



Universiteit
Leiden
The Netherlands

Learning trajectories in analogical reasoning : exploring individual differences in children's strategy paths

Pronk, C.M.E.

Citation

Pronk, C. M. E. (2014, February 19). *Learning trajectories in analogical reasoning : exploring individual differences in children's strategy paths*. Retrieved from <https://hdl.handle.net/1887/24301>

Version: Corrected Publisher's Version

License: [Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden](#)

Downloaded from: <https://hdl.handle.net/1887/24301>

Note: To cite this publication please use the final published version (if applicable).

Cover Page



Universiteit Leiden



The handle <http://hdl.handle.net/1887/24301> holds various files of this Leiden University dissertation

Author: Pronk, C.M.E.

Title: Learning trajectories in analogical reasoning : exploring individual differences in children's strategy paths

Issue Date: 2014-02-19

CHAPTER 2

Inter- and intra-individual variability in the process of change in the use of analogical strategies to solve geometric tasks in children: A microgenetic analysis



Tunteler, E., Pronk, C.M.E., & Resing, W.C.M. (2008). Inter- and intra-individual variability in the process of change in the use of analogical strategies to solve geometric tasks in children: A microgenetic analysis. *Learning and Individual Differences*, 18, 44–60.

Abstract

This study focused on unprompted changes in children's analogical reasoning on geometric tasks and the additional effect of a short training procedure. Participants were 36 grade 1 level children (M=6;8 years) divided over a not-trained and a trained condition. The study was a 5-sessions microgenetic procedure, with a follow-up test session after 3 months. The results showed considerable inter-and intra-individual variability in the process of change in the use of analogical strategies in both not-trained and trained children. Repeated practice, without explicit prompting, caused a spontaneous improvement in analogical reasoning. This improvement was mainly due to an increase in implicit analogical reasoning. The short training procedure caused an improvement above and beyond that of practice alone (Estrained/not-trained=.96), inducing in 9 children a continuation of a gradual process of change, while in 4 other children it caused a rather rapid change in analogical performance. The training effect was greatly due to an increase in explicit analogical reasoning. Both effects were still visible after a period of 3 months. Because the study may have implications for geometric learning with young children, the authors recommend further investigations of young children's use of analogies on tasks involving geometric transformations. The authors also recommend further research into transfer to other mathematical competencies to investigate implications for mathematics besides geometry.

2.1 Introduction

Our primary goal in this study was to gain insight into the nature of young children's analogical reasoning ability by investigating whether children's analogical performance changes due to practice alone, without explicit prompting, and whether a short training procedure that provides children with some explicit modeling and feedback improves their performance. Unlike other studies on young children's analogical ability (e.g., Alexander et al., 1989; Brown, 1989; Goswami & Brown, 1989; Hosenfeld, Van der Maas, & Van den Boom, 1997a,b; Tunteler & Resing, 2002), this study investigated children's unprompted analogical performances over a period of weeks both before and after a short training in analogical reasoning. This was compared with the performances of children of the same age who were given multiple practice opportunities over time, but no instructions or explicit prompting.

The ability to reason by analogy has long been regarded as central to human cognition (Goswami, 1991, 1992; Halford, 1993) and as an important skill for classroom learning (e.g., Csapó, 1997; Goswami, 1992; Vosniadou, 1989) and instruction (e.g., Kolodner, 1997). During the past few decades, a considerable number of researchers have focused on understanding the development of this reasoning ability in children (e.g., Alexander et al., 1989; Brown, 1989; Gentner, 1989; Goswami & Brown, 1989; Halford, 1993; Hosenfeld, Van der Maas et al., 1997a,b; Singer-Freeman, 2005; Singer-Freeman & Goswami, 2001). Although these studies have resulted in much information on children's analogical reasoning competency under various circumstances, there is still no consensus about the nature of this reasoning ability in young children. An increasing number of studies, in which a variety of analogy tasks were used, showed that very young children can already reason analogically after a certain amount of help on the condition that they understand the relationships on which the analogies are based (e.g., Brown, 1989; Chen, 1996; Chen & Daehler, 1989, 1992; Singer-Freeman, 2005; Singer-Freeman & Goswami, 2001). In this research tradition, developmental changes in children's analogical reasoning ability is generally assumed to be gradual and quantifiable, and driven by a growing knowledge base or increasing metacognitive skills (Brown, 1989; Goswami, 1991). However, other researchers (e.g., Halford & McCredde, 1998; Halford, Wilson & Phillips, 1998; Hosenfeld, Van der Maas et al., 1997a,b) are more apprehensive about young children's analogical capacity; they posit that developmental changes in analogical reasoning is a matter of changes in global competence. This lack of consensus may cause one to question whether the claim for analogical reasoning at an early age made in some studies might be an artifact of the experimental manipulations in these studies.

Review of the literature on analogical reasoning showed that the conclusions with respect to the nature of changes in the ability to reason by analogy described above were frequently drawn on the basis of results from cross-sectional training studies (e.g., Brown, 1989; Gholson, Morgan, Dattel, & Pierce, 1990; Gentner, 1989; Chen, 1996; Chen & Daehler, 1989, 1992). Yet, Bjorklund, Miller, Coyle and Slawinsky (1997) asserted that natural, unprompted changes, as opposed to changes induced by training, may show a different path. Moreover, various other authors stressed that such single-occasion assessments could produce an incomplete or even over-optimistic picture of the process of change of the cognitive strategy under investigation because they address changes indirectly (e.g., Granott & Parziale, 2002; Kuhn, 1995; Siegler, 1995, 2006).

Despite the many studies in the field of analogical reasoning conducted in the past, very few of them have focused on a comparison of changes over time in children's analogical reasoning performance induced by practice and changes induced by a training procedure.

Two exceptions worth mentioning are the longitudinal studies conducted by Alexander et al. (1989) and Hosenfeld, Van der Maas et al., (1997b). Alexander et al. (1989) used simple 3-dimensional geometric analogical tasks of type A:B::C:D, and monitored the analogical performances of trained 4–5 year-old children and that of not-trained children of the same age over a period of months. They showed that children of this age were able to benefit from an extensive training in analogical reasoning skills, but revealed little about the paths of change in the two conditions. Moreover, it should be noted that the not-trained children in the Alexander et al. (1989) study were repeatedly given explicit instructions to the tasks before and during testing, and explicit instructions may also be seen as a form of training. Hosenfeld, Van der Maas et al. (1997b) observed 6–8 year-old children's analogical performance on paper and pencil classical geometric tasks over a period of months. These authors posited an age-related transition in analogical reasoning on geometric tasks in children of this age. However, the children in their study were given extensive instructions for the tasks, both before and during testing so that we are unable to determine the natural and unprompted analogical reasoning of those children. Such natural reasoning might not proceed in the same way suggested by the sequence of instructions given by Hosenfeld, Van der Maas et al.

