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## Effects of heavy fields on inflationary cosmology

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# Sgoldstino inflation

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In this chapter we continue the discussion on the effect of additional fields during inflation, but in the context of supergravity theories. Also, we will not discuss the explicit integration of heavy fields and their effect from an EFT point of view, but rather the decoupling between different sectors of the theory. In particular, we will discuss the decoupling between supersymmetric and non-supersymmetric sectors for a special type of couplings during the inflationary stage. We will see that even when a heavy sector is decoupled and stable, its influence on the dynamical sector can be translated into constraints. In chapter 5 we will discuss more general couplings and also the possibility of finding stable vacua. Meanwhile, in this chapter we discuss the possibility that inflation is driven by the sgoldstino, the superpartner of the goldstino. Unlike in generic supergravity scenarios, the sgoldstino decouples from all other fields in the theory, which allows for a simple and robust inflationary model. We argue that the two-field model given by this single complex scalar correctly captures the full multi-field inflationary phenomenology, that is, the non-inflationary sector is consistently truncated. On the other hand, the assumption of stability, along the entire inflationary trajectory, of the supersymmetry-preserving sector that is integrated out leads to supplementary constraints on the parent supergravity. We investigate small field, large field and hybrid sgoldstino inflation scenarios and provide some working examples. They are subject to the usual fine-tuning issues that are common to all supergravity models of inflation. We comment on some other proposed sgoldstino inflation models.

## 4.1 Introduction

Scalar fields are abundant in supersymmetric theories. They all couple to each other with at least gravitational strength interactions. Planck-suppressed

couplings are generically unimportant when describing processes at low energies, but such a decoupling does not work for inflation. This can be most easily inferred from the slow roll parameters, which get contributions from dimension five and six operators that are unsuppressed. Describing inflation in a generic supergravity model is thus a challenging task, as generically the scalar field dynamics pose a complicated multi-field problem, as already explained in sections 1.4 and 1.5.

There are good reasons why a single-field description is desirable. In line with Ockham’s razor, it is the simplest model that fits the data. Multi-field slow-roll inflation with several (real) light fields has been studied for over a decade [88–91] (see [92, 96] and references therein), and is very constrained by the observations, in particular through the tight limits on isocurvature modes and non-gaussianity that we have reviewed in section 1.3. However, it is technically challenging to obtain single-field behavior in a full multi-field set-up.

As we have extensively described in the previous chapters, if there are turns in the inflationary trajectory, derivative interactions between the inflaton and the heavy fields can become transiently strongly coupled. These lead to features and non-gaussianity in the spectrum of primordial perturbations that would not be inferred from the naive EFT. Careful integration of the heavy fields recovers the general low energy effective field theory of inflation including a variable speed of sound for the adiabatic perturbations [98–101, 107, 176]. These interactions are unavoidable whenever the potential “valley” provided by the multi-field potential deviates from a geodesic of the multi-field sigma model metric. A corollary from the point of view of inflationary model building is that, when it comes to precision cosmology, the field space geometry of the “spectator” heavy fields (that are supposed to sit at their adiabatic minima during slow-roll inflation) is as important as their masses and couplings inferred from the potential alone.

Among the many scalars in a supersymmetric theory, the sgoldstino field stands out. The sgoldstino is the scalar partner of the goldstino, and belongs to the chiral superfield whose non-zero F-term breaks supersymmetry. It has the special property [82, 102, 177] that it decouples from all other fields in the theory. More precisely, setting all other superfields at the minimum of their potential is a consistent truncation from the  $\mathcal{N} = 1$  supergravity multi-field parent theory to an effective  $\mathcal{N} = 1$  supergravity with a single chiral superfield, the sgoldstino. In particular, the (real, two-dimensional) sgoldstino plane is a geodesically generated surface of the Kähler metric in the parent theory, so there are no derivative interactions with the truncated heavy fields: all turns in the inflationary trajectory are entirely confined to the sgoldstino plane. The effects of the fluctuations of the heavy fields are suppressed by their mass squared exactly as one would expect from an EFT expansion. This makes the sgoldstino an ideal inflaton candidate, for it allows for a description of inflation in terms of

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a single complex field. From the point of view of inflationary modeling this is still multi-field inflation (with two real fields), but this two-field model is not a toy model, it really is the correct effective description for the full multi-field system.

If inflation is effectively driven by a single real scalar field, the inflaton, this can be identified with a suitable linear combination of the real and imaginary parts of the sgoldstino field. Meanwhile, the orthogonal combination is to remain stabilized at a local minimum of the potential during inflation. The effect of turns in the trajectory on the spectrum of primordial perturbations have to be taken into account when comparing to observations, but at least they can be calculated from the two-field model (see [93–95, 103, 178] and references therein).

Needless to say, this does not mean that all other scalars in the theory (from the supersymmetry-preserving superfields) can be completely neglected, as they have to be stabilised in a minimum of the potential during inflation. Even though the sgoldstino decouples from these fields, vice versa this is not true: the masses and couplings of all other fields generically depend on the field value of the sgoldstino field. As during inflation the sgoldstino evolves along its inflationary trajectory, the masses of the scalars change. If the inflaton is the sgoldstino, they will remain at the critical points, but they may become light or even tachyonic, triggering a waterfall-type exit from inflation that is not seen in the two-field model, and that most likely would ruin inflation. Although it is still a complicated task to determine the minimum of the multi-field potential along the inflationary trajectory, it is much simpler than the full multi-field *dynamics* needed for a generic inflationary model in supergravity.

The potential energy density driving inflation breaks supersymmetry spontaneously [179, 180]. This source of supersymmetry breaking in the inflaton sector is always present during inflation, and is in principle independent of the source of vacuum supersymmetry breaking. For sgoldstino inflation there are two possibilities. First, the same superfield that drives inflation is also responsible for low energy supersymmetry breaking<sup>1</sup>. This would be the ideal situation. Not only does inflation decouple from all other fields in the theory, it also links the scale of inflation to the scale of supersymmetry breaking. The second possibility is that the two sources of supersymmetry breaking are due to different fields. Both sources may be operative during inflation, or alternatively, it may be that only after inflation has ended, a phase transition takes place generating our present-day supersymmetry breaking. In both cases the present day sgoldstino field is not the sgoldstino during inflation.

The decoupling of the sgoldstino from the other fields fits in with recent

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<sup>1</sup>This possibility has been discussed in [181–183] but as we will show it is difficult to implement in practice.

work on how to incorporate different fields, or sets of fields, in a supergravity set-up minimizing the coupling between them [82–85, 184–191]. Quite commonly different sectors – e.g. the fields and couplings responsible for supersymmetry breaking, for inflation, for moduli stabilisation, or making up the standard model – are combined by simply adding their respective Kähler- and superpotentials. However, following this procedure one cannot completely decouple these sectors. Even if the Kähler and superpotential do not contain direct interaction terms between fields in different sectors, the resulting scalar potential does. There are always at least Planck-suppressed interactions between the fields, and generically the mass matrix is not block diagonal in the different sectors. This complicates the analysis of the full model enormously. Sectors are affected by the presence of others, and although they work in isolation, they may no longer do so in the full set-up. Moreover, heavy fields generically cannot be integrated out in a consistent supersymmetric way.

