct product of these two sectors, so that $G_{\text{total}} = G_{\text{SUSY}} + G_{\not{\text{SUSY}}}$. In A.3 we rewrite these formulas in terms of the Kähler potential K and the superpotential W, which is especially useful in the case of a vanishing superpotential, since in that case the Kähler function G is ill defined. In

Symbol	Meaning	Chapters
$\mu = 0, 1, 2, 3$	Space-time indices, x^0 being the time coordinate	All
$i = 1, 2, 3$	Spatial indices	All
τ	Conformal time	1-3
ℓ	Multipole, angular scale	1-3
c_s	Speed of sound of the adiabatic mode	1-3
$s = \dot{c}_s / c_s H$	Rate of change of the speed of sound	1-3
X^i	Supersymmetry breaking fields, usually only X	4-5
z^i	Supersymmetry preserving fields	4
$A = (X, i)$	Index running over all scalar fields	4
$b = G^A G_A - 3$	Amount of supersymmetry breaking	4
$3\gamma = G^A G_A - 3$	Amount of supersymmetry breaking	5
$\lambda = 1, \ldots, \mathrm{N}$	Supersymmetric directions	5
m	Chiral fermion mass	5
μ	Eigenvalue of the Hessian of the potential (scalar mass)	5
m_h	Standard deviation of chiral fermion masses (largest mass in the typical spectrum)	5

Table A.1 – *Summary of indices and symbols.*

A.4 we focus on relevant quantities for inflation such as the scalar potential and its derivatives, and in A.5 we take the limit of vanishing superpotential, where we will see that the physical quantities are well defined in the limit $W \to 0$, as it should be. The generic expressions for the Christoffel symbols, the Riemann tensor, and other useful geometrical quantities derived from the Kähler function G are the following:

$$\Gamma^a_{bc} = G^{a\bar{d}}(\partial_b G_{c\bar{d}}) \ , \tag{A.0.3}$$

$$\partial_a G^{b\bar{c}} = -G^{b\bar{d}} G^{e\bar{c}} (\partial_a G_{e\bar{d}}) = -G^{e\bar{c}} \Gamma^b_{ae} \ , \tag{A.0.4}$$

$$\partial_a \Gamma^d_{bc} = G^{d\bar{e}}(\partial_a \partial_b G_{c\bar{e}}) - \Gamma^d_{ae}\Gamma^e_{bc} \ , \tag{A.0.5}$$

$$\nabla_{\bar{a}} \Gamma^d_{bc} = \partial_{\bar{a}} \Gamma^d_{bc} = G^{d\bar{e}}(\partial_{\bar{a}} \partial_b G_{c\bar{e}}) - \Gamma^e_{bc} G^{d\bar{f}}(\partial_{\bar{a}} G_{e\bar{f}}) \ , \tag{A.0.6}$$

$$R_{a\bar{b}c\bar{d}} = R_{c\bar{b}a\bar{d}} = R_{a\bar{d}c\bar{b}} = G_{e\bar{d}}(\partial_{\bar{b}}\Gamma^e_{ac}) = \partial_a \partial_{\bar{b}} G_{c\bar{d}} - \Gamma^e_{ac}(\partial_{\bar{b}} G_{e\bar{d}}) \ , \tag{A.0.7}$$

$$\begin{aligned} \nabla_a \nabla_b G_c = {} & \partial_a \partial_b G_c - G^{\bar{d}}(\partial_a \partial_b G_{c\bar{d}}) - \Gamma^d_{ab}(\partial_c G_d) - \Gamma^d_{bc}(\partial_a G_d) - \Gamma^d_{ac}(\partial_b G_d) \\ & + G_d(\Gamma^e_{ab}\Gamma^d_{ce} + \Gamma^e_{bc}\Gamma^d_{ae} + \Gamma^e_{ac}\Gamma^d_{be}) \ . \end{aligned} \tag{A.0.8}$$

A.1 Supersymmetric and sGoldstino directions

Here we specialise to situations where there is a supersymmetric sector embedded in a theory with supersymmetry breaking, which applies to chapters 4 and 5. Let us assume that the supersymmetry breaking direction is aligned in the X-field (sGoldstino) direction, as in sGoldstino inflation, while all the other fields (z_i) preserve supersymmetry. This translates into the conditions:

$$G_i(X, \bar{X}, z_i^{(0)}, \bar{z}_{\bar{i}}^{(0)}) = 0 \quad , \quad G_X(X, \bar{X}, z_i^{(0)}, \bar{z}_{\bar{i}}^{(0)}) \neq 0 \tag{A.1.1}$$

where $(z_i^{(0)}, \bar{z}_{\bar{i}}^{(0)})$ is the so-called supersymmetric critical point. This means that any term containing one single derivative of G with respect to the z_i fields will vanish. We introduce the index notation (A, i), where A runs over all fields (X, z_i) and i runs only over the z_i fields. As in chapters 4 and 5, we will consider the case where supersymmetric sector is truncated. In this section we compute all the possible elements of the second and third covariant derivatives, Christoffel symbols and Riemann tensor for the particular case described above. The reader should keep in mind that all expressions are evaluated in the supersymmetric critical point.

- Second covariant derivatives:

$$\begin{aligned} \nabla_X G_X &= \partial_X G_X - \Gamma^X_{XX} G_X \\ \nabla_X G_i &= \nabla_i G_X = 0 \\ \nabla_i G_j &= \partial_i G_j - \Gamma^X_{ij} G_X \end{aligned} \tag{A.1.2}$$

- Christoffel symbols:

$$\begin{aligned} \Gamma^X_{XX} &= G^{X\bar{A}}(\partial_X G_{X\bar{A}}) = G^{X\bar{X}}(\partial_X G_{X\bar{X}}) \\ \Gamma^i_{XX} &= G^{i\bar{A}}(\partial_X G_{X\bar{A}}) = 0 \\ \Gamma^X_{Xi} &= G^{X\bar{A}}(\partial_X G_{i\bar{A}}) = 0 \\ \Gamma^i_{Xj} &= G^{i\bar{A}}(\partial_X G_{j\bar{A}}) = G^{i\bar{k}}(\partial_X G_{j\bar{k}}) \\ \Gamma^X_{ij} &= G^{X\bar{A}}(\partial_i G_{j\bar{A}}) = G^{X\bar{X}}(\partial_i G_{j\bar{X}}) \\ \Gamma^k_{ij} &= G^{k\bar{A}}(\partial_i G_{j\bar{A}}) = G^{k\bar{l}}(\partial_i G_{j\bar{l}}) \end{aligned} \tag{A.1.3}$$

Note that the Christoffel symbols with a single spectator index vanish.