More recently Tunteler and Resing (2007a) microgenetically investigated the performances on problem analogy tasks over a period of weeks of 5–7 year-old children who were given repeated practice opportunities without any instruction or feedback in comparison to the performances of children who were previously given a short training consisting of some instructions in how to use analogies. A microgenetic procedure allows close observation of change mechanisms over a relatively short period of time, as well as the identification of the conditions and transition strategies leading up to change (Siegler & Crowley, 1991). The microgenetic procedure used in the Tunteler and Resing study allowed the authors to distinguish three groups of reasoners: 1) children showing consistent analogical reasoning over trials; 2) children showing consistent inadequate, non-analogical reasoning; and 3) children showing variable, adequate and inadequate, reasoning. Some children had difficulty with using analogies despite of the training, while other children of the same age and even some younger children consistently used analogies over trials without reminding. Over time, an increasing number of children, particularly in the trained group, showed very consistent analogical reasoning, while a decreasing number demonstrated inadequate, non-analogical reasoning. However, variable and diverse strategy use over trials existed in a considerable number of both the trained and not-trained children of the two age groups. The authors concluded that variability in strategy use on problem analogy tasks is not only common in situations in which children are not explicitly given instructions as they demonstrated earlier (Tunteler & Resing, 2002, 2007b), but apparently exists in trained children as well.

According to Tunteler and Resing (2007a), this pervasiveness in variability in children's strategy use on analogical problem solving tasks indicates that the ability to reason by analogy on this type of analogy tasks develops over a protracted age range. It also underlines the importance of a microgenetic research method in studying the process of change in the domain of analogical reasoning. Therefore, we realized that in order to gain more insight into the nature of young children's analogical reasoning ability, we needed, in addition to the Tunteler and Resing study, to microgenetically examine changes in young children's analogical reasoning under different conditions—trained and not-trained—on another type of analogy task. In this study we used classical geometric analogy tasks. This type of analogy tasks is said to measure analogical reasoning more purely than verbal analogical tasks, since they need no vocabulary and domain specific knowledge (Goswami, 1992).

The advantages of the microgenetic approach have been extensively described elsewhere (e.g., Kuhn, 1995; Siegler & Crowley, 1991; Siegler, 2006). It should however be noted that even though most older microgenetic studies sought to accelerate the natural process of developmental changes by increasing the density of exercises within the domain under investigation, this research method is not restricted to this purpose (Siegler, 2006). Adding an element of training is assumed to be informative regarding the development of the skills examined (Kuhn, 1995; Opfer & Siegler, 2004). Recently, microgenetic studies have increasingly focused on the effectiveness of various learning experiences (see Siegler, 2006). Our study is similar to this latter type, because it observed changes in children's analogical reasoning performance as a result of practice alone, as well as changes that occurred as a consequence of a short training procedure. Siegler (2006) asserted that change as a result of practice alone, without explicit instructions or prompting, may be considered as natural because it does not arise from explicit interventions. We therefore considered a study that couples observations of children's unprompted analogical performances over time with that as a result of a short training procedure as a valuable tool to increase knowledge of the nature of analogical reasoning in young children.

Because the type of intra-individual variability in strategy use described earlier has been shown to predict later learning substantially (Siegler, 2006), we thought it might be useful to investigate whether a short training procedure would have a greater effect on children showing variable, inconsistent analogical reasoning over trials than in children not showing this kind of behavior. Participants in the study were 6–8 year-old children from first grade. Results of prior studies on analogical reasoning on various types of analogy tasks showed that changes in analogical reasoning are most prominent at this age (e.g., Goswami & Brown, 1989; Hosenfeld, Van der Maas et al., 1997a,b; Tunteler & Resing, 2007a,b).

In the current study, analogical reasoning was measured by a combination of both children's overt solutions to the problems and their verbal explanations for their solutions. Siegler (2006) stated the advantage of using retrospective self-reports of strategy use next to overt solution behavior in studies on strategy development, because self-reports may give additional information about the strategy used. According to Siegler (2006), this additional information would enhance the accuracy of classification of strategy use substantially. We therefore assumed that a combination of both an overt behavior and a verbal measure for analogical reasoning in our study would reveal more about the development of analogical reasoning on classical geometric analogy tasks, than just one measure. Because several authors (e.g., Halford, 1993; Halford et al., 1998) stated that analogical reasoning may depend on memory capacity, we also collected data on children's memory capacity prior to the study. These data provided a means of assessing the role memory may have in the development of analogical reasoning on geometric tasks in children.

The short training procedure in the current study consisted of a standardized step by step procedure, which prompted the child to explain the reasoning behind the experimenters' correct analogical solution. Explaining correct solutions of a more knowledgeable person has been found to induce learning (Siegler, 1995; Siegler, 2002; Rittle-Johnson, 2006). The step by step procedure was based on the component processes of analogical reasoning put forward by Sternberg and Rifkin (1979): encoding, inference, mapping and application. These component processes have also been successfully used by others in training young children in analogical reasoning (e.g., Alexander et al., 1989; Resing, 1990, 2000; White & Caropreso, 1989).

Due to the microgenetic research design, in combination with several experimental conditions we used, we could examine possible changes in children's analogical performances both before and after the short training procedure, as well as differences between conditions. Furthermore, after three months we conducted a final testing to investigate whether a progress in analogical performance lasted over a more extended period of time, over both conditions in the study.

In sum, the present study aimed to answer the questions whether: 1) children's analogical performance changes due to practice alone, without explicit prompting, and whether a short training procedure adds to this effect; 2) the short training procedure has a greater effect on children showing variable, inconsistent analogical reasoning over trials than on children not showing this kind of behavior; and 3) changes in analogical reasoning either because of repeated practice alone or because of the short training procedure lasts over a period of 3 months. In addition, in an attempt to investigate the role memory capacity and inductive reasoning skills plays in the development of analogical reasoning we exploratively investigated whether children's analogical reasoning performance was related to their memory and inductive reasoning scores.

2.2 Method

Participants

Participants in this study were 36 children, 17 girls and 19 boys, from grade 1 of two elementary schools located in a midsize town in the Netherlands. From each school, eighteen children were randomly selected. For all children Dutch was the primary language in their home. Parental permission to participate was obtained for all children. At the start of testing, the children ranged in age from 5 years and 11 months to 8 years ($M=6$ years and 8 months, $SD=4.9$ months). Both genders were approximately equally represented within each condition. No child dropped out during the extended period of testing.