The cross-couplings between sectors can be minimised if instead of adding Kähler and superpotentials, one adds the Kähler invariant functions  $G = K + \ln |W|^2$  for the two sectors [83, 192]. This approach allows to integrate out fields in a supersymmetry preserving way [82]. In Ref. [83] the addition of Kähler functions was used to couple a supersymmetry breaking moduli sector (fields  $X^i$ ) to a supersymmetry preserving sector, for example the standard model (fields  $z_i$ ):

$$G^{\text{tot}}(X^i, \bar{X}^{\bar{i}}, z_i, \bar{z}_{\bar{i}}) = g(X^i, \bar{X}^{\bar{i}}) + G^{\text{other}}(z_i, \bar{z}_{\bar{i}}). \quad (4.1.1)$$

In this work we use the same idea to couple a supersymmetry breaking inflationary sector (fields  $X^i$ ) to a supersymmetry preserving sector ( $z_k$ )<sup>2</sup>. For simplicity we restrict to effectively single field inflation, and models with a single inflaton field  $X$ . As supersymmetry is broken during inflation, the inflaton is then the sgoldstino. As it turns out, the ansatz (4.1.1) is actually too strict. We can allow for explicit couplings between the inflaton and the other fields, of the form

$$G(X, \bar{X}, z_k, \bar{z}_{\bar{k}}) = g(X, \bar{X}) + \frac{1}{2} \sum_{i \geq j} \left[ (z_i - (z_i)_0)(z_j - (z_j)_0) f^{(ij)}(X, \bar{X}, z_k, \bar{z}_{\bar{k}}) \right. \\ \left. + (z_i - (z_i)_0)(\bar{z}_{\bar{j}} - (\bar{z}_{\bar{j}})_0) h^{(ij)}(X, \bar{X}, z_k, \bar{z}_{\bar{k}}) + \text{h.c.} \right] \quad (4.1.2)$$

with  $f, h$  arbitrary functions of their arguments. This is the most general ansatz consistent with  $X$  being the sgoldstino. The explicit  $X$ -dependence in the second term does not spoil the decoupling of the inflaton field, because the mass matrix remains block diagonal in the two sectors as long as the fields  $z_i$  sit at the supersymmetric critical point  $(z_i)_0$  during inflation (recall section 1.4.2). As we will show, during sgoldstino inflation the Kähler function  $G$  is well defined,

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<sup>2</sup>In [193] the separable form (4.1.1) was used to combine hybrid inflation with a supersymmetry breaking moduli sector in a successful way. In this set-up the inflaton is not the sgoldstino.

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i.e. the superpotential is non-zero, maybe except from isolated points in field space.

Single field inflation can be divided into three main classes: large field, small field and hybrid inflation. We discuss whether and how sgoldstino inflation might work in these three regimes. Any supergravity model of inflation has to address the  $\eta$ -problem, as explained in section 1.4.1; this puts bounds on the Kähler geometry [194–196], since the spectator sector must remain stabilised in order to keep inflation going. As a summary of our findings, we find for these three different regimes the following:

- Large field sgoldstino inflation does not work, at least not for potentials that grow at most polynomial.
- Hybrid inflation provides the most natural embedding for sgoldstino inflation. Indeed, usual F-term hybrid inflation is an example of having a sgoldstino inflaton. In this set-up supersymmetry is restored in the vacuum, and there is no relation with low energy supersymmetry breaking. More complicated waterfall regimes may be devised, such that supersymmetry is broken in the minimum after inflation. However, such an analysis is multi-field, and complicated multi-field dynamics enters via the back door again.
- Small field inflation offers the best possibility to link inflation to supersymmetry breaking. Naively all that is needed is finding and tuning a saddle or maximum in a single field potential with a supersymmetry breaking Minkowski minimum. We only find inflection points suitable for inflation rather than a maximum or saddle point. Inflection point inflation yields [197, 198] a low spectral index  $n_s \leq 0.92 - 0.93$  (for  $N = 50 - 60$  e-folds), which is ruled out by the CMB data, cf. eq. (1.2.22). However, in principle one may add non-canonical kinetic terms to alter this prediction, at the cost of tuning more coefficients. Interestingly enough, models in which supersymmetry is broken after inflation are much easier to embed in a multi-field set-up than models with a supersymmetry preserving vacuum. Finally, we comment on some claims in the literature for small field sgoldstino inflation [182, 183, 199] with no or very little fine-tuning. We will explain why these models cannot work.

Summarising, in this work we provide a systematic approach in which the additional sector of fields is consistently truncated in a supersymmetric way, and we provide working examples which illustrate the fact that fine-tuning is necessary in generic supergravity models in order to achieve successful inflation. As emphasised along this manuscript, it is of great importance to provide mechanisms that explain the consistent truncation of additional non-inflating degrees of freedom, especially in high-energy theories where their presence is inevitable. Additionally, the requirement of stabilisation imposes further constraints in the parent supergravity, which will be generalised in the next chapter.

## 4.2 Decoupling of the sgoldstino

In this section we show the decoupling of the sgoldstino field explicitly. In the first subsection we derive the mass matrix, which is block diagonal along the sgoldstino inflation trajectory. We will argue in subsection 4.2.2 that the Kähler function for a dynamical sgoldstino field can always be put in the form (4.1.2). In subsection 4.2.3 we show that this sgoldstino trajectory is independent of the field values of all the other fields. Vice versa that is not the case: the dynamics of the supersymmetry-preserving fields does depend on the sgoldstino field. Care must be taken so that these fields remain stabilised along the full inflationary trajectory. Finally, in subsection 4.2.4 we discuss the special limit of separable Kähler functions (4.1.1), which is a non-generic type of coupling, where the results of [83] are retrieved. In the next chapter we will discuss physical frameworks for which quasi-separable Kähler functions naturally arise.

### 4.2.1 Mass matrix

We start with the general formula for the mass matrix, then specialise to sgoldstino inflation. For the sake of clarity, we will repeat some of the formulas and statements established in section 1.4, where the basic notation is outlined, and refer to the reader to that section for conventions, and to appendix A for a translation of quantities in terms of  $G$  to quantities in terms of  $K$  and  $W$ . The scalar potential can be expressed solely in terms of the Kähler function<sup>3</sup>  $G = K + \ln |W|^2$ :

$$V_F = e^G [G_I G^{I\bar{J}} G_{\bar{J}} - 3], \quad (4.2.1)$$

with  $I, J$  running over all fields  $\Phi_I$ . The mass matrix is given by

$$\mathcal{M} = \begin{pmatrix} M_J^I & M_{\bar{J}}^I \\ M_{\bar{J}}^I & M_J^I \end{pmatrix}, \quad M_J^I = G^{I\bar{K}} \nabla_{\bar{K}} \nabla_J V, \quad M_{\bar{J}}^I = G^{I\bar{K}} \nabla_{\bar{K}} \nabla_{\bar{J}} V, \quad (4.2.2)$$

where  $\nabla_K v_L = \partial_K v_L - \Gamma_{KL}^M v_M$  is the covariant derivative of some vector  $v_L$ . The eigenvalues and eigenvectors of the mass matrix correspond to the  $m^2$ -values and mass eigenstates respectively. The first derivative of the potential is

$$V_K = G_K V + e^G [G^I \nabla_K G_I + G_K] \quad (4.2.3)$$

where we used metric compatibility  $\nabla_K G_{I\bar{J}} = 0$ ,  $\nabla_K G^I = \delta_K^I$  and introduced the notation  $V_K = \partial_K V$ ,  $G^I = G^{I\bar{J}} G_{\bar{J}}$ . Stationarity is not assumed, as the inflaton field is displaced from its minimum during inflation. The second derivatives of

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<sup>3</sup>This procedure is ill defined for  $W = 0$ . To cure this, one can use the variable  $\phi \equiv e^G$  instead, which remains well defined [200]. However, in the next section we show that  $W = 0$  at most in isolated points in field space.