- Third covariant derivatives:

$$\begin{aligned}
\nabla_X\nabla_X G_X &= \partial_X\partial_X G_X - G^{\bar{X}}(\partial_X\partial_X G_{X\bar{X}}) - 3\Gamma^X_{XX}(\partial_X G_X) + 3G_X(\Gamma^X_{XX})^2 \\
\nabla_X\nabla_X G_i &= \nabla_X\nabla_i G_X = \nabla_i\nabla_X G_X = 0 \\
\nabla_X\nabla_i G_j &= \partial_X\partial_i G_j - G^{\bar{X}}(\partial_X\partial_i G_{j\bar{X}}) - \Gamma^k_{Xi}(\partial_j G_k) - \Gamma^X_{ij}(\partial_X G_X) - \Gamma^k_{Xj}(\partial_i G_k) \\
&\quad + G_X(\Gamma^k_{Xi}\Gamma^X_{jk} + \Gamma^X_{ij}\Gamma^X_{XX} + \Gamma^k_{Xj}\Gamma^X_{ik}) \\
\nabla_i\nabla_j G_k &= \partial_i\partial_j G_k - G^{\bar{X}}(\partial_i\partial_j G_{k\bar{X}}) - \Gamma^l_{ij}(\partial_k G_l) - \Gamma^l_{jk}(\partial_i G_l) - \Gamma^l_{ik}(\partial_j G_l) \\
&\quad + G_X(\Gamma^l_{ij}\Gamma^X_{kl} + \Gamma^l_{jk}\Gamma^X_{il} + \Gamma^l_{ik}\Gamma^X_{jl})
\end{aligned} \tag{A.1.4}$$

- Riemann tensor:

$$\begin{aligned}
R_{X\bar{X}X\bar{X}} &= \partial_X\partial_{\bar{X}} G_{X\bar{X}} - \Gamma^X_{XX}(\partial_{\bar{X}} G_{X\bar{X}}) \\
R_{X\bar{X}X\bar{i}} &= R_{X\bar{X}i\bar{X}} = R_{X\bar{i}X\bar{X}} = R_{i\bar{X}X\bar{X}} = 0 \\
R_{X\bar{X}i\bar{j}} &= R_{X\bar{j}i\bar{X}} = R_{i\bar{X}X\bar{j}} = R_{i\bar{j}X\bar{X}} = \partial_X\partial_{\bar{X}} G_{i\bar{j}} - \Gamma^k_{Xi}(\partial_{\bar{X}} G_{k\bar{j}}) \\
R_{X\bar{i}j\bar{k}} &= R_{j\bar{i}X\bar{k}} = R_{X\bar{k}j\bar{i}} = R_{j\bar{k}X\bar{i}} = \partial_X\partial_{\bar{i}} G_{j\bar{k}} - \Gamma^l_{Xj}(\partial_{\bar{i}} G_{l\bar{k}}) \\
R_{i\bar{X}j\bar{k}} &= R_{j\bar{X}i\bar{k}} = R_{i\bar{k}j\bar{X}} = R_{j\bar{k}i\bar{X}} = \partial_i\partial_{\bar{X}} G_{j\bar{k}} - \Gamma^l_{ij}(\partial_{\bar{X}} G_{l\bar{k}}) \\
R_{i\bar{j}k\bar{l}} &= R_{k\bar{j}i\bar{l}} = R_{i\bar{l}k\bar{j}} = R_{k\bar{l}i\bar{j}} = \partial_i\partial_{\bar{j}} G_{k\bar{l}} - \Gamma^m_{ik}(\partial_{\bar{j}} G_{m\bar{l}}) - \Gamma^X_{ik}(\partial_{\bar{j}} G_{X\bar{l}})
\end{aligned} \tag{A.1.5}$$

A.2 Separable Kähler function $G_{\textbf{total}} = G_{\textbf{SUSY}} + G_{\not{\textbf{SUSY}}}$

When the Kähler function is of the separable form

$$G(X,\bar{X},z_i,\bar{z_{\bar{i}}}) = g(X,\bar{X}) + \tilde{g}(z_i,\bar{z_{\bar{i}}})\ , \tag{A.2.1}$$

is clear that all mixed derivatives vanish in every point. Furthermore, we impose the condition (A.1.1) at the supersymmetric critical point. This leads to a enormous simplification of our equations. We rewrite (A.1.2)-(A.1.5) for this simple case:

- Second covariant derivatives:

$$\begin{aligned}
\nabla_X G_X &= \partial_X G_X - \Gamma^X_{XX} G_X \\
\nabla_X G_i &= \nabla_i G_X = 0 \\
\nabla_i G_j &= \partial_i G_j
\end{aligned} \tag{A.2.2}$$

• Christoffel symbols:

$$
\begin{aligned}
\Gamma^X_{XX} &= G^{X\bar{X}}(\partial_X G_{X\bar{X}}) \\
\Gamma^i_{XX} &= \Gamma^X_{Xi} = \Gamma^i_{Xj} = \Gamma^X_{ij} = 0 \\
\Gamma^k_{ij} &= G^{k\bar{l}}(\partial_i G_{j\bar{l}})
\end{aligned}
\tag{A.2.3}
$$

• Third covariant derivatives:

$$
\begin{aligned}
\nabla_X\nabla_X G_X &= \partial_X\partial_X G_X - G^{\bar{X}}(\partial_X\partial_X G_{X\bar{X}}) - 3\Gamma^X_{XX}(\partial_X G_X) + 3G_X(\Gamma^X_{XX})^2 \\
\nabla_X\nabla_X G_i &= \nabla_X\nabla_i G_j = 0 \\
\nabla_i\nabla_j G_k &= \partial_i\partial_j G_k - \Gamma^l_{ij}(\partial_k G_l) - \Gamma^l_{jk}(\partial_i G_l) - \Gamma^l_{ik}(\partial_j G_l)
\end{aligned}
\tag{A.2.4}
$$

• Riemann tensor:

$$
\begin{aligned}
R_{X\bar{X}X\bar{X}} &= \partial_X\partial_{\bar{X}} G_{X\bar{X}} - \Gamma^X_{XX}(\partial_{\bar{X}} G_{X\bar{X}}) \\
R_{X\bar{X}X\bar{\imath}} &= R_{X\bar{X}i\bar{X}} = R_{X\bar{\imath}X\bar{X}} = R_{i\bar{X}X\bar{X}} = 0 \\
R_{X\bar{X}i\bar{\jmath}} &= R_{X\bar{\jmath}i\bar{X}} = R_{i\bar{X}X\bar{\jmath}} = R_{i\bar{\jmath}X\bar{X}} = 0 \\
R_{X\bar{\imath}j\bar{k}} &= R_{j\bar{\imath}X\bar{k}} = R_{X\bar{k}j\bar{\imath}} = R_{j\bar{k}X\bar{\imath}} = 0 \\
R_{i\bar{X}j\bar{k}} &= R_{j\bar{X}i\bar{k}} = R_{i\bar{k}j\bar{X}} = R_{j\bar{k}i\bar{X}} = 0 \\
R_{i\bar{\jmath}k\bar{l}} &= R_{k\bar{\jmath}i\bar{l}} = R_{i\bar{l}k\bar{\jmath}} = R_{k\bar{l}i\bar{\jmath}} = \partial_i\partial_{\bar{\jmath}} G_{k\bar{l}} - \Gamma^m_{ik}(\partial_{\bar{\jmath}} G_{m\bar{l}})
\end{aligned}
\tag{A.2.5}
$$

A.3 Geometric quantities in terms of K and W

It can be useful to have the expressions obtained along this thesis in terms of the Kähler potential K and the superpotential W, especially if we want to analyse the case where $W = 0$. Is a critical case in the sense that many quantities diverge, but not the physical ones, as we will see. Moreover, some models only work in this case (see [86]), so it convenient to have the expressions displayed above in terms of K and W, since the Kähler function G is not well defined in that case.