Material

Two pretests were used in the study: Exclusion and Memory Span. Both tests are subtests of a Dutch child intelligence test (Revisie Amsterdamse Kinder Intelligentie Test, Bleichrodt, Drenth, Zaal, & Resing, 1984). The exclusion test is a visual inductive reasoning test. It calls upon children's ability to infer rules, an ability that is assumed to be important for successful analogical reasoning. The test consists of 50 items of four abstract figures. Three figures belong together according to a rule, the child's task is to discover the rule and select the figure that does not satisfy the rule. The test served grouping purposes, but was also used to investigate whether analogical reasoning was related to inductive reasoning skills. The memory span test measures children's visual memory capacity. The testing material consists of two small booklets and two sets of small blocks with pictures. One set measures concrete visual memory and contains pictures like a fish and a flower. The other set measures abstract visual memory and contains abstract pictures with undefined forms. The pictures in the booklet are given in a certain order, which the child needs to remember and reproduce with the blocks. The amount of pictures increases steadily, making it harder to remember and reproduce the sequence. The test was used to examine whether analogical reasoning was related to memory capacity.

Testing material consisted of five parallel sets of open-ended paper and pencil geometric analogy tasks of the type A::B::C:D. Each of the five sets contained 20 items of various levels

of difficulty. The original set of items represented a selection of 20 items from a highly homogeneous scale of 36 items created by Hosenfeld, Van den Boom and Resing (1997). These items were constructed out of six basic geometrical shapes – circles, squares, triangles, pentagons, hexagons, and ellipses – and five possible transformations – adding an element, changing size, halving, doubling, and changing position (Hosenfeld, Van den Boom, et al., 1997; Mulholland, Pellegrino, & Glaser, 1980). Earlier research (Mulholland et al., 1980) showed that the number of elements and transformations the item contained could satisfactorily predict its level of difficulty. However, it should be noted that although the difficult items contained more information than the easier ones, they could be reduced to smaller sets of information and consequently be solved in small steps. In this way, the amount of information that had to be processed in parallel remained small (Hosenfeld, Van der Maas, et al., 1997a). Some examples of items of various difficulty levels used in the current study are displayed in Appendix 2A.

Because of the repeated testings required in the current study, we developed 4 additional tests with parallel test items. The parallel items contained different geometric shapes, but were constructed according to comparable construction rules as were the items in the original set. The five sets were therefore supposed to be highly comparable. Each set of 20 analogical items was presented in a separate booklet. The order of presentation of difficult and easy items in each set was mixed, but remained the same over participants and sessions. Every analogical item was printed on a separate sheet of paper and presented in an open-ended format, which gave children the opportunity to come up with their own solution. Children had to draw their solution with a pencil in the last, empty box.

Additionally, a set of 6 analogical items of various difficulty levels was used during the short training procedure. These training items were constructed so that all of the five possible transformations were presented at least once.

Design

One week before the experiment started, children were pretested on the Exclusion and Memory Span tests. They were randomly allocated to either a trained or a not-trained condition based on blocked scores on the exclusion test. As can be seen in Table 1, children in both conditions were presented with the analogical reasoning tasks at weekly intervals over a period of 5 consecutive weeks. During test sessions 1 and 2 children in both conditions were administered a set of 20 geometrical analogies without any instruction or feedback concerning the correctness of their responses. During the training session, a short 15-minute-training in analogical reasoning was delivered to the children in the trained condition, while the not-trained children were presented with the same tasks, but without explicit prompting or any instructions. During test sessions 3 and 4, all children were again given a set of 20 geometrical analogies without any instructions or feedback. In addition, three months after test session 4 all children—trained and not-trained—were given a follow-up test during which they were presented again with a set 20 geometrical analogies without explicit prompting or instructions.

Table 1. Research design

Condition	Pretests: exclusion memory span	Test session 1	Test session 2	Training session ¹	Test session 3	Test session 4	Follow-up test session ²
Trained	x	x	x	x	x	x	x
Not-trained	x	x	x	-	x	x	x

Note: ¹During the Training session, children in the not-trained condition needed to solve the same analogical items as children in the trained condition, but without receiving any instruction or feedback.

²Follow-up test session was administered after a period of 3 months.

General procedure

Children were tested individually. Testing took place in a separate room in the child's own school during weekly sessions lasting approximately 15–20 minutes each. The testing procedure was basically the same for both trained and not-trained conditions, during all, but the training session. Children were presented with the booklets with the geometric analogy items. The six basic geometric shapes were displayed on the cover sheet of each booklet. At the beginning of each test session, the instructor named each geometric figure in a way that the child could understand, and asked the child to copy it underneath the corresponding figure printed on the sheet. This procedure served two purposes. It gave the child the opportunity to get familiar with the testing material prior to the test. In addition, it allowed the experimenter to observe how a child drew a particular figure. This served the purpose of controlling for drawing ability, because it allowed the experimenter to take possible difficulties with drawing into account.

Subsequently, the analogy items were presented one by one with a minimum of instruction; the child was merely told that this was a kind of 'puzzle' with four boxes, the first three containing figures (the experimenter pointed to the A, B, and C boxes), but not the fourth one (the experimenter pointed to the empty D box). The child was then asked to draw what needed to be drawn in the fourth, empty box in order to solve the 'puzzle'. Then the child was given the opportunity to draw his own solution to the problem, and asked to verbally explain his way of solving the 'puzzle'.

Both the overt solution drawn on the paper and the verbal explanation provided by the child were recorded on video and analyzed afterwards. In order to ensure that the child would not just solve the analogy items over sessions on the basis of mere recognition, items consisting of the same level of difficulty but containing different geometrical shapes, were alternately administered.

Training procedure

During a short training session of approximately 15 minutes, children were given similar analogical items as they received during the test sessions. This time however, children in the trained condition were given a short standardized training procedure in which they were step by step prompted to explain the way they thought that the experimenter had found the correct solution to the so-called 'puzzle'.

First, they were presented with an analogy item of medium difficulty. They were given comparable instructions as they were given before during the test sessions. Then, they were

told that the experimenter would help them this time and show them the correct solution after they finished their drawing. When the child completed the first analogy item, the experimenter revealed the correct solution and gave feedback concerning the correctness of the child's own solution. Subsequently, regardless of the correctness of the child's solution, the child was asked to explain how he thought that the experimenter had arrived at his solution. Prompting children to explain the correct solutions of someone more experienced has been shown to improve subsequent learning even more than self-explanations of their own solutions or corrective feedback (Siegler, 1995, 2002; Rittle-Johnson, 2006). Children who subsequently put forward a correct analogical explanation were still given all the steps of the analogical reasoning process in order to avoid a disadvantage for the more advanced children when new, or more difficult items were presented.

However, these children were not asked any more questions about the analogy for which they gave a correct analogical explanation. Children who put forward incorrect reasoning or partly incorrect reasoning, were given the necessary step by step questions, prompting them to correct analogical reasoning. These questions were based on the component processes put forward in studies by Sternberg and Rifkin (1979) and Resing (1990, 2000): encoding, inference, mapping and application (see Appendix 2B).