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the potential are

$$\begin{aligned} \nabla_{\bar{L}}\nabla_K V &= (G_{K\bar{L}} - G_K G_{\bar{L}})V + 2G_{(K}V_{\bar{L})} \\ &\quad + e^G [G^{I\bar{J}}(\nabla_{\bar{L}}G_{\bar{J}})(\nabla_K G_I) - R_{I\bar{J}K\bar{L}}G^I G^{\bar{J}} + G_{K\bar{L}}] , \end{aligned} \quad (4.2.4)$$

$$\nabla_L\nabla_K V = (\nabla_{(L}G_{K)} - G_{(K}G_{L)})V + 2G_{(K}V_{L)} + e^G [2\nabla_{(K}G_{L)} + G^I\nabla_{(L}\nabla_{K)}G_I] ,$$

where round brackets denote symmetrisation. We used that  $[\nabla_{\bar{L}}, \nabla_K]G_I = \nabla_{\bar{L}}\nabla_K G_I = -R_{K\bar{L}I\bar{J}}G^{\bar{J}}$ .

Now consider F-term breaking of supersymmetry, signalled by a non-zero  $G_X \neq 0$ . Here  $X$  is the scalar component of the chiral superfield which also contains the goldstino. Note that one can always make a field redefinition such that only one linear combination of fields breaks supersymmetry. All other fields in the theory, denoted by  $z_i$  (indexed by lower case latin letters), do not break supersymmetry. Hence, we split the fields in  $\Phi_I = \{X, z_i\}$ , with

$$G_X|_{z_0} \neq 0, \quad G_i|_{z_0} = 0 \quad (4.2.5)$$

at some point in field space  $z_0 = \{(z_1)_0, (z_2)_0, \dots\}$ , the so-called supersymmetric critical point.

We are interested in a cosmological situation, in which  $X(t)$  is the inflaton rolling along some trajectory with  $V_X \neq 0$ . While the inflaton rolls in the  $X$ -direction, we want all orthogonal fields  $z_i$  to remain extremised at  $z_0$ . To that end we demand that

$$(\partial_X)^m (\partial_{\bar{X}})^n G_i|_{z_0} = 0, \quad \forall m, n \in \mathbb{N}. \quad (4.2.6)$$

Indeed, from (4.2.3), we then have that

$$V_i|_{z_0} = G_i V + e^G [G^P \nabla_i G_P + G_i] = e^G G^X \nabla_i G_X = 0. \quad (4.2.7)$$

For notational convenience we dropped the  $|_{z_0}$  on the right hand side, but the reader should keep in mind that all expressions should be evaluated at  $z = z_0$ .

Thus  $z_i = (z_i)_0$  is an extremum of the potential. To see whether this is a maximum, minimum or saddle point, we must calculate the eigenvalues of the mass matrix, which need to be positive definite for a stable minimum. This analysis is simplified because (4.2.5) assures the mass matrix is in block diagonal form, i.e.  $M_i^X|_{z_0} = M_i^{\bar{X}}|_{z_0} = 0$ . To prove this last statement, it is enough to show the block diagonal form of the second covariant derivatives, as it follows from (4.2.6) that the field metric  $G_{I\bar{J}}|_{z_0}$  is block diagonal as well. The first equation in (4.2.4) gives for mixed indices

$$\begin{aligned} \nabla_{\bar{i}}\nabla_X V|_{z_0} &= (G_{X\bar{i}} - G_X G_{\bar{i}})V + 2G_{(X}V_{\bar{i})} \\ &\quad + e^G [G^{K\bar{L}}(\nabla_{\bar{i}}G_{\bar{L}})(\nabla_X G_K) - R_{K\bar{L}X\bar{i}}G^K G^{\bar{L}} + G_{X\bar{i}}] \\ &= -e^G G^X G^{\bar{X}} R_{X\bar{X}X\bar{i}} = 0 . \end{aligned} \quad (4.2.8)$$

In the first step we used (4.2.5, 4.2.6) and that  $\nabla_i G_X|_{z_0} = \nabla_X G_i|_{z_0} = 0$ ; in the second step that  $R_{X\bar{X}X\bar{i}}|_{z_0} = G_{j\bar{i}}\partial_{\bar{X}}\Gamma_{XX}^j = 0$  as well, which also follows from (4.2.6). The second equation in (4.2.4) likewise vanishes for mixed indices:

$$\nabla_i \nabla_X V|_{z_0} = (\nabla_{(i} G_{X)} - G_{(X} G_{i)})V + 2G_{(X} V_{i)} + e^G [2\nabla_{(X} G_{i)} + G^K \nabla_{(i} \nabla_{X)} G_K] = 0. \quad (4.2.9)$$

## 4.2.2 Kähler invariant function for sgoldstino inflation

Let us quickly comment on our use of the Kähler function  $G = K + \ln |W|^2$ , rather than expressing results in terms of the Kähler potential and superpotential. The potential danger in using  $G$  is that it becomes undefined when  $W = 0$ . However, it is easy to show that for sgoldstino inflation we nowhere have  $W = 0$ , except maybe for isolated points in field space. Therefore the Kähler function  $G$  is well defined. To illustrate this, consider a theory with two chiral fields – the extension to many fields is straightforward – with a superpotential  $W = W(X, Z)$ . For sgoldstino inflation, with  $X$  the goldstino superfield, we have

$$D_X W|_{\{X(t), Z_0\}} \neq 0, \quad D_Z W|_{\{X(t), Z_0\}} = 0, \quad (4.2.10)$$

with  $D_X W = K_X W + W_X$  the Kähler covariant derivative. Setting  $W = 0$  along the *whole* trajectory implies

$$W|_{\{X(t), Z_0\}} = 0 \quad \Rightarrow \quad W_X|_{\{X(t), Z_0\}} = 0 \quad \Rightarrow \quad D_X W|_{\{X(t), Z_0\}} = 0 \quad (4.2.11)$$

in contradiction with (4.2.10). Therefore the superpotential can only vanish for sgoldstino inflation at accidental zeroes at isolated points in field space (possibly on the trajectory, but this does not change our conclusions).

As a side remark, note that when the inflaton is identified with the  $Z$  field rather than the sgoldstino field  $X$ , as for example in the models of Ref. [86], it is possible to have  $W = 0$ ,  $D_X W|_{\{X_0, Z(t)\}} \neq 0$  and  $D_Z W|_{\{X_0, Z(t)\}} = 0$  along the whole trajectory  $\{X_0, Z(t)\}$ , as already mentioned in section 1.4. In this case the Kähler invariant function is not well defined, and a description in terms of  $K$  and  $W$  is needed. Despite this, let us stress that the physical quantities, such as the scalar potential and its derivatives, are well defined in any case.