We introduce the Kähler covariant derivative $D_a W \equiv W_a + K_a W$. Notice that we are not specifying any sectors, so lower case letters run over all possible values. When we distinguish between supersymmetric and non-supersymmetric sectors, as in section A.1, we will reintroduce the notation with capital letters $A = (X, i)$. The Kähler function, first derivatives, and metric are given by:

$$G = K + \ln |W|^2 \ , \ e^G = e^K |W|^2 \tag{A.3.1}$$

$$G_a = K_a + \frac{1}{W} W_a = \frac{1}{W} D_a W \ , \ G_{\bar{b}} = K_{\bar{b}} + \frac{1}{\bar{W}} \bar{W}_{\bar{b}} = \frac{1}{\bar{W}} D_{\bar{b}} \bar{W} \tag{A.3.2}$$

$$G_{a\bar{b}} = K_{a\bar{b}} \ , \ G^{a\bar{b}} = K^{a\bar{b}} \tag{A.3.3}$$

$$G^{\bar{b}} = G^{a\bar{b}} G_a = K^{a\bar{b}} \left(K_a + \frac{1}{W} W_a \right) \tag{A.3.4}$$

Using the previous equations, we can already rewrite the scalar potential:

$$\begin{aligned} V &= e^K \left(K^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} - 3|W|^2 \right) \\ &= e^K \left(K^{a\bar{b}} \left(W_a + K_a W \right) \left(\bar{W}_{\bar{b}} + K_{\bar{b}} \bar{W} \right) - 3|W|^2 \right) \end{aligned} \tag{A.3.5}$$

The Christoffel symbols and higher derivatives of the Kähler function read:

$$\Gamma^a_{bc} = K^{a\bar{d}} K_{bc\bar{d}} \tag{A.3.6}$$

$$\partial_a G_b = G_{ab} = K_{ab} + \frac{1}{W} W_{ab} - \frac{1}{W^2} W_a W_b \tag{A.3.7}$$

$$\begin{aligned} G_{abc} = K_{abc} &+ \frac{1}{W} W_{abc} - \frac{1}{W^2} \left(W_{ab} W_c + W_{ac} W_b + W_{bc} W_a \right) \\ &+ \frac{2}{W^3} W_a W_b W_c \end{aligned} \tag{A.3.8}$$

Using the above expressions we can compute the second and third covariant derivatives, given by:

$$\nabla_a G_b = K_{ab} + \frac{1}{W} W_{ab} - \frac{1}{W^2} W_a W_b - K^{c\bar{d}} K_{ab\bar{d}} \left(K_c + \frac{1}{W} W_c \right) \tag{A.3.9}$$

$$\begin{aligned} \nabla_a \nabla_b G_c = K_{abc} &+ \frac{1}{W} W_{abc} - \frac{1}{W^2} \left(W_{ab} W_c + W_{ac} W_b + W_{bc} W_a \right) + \frac{2}{W^3} W_a W_b W_c \\ &- K^{e\bar{d}} K_{abc\bar{d}} \left(K_e + \frac{1}{W} W_e \right) - K^{d\bar{e}} K_{ab\bar{e}} \left(K_{cd} + \frac{1}{W} W_{cd} - \frac{1}{W^2} W_c W_d \right) \\ &- K^{d\bar{e}} K_{bc\bar{e}} \left(K_{ad} + \frac{1}{W} W_{ad} - \frac{1}{W^2} W_a W_d \right) \\ &- K^{d\bar{e}} K_{ca\bar{e}} \left(K_{bd} + \frac{1}{W} W_{bd} - \frac{1}{W^2} W_b W_d \right) \\ &+ \left(K_d + \frac{1}{W} W_d \right) K^{e\bar{f}} K^{d\bar{g}} \left(K_{ab\bar{f}} K_{ce\bar{g}} + K_{bc\bar{f}} K_{ae\bar{g}} + K_{ca\bar{f}} K_{be\bar{g}} \right) \end{aligned} \tag{A.3.10}$$

Last, the Riemann tensor is simply written as:

$$R_{a\bar{b}c\bar{d}} = K_{a\bar{b}c\bar{d}} - K^{e\bar{f}} K_{ac\bar{f}} K_{\bar{b}\bar{d}e} \ . \tag{A.3.11}$$

A.4 Physical quantities relevant for inflation

In this section we compute several quantities that are relevant for the inflationary dynamics, such as the scalar potential and the elements of the mass matrix. In this section we will use the index notation $A = (X, i)$, where X represents the sGoldstino and i runs over the spectator fields z_i. First we will write explicitly the scalar potential and its first and second derivatives, and then we evaluate the scalar potential and its first derivative at the supersymmetric critical point, where the spectator sector satisfies $G_i(X, z_0^i) = 0$. These expressions can be used to calculate, for instance, the 'potential' slow-roll parameters, and get an estimate of the viability and amount of inflation for a given model. One can also study the stability of a given model as we did in chapters 4 and 5 by using the elements of the mass matrix. In any case, it is convenient to have the explicit expressions for cases where the superpotential vanishes at one or more points of the inflationary trajectory. The generic lengthy expressions for the scalar potential and its derivatives are the following:

$$V = e^K \left(K^{A\bar{B}} \left(W_A + K_A W \right) \left(\bar{W}_{\bar{B}} + K_{\bar{B}} \bar{W} \right) - 3|W|^2 \right) \,, \tag{A.4.1}$$

$$\begin{aligned} V_A = {} & e^K \left(K_A + \frac{1}{W} W_A \right) \left(K^{B\bar{C}} \left(W_B + K_B W \right) \left(\bar{W}_{\bar{C}} + K_{\bar{C}} \bar{W} \right) - 2|W|^2 \right) \\ & + e^K K^{B\bar{C}} \left(K_{\bar{C}} \bar{W} + \bar{W}_{\bar{C}} \right) \Big(W K_{AB} + W_{AB} \\ & - \frac{1}{W} W_A W_B - K^{D\bar{E}} K_{AB\bar{E}} \left(K_D W + W_D \right) \Big) \,, \end{aligned} \tag{A.4.2}$$

$$\begin{aligned} \nabla_A \nabla_{\bar{B}} V = {} & \left[K_{A\bar{B}} - \left(K_A + \frac{1}{W} W_A \right) \left(K_{\bar{B}} + \frac{1}{\bar{W}} \bar{W}_{\bar{B}} \right) \right] V \\ & + \left(K_A + \frac{1}{W} W_A \right) V_{\bar{B}} + \left(K_{\bar{B}} + \frac{1}{\bar{W}} \bar{W}_{\bar{B}} \right) V_A + e^K |W|^2 K^{C\bar{D}} \\ & \times \left[K_{AC} + \frac{1}{W} W_{AC} - \frac{1}{W^2} W_A W_C - K^{E\bar{F}} K_{AC\bar{F}} \left(K_E + \frac{1}{W} W_E \right) \right] \\ & \times \left[K_{\bar{B}\bar{D}} + \frac{1}{\bar{W}} \bar{W}_{\bar{B}\bar{D}} - \frac{1}{\bar{W}^2} \bar{W}_{\bar{B}} \bar{W}_{\bar{D}} - K^{E\bar{F}} K_{\bar{B}\bar{D}E} \left(K_{\bar{F}} + \frac{1}{\bar{W}} \bar{W}_{\bar{F}} \right) \right] \\ & + e^K |W|^2 \Bigg[\left(-K_{A\bar{B}C\bar{D}} + K^{E\bar{F}} K_{AC\bar{F}} K_{\bar{B}\bar{D}E} \right) K^{C\bar{H}} K^{I\bar{D}} \\ & \times \left(K_{\bar{H}} + \frac{1}{\bar{W}} \bar{W}_{\bar{H}} \right) \left(K_I + \frac{1}{W} W_I \right) + K_{A\bar{B}} \Bigg] \,, \end{aligned}$$