After this procedure for the first item, children were presented with a second item, which was easier than the one they just solved. Here they first had to explain what they were going to do and why, before they were allowed to draw. If necessary, they were systematically helped towards the right analogical answer, using the above mentioned procedure. In this way they could practice out loud what they had just learned and could be corrected if they would fall back into any incorrect old pattern of solving the analogical item. The third training item was more difficult again and children were taken through the same procedure as they were during the first training item. The fourth training item was about as difficult as the third one, but followed the same procedure as the second training item. The fifth training item was the most difficult kind of analogical item they would encounter during testing. This item contained three elements and five different transformations. The procedure was equal to that for the first training item. The sixth and last training item was once again an easier item in order not to discourage children who found the fifth item too difficult. The procedure followed for the sixth training item was the same as for the first one.

Scoring

Test items were scored on the basis of a combination of both children's drawing and their verbal explanations. Some children sometimes experienced difficulties with drawing the geometrical shapes. However, this did not cause any problems, because the experimenter had seen how the child had drawn all of the figures on the cover sheet and therefore usually knew what the child meant by a particular drawing. Whenever she doubted, she asked the child to point out and explain which figure(s) he meant by that (e.g., can you point out or tell me which figure this one is supposed to be?).

The scoring-system that we used was adapted from a scoring-system used by Hosenfeld, Van der Maas, et al. (1997b) for comparable geometric analogy tasks and earlier by Resing (1990) for verbal analogies. This scoring-system consisted of 4 types of solutions: explicit correct analogical (Category 1); implicit correct analogical (Category 2); incomplete analogical (Category 3); and non-analogical, associative (Category 4). Explicit and implicit solutions (Categories 1 and 2) are both considered correct analogical solutions. However, an explicit solution indicates that the child has drawn and explicitly stated verbally all the

transformations the analogy item contained, while an implicit solution indicates that the drawing looked correct, but the child has not explicitly stated all the transformations that the analogy item contained. An incomplete analogical solution indicates that analogical reasoning is present but only partially, for either one or more, but not all, of the transformations the item contained was drawn and mentioned verbally. The non-analogical, associative type of solution indicates that the solution was produced by an association strategy instead of analogical reasoning. Solutions scored in this category are for example: a copy of elements from the A, B or C terms; a complete or partial copy of the A, B or C term; a copy of A, B or C with horizontal or vertical position change. Some examples of children's descriptions of their own reasoning are displayed in Table 2.

Inter-rater reliability

All data were coded by the second author. To estimate coding reliability, data for the first two weeks (40%) were coded by the first author who was blind to the child's condition. Inter-rater agreement was 97%, indicating that the data were scored reliably.

2.3 Results

Before investigating the research questions, we examined children's initial level of inductive reasoning. The result of a one-way ANOVA, with the independent variable being condition with two levels—not trained and trained—and the dependent variable being children's score on the inductive reasoning Exclusion test, showed no significant effect for condition. This finding indicates that the two conditions did not have different levels of inductive reasoning prior to the first test session. The first research question concerned whether children's analogical performance changed due to practice alone, without explicit prompting, and whether a short training procedure added to this effect. In order to answer this question, we analyzed the data at both the group level and the individual level. The analyses at the group level were expected to provide general information on changes in the analogical performances of the two experimental conditions. The analyses at the individual level were expected to provide more detailed information on how changes occurred.

Analyses at the group level

First, it was important to investigate whether the two conditions had comparable levels of analogical reasoning at the start. A one-way ANOVA was conducted, with the independent variable being condition with two levels – not trained and trained – and the dependent variable being the number of correct analogical – explicit and implicit – solutions on test session 1. We included both Category 1 and Category 2 types of solutions in this analysis because these two types of solutions are both considered correct analogical solutions. An overview of the mean number of correct analogical solutions per session and condition is displayed in Table 3. The results showed no significant effect for condition, indicating that children in the two conditions did not differ with respect to their levels of analogical reasoning at the start.

Next, we conducted a one within (Session: test sessions 1–4) and one between (Condition: not trained and trained) repeated-measures ANOVA to investigate whether the two conditions changed their use of analogical strategies over time. The dependent variable in the analysis was the number of correct analogical—explicit and implicit—solutions (Category 1 and 2) for each test session (see Table 3). The results showed a statistically significant session

Table 2. Examples of children's descriptions of their own reasoning

Category	Example	Description
Non-analogical	1	Here you can see a hexagon (points to C). Therefore I draw the same in here (points to D) [child copied the C-term]
	2	Here I see a square and there I see a pentagon (points to the figures in A). I draw the same in here (points to the figures in D) [child copied the A-term]
	3	A circle with a pentagon in it (points to C). I did the same here (points to D) [child copied the C-term]
Incomplete	1	Here circle (points to A) and half circle (points to B). There hexagon (points to C) and a bowl (points to D) [child draw a half hexagon; lower half]
	2	Look, here (points to B) you see a square with a star in it and there (points to A) you can see a square with no star. And there (points to the pentagons in B) you can see 2, and there (points to pentagon in A) you see only one. Here (points to oval in C) is also only one. Therefore, I have 2 of them (points to 2 ovals in D) [child draw the correct transformations, but no star in the triangle]
	3	Look, here is an oval in a hexagon (points to the figures A). And there (points to the figures in B), there are two of those things in a big oval. That is why I have drawn 2 circles in a big pentagon [child draw only one of the transformations]
Implicit	1	Circle (points to A), half circle, half circle (points to B). There is a hexagon (points to C). Half hexagon, half hexagon (points to D) [child draw the correct figures]
	2	Square, pentagon (points to the figures in A). Square, star, pentagon, pentagon (points to the figures in B). Triangle, oval (points to the figures C). therefore, triangle, oval, oval [Child draw the correct figures; with a star in the triangle]
	3	Here I see a hexagon with an oval in it, and there a triangle (points to the figures in A). And here I see another triangle, but there is an arrow in it (points to B). And here I also see 2 small hexagons with a big oval around it (points to B). Then I did the same with these figures (points to C) and got this (points to D) [Child draw the correct figures]
Explicit	1	Here is a circle (points to A), and there are two half circles (points to the figures in B). And there (points to C) I can see a hexagon. Therefore, I did the same here and draw 2 of these (points to the 2 halves of the hexagon drawn in D) [Child draw the correct figures]
	2	Here is a square and a pentagon (points to the figures in A). Here is also a square, but with a star in it (points to B). And there are also two pentagons. Here is a triangle and there one oval (points to the figures in C) and look there is no star in it (points to the triangle in C). That is why I draw a triangle with a star in it here, and there 2 ovals (points to the figures in D) [Child draw the correct figures]
	3	An oval with a hexagon around it, and a triangle (points to the figures in A). Circle with a pentagon and a square (points to the figures in C). Here is an arrow in the triangle and a big oval around the two hexagons (points to the figures in B). That is why (points to D) square with an arrow in it here, and pentagon with 2 circles in it there [Child draw the correct figures]

Note: Examples 1, 2 and 3 refers to the examples in Appendix 2A.

effect, Wilks' $\lambda=.33$, $F(3, 32)=21.28$, $p<.001$, partial $\eta^2=.67$, but, more important, there was also a statistically significant interaction effect between session and condition, $\lambda=.51$, $F(3, 32)=10.44$, $p<.001$, partial $\eta^2=.50$.