Expanding the Kähler function around the supersymmetry critical point  $z^i = z_0^i$ , the most general form for sgoldstino inflation – satisfying (4.2.5) and (4.2.6) – can be written as in eq. (4.1.2).

## 4.2.3 Inflationary trajectory

We have seen in subsection 4.2.1 that along the inflationary trajectory all supersymmetry preserving fields are extremised at  $z^i = z_0^i$ . Since the mass matrix

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is block diagonal, we can determine the stability of the  $z_i$  extremum from the sub-block of  $\mathcal{M}$  with  $z_i$  indices. It can easily be shown that the inflaton trajectory itself is independent of the field values of the other fields. Indeed, the potential along the inflationary trajectory only depends on the function  $g(X, \bar{X})$  in (4.1.2), and is thus independent of the field values of all other fields. The height  $V_0 \equiv V|_{z_0}$ , slope and second derivatives of the inflaton potential are given by (4.2.1, 4.2.3, 4.2.4) with  $I, J$  only running over  $X$  and  $G \rightarrow g$ . For example we have

$$V_0 = e^g \left[ g_X g^{X\bar{X}} g_{\bar{X}} - 3 \right], \quad (4.2.12)$$

$$V_X|_{z_0} = g_X V_0 + e^g \left[ g^X \nabla_X g_X + g_X \right]. \quad (4.2.13)$$

In contrast, the mass matrix along the orthogonal directions does depend on the inflaton field value. We find

$$\begin{aligned} M_j^i|_{z_0} &= G^{i\bar{k}} \nabla_{\bar{k}} \nabla_j V \\ &= G^{i\bar{k}} \left[ G_{j\bar{k}} V_0 + e^G [G^{l\bar{m}} (\nabla_{\bar{k}} G_{\bar{m}}) (\nabla_j G_l) - R_{X\bar{X}j\bar{k}} G^X G^{\bar{X}} + G_{j\bar{k}}] \right] \\ &= e^g \left[ \delta_j^i (b+1) + x_{\bar{m}}^i x_j^{\bar{m}} + w_j^i \right] \end{aligned} \quad (4.2.14)$$

and

$$\begin{aligned} M_j^{\bar{i}}|_{z_0} &= G^{\bar{i}k} \nabla_k \nabla_j V \\ &= G^{\bar{i}k} \left[ \nabla_{(k} G_{j)} V_0 + e^G [2\nabla_{(j} G_{k)} + G^X \nabla_{(k} \nabla_{j)} G_X] \right] \\ &= e^g \left[ x_j^{\bar{i}} (b+2) + y_j^{\bar{i}} \right]. \end{aligned} \quad (4.2.15)$$

Here we introduced the notation

$$b = V_0 e^{-g} = g_X g^X - 3 \quad (4.2.16)$$

$$x_j^{\bar{i}} = G^{\bar{i}k} \nabla_k G_j = G^{\bar{i}k} \nabla_j G_k \quad (4.2.17)$$

$$w_j^i = -G^{i\bar{k}} G^X G^{\bar{X}} R_{X\bar{X}j\bar{k}} \quad (4.2.18)$$

$$y_j^{\bar{i}} = G^{\bar{i}k} G^X \nabla_{(k} \nabla_{j)} G_X. \quad (4.2.19)$$

Note that  $b = V_0/m_{3/2}^2$  gives the height of the potential in units of the gravitino mass. During slow-roll this is approximately  $b \simeq 3H^2/m_{3/2}^2$ .

The functions  $b, x, y, w$  can be expressed in terms of the functions  $f, g, h$  appearing in the Kähler function (4.1.2). In general, the constraint that the squared masses should be positive is complicated, but there is a situation in which it simplifies considerably. As discussed in the next section, if the Kähler function is separable [83, 84], the matrices  $y$  and  $w$  vanish and the constraint only involves the eigenvalues of the  $x$  matrix. The diagonalisation for separable and quasi-separable Kähler functions is performed in detail in appendix B.

#### 4.2.4 Separable Kähler function

Let us consider a set-up with separable Kähler functions [83–85]:

$$G(X, \bar{X}, z_i, \bar{z}_i) = g(X, \bar{X}) + \tilde{g}(z_i, \bar{z}_i), \quad (4.2.20)$$

For the separable Kähler function above (4.2.20) all mixed derivatives of  $G$ , such as  $G_{zzX}$ , cancel. With this simplification

$$b = g_X g^X - 3, \quad x_m^{\bar{i}} = \tilde{g}^{\bar{i}k} \tilde{g}_{km}, \quad y_j^{\bar{i}} = w_j^i = 0. \quad (4.2.21)$$

We now consider the case with only one  $z$  field, which turns  $x_j^{\bar{i}}$  into a scalar. As one can always diagonalise the matrix  $x$ , this simplification precisely gives the result along one of the eigenvectors, and thus can be straightforwardly generalised to several  $z$  fields. We recover the system studied in [83]<sup>4</sup>:

$$M_z^z|_{z_0} = e^g [(b+1) + |x|^2], \quad M_z^{\bar{z}}|_{z_0} = e^g (b+2)x, \quad (4.2.22)$$

which has eigenvalues<sup>5</sup>

$$m_{\pm}^2|_{z_0} = e^g [1 + b + |x|^2 \pm |(2+b)x|] = e^g \left[ \left( |x| \pm \frac{1}{2}|b+2| \right)^2 - \frac{b^2}{4} \right]. \quad (4.2.23)$$

The function  $b$  is bigger, equal or smaller than zero for a dS, Minkowski or AdS universe, respectively. Take  $b \geq 0$ ; in the opposite limit the masses  $m_-^2$  and  $m_+^2$  are exchanged. The smallest mass eigenstate is positive  $m_-^2 > 0$ , i.e., the  $z$ -field is stabilized along the inflationary trajectory, for  $|x| < 1$  or  $|x| > (1+b)$ . We will put this analysis in practice for sgoldstino inflation in subsection 4.3.2 (hybrid inflation) and 4.3.3 (small field inflation).

Close to the instability bounds  $|x| \lesssim 1$  or  $|x| \gtrsim (1+b)$  the spectator field  $z$  is lighter than the Hubble scale, and cannot be integrated out. In a Minkowski vacuum after inflation either  $b = 0$  or  $b \rightarrow \infty$ ; the latter case may occur in a supersymmetric vacuum with  $W \rightarrow 0$ . For  $b = 0$ , the masses reduce to  $m_{\pm}^2 = m_{3/2}^2 (1 \pm |x|)^2$ , with  $m_{3/2}$  the gravitino mass. For  $|x| > 1$ , the lightest scalars from the supersymmetric sector are heavier than the gravitino. However, for  $|x| < 1$  the lightest of the two eigenstates is lighter than the gravitino and cannot be neglected from a low-energy description. This will play an important role later. In the supersymmetric vacuum with  $b \rightarrow \infty$  we find  $m_{\pm}^2 \approx V_0(1 \pm |x|) \rightarrow 0$ , and the spectators are massless. To avoid a plethora of massless fields in the theory, one has to either break the supersymmetry, or else go beyond the simple separable form of the Kähler function (4.2.20).