$$
\begin{aligned}
\nabla_A\nabla_B V = &\left[K_{AB} + \frac{1}{W}W_{AB} - \frac{1}{W^2}W_AW_B - K^{E\bar{F}}K_{AB\bar{F}}\left(K_E + \frac{1}{W}W_E\right)\right.\\
&\left. - \left(K_A + \frac{1}{W}W_A\right)\left(K_B + \frac{1}{W}W_B\right)\right]V\\
&+ \left(K_A + \frac{1}{W}W_A\right)V_B + \left(K_B + \frac{1}{W}W_B\right)V_A + 2e^K|W|^2\\
&\times\left[K_{AB} + \frac{1}{W}W_{AB} - \frac{1}{W^2}W_AW_B - K^{E\bar{F}}K_{AB\bar{F}}\left(K_E + \frac{1}{W}W_E\right)\right]\\
&+ e^K|W|^2K^{C\bar{D}}\left(K_{\bar{D}} + \frac{1}{\bar{W}}\bar{W}_{\bar{D}}\right)\\
&\times\left[K_{ABC} + \frac{1}{W}W_{ABC} - \frac{1}{W^2}(W_{AB}W_C + W_{AC}W_B + W_{BC}W_A)\right.\\
&+ \frac{2}{W^3}W_AW_BW_C - K^{E\bar{D}}K_{ABC\bar{D}}\left(K_E + \frac{1}{W}W_E\right)\\
&- K^{D\bar{E}}K_{AB\bar{E}}\left(K_{CD} + \frac{1}{W}W_{CD} - \frac{1}{W^2}W_CW_D\right)\\
&- K^{D\bar{E}}K_{BC\bar{E}}\left(K_{AD} + \frac{1}{W}W_{AD} - \frac{1}{W^2}W_AW_D\right)\\
&- K^{D\bar{E}}K_{CA\bar{E}}\left(K_{BD} + \frac{1}{W}W_{BD} - \frac{1}{W^2}W_BW_D\right) + \left(K_D + \frac{1}{W}W_D\right)\\
&\left.\times K^{E\bar{F}}K^{D\bar{G}}\left(K_{AB\bar{F}}K_{CE\bar{G}} + K_{BC\bar{F}}K_{AE\bar{G}} + K_{CA\bar{F}}K_{BE\bar{G}}\right)\right]
\end{aligned}
$$

When there is a spectator sector sitting on a supersymmetric critical point $z_i^{(0)}$, this configuration is also a critical point of the scalar potential, $V_i\big|_0 = 0$, as it has been extensively reviewed along this thesis. However, this is not the case for the supersymmetry breaking sector. As we have done previously, considering the supersymmetry breaking aligned with the sGoldstino direction X, the scalar potential and its first derivative in the sGoldstino direction will be:

$$
V\big|_0 = e^K\left(K^{X\bar{X}}\left(W_X + K_XW\right)\left(\bar{W}_{\bar{X}} + K_{\bar{X}}\bar{W}\right) - 3|W|^2\right) \tag{A.4.3}
$$

$$
\begin{aligned}
V_X\big|_0 = &\, e^K\left(K_X + \frac{1}{W}W_X\right)\left(K^{X\bar{X}}\left(W_X + K_XW\right)\left(\bar{W}_{\bar{X}} + K_{\bar{X}}\bar{W}\right) - 2|W|^2\right)\ ,\\
&+ e^KK^{X\bar{X}}\left(K_{\bar{X}}\bar{W} + \bar{W}_{\bar{X}}\right)\left(WK_{XX} + W_{XX} - \frac{1}{W}W_XW_X\right.\\
&\left. - K^{X\bar{X}}K_{XX\bar{X}}\left(K_XW + W_X\right)\right)\ .
\end{aligned} \tag{A.4.4}
$$

A.5 Vanishing superpotential

Now we will take the limit $W \to 0$. This does not make sense in sGoldstino inflation, since in that limit we would also have $W_X = 0$, which implies that the field X preserves supersymmetry. Hence, we cannot have sGoldstino inflation when $W = 0$. But this limit might be very interesting to analyse models for which the superpotential vanishes during inflation. Notice that once we fix the superpotential, we are fixing (some of) the fields. If the superpotentials of different sectors are combined by multiplication, the dynamical sector can be used to stabilise the inflationary trajectory, and this is precisely the functional freedom claimed in [86]. We assume that supersymmetry is broken in at least one of the sectors, which is necessary in order to achieve inflation or stable dS vacua.

Then, in the special case of $W \to 0$ the divergent terms of the expressions in section A.4 cancel with each other. The expressions for the scalar potential and its first derivative in this case read:

$$V \xrightarrow{W\to 0} e^K K^{A\bar{B}} W_A \bar{W}_{\bar{B}} \ , \tag{A.5.1}$$

$$V_A \xrightarrow{W\to 0} e^K K^{B\bar{C}} \bar{W}_{\bar{C}} \left[W_A K_B + K_A W_B + W_{AB} - K^{D\bar{E}} K_{AB\bar{E}} W_D \right] \ . \tag{A.5.2}$$

$\mathcal{B}$

Small spectral index for inflection point inflation

In this appendix we derive the spectral index and power spectrum for inflection point inflation, following the work of Refs. [197, 198]. To a very good approximation the inflationary observables only depend on the η-parameter at the extremum and on the number of efolds.

Expanding the potential around the inflection point gives:

$$V = V_0(1 + 1/2\eta_0\phi^2 + C_3\phi^3 + C_4\phi^4 + ...), \tag{B.0.1}$$

with $\eta, C_3 < 0$ so that the field rolls towards the minimum at positive ϕ values. Inflation ends when the C_3 term becomes important, and $\epsilon \approx 1$, which occurs for field values $\phi_f^2 \sim \sqrt{2}/(3|C_3|)$. We can calculate the number of efolds

$$N \approx \int_{\phi_f}^{\phi_N} \frac{V}{V'} = \frac{1}{\eta} \log\left[\frac{\phi}{3C_3\phi + \eta}\right]_{\phi_f}^{\phi_N}, \tag{B.0.2}$$

where we used $V \approx V_0$ above. The above expression can be inverted to obtain the value of the inflaton field N efolds before the end of inflation ϕ_N:

$$\phi_N = \frac{e^{N\eta_0}\eta_0/C3}{-3(e^{N\eta_0} - 1) - \eta_0/(\phi_f C_3)} \approx \frac{e^{N\eta_0}\eta_0}{-3C_3(e^{N\eta_0} - 1)}, \tag{B.0.3}$$

where in the second step we used $\eta_0/(\phi_f|C_3|) \ll 1$. This is a good approximation as $\eta_0 \ll 1$ is fine-tuned, whereas C_3, and thus ϕ_f, is naturally of order one[1]. Note

[1] To be precise, $C_3 = \mathcal{O}(1)$ for $\phi_0 \sim 1$. For minima at smaller field values generically C_3 increases, as a sharper turnover of the potential is needed. We do not find valid solutions for minima for $\phi_0 \gg 1$ much larger, as then other local minima at smaller field values appear.

that in this limit, the number of efolds is independent of the end of inflation, as ϕ_f has dropped out of the equation. As a result the inflationary observables are insensitive to the precise coefficients of the higher order terms in (B.0.1). The spectral index is

$$n_s \approx 1 + 2\eta \approx 1 + 2\eta_0 + 12C_3\phi_N \approx 1 - 2\eta_0 \frac{(e^{\eta_0 N} + 1)}{(e^{\eta_0 N} - 1)}, \tag{B.0.4}$$

where we used that $\epsilon \ll \eta$. For $N < 50 - 60$ one finds $n_s < 0.92 - 0.93$ for the whole range of $|\eta_0| \lesssim 10^{-2}$. The power spectrum is

$$P_\zeta = \frac{V}{150\pi^2\epsilon} = \frac{3C_3^2 e^{-4N\eta_0}(e^{N\eta_0} - 1)^4 V_0}{25\pi^2\eta_0^4} \tag{B.0.5}$$

with $P_\zeta = 4 \times 10^{-10}$ measured by WMAP.