Table 3. Means and standard deviations of the number of analogical responses per condition and session

Condition	Session			
	1	2	3	4
<i>Not trained</i>				
M	3.17	4.83	5.17	5.06
(SD)	(4.73)	(6.46)	(6.95)	(6.70)
<i>Trained</i>				
M	3.11	5.78	12.06	12.00
(SD)	(3.41)	(6.39)	(6.22)	(6.22)

The results of follow-up repeated contrasts evaluating the session effect revealed that there was a difference in the number of analogical solutions between test sessions 1 and 2, and between test sessions 2 and 3, $F(1, 34)=17.85$, $p<.001$, partial $\eta^2=.34$ and $F(1, 34)=28.85$, $p<.001$, partial $\eta^2=.46$, respectively. As can be seen in Table 3, the means for the second test session were higher relative to the means for the first test session, and the means for the third test session were higher relative to the means for the second test session, indicating an improvement in analogical solutions from test sessions 1 to 3. No significant difference was found between test sessions 3 and 4, indicating that children's analogical performance stabilized from test sessions 3 to 4.

Investigation of the session by condition interaction effect using repeated contrasts follow-up analyses, revealed that there was a difference among conditions between test sessions 2 and 3, $F(1, 34)=23.32$, $p<.001$, partial $\eta^2=.41$. The trained children showed greater improvement in their use of analogical strategies between test sessions 2 and 3 relative to the not-trained children, who hardly improved their analogical performance between these two test sessions ($ES_{\text{trained/not-trained}}=.96$)¹, as can be seen in Figure 1. These findings led us to conclude that practice alone led to an increase in the use of analogical strategies between test sessions 1 and 2. However, the short training procedure led to an improvement in analogical reasoning between test sessions 2 and 3 over and above that of practice alone.

Because averaging data over strategies may distort conclusions about several aspects of children's performance (Siegler, 1987), we also examined the performances of the two conditions generated by each strategy separately. On this account, we conducted 4 separate one within (Session: test sessions 1–4) and one between (Condition: not trained and trained) repeated-measures ANOVAs. The dependent variables in the separate analyses were the number of explicit correct analogical (category 1), implicit correct analogical (category 2), incomplete analogical (category 3) and non-analogical, associative (category 4) solutions on each of the 4 test sessions. The course of each strategy over test sessions can be seen in Figure 2.

1 $ES_{\text{trained/not-trained}}$ represents the standardized mean difference (d) between the scores of test sessions 3 (after the training) and 2 (before the training) for the trained group minus the standardized mean difference (d) between the scores of test sessions 3 and 2 for the not-trained group.

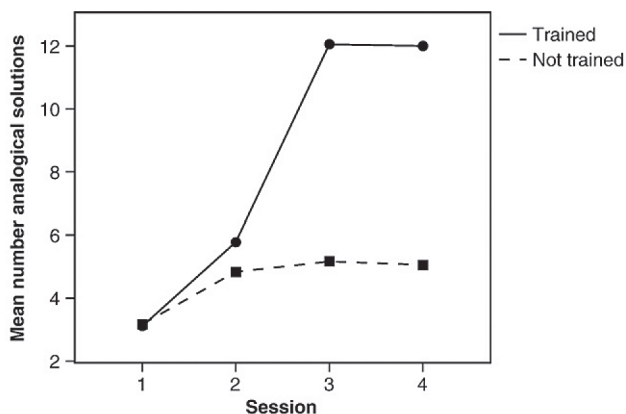


Figure 1. Changes in the number of correct analogical solutions by condition.

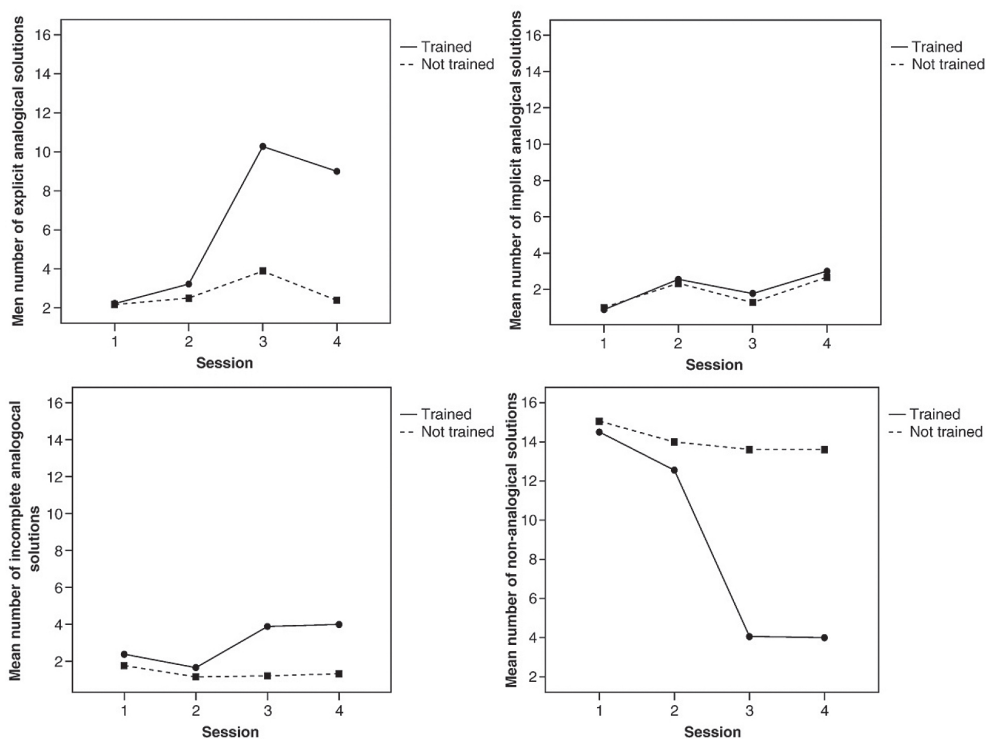


Figure 2. Changes in the number of explicit analogical, implicit analogical, incomplete analogical, and non-analogical solutions by condition and session.