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<sup>4</sup>The definition of  $b$  is different from [83], which has  $b \leftrightarrow b - 3$ .

<sup>5</sup>See appendix B for further details.

## 4.3 Single field sgoldstino inflation

In this work we focus on effectively single field inflation models, for simplicity. The inflaton, a real scalar, is identified with a suitable linear combination of the real and imaginary parts of the sgoldstino field; the orthogonal combination is to remain stabilised at a local minimum of the potential during inflation. For the purpose of this chapter, we will not take into account the dynamics of the stabilised field, since the aim of this chapter is to show the restrictions on the parent supergravity, but for a complete and consistent description one should also take into account the dynamics of the stabilized field, as stressed along this thesis.

Single field inflation can be divided into three classes: small field, large field and hybrid inflation. In the first two cases, if the model only contains a single chiral superfield, the inflaton is automatically the sgoldstino. If several fields are present, as is the case for hybrid inflation, one has to be more careful, as the sgoldstino does not have to coincide with the inflaton direction.

As is well known any supergravity model of inflation has to address the  $\eta$ -problem [179, 201, 202], which has been explained in section 1.4.1, together with its possible solutions. In the remainder of this section we will discuss large field, small field and hybrid sgoldstino inflation, and how the  $\eta$ -problem may or may not be addressed in each case.

### 4.3.1 Large field inflation

In models of large field inflation [203], the inflaton field traverses super-Planckian distances in field space during inflation. For a potential dominated by a single monomial during inflation,  $V \sim \lambda\phi^n$ , the slow roll parameters

$$\epsilon = \frac{1}{2} \left( \frac{V_\phi}{V} \right)^2, \quad \eta = \frac{V_{\phi\phi}}{V}, \quad (4.3.1)$$

both scale as  $\eta, \epsilon \sim 1/\phi^2$ , and are automatically suppressed for super-planckian field values. At first sight, no tuning of the potential is needed. However, the problem is that for such large field values *all* non-renormalisable operators are unsuppressed. Therefore, an explicit UV completion of the model is needed to determine whether inflation is possible.

Embedding large field inflation in supergravity provides a better control over the UV behavior of the theory. Because of the  $\eta$ -problem such an embedding is far from straightforward, as the potential grows exponentially rather than polynomial. Fine-tuning  $\eta$  is not an option, as  $\eta$  has to be small along the whole inflationary trajectory, which spans super-Planckian distances in field space  $\Delta\phi > 1$  (in

Planck units). This is in contrast with small field inflation, discussed in subsection 4.3.3, where the  $\eta$ -problem can be solved by tuning  $\eta$  at a single point in field space.

Instead of fine-tuning, we can try to solve the  $\eta$ -problem by invoking a shift symmetry [80]. Consider a Kähler function  $G = \mathcal{K}(X - \bar{X})$ , which is symmetric under a shift  $X \rightarrow X + c$  with  $c$  a real constant. Since  $G$  does not depend explicitly on  $\phi \propto \text{Re}(X)$ , the exponent in the scalar potential is independent of  $\phi$  and there is no  $\eta$ -problem. However, since we need a slope for the potential in order to obtain inflation, the shift symmetry needs to be weakly broken. To assure the breaking does not reintroduce exponential terms that ruin inflation, we add a logarithmic term  $G = \mathcal{K}(X - \bar{X}) + \ln |W(X)|^2$  with  $W$  not growing faster than power law. Let us, for instance, consider a canonical Kähler potential<sup>6</sup> with a shift symmetry and a polynomial superpotential:

$$K = \frac{1}{2} (\phi + \bar{\phi})^2, \quad W = \sum_n \lambda_n \phi^n \quad (4.3.2)$$

Let us assume that the real part of the field  $\alpha = (\phi + \bar{\phi})/2$  is stabilised at the critical point  $\alpha = 0$ . The scalar potential and the potential slow-roll parameters for the imaginary part  $\beta = (\phi - \bar{\phi})/2i$  are the following:

$$V(\beta) = \sum_n |\lambda_n|^2 \beta^{2n} \left( \frac{n^2}{\beta^2} - 3 \right), \quad (4.3.3)$$

$$\epsilon_V = \frac{2 \sum_{n,m} n m |\lambda_n|^2 |\lambda_m|^2 \beta^{2(n+m)} \left[ \frac{n(n-1)}{\beta^2} - 3 \right] \left[ \frac{m(m-1)}{\beta^2} - 3 \right]}{\sum_{n,m} |\lambda_n|^2 |\lambda_m|^2 \beta^{2(n+m)} \left( \frac{n^2}{\beta^2} - 3 \right) \left( \frac{m^2}{\beta^2} - 3 \right)}, \quad (4.3.4)$$

$$\eta_V = \frac{2 \sum_n n |\lambda_n|^2 \beta^{2n} \left[ \frac{n(2n-3)(n-1)}{\beta^2} - 3(2n-1) \right]}{\sum_n |\lambda_n|^2 \beta^{2n} \left( \frac{n^2}{\beta^2} - 3 \right)}. \quad (4.3.5)$$

We now distinguish two situations:

- If all the coefficients  $\lambda_n$  are of the same order, the highest monomial will dominate for values of the field  $\beta > M_{\text{P}}$ . Given this, we must require that  $n^2 > 3\beta^2$  along the whole trajectory in view of eq. (4.3.3), so  $n \gg 1$  in any case. The slow-roll parameters are approximately given by:

$$\epsilon_V \approx \frac{2n^2}{\beta^2} > 6, \quad \eta_V \approx \frac{4n^2}{\beta^2} > 12 \quad (4.3.6)$$

Clearly we cannot achieve inflation in this situation.

<sup>6</sup>Generalising to non-canonical terms is straightforward, since along the inflaton trajectory these become constant and can therefore be absorbed in a redefinition of the fields.

### 4.3. Single field sgoldstino inflation

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- If we tune the coefficients such that all the terms are equally important in the large field regime, we need:

$$|\lambda_0| \gg |\lambda_1| \gg \dots \gg |\lambda_n| \quad (4.3.7)$$

At the same time, the highest monomials must cancel the negative contributions to the scalar potential coming from the lowest monomials, that is:

$$|\lambda_{n-p}|^2 \geq \frac{|\lambda_p|^2}{\beta^{2(n-2p)}} \left| \frac{p^2 - 3\beta^2}{(n-p)^2 - 3\beta^2} \right|, \quad p < n/2 \quad (4.3.8)$$

As argued before, fine-tuning along the whole (large field) trajectory is not a viable option.

Although we did the analysis for a single field, this straightforwardly generalises to the multi-field case. If the inflaton is the sgoldstino, it decouples from the other fields, and its potential can be analysed independently and will always be of the form (4.3.3). We conclude that large field sgoldstino inflation in a supergravity model does not work as it is plagued by an instability in the scalar potential.

We note that it is certainly not impossible to have large field inflation in supergravity, only that it does not work with a single chiral superfield. Two-field models have been constructed that avoid the instability [80, 86], employing a shift symmetry to address the  $\eta$ -problem. However, in these models the inflaton is *not* the sgoldstino (rather the sgoldstino is the orthogonal field).