For the first example (4.3.12) in the text $\eta_0 = 0$ and $C_3 = -2.39$. For $\eta_0 = 0$, the expressions simplify to

$$n_s - 1 = -\frac{4}{N}, \quad P_\zeta = \frac{3C_3^2 N^4 V_0}{25\pi^2}, \quad (\text{for } \eta_0 = 0). \tag{B.0.6}$$

Choosing $N = 50$ this gives $n_s = 0.92$ and $V_0 = 9 \times 10^{-16}$. The second example (4.3.13) has $C_3 = -3.69$, and gives the same spectral index and similar $V_0 = 4 \times 10^{-16}$. The gravitino mass today is related to the inflationary scale via $m_{3/2} = e^{K/2}W|_{\min} \sim 10^2\sqrt{V_0} \sim 10^{-7}$, far above the electroweak scale.

C

Mass spectrum for quasi-separable Kähler functions

In this appendix we derive in full detail the result in (5.3.8), which refers to the eigenvalues of the mass matrix for Kähler functions with small coupling between the heavy and light sectors. On our way, we will also derive the result (5.3.15) for separable Kähler functions. We will briefly review eigenvalue perturbation theory and afterwards we will use this to calculate the perturbed eigenvalues for a Kähler function with a small mixing between sectors.

Perturbation theory

Consider a $n \times n$ square matrix $\mathcal{H} = \mathcal{H}_0 + \delta\mathcal{H}$, where the elements of $\delta\mathcal{H}$ are much smaller than those of $\mathcal{H}_0$. Let us denote by $\lambda_{0,i}$ the eigenvalues of $\mathcal{H}_0$, where $i = 1, \ldots, n$. The eigenvectors corresponding to those eigenvalues form a orthonormal basis with which one can build the matrix A that diagonalises $\mathcal{H}_0$, that is, $A^\dagger \mathcal{H}_0 A = \mathrm{diag}(\lambda_{0,1}, \ldots, \lambda_{0,n})$.

Then, to first order in perturbation theory, the eigenvalues of the full matrix $\mathcal{H}$ will be given by

$$\lambda_i = \left(A^\dagger \mathcal{H} A\right)_{ii} = \lambda_{0,i} + \left(A^\dagger \delta\mathcal{H} A\right)_{ii} \ , \quad i = 1, \ldots, n \ . \tag{C.0.1}$$

In other words, the perturbation over the 'bare' eigenvalues is given by the diagonal elements of the matrix perturbation in the basis that diagonalises the 'bare' matrix.

Perturbed eigenvalues

Let us consider a Kähler function with a small interaction term:

$$G(H, \bar{H}, L, \bar{L}) = A(H, \bar{H}) + B(L, \bar{L}) + \epsilon\, G_{\rm int}(H, \bar{H}, L, \bar{L}) \ , \tag{C.0.2}$$

where the heavy fields H^α are consistently truncated at the supersymmetric critical point such that[1]

$$A_\alpha|_{H_0} = G_{\text{int},\alpha}|_{H_0} = 0 \; . \tag{C.0.3}$$

We take the Hessian matrix in section 5.6.3, given by

$$\mathcal{H} = \begin{pmatrix} \nabla_\alpha \nabla_{\bar\beta} V & \nabla_{\bar\alpha} \nabla_{\bar\beta} V \\ \nabla_\alpha \nabla_\beta V & \nabla_{\bar\alpha} \nabla_\beta V \end{pmatrix} \; , \tag{C.0.4}$$

where the elements are

$$\begin{aligned} \nabla_\alpha \nabla_\beta V|_{H_0,L_0} &= \mathrm{e}^G \left[(3\gamma+2)\nabla_\alpha G_\beta + G^i\, \nabla_i(\nabla_\alpha G_\beta)\right] \; , \qquad \text{(C.0.5)} \\ \nabla_\alpha \nabla_{\bar\beta} V|_{H_0,L_0} &= \mathrm{e}^G \left[\delta_{\alpha\bar\beta}\,(3\gamma+1) + \delta^{\gamma\bar\sigma}\,(\nabla_\gamma G_\alpha)\left(\nabla_{\bar\sigma} G_{\bar\beta}\right) - R_{i\bar{j}\alpha\bar\beta}\, G^i G^{\bar{j}}\right] \; . \end{aligned}$$

We define the following quantities

$$\begin{aligned} M_{\alpha\beta} &\equiv \nabla_\alpha G_\beta = M_{\beta\alpha} \; , & \text{(C.0.6)} \\ Y_{\alpha\beta} &\equiv G^i\, \nabla_i\,(\nabla_\alpha G_\beta) = Y_{\beta\alpha} \; , & \text{(C.0.7)} \\ \Omega_{\alpha\beta} &\equiv -R_{i\bar{j}\alpha\bar\beta}\, G^i G^{\bar{j}} = \Omega^*_{\beta\alpha} \; . & \text{(C.0.8)} \end{aligned}$$

One can recast the mass matrix $\mathcal{H}$ in terms of the quantities above as follows:

$$\mathcal{H} = \mathcal{H}_0 + \delta\mathcal{H} \tag{C.0.9}$$

$$= \begin{pmatrix} \mathrm{e}^G[(3\gamma+1)\mathbb{I} + MM^*] & \mathrm{e}^G(3\gamma+2)M^* \\ \mathrm{e}^G(3\gamma+2)M & \mathrm{e}^G[(3\gamma+1)\mathbb{I} + M^*M] \end{pmatrix} + \begin{pmatrix} \mathrm{e}^G\Omega & \mathrm{e}^G Y^* \\ \mathrm{e}^G Y & \mathrm{e}^G\Omega^T \end{pmatrix} \; ,$$

where the terms in $\delta\mathcal{H}$ are at least $\mathcal{O}(\epsilon)$. It is possible to perform a transformation of the fields such that

$$M \to \tilde{M} = UMU^t \; , \tag{C.0.10}$$

where U is a unitary matrix. Thanks to the symmetry properties of M, we can easily see that MM^* is hermitian and hence it can be diagonalised by a unitary transformation. In fact, given the transformation of M, it follows that

$$\tilde{M}\tilde{M}^* = U(MM^*)U^\dagger = \mathrm{diag}\left(|\lambda_1|^2\, \mathbb{I}_{n_1}, \ldots, |\lambda_p|^2\, \mathbb{I}_{n_p}\right) = \tilde{M}^*\tilde{M} \; , \tag{C.0.11}$$

where n_p is the degeneracy of the p^{th} eigenvalue. A direct consequence of the above is that $[\tilde{M}, \tilde{M}\tilde{M}^*] = 0$, which means that $\tilde{M}$ is block diagonal in the subspaces of dimension n_p. We will denote each of those matrices by $\tilde{M}_p$, satisfying

$$\tilde{M}_p\tilde{M}_p^* = |\lambda_p|^2\, \mathbb{I}_{n_p} \; . \tag{C.0.12}$$

[1] Since we want to impose this condition for any value of ϵ, both functions A and G_{int} must satisfy this requirement.

Now, since $\tilde{M}_p$ is complex and symmetric, we can always rewrite it using Takagi's factorisation, i.e. $\tilde{M}_p = V_p D_p V_p^t$, where V_p is unitary and D_p is diagonal and contains the non-negative square roots of the eigenvalues of $\tilde{M}_p \tilde{M}_p^\dagger$. Therefore, we may write $\tilde{M}_p = V_p V_p^t |\lambda_p|$.