Explicit analogical solutions

The results of the first ANOVA, the analysis on the number of explicit correct analogical solutions, showed a statistically significant effect for both session and the interaction between session and condition, Wilks' $\lambda=.46$, $F(3, 32)=12.30$, $p<.001$, partial $\eta^2=.54$ and Wilks' $\lambda=.60$, $F(3, 32)=7.04$, $p=.001$, partial $\eta^2=.54$, respectively. Evaluation of the session effect with repeated

contrast analyses showed that there was a difference among the means for test sessions 2 and 3 and for test sessions 3 and 4, $F(1, 34)=31.57$, $p<.001$, partial $\eta^2=.48$ and $F(1, 34)=6.94$, $p=.013$, partial $\eta^2=.17$, respectively. The difference between test sessions 1 and 2 was almost significant, $F(1, 34)=3.63$, $p=.065$, partial $\eta^2=.10$. Investigation of the session by condition interaction effect revealed a difference among conditions only between test sessions 2 and 3, $F(1, 34)=14.22$, $p=.001$, partial $\eta^2=.30$. As can be seen in Figure 2, these results suggest that children, regardless of condition, used the explicit analogical strategy slightly more frequently from test sessions 1 to 2. From tests sessions 2 to 3 both conditions further improved their use of explicit solutions, but the improvement was considerably greater for the trained condition. From test sessions 3 to 4 the two conditions showed a slight decrease in their use of the explicit analogical strategy.

Implicit analogical solutions

The results for the second ANOVA, the analysis on the number of implicit correct analogical solutions, also showed a statistically significant effect for session, Wilks' $\lambda=.75$, $F(3, 32)=3.63$, $p=.023$, partial $\eta^2=.25$, but not for the interaction between session and condition. Evaluation of the session effect using repeated contrast analyses indicated that, regardless of condition, there was a statistically significant difference in the number of implicit analogical solutions between test sessions 1 and 2, and between test sessions 3 and 4, $F(1, 34)=7.65$, $p=.009$, partial $\eta^2=.18$ and $F(1, 34)=6.66$, $p=.014$, partial $\eta^2=.16$, respectively. These results suggest that the use of the implicit analogical strategy improved through practice alone; the short training procedure had little additional effect on its use since both conditions changed their number of implicit analogical solutions approximately the same during the period of the study.

Incomplete analogical solutions

The results of the ANOVA on the number of incomplete analogical solutions showed a statistically significant effect for session only, Wilks' $\lambda=.72$, $F(3, 32)=4.22$, $p=.013$, partial $\eta^2=.39$. The results of repeated contrast analyses evaluating this session effect revealed that the effect was due to an improvement in the number of incomplete analogical solutions between test sessions 2 and 3, $F(1, 34)=7.02$, $p=.012$, partial $\eta^2=.17$. As can be seen in Figure 2, the trained condition showed more frequently incomplete solutions than the not trained condition, particularly during test sessions 3 and 4. Further inspection of the data revealed that with this progression, children in the trained condition decreased their use of non-analogical, associative strategies even more.

Non-analogical solutions

The results of the ANOVA on the number of non-analogical solutions revealed a statistically significant effect for session as well as for the interaction between session and condition, Wilks' $\lambda=.41$, $F(3, 32)=15.19$, $p<.001$, partial $\eta^2=.59$ and Wilks' $\lambda=.56$, $F(3, 32)=8.35$, $p=.001$, partial $\eta^2=.44$, respectively. Evaluation of the session effect showed that there was a difference in the use of non-analogical strategies between test sessions 1 and 2 and between test sessions 2 and 3, but not between test sessions 3 and 4, $F(1, 34)=9.63$, $p=.004$, partial $\eta^2=.22$ and $F(1, 34)=23.87$, $p<.001$, partial $\eta^2=.41$, respectively. The results of follow-up analyses evaluating the session by condition interaction effect revealed that the effect was due to a difference among conditions between test sessions 2 and 3 only, $F(1, 34)=19.88$, $p<.001$, partial $\eta^2=.37$. This result indicates that practice alone led to a rather continuous decrease in the use of non-analogical strategies from test sessions 1 to 3, as can be seen in Figure 2. However, the short

training procedure decreased the use of these inappropriate, non-analogical strategies even more. After the third session the use of non-analogical strategies stabilized.

Analyses at the individual level

In order to gain more detailed information on the process of change in the use of analogical strategies, we also investigated the data on analogical reasoning at the individual level. We were particularly interested in whether the data for the individual children of both conditions revealed specific patterns of change over the 4 test sessions. We therefore observed for each individual child the number of correct analogical – implicit and explicit – responses on each of the 4 test sessions. Next, we investigated change in the distributions of their response categories from test sessions 1 to 4. If a child progressed 15% (3 out of the 20 items) or more from one test session to another, this was considered an improvement in analogical reasoning. Various patterns of improvement in analogical reasoning were then identified within the two conditions. Children with a similar pattern of improvement were grouped together. These subgroups of children took varying routes in the acquisition of analogical strategies to solve geometric tasks (see Table 4).

As can be seen in Table 4, within the not-trained condition, three subgroups of children with varying patterns of change in analogical reasoning were identified. The first subgroup consisted of 10 children who made no progression at all during the period of the study. These children practically only used non-analogical strategies across the 4 test sessions. A second subgroup consisted of 7 children who progressed from test sessions 1 to 2 only. Finally, there was 1 child who made no progression because he scored high from the beginning.

Table 4 also displays that within the trained condition 4 subgroups of children with varying patterns of change in the use of analogical strategies were identified. The first subgroup consisted of 2 children who, despite the short training procedure, only used inadequate, non-analogical strategies during the 4 test sessions. The second subgroup consisted of 3 children who improved considerably through practice from test sessions 1 to 2, but did not progress any further after the short training procedure. The third subgroup consisted of 9 children who made no progression from test sessions 1 to 2. However, after the short training procedure this subgroup made considerable progression during the third test session. The fourth subgroup consisted of 4 children who progressed from test sessions 1 to 2, and after the short training procedure made additional progress during test session 3.

Table 4. Number of children per subgroup and condition

Condition ¹	No improvement/ low scores	Improvement between sessions 1–2 ²	Improvement between sessions 2–3 ³	Improvement between sessions 1–2–3 ⁴	No improvement/ high scores
Not trained	10	7	-	-	1
Trained	2	3	9	4	-

Note: ¹N=18 per condition, ²Improvement due to practice alone, ³Improvement due to training alone, ⁴Improvement due to practice (1–2) and training (2–3).