#### 4.3.2 Hybrid inflation

Hybrid inflation is a multi-field model of inflation which in addition to the inflaton contains one or more so-called waterfall fields, which serve to end inflation [204]. During inflation the waterfall fields are stabilised in a local minimum, and inflation is effectively single field. If the inflaton field drops below a critical value, one of the waterfall fields becomes tachyonic, and inflation ends with a phase transition.

Standard F-term hybrid inflation [205, 206] is an example of sgoldstino inflation. The Kähler function is of the separable form (4.2.20) discussed in section 4.2.4.

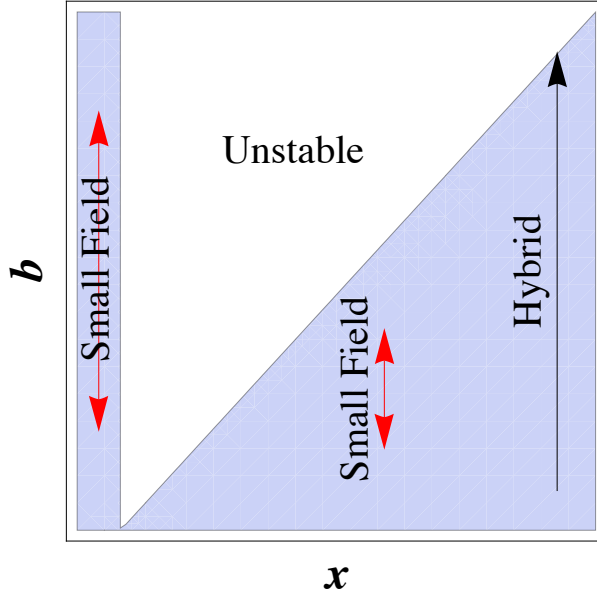
$$G = g(X, \bar{X}) + \tilde{g}(\chi_1, \bar{\chi}_1, \chi_2, \bar{\chi}_2), \quad (4.3.9)$$

with<sup>7</sup>

$$g = X\bar{X} + k_s(X\bar{X})^2 + \ln |\kappa X|^2 + \dots, \quad \tilde{g} = \chi_1\bar{\chi}_1 + \chi_2\bar{\chi}_2 + \ln |\chi_1\chi_2 - \mu^2|^2 + \dots$$

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<sup>7</sup>To see that this setup is indeed of the general form (4.1.2), one can move a factor of  $\ln |\mu^2|^2$  from  $\tilde{g}$  to  $g$  and Taylor expand the remaining  $\ln \left| \frac{\chi_1\chi_2}{\mu^2} - 1 \right|^2$ .



**Figure 4.1** – (Figure adapted from [83, 85].) Stability diagram for the separable case  $G = g(X, \bar{X}) + \tilde{g}(z, \bar{z})$ . The variables on the axes  $b, x$  are defined in (4.2.21), with  $x$  one of the eigenvalues of the  $x_j^i$  matrix. The masses of the spectator fields are positive in the shaded region, while the unstable region signals a tachyonic mode. The black arrow represents the inflationary trajectory for the proposed hybrid set-up, which ends when one of the spectator fields (the waterfall fields) becomes tachyonic. Also shown are possible inflationary trajectories for small field inflation (red arrows).

The inflaton  $\phi$  is identified with the real direction via the decomposition  $X = (\phi + i\theta)/\sqrt{2}$ . Inflation takes place for large  $\phi > \phi_c = \sqrt{2}\mu$ , and all other fields stabilised at zero field value. The potential along the inflationary trajectory is

$$V = \kappa^2 \mu^4 (1 - 2k_s \phi^2 + \dots) \quad (4.3.10)$$

The  $\eta$ -problem is solved via a moderate fine-tuning of  $k_s \lesssim 10^{-2}$ . During inflation  $G_X = \frac{\sqrt{2}}{\phi} + \frac{\phi}{\sqrt{2}} + \frac{k_s \phi^3}{\sqrt{2}}$  and  $G_{\chi_1} = G_{\chi_2} = 0$ . Hence  $\phi$  is indeed the (real part of the) sgoldstino field. The Minkowski minimum after inflation is at  $X = 0$ , and  $|\chi_1| = |\chi_2| = \mu$ . In the minimum  $G_X = G_{\chi_{\pm}} = 0$  and supersymmetry is restored, and therefore there is no relation between inflation and low energy supersymmetry breaking.

The masses of waterfall fields along the inflationary trajectory can be found using the results of section 4.2.4. The mass eigenstates are the linear combinations  $\chi_{\pm} = (\chi_1 \pm \chi_2)/\sqrt{2}$ . Using these as a basis the matrix  $x_m^i$  becomes



### 4.3. Single field sgoldstino inflation

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diagonal during inflation. This shows that we can restrict our attention to only one of the complex fields  $\chi_{\pm}$ , the other field will give the same masses for its two real degrees of freedom. Now we can directly compute the masses from (4.2.23). The stability region as a function of  $b$  and  $|x|$  is plotted in Fig 4.1. The inflationary trajectory corresponds to a vertical trajectory in the plot, going upwards as the field rolls down. When it irrevocably hits the instability region (i.e. when the lower mass eigenvalue becomes negative), inflation ends.

We note that a similar stability analysis can be done for all models of sgoldstino inflation. Whereas hybrid inflation critically makes use of the instability regions, for any non-hybrid scenario — being it small or large field inflation — the inflationary trajectory would have to stop before reaching the instability region. This is automatic for  $|x| < 1$ , otherwise the stability conditions place an upper bound on  $b$  during inflation. We will return to this point shortly when discussing small field inflation.

#### 4.3.3 Small field inflation

Inflation in small field models [9, 10] takes place for sub-Planckian values of the inflaton field. This allows for a Taylor expansion of the inflaton potential around its Minkowski minimum. If one term in the polynomial expansion dominates during inflation, the slow roll parameters blow up:  $\epsilon, \eta \sim 1/\phi^2$  in the small field limit, prohibiting inflation. The only way to get around this conclusion is that several terms in the expansion conspire together to nearly cancel, thus obtaining small slow-roll parameters.

This motivates to consider inflation near an extremum – a maximum, saddle point or inflection point – of the potential. This assures that the first slow roll parameter  $\epsilon$  vanishes. The  $\eta$ -parameter can be made small by tuning the parameters in the potential. Since the inflaton field traverses only small, sub-Planckian distances in field space, tuning the curvature of the potential at a single point (the extremum) suffices, in contrast with large field inflation.