Given this, let us transform the fields once more, in such a way that the resulting transformation of $\tilde{M}_p$ is the following:

$$\tilde{M}_p \to M'_p = V_p^\dagger \tilde{M}_p V_p^* = |\lambda_p| \, \mathbb{I}_{n_p} \ . \tag{C.0.13}$$

After this, the unperturbed mass matrix $\mathcal{H}_0$ has been rewritten in a new basis as $\mathcal{H}'_0$ and it has four blocks of size $n_1 + \cdots + n_p$ each, which are diagonal:

$$\mathcal{H}'_0 = \mathrm{e}^G \left(\begin{array}{cc|cc} \left[(3\gamma+1)+|\lambda_1|^2\right]\mathbb{I}_{n_1} & \mathbf{0} & (3\gamma+2)|\lambda_1|\mathbb{I}_{n_1} & \mathbf{0} \\ & \ddots & & \ddots \\ \mathbf{0} & \left[(3\gamma+1)+|\lambda_p|^2\right]\mathbb{I}_{n_p} & \mathbf{0} & (3\gamma+2)|\lambda_p|\mathbb{I}_{n_p} \\ \hline (3\gamma+2)|\lambda_1|\mathbb{I}_{n_1} & \mathbf{0} & \left[(3\gamma+1)+|\lambda_1|^2\right]\mathbb{I}_{n_1} & \mathbf{0} \\ & \ddots & & \ddots \\ \mathbf{0} & (3\gamma+2)|\lambda_p|\mathbb{I}_{n_p} & \mathbf{0} & \left[(3\gamma+1)+|\lambda_p|^2\right]\mathbb{I}_{n_p} \end{array} \right) . \tag{C.0.14}$$

We can always solve the eigenvalue problem by rearranging rows and columns to make the mass matrix block diagonal, with blocks of dimension $2n_1, \ldots, 2n_p$ given by the matrices

$$\mathcal{H}'^{(p)}_0 = \mathrm{e}^G \begin{pmatrix} \left(|\lambda_p|^2+3\gamma+1\right)\mathbb{I}_{n_p} & (3\gamma+2)|\lambda_p|\,\mathbb{I}_{n_p} \\ (3\gamma+2)|\lambda_p|\,\mathbb{I}_{n_p} & \left(|\lambda_p|^2+3\gamma+1\right)\mathbb{I}_{n_p} \end{pmatrix} \ ; \ \mathcal{H}'_0 = \begin{pmatrix} \mathcal{H}'^{(1)}_0 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathcal{H}'^{(p)}_0 \end{pmatrix} . \tag{C.0.15}$$

Although the previous step is not strictly necessary, it makes the eigenvalue problem more visual to solve it for each subspace. We easily find the eigenvalues, which have degeneracy n_p each, and are given by:

$$m^2_{p\pm} = \mathrm{e}^G \left[|\lambda_p|^2 + (3\gamma+1) \pm |\lambda_p|(3\gamma+2)\right] = \mathrm{e}^G \left[\left(|\lambda_p| \pm \tfrac{1}{2}(3\gamma+2)\right)^2 - \tfrac{9}{4}\gamma^2\right] . \tag{C.0.16}$$

This is the result displayed in (5.3.15). The eigenvectors are easily found through the equation:

$$\begin{pmatrix} \mp(3\gamma+2)|\lambda_p|\,\mathbb{I}_{n_p} & (3\gamma+2)|\lambda_p|\,\mathbb{I}_{n_p} \\ (3\gamma+2)|\lambda_p|\,\mathbb{I}_{n_p} & \mp(3\gamma+2)|\lambda_p|\,\mathbb{I}_{n_p} \end{pmatrix} \begin{pmatrix} \vec{a} \\ \vec{b} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \vec{u}_\pm = \frac{1}{\sqrt{2}|\vec{a}|} \begin{pmatrix} \vec{a} \\ \pm\vec{a} \end{pmatrix} . \tag{C.0.17}$$

Since they are n_p times degenerate, we can choose n_p linearly independent vectors with a 1 in the p^{th} position and 0 in all the others. The matrix of change of basis is then:

$$A_p = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbb{I}_{n_p} & \mathbb{I}_{n_p} \\ \mathbb{I}_{n_p} & -\mathbb{I}_{n_p} \end{pmatrix} \Rightarrow A_p^\dagger \mathcal{H}'^{(p)}_0 A_p = \mathrm{diag}\left(m^2_{p+}\mathbb{I}_{n_p}, m^2_{p-}\mathbb{I}_{n_p}\right) \ . \tag{C.0.18}$$

We can repeat the process for every block, which leads to our final result for the unperturbed mass matrix:

$$A = \begin{pmatrix} A_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & A_p \end{pmatrix} \quad \Rightarrow \quad A^\dagger \mathcal{H'}_0 A = \begin{pmatrix} m^2_{1+}\mathbb{I}_{n_1} & & & & \mathbf{0} \\ & m^2_{1-}\mathbb{I}_{n_1} & & & \\ & & \ddots & & \\ & & & m^2_{p+}\mathbb{I}_{n_p} & \\ \mathbf{0} & & & & m^2_{p-}\mathbb{I}_{n_p} \end{pmatrix} . \tag{C.0.19}$$

We retrieve the results of [3, 83, 84]. Now we just have to express the perturbed matrix $\delta\mathcal{H}'$ in the basis that diagonalises $\mathcal{H}'_0$. In order to do that, we first have to undo the rearranging of rows and columns we did to get to (C.0.15) (which was only done to facilitate the discussion). Instead of rearranging, it is easy to realise that the matrix that diagonalises $\mathcal{H}'_0$ (C.0.14) is

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbb{I}_{n_1+\cdots+n_p} & \mathbb{I}_{n_1+\cdots+n_p} \\ \mathbb{I}_{n_1+\cdots+n_p} & -\mathbb{I}_{n_1+\cdots+n_p} \end{pmatrix} \Rightarrow \tag{C.0.20}$$

$$\Rightarrow A^\dagger \mathcal{H'}_0 A = \operatorname{diag}\left(m^2_{1+}\mathbb{I}_{n_1}, \ldots, m^2_{p+}\mathbb{I}_{n_p}, m^2_{1-}\mathbb{I}_{n_1}, \ldots, m^2_{p-}\mathbb{I}_{n_p}\right) \tag{C.0.21}$$

Therefore, the leading order correction to the eigenvalues (C.0.16) due to $\delta\mathcal{H}$ in (C.0.9) is given by the diagonal elements of the matrix

$$A^\dagger \delta\mathcal{H}\, A = \frac{1}{2} \begin{pmatrix} \mathrm{e}^G\left[(\Omega+\Omega^T)+(Y+Y^*)\right] & \mathrm{e}^G\left[(\Omega-\Omega^T)+(Y-Y^*)\right] \\ \mathrm{e}^G\left[(\Omega-\Omega^T)-(Y-Y^*)\right] & \mathrm{e}^G\left[(\Omega+\Omega^T)-(Y+Y^*)\right] \end{pmatrix} , \tag{C.0.22}$$

where Ω and Y have been already transformed according to (C.0.10) and (C.0.13). To first order in perturbation theory, the eigenvalues then read

$$m^2_{p\pm} + \delta m^2_{p\pm} = \mathrm{e}^G \left\{ |\lambda_p|^2 + (3\gamma+1) + \omega_p \pm \left[|\lambda_p|(3\gamma+2) + y_p\right] \right\} , \tag{C.0.23}$$

where $\omega_p \equiv \Omega_{pp} = -R_{i\bar{j}p\bar{p}} G^i G^{\bar{j}}$ and $y_p \equiv \mathrm{Re}(Y_{pp}) = \mathrm{Re}(G^i \nabla_i (\nabla_p G_p))$. This result was derived in [3] for the simplest case of one light field and one heavy field[2]. We emphasise that the quantities y_p and ω_p are $\mathcal{O}(\epsilon)$ plus subleading corrections.