Next, it was important to investigate the distributions of the various response categories of the individual children within each of the subgroups more closely in order to say something about the pattern of changes in the use of analogical strategies within individuals. For the not-trained condition only the distribution of the response categories of the second subgroup was examined, because the children in the other two subgroups made no improvement. For the trained condition, we examined only the distributions of response categories of subgroups 3 and 4, since the first subgroup made no improvement and the second subgroup only consisted of 3 children of which two children reached near maximum score during test sessions 2 to 4.

Not-trained condition

The distributions of the response categories of the individual children in the second subgroup (increases from test sessions 1 to 2) of the not-trained condition are displayed in Figure 3. As can be seen in this figure, there was much within-child variability within this subgroup. Moreover, the children who relatively often showed incomplete analogical responses during

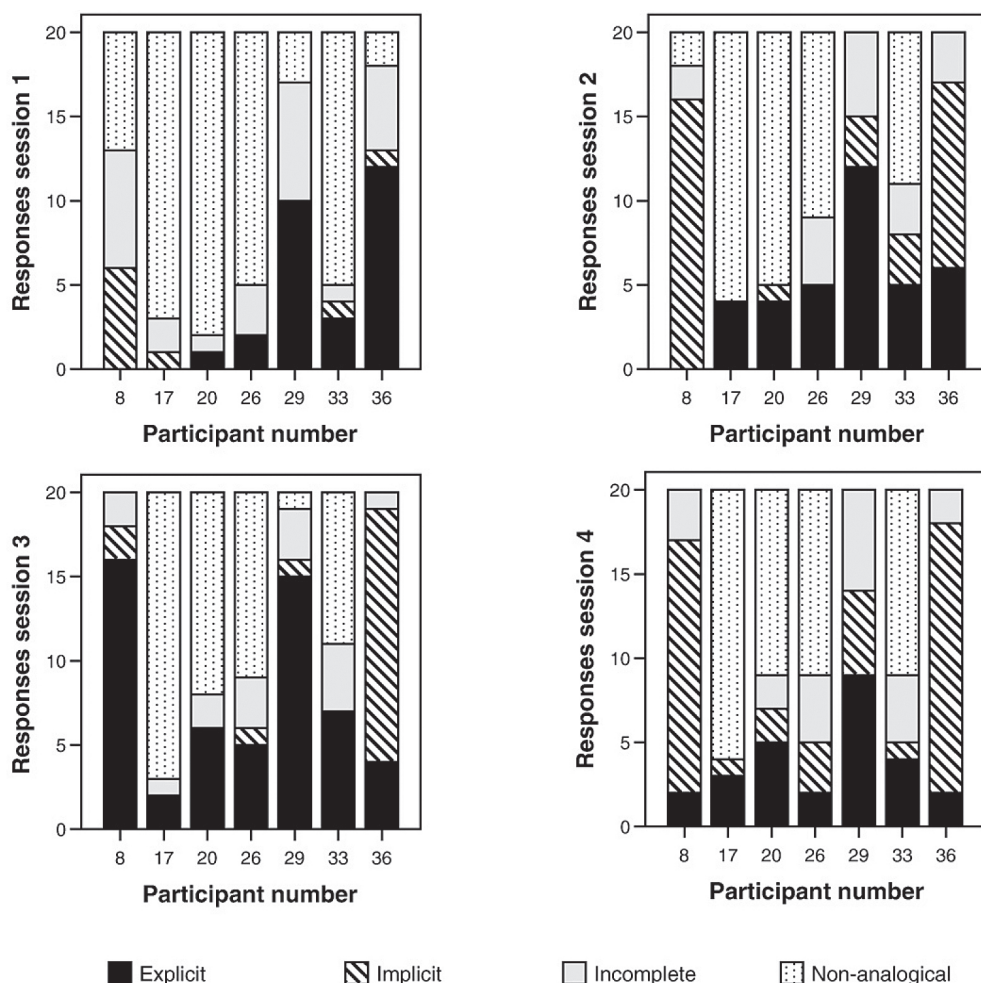


Figure 3. Number of responses per category, session and child of subgroup 2 of the not-trained condition.

the first test session, made the most improvement in test session 2. This observation suggests that the earlier described practice effect between these two test sessions was mostly due to a progression from incomplete to complete – either explicit or implicit – analogical solutions.

Trained condition

The distributions of the response categories of the individual children in the third subgroup (increases from test session 2 to 3) of the trained condition are displayed in Figure 4. This figure reveals that the 5 children (6, 12, 13, 22, and 31) who showed some analogical reasoning in the first 2 test sessions showed a considerable increase in analogical – either complete or incomplete – solutions from the second to the third test session. After test session 3 their response patterns stabilized. However, the remaining 4 children (3, 4, 24 and