We were able to construct a fine-tuned small field inflation model in supergravity containing only a single chiral field. In such a set-up the inflaton is automatically the sgoldstino, and our example is proof of principle for small field sgoldstino inflation. Consider a model with<sup>8</sup>

$$K = \sum_n \alpha_n (X \bar{X})^n, \quad W = \sum_n \lambda_n X^n. \quad (4.3.11)$$

We decompose the complex scalar  $X = (\phi + i\theta)/\sqrt{2}$  with  $\phi$  the inflaton field. The model parameters  $\lambda_n, \alpha_n$  can be tuned in such a way that the potential

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<sup>8</sup> This ansatz (4.3.11) is equivalent to  $G = \sum_{n=1} \alpha_n (X \bar{X})^n + \log |\sum_{n=0} \lambda_n X^n|^2$ .

allows for inflation near an inflection point which, without loss of generality, is located at the origin  $(\phi, \theta) = (0, 0)$ , and a Minkowski minimum at finite field value  $(\phi, \theta) = (\phi_0, 0)$ . In particular, we demand

- Vanishing slope and curvature of the potential at the origin 1)  $V_\phi|_{(0,0)} = 0$  and 2)  $V_{\phi\phi}|_{(0,0)} = 0$ .
- The height 3)  $V|_{(0,0)} \equiv V_0$  of the potential at the origin is fixed by the normalisation of the power spectrum given by the data.
- After inflation the inflaton settles in a local Minkowski minimum with 4)  $V|_{(\phi_0,0)} = 0$  and 5)  $V_\phi|_{(\phi_0,0)} = 0$ . Moreover, the masses are positive definite 6)  $m_i^2|_{(\phi_0,0)} > 0$ .
- Along the whole trajectory, from the extremum to the minimum, the orthogonal field is stabilized 7)  $V_\theta = V_{\phi\theta} = 0$  and 8)  $m_\theta^2 \gtrsim H^2$ .

We consider solutions with canonical kinetic terms, i.e. we set  $\alpha_1 = 1$  and  $\alpha_i = 0$  for  $i \neq 1$ . To satisfy conditions 1-5 we need at least five parameters and choose them accordingly. We take all  $\lambda_i$  real, and consider the first five in the expansion. Tuning is required to satisfy conditions (2) and (4) — the smallness of  $\eta$  parameter and of the cosmological constant — in the usual sense that large contributions should nearly cancel. Conditions 6-8 are then checked for consistency, but do not require any new input. Setting the minimum at  $\phi_0 = 1$  we find two inflationary inflection point solutions<sup>9</sup>

$$\{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4\} = \sqrt{\frac{V_0}{23}} \times \{3, -5\sqrt{2}, 3, 0, 2\}, \quad (4.3.12)$$

and

$$\begin{aligned} \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4\} = & \frac{\sqrt{V_0}}{19\sqrt{73}} \times \\ & \left\{ 3\sqrt{39287 - 1464\sqrt{6}}, \sqrt{2(543551 - 19764\sqrt{6})}, \right. \\ & \left. 3\sqrt{39287 - 1464\sqrt{6}}, 0, -2\sqrt{4943 - 1152\sqrt{6}} \right\}, \end{aligned} \quad (4.3.13)$$

and all other  $\lambda_i$  are zero.

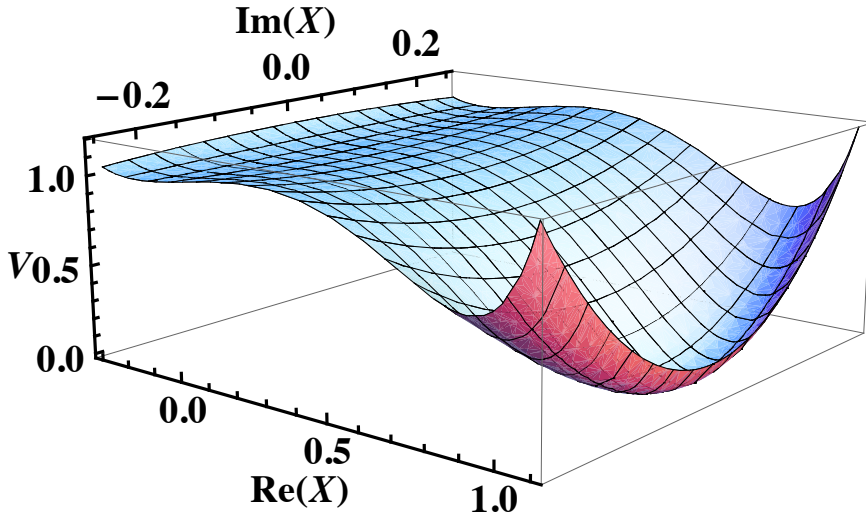
Both examples above correspond to inflection point inflation, rather than to inflation near a maximum or saddle point. This is unfortunate, as for inflection point inflation the spectral index is bounded to be  $n_s \simeq 0.92$ , which is ruled out. We review this argument in appendix C. However, the spectral index can

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<sup>9</sup> $\lambda_3 = 0$  only vanishes for  $\phi_0 = 1$ , but is non-zero for other positions of the minima.

### 4.3. Single field sgoldstino inflation

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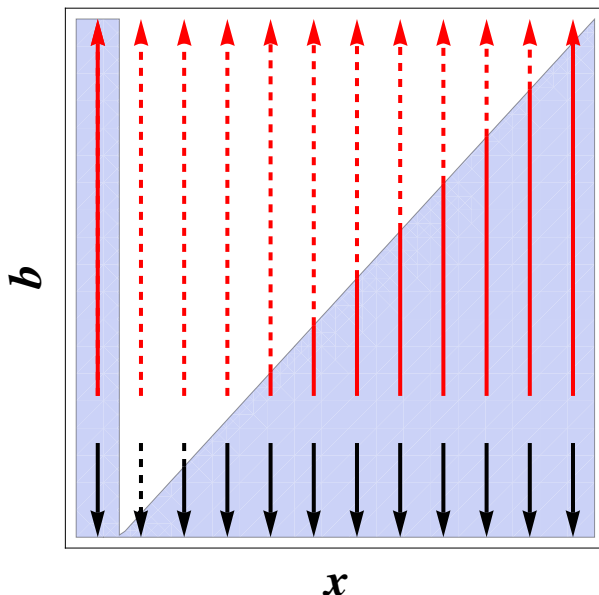


*Figure 4.2* – Scalar potential for small field inflation corresponding to the first solution (4.3.12).

be larger if the cubic term is absent or unnaturally small, as is the case for inflation at a maximum rather than an inflection point. Then the correction to the spectral index (B.0.4) is set by the quartic term in the Taylor expansion around the extremum, rather than by cubic term, with an upper bound  $n_s \lesssim 0.95$ . In our set-up this would require an extra tuning condition  $V_{\phi\phi\phi} \approx 0$ ; without it we always find a saddle point.

The first solution above (4.3.12) has a supersymmetric Minkowski minimum. In this scenario the supersymmetry breaking observed today is not related to the supersymmetry breaking during inflation. The second solution (4.3.13), however, does end in a supersymmetry breaking minimum, and the gravitino mass today can be related to the inflationary scale. The gravitino mass is  $m_{3/2} \sim 10^{-7}$  in Planck units, see appendix C.