The matrix Y is proportional to the derivative of the fermion mass matrix along the sGoldstino direction. Thus, in the basis that diagonalises M, it is possible to show that $\tilde{Y} \equiv V^\dagger Y V^*$ has the following form:

$$\tilde{Y} = V(G^i\partial_i X)V^t = G^i \partial_i D + G^i \left(V^\dagger \partial_i V D - D\, V^t \partial_i V^*\right), \tag{C.0.24}$$

where the unitary matrix V and the diagonal matrix D are the ones appearing in the Takagi's factorisation of $M = VDV^t$. Due to the unitarity of V the

[2]The notation in [3] is slightly different, it corresponds to $\gamma \to b/3$.

matrix $V^\dagger \partial_i V$ is anti-hermitian, and therefore its diagonal elements have to be purely imaginary $(V^\dagger \partial_i V)_{pp} = \mathrm{i}\theta_p$, with $\theta_p \in \mathbb{R}$. Then, in this basis, the diagonal elements of Y read

$$\tilde{Y}_{pp} = G^i \left(\partial_i |\lambda_p| + \mathrm{i}2|\lambda_p|\theta_p\right), \qquad \Longrightarrow \qquad y_p = G^i \partial_i |\lambda_p|, \tag{C.0.25}$$

implying that the perturbation y_p is just proportional to the derivative of the eigenvalues of the matrix M along the sGoldstino direction. In order to reduce the dependence on $|\lambda_p|$ of the perturbation parameters appearing in the Hessian, it is convenient to write it in terms of $\tilde{y}_p \equiv y_p/|\lambda_p|$, which gives

$$\tilde{y}_p = G^i \partial_i \log(|\lambda_p|). \tag{C.0.26}$$

$\mathcal{D}$

Random matrix theory: atypical minima and fluctuated spectra

In this appendix we review the expressions for the probability of occurrence of atypical fluctuations of the fermionic mass spectra, and in particular we will discuss the probability distribution of the lightest and largest fermion masses. As we have discussed in the main text, the CI-ensemble describes the statistical properties of the fermion mass matrix $\mathcal{M}_h$ for a generic supersymmetric sector. The CI-ensemble is closely related to the set of Wishart ensemble [212] for which there are many results in the literature regarding fluctuated spectra. For this reason we will first discuss known results for the Wishart ensemble, and then we will translate them into properties of the fermion mass spectrum in a generic supersymmetric sector.

D.1 Typical spectral density in the Wishart and CI-ensembles

The Wishart ensemble is composed of matrices of the form $\mathcal{W} = AA^\dagger$, where A is an $\textsc{n} \times \textsc{m}$ real or complex matrix, (with $\textsc{m} \geq \textsc{n}$), whose entries are independent and identically distributed (i.i.d.) random variables drawn from a statistical distribution with zero mean and variance σ^2: $A_{IJ} \in \Omega(0, \sigma)$. When $\Omega = N(0, \sigma)$ is a normal distribution, the joint probability distribution for the ordered eigenvalues $\lambda_1 \leq \lambda_2, \ldots, \leq \lambda_\textsc{n}$ is [237]:

$$f(\lambda_1, \ldots, \lambda_\textsc{n}) = \mathcal{C}\ \exp\left(-\frac{\beta}{2}\left(\frac{1}{\sigma^2}\sum_{a=1}^{\textsc{n}}\lambda_a\ - 2\sum_{a<b}^{\textsc{n}}\ln|\lambda_b - \lambda_a| - \xi\ \sum_{a=1}^{\textsc{n}}\ln\lambda_a\right)\right)\,, \tag{D.1.1}$$

where $\xi = \text{M} - \text{N} + 1 - 2/\beta$, and $\beta = 1, 2$ for real and complex matrices, respectively. The eigenvalue density function for the eigenvalues of $\mathcal{W}$ is given by the Marčenko-Pastur law [231],

$$\rho_{\text{MP}}(\lambda)\, d\lambda = \frac{1}{2\pi\sigma^2\lambda}\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}\, d\lambda\,, \tag{D.1.2}$$

with support $\lambda \in [\lambda_-, \lambda_+]$, where

$$\lambda_\pm = \text{N}\sigma^2(1 \pm \sqrt{\eta})^2\,, \qquad \text{and} \qquad \eta = \text{M}/\text{N} \geq 1. \tag{D.1.3}$$

The joint probability distribution for the eigenvalues of a matrix from the CI-ensemble was given in eq. (5.4.8). As was pointed out in [212], the p.d.f. of the eigenvalues of a Wishart matrix (D.1.1) reduces to (5.4.8) for $\beta = 1$ and $\text{M} = \text{N}+1$ after doing the identification $\lambda_a \leftrightarrow m_\lambda^2$. Moreover, as we are interested in results to leading leading order in $1/\text{N}$, it will be sufficient to discuss square Wishart matrices $\text{N} \approx \text{M}$. Thus, since the fermion mass matrix of a generic supergravity theory can be identified with an element of the CI ensemble, the typical spectral density of the fermion masses m_λ is also given by the Marčenko-Pastur law (D.1.2) with $\lambda = m^2$. Defining $m_h \equiv 2\sqrt{\text{N}}\sigma$, we have that to leading order in $1/\text{N}$ the fermion mass density function reads:

$$\rho_{\text{MP}}(m^2)dm^2 = \frac{2\text{N}}{\pi m_h^2 m}\sqrt{m_h^2 - m^2}\; dm^2, \tag{D.1.4}$$

which has support in $m^2 \in \left[0, m_h^2\right]$. In the limit $\text{N} \to \infty$ the bounds of the support coincide with the expectation value of the smallest and largest fermionic masses squared, m_1^2 and m_N^2 respectively [238]:

$$\mathbb{E}[m_1^2] = 0, \qquad\qquad \mathbb{E}[m_\text{N}^2] \approx m_h^2. \tag{D.1.5}$$

D.2 Probability distributions of the limiting eigenvalues

Let us first discuss the probability distribution of the largest eigenvalue λ_N of a real, almost square Wishart matrix, $\beta = 1$, $\text{M} \approx \text{N}$. The probability distribution of large $\mathcal{O}(\sigma^2\text{N})$ fluctuations of λ_N far to the right and left of its mean value λ_+ was calculated in [239] and [240], respectively, and are given by:

$$\begin{aligned}
t > \lambda_+ : \quad & \lim_{\text{N}\to\infty} \mathbb{P}(\lambda_\text{N} \leq t) \;\approx\; 1 - \left(\sqrt{x+1} + \sqrt{x}\right)^{2\text{N}} \mathrm{e}^{-2\text{N}\sqrt{x(x+1)}}, \\
t < \lambda_+ : \quad & \lim_{\text{N}\to\infty} \mathbb{P}(\lambda_\text{N} \leq t) \;\approx\; \left(\frac{x+1}{\sqrt{\mathrm{e}}}\right)^{\frac{\text{N}^2}{2}} \mathrm{e}^{-\frac{\text{N}^2}{4}(1-x)^2}\,,
\end{aligned} \tag{D.2.1}$$

where $x \equiv (t - \lambda_+)/\lambda_+$. For large but finite values of N, the maximum value of a Wishart matrix, λ_N, *typically* fluctuates over a region of size $\mathcal{O}(\sigma^2\,\text{N}^{-1/3})$, and the corresponding probability distribution for these small fluctuations can be approximated by the Tracy-Widom distribution $F_1(x)$ [232, 238, 239]:

$$\mathbb{P}(\lambda_{\rm N} \leq t) \approx F_1\left(\frac{\eta^{\frac{1}{12}}\,{\rm N}^{\frac{1}{3}}\,(t-\lambda_+)}{\sigma^{\frac{2}{3}}\lambda_+^{\frac{2}{3}}}\right) \approx F_1\left(2^{\frac{2}{3}}\,{\rm N}^{\frac{2}{3}}\,\frac{(t-\lambda_+)}{\lambda_+}\right)\,, \qquad \text{(D.2.2)}$$

where we have used the leading order approximation $\eta = 1$ and $\lambda_+ \approx 4{\rm N}\sigma^2$ for large N in the last step. For the asymptotic values of the probability (D.2.2), see [241] and references therein. In particular, to leading order in $1/{\rm N}$, the cumulative probability distribution for the largest eigenvalue $\lambda_{\rm N}$ is:

$$\begin{aligned} t > \lambda_+: \quad & \lim_{{\rm N}\to\infty} \mathbb{P}_{\rm N}(\lambda_{\rm N} \leq t) && \approx && 1 - \frac{{\rm e}^{-\frac{4}{3}{\rm N}\,x^{\frac{3}{2}}}}{8\sqrt{\pi}\,{\rm N}\,x^{\frac{3}{2}}} - \frac{{\rm e}^{-\frac{8}{3}{\rm N}\,x^{\frac{3}{2}}}}{64\pi{\rm N}\,x^{\frac{3}{2}}}\,, \\ t < \lambda_+: \quad & \lim_{{\rm N}\to\infty} \mathbb{P}_{\rm N}(\lambda_{\rm N} \leq t) && \approx && \tau_1\,\frac{{\rm e}^{-\frac{1}{6}|x|^3{\rm N}^2}}{2^{\frac{1}{24}}\,{\rm N}^{\frac{1}{24}}\,|x|^{\frac{1}{16}}}\,, \end{aligned} \qquad \text{(D.2.3)}$$

where $\tau_1 \equiv 2^{-11/48}\,{\rm e}^{\frac{1}{2}\zeta'(-1)}$, and $\zeta'(-1) = -0.16542\ldots$ is the derivative of the Riemann zeta function evaluated at -1. It is easy to check that, to leading order in $\mathcal{O}(1/{\rm N})$, the probability distributions in (D.2.1) match the tail behaviour of the Tracy-Widom distributions in the limit $t \to \lambda_+$, which describes small fluctuations.

The probability distribution of the smallest eigenvalue λ_1 of a real square Wishart matrix was derived in [233]. To leading order in $1/{\rm N}$ it is given by:

$$\lim_{{\rm N}\to\infty} \mathbb{P}(\lambda_1 \geq t) \approx \frac{\lambda_+}{4{\rm N}^2}\,{\rm e}^{-\frac{2{\rm N}^2}{\lambda_+}t}. \qquad \text{(D.2.4)}$$

D.3 Probability of atypical field configurations

In chapter 5, where we study the stability of a consistently truncated supersymmetric sector in models with a separable Kähler function, we estimated the probability of occurrence of critical points with light scalar fields, i.e. with a mass $\mu^2|_{min} \leq \alpha^2$, in the regime $m_h < 1-\alpha$. We argued that, due to the relation between scalar and fermion masses, this would require the largest fermion mass fermion to be above its expectation value $m_{\rm N} \geq 1-\alpha > m_h$. Using the first equation in (D.2.1), and taking into account the relation between the Wishart and CI-ensembles, we find

$$1-\alpha > m_h: \quad \lim_{{\rm N}\to\infty} \mathbb{P}(m_{\rm N} \geq 1-\alpha) \approx \left(\sqrt{x+1}+\sqrt{x}\right)^{2{\rm N}}\,{\rm e}^{-2{\rm N}\sqrt{x(x+1)}}, \quad \text{(D.3.1)}$$

with $x = (1-\alpha)^2/m_h^2 - 1$. The Tracy-Widom distribution gives an accurate description in the limit $m_h \to (1-\alpha)^-$, where the deviations of $m_{\rm N}$ from its expectation value are small.

Now we turn to the stability of the truncated sector, when the sector surviving the truncation is driving a period of inflation, and the Kähler function is also separable in the two sectors. In general, in the regime where the mass scale m_h is larger than the gravitino mass ($m_h > 1$), the typical spectrum contains tachyons (see right plot in Fig. 5.5). However, as illustrated by (5.6.7), there is an exponentially suppressed probability that the fermionic spectrum fluctuates in such a way that the scalar spectrum is free of tachyons. There are two possible types of configurations which are non-tachyonic: when the fermion masses are confined to $m_\lambda < 1$, or to $m_\lambda > 3\gamma + 1$, for all λ. It is interesting to check, for a configuration with a Hubble parameter given in terms of γ, in what regime of parameters one of these types of critical points becomes more abundant than the other. The probability that the fermion masses are bounded below as $m_\lambda \geq 3\gamma+1$, can be calculated from (D.2.4):

$$\lim_{\mathrm{N}\to\infty} \mathbb{P}\Big(m_1 \geq 3\gamma + 1\Big) \approx \frac{m_h^2}{4\mathrm{N}^2}\, \mathrm{e}^{-\frac{2\mathrm{N}^2}{m_h^2}(3\gamma+1)^2} \; . \tag{D.3.2}$$

On the other hand, the probability that the fermion masses are bounded above by $m_\mathrm{N} = 1$, also to leading order in $1/\mathrm{N}$, can be derived from the second equation in (D.2.1):

$$\lim_{\mathrm{N}\to\infty} \mathbb{P}\Big(m_\mathrm{N} \leq 1\Big) \approx \frac{\mathrm{e}^{-\mathrm{N}^2/4}}{m_h^{\mathrm{N}^2}}\, \mathrm{e}^{-\frac{\mathrm{N}^2}{4}\left(2-\frac{1}{m_h^2}\right)^2} \; . \tag{D.3.3}$$

From the above expressions it can be seen that a fluctuation to a minimum of the Kähler function, where all $m_\lambda < 1$ (D.3.2), becomes more likely than a fluctuation to large fermionic masses (D.3.3) when the following condition is satisfied:

$$(2+\mathrm{N}^2)\log m_h - 2\log(2\mathrm{N}) - \frac{2\mathrm{N}^2}{m_h^2}(1+3\gamma)^2 + \frac{\mathrm{N}^2}{4} + \frac{\mathrm{N}^2}{4}\left(2 - \frac{1}{m_h^2}\right)^2 < 0 \; . \tag{D.3.4}$$

In the regime $\mathrm{N} \gg m_h \gg 1$ and with γ comparable to m_h the expression above simplifies to

$$\gamma^2 \geq \frac{m_h^2}{18}\,(\log m_h + \tfrac{5}{4}) \; , \tag{D.3.5}$$

which can be rewritten in terms of the Hubble parameter as follows:

$$H^2 \gtrsim \frac{m_h m_{3/2}}{3\sqrt{2}}\sqrt{\log m_h + \frac{5}{4}} \; . \tag{D.3.6}$$

We can see that regardless of the value of the mass scale of the supersymmetric sector m_h, there is always a value of the Hubble parameter (D.3.6) above which the largest fraction of stable critical points corresponds to a minimum of the Kähler function. This is particularly important for cosmological models which involve a large inflationary scale H and a low supersymmetry breaking scale. This result depends strongly on the value of the mass scale of the supersymmetric sector m_h which, as we discussed in section 5.4.2, should also be regarded as a random variable depending on the value of the gravitino mass $m_{3/2}$, and its statistical properties should be derived from a realistic characterisation of the String Theory Landscape.

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