There is a huge difference between the two solutions when combined with other spectator fields. The first solution has a supersymmetry preserving vacuum in which  $W \rightarrow 0$ . Although at this exact point our description in terms of a Kähler function  $G$  breaks down, we can nevertheless describe the behavior of



**Figure 4.3** – Stability plot of the spectator  $z$ -fields for a separable Kähler function  $G = g(X, \bar{X}) + \tilde{g}(z, \bar{z})$ . The trajectories for small field inflation are vertical lines, going upward (red) to infinity for solution (4.3.12) which has a supersymmetry preserving vacuum, and downward (black) to zero for (4.3.13) which has a supersymmetry breaking vacuum. Dashed lines indicate unstable trajectories. The position on the horizontal axis depends on the specifics of the spectator sector. Solution (4.3.12) always leads to an instability for  $|x| > 1$ .

the potential as we approach this singular limit. We find that  $b \propto V_0/W_0 \rightarrow \infty$ , which implies that if we draw the stability diagram for the simplified case of separable Kähler functions (4.2.20), see Fig. 4.3, this inflationary model corresponds to vertical trajectories going upwards to infinity. The position on the horizontal axis given by  $|x|$  depends on the specifics of the spectator sector, but it is clear that for all  $|x| > 1$  one of the fields becomes tachyonic as the inflaton approaches its minimum, and the potential is unstable. Hence, solution (4.3.12) with a supersymmetry vacuum can only be combined with different fields if this extra sector has  $|x| < 1$  (for several fields the eigenvalues of the  $|x|^2$  matrix should all be less than unity, and in fact in chapter 5 we will consider the statistical properties of a truncated sector with a large number of fields). This puts enormous limitations on the spectator sector. For  $|x| < 1$  the masses of the spectator fields vanish in the vacuum, as discussed at the end of section (4.2.4). However, in a subsequent supersymmetry breaking phase transition they may pick up a soft mass term. This disastrous conclusion may be avoided by taking more generic Kähler functions.

### 4.3. Single field sgoldstino inflation

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In contrast, solution (4.3.13) has a supersymmetry breaking vacuum, and the parameter  $b = V_0/W = 0$  vanishes in the minimum. The inflaton trajectory again corresponds to a vertical trajectory in the stability diagram, but now going downwards. Except for a small region near  $|x| = 1$  there are no instabilities in the potential, and at least for the separable Kähler function (4.2.20) sgoldstino inflation can straightforwardly be combined with a spectator sector. In the region  $|x| > 1$  the spectator fields are heavy in the vacuum and can be integrated out to get a low energy EFT. In the other limit  $|x| < 1$  the spectator fields are of the same order as the gravitino mass (see the discussion at the end of section 4.2.4), and are relatively light.

#### Other proposals for small field sgoldstino inflation

In the recent literature there have been claims for small field sgoldstino inflation, with no or very little fine-tuning of the parameters in the potential. As argued in this paper, unless some symmetry principle is invoked, this is not possible as the slow roll parameters generically blow up in the small field limit. Indeed we find that these proposals do not work, although the devil is sometimes in the details. I will describe one of the setups, which illustrates how to not deal with multi-field dynamics, and refer to our original paper [3] for further details.

Refs. [182, 183] propose a model of sgoldstino inflation in a single field set-up without tuning of parameters. To address the  $\eta$  problem they add a logarithmic term to the Kähler potential

$$\begin{aligned} K &= X\bar{X} + aX\bar{X}(X + \bar{X}) + b(X\bar{X})^2 + \dots - 2\ln(1 + X + \bar{X}), \\ W &= fX + f_n M. \end{aligned} \tag{4.3.14}$$

However, in the small field regime the logarithm can simply be expanded and does not alter the qualitative structure of the potential. It also does not enhance the symmetry.

Taking arbitrary parameters, except for the constraint that the minimum at the origin is stable and has zero cosmological constant, both the epsilon and eta-parameter exceed unity throughout the whole field space  $|X| < 1$ . Slow roll inflation cannot happen. In [182] it is actually claimed that  $\epsilon < 1$ , but what they calculate is  $\epsilon_\theta = g^{\theta\theta}(V_\theta/V)^2$ , where we again decomposed the field  $X = (\phi + i\theta)/\sqrt{2}$  and  $g_{ij}$  is the metric in field space. However, in a situation where the potential falls much steeper in the  $\phi$ -direction than in the  $\theta$ -direction, this is not the relevant slow roll parameter. Instead, one should use the more general multi-field generalization  $\epsilon = g^{ij}V_i V_j / V^2$ .

Ref. [183] shows inflationary trajectories with a large number of e-folds  $N > 60$ . However, their trajectories are calculated in the – non-applicable – slow roll approximation. For all initial points in field space proposed in [182, 183] we have solved the full two-dimensional field equations and the slow-roll approximations to them. In all cases the slow roll solutions wildly diverge from the full solutions, which can only give inflation for less than an e-fold, confirming once more that this setup does not provide a slow roll regime. The only way to get inflation in the set-up of [182, 183], in any case, is to tune parameters near an extremum, along the lines of our example (4.3.11).

## 4.4 Conclusions

Inflationary models in supergravity, where the inflaton sits in a complex scalar superfield, necessarily involve a multi-field analysis. Any extra fields present during inflation must be integrated out to give an effective single-field slow-roll dynamics that is consistent with the CMB. However, even very heavy fields can leave a detectable imprint in the spectrum of primordial perturbations, in particular through a reduction in the speed of sound of the adiabatic perturbations, as deeply explained along the previous chapters of this thesis. The correct effective field theory for the adiabatic mode has a variable speed of sound that depends on the background trajectory. A necessary condition to recover the standard single-field slow roll description is that the trajectory should have no turns into the heavy directions. In this case, the speed of sound is unity, equal to the speed of light, and integrating out the extra fields gives the same effective action as truncating the heavy fields at their adiabatic minima.

In supersymmetric models there is an extra complication. One has to integrate out whole supermultiplets in order to obtain an effective supergravity description for the remaining superfields. This is only possible if the superfields that are being integrated out are in configurations that do not contribute to supersymmetry breaking.

Sgoldstino inflation naturally implements these two conditions. The full inflationary dynamics is confined to the sgoldstino plane. Putting the scalar components of all other superfields at their minima is a consistent truncation of the parent theory. This makes sgoldstino inflationary models extremely attractive, because of their simplicity and robustness.

We have analysed sgoldstino inflation scenarios exploiting the fact that the Kähler function  $G = K + \log |W|^2$  has a relatively simple separable form which allows some aspects to be analysed in a model-independent way. We derived a necessary and sufficient condition on the Kähler function for the stability of the supersymmetry-preserving sector, the spectator fields that are integrated out.

#### 4.4. Conclusions

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Figure 4.1 shows the constraint for a separable Kähler function, in particular for hybrid F-term inflation (which is a well studied case of goldstino inflation) and small field inflation.

In the case of small field goldstino inflation we were able to provide some viable fine-tuned examples around inflection points. The spectral index is rather low, but a higher spectral index would be possible with additional fine-tuning. Rather surprisingly, the inflationary model can only be straightforwardly combined with a spectator sector if the minimum after inflation breaks supersymmetry. In our inflation example with a supersymmetry preserving Minkowski vacuum the spectator sector is very constrained by the condition that there should be no tachyonic modes in the system. This is illustrated in figure 4.3.

Summarising, in this work we provide proof of principle for the viability of inflationary models where the supersymmetry breaking field is the inflaton. More importantly, the presence of an additional sector that preserves supersymmetry can be incorporated in the description if the decoupling and stability constraints are satisfied.

Here we restricted the analysis to inflationary dynamics and a separable Kähler function, but in the next chapter we extend the analysis to more general Kähler functions, and we derive constraints using the supersymmetric and non-supersymmetric directions, not only for the inflationary dynamics but also for a stationary vacuum. Moreover, we will give an example in which the quasi-separable (non-generic) structure of the Kähler function is naturally realised. In order to study systems with a large number of fields in the supersymmetric sector, we also use random matrix theory techniques to describe their statistics. This completes the picture of constraints on inflation in supergravity due to the presence of additional fields.