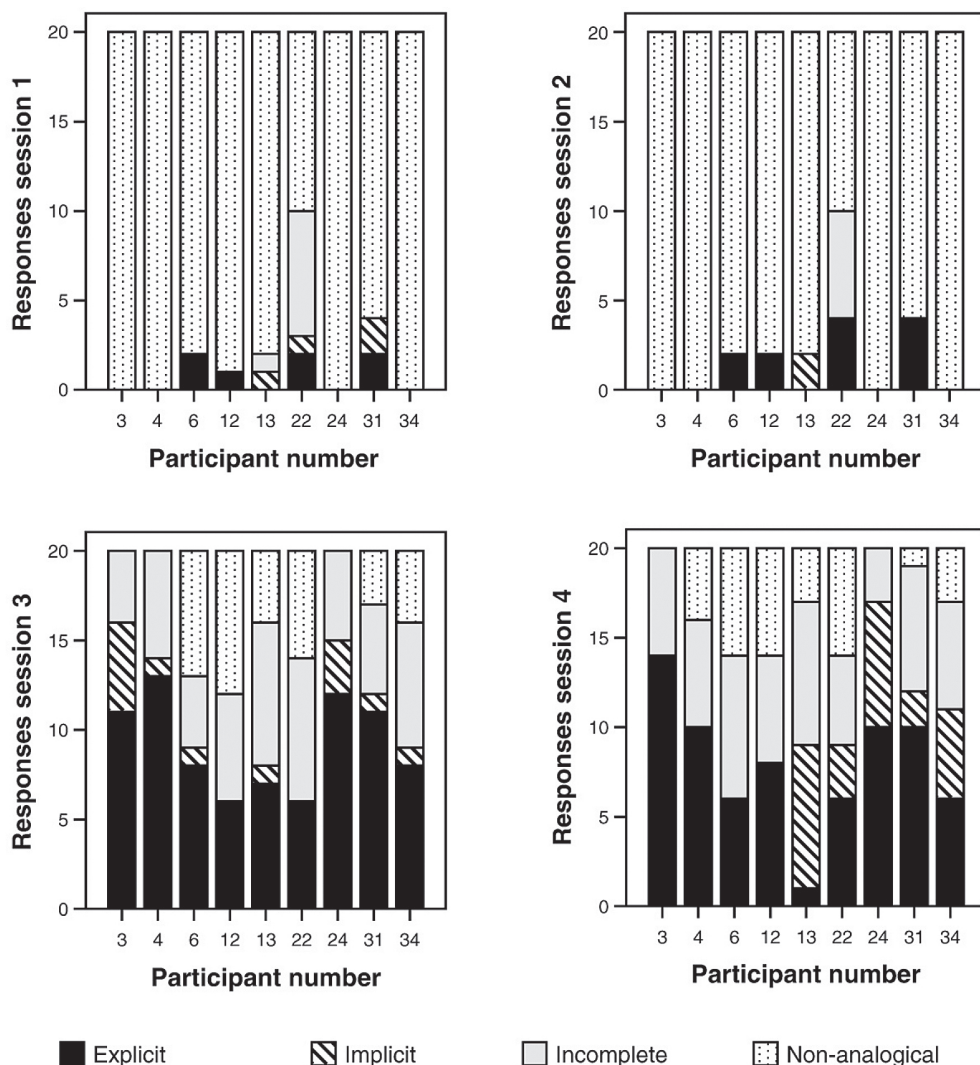


Figure 4. Number of responses per category, session and child of subgroup 3 of the trained condition.

34) in this subgroup went from completely non-analogical to approximately 15 complete plus 5 incomplete analogical solutions. A rather rapid change in analogical thinking was thus made at one time. Therefore it can be concluded that the short training procedure had a different effect on the process of change in the use of analogical strategies in children at this age.

The distributions of the response categories of the individual children in the fourth subgroup (increases from test session 1 to 2 and test sessions 2 to 3) of the trained condition are displayed in Figure 5. This figure shows that all children in the fourth subgroup increased in all analogical – either complete or incomplete – response categories from test sessions 1 to 2.

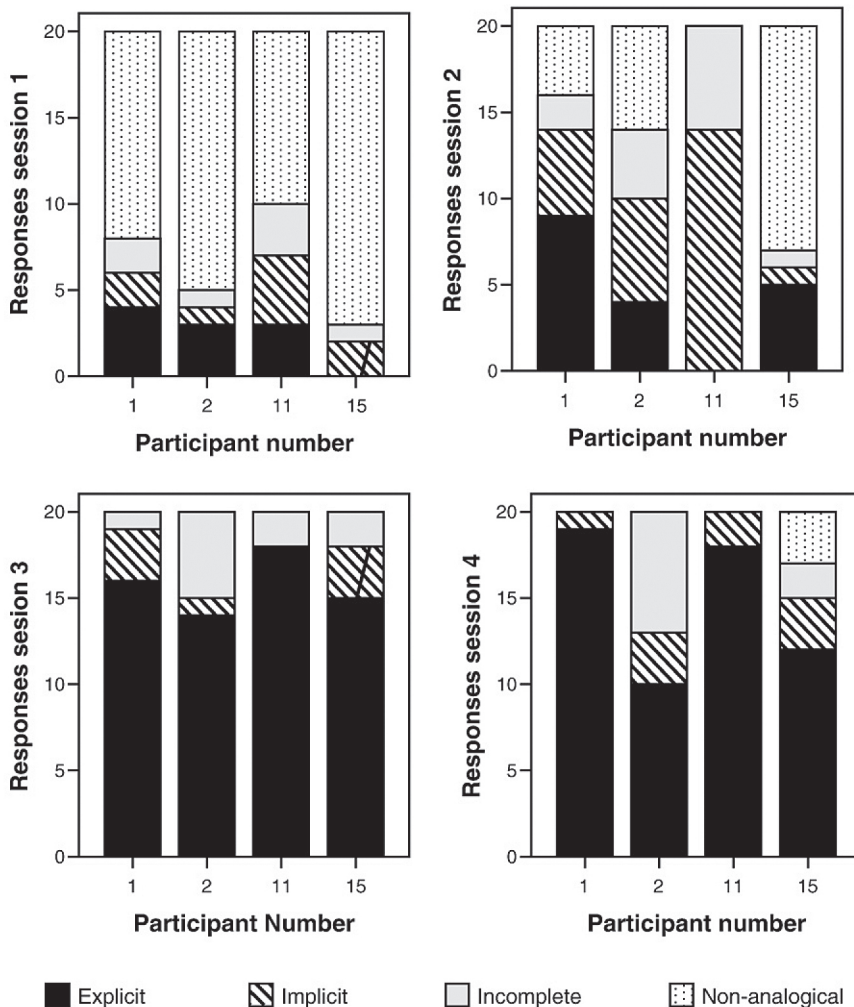


Figure 5. Number of responses per category, session and child of subgroup 4 of the trained condition.

This improvement can be attributed to practice. From test sessions 2 to 3 they particularly increased their number of explicit analogical solutions. Apparently, the short training procedure led these children to become more explicitly analogical. It can also be seen that these children did not give non-analogical responses any longer during this third test session; they gave incomplete analogical responses instead. During the fourth test session, the facilitating effect of the short training procedure continued for 3 of the 4 children.

Because the age range of the participants seems a bit wide, we conducted several Mann-Whitney U tests to evaluate whether the observed differences in response patterns over sessions among the various subgroups were due to age. For the not-trained condition, we compared subgroups 1 versus 2 with respect to age. For the trained condition, we compared the ages of subgroups 1 versus 2, subgroups 1+2 versus 3, subgroups 3 versus 4, and within subgroup 3 for children who showed some analogical reasoning on the test sessions 1 and 2 versus children who showed no analogical reasoning on the first two test sessions. We did not find any significant difference among the various subgroups with respect to age, indicating that the observed differences in responding over time among the various subgroups were not due to age.

Next, we conducted several Mann-Whitney U tests to examine possible differences among the subgroups with respect to inductive reasoning and memory (concrete and abstract) capacity. Results of all comparisons were not significant, indicating that the observed differences between the various subgroups in responding over time were not due to children's inductive reasoning or memory capacity either.

Change in inconsistent analogical reasoners

Next, we examined the question whether the short training procedure had a greater effect on children showing variable, inconsistent analogical reasoning over trials than in children not showing this kind of behavior. To answer this question we divided the trained children, based on their scores in test session 2 (before the training), into three groups: non-/weak-analogical reasoners, children who had less than 25% of the items correct; variable, inconsistent analogical reasoners, children who had 25%–75% of the items correct; and strong analogical reasoners, children who had more than 75% of the items correct. Then, we conducted an ANCOVA, with the dependent variable being the number of correct analogical responses (category 1 and 2) in test session 3 (after the training) and the covariate being the number of correct analogical responses (category 1 and 2) in session 2 (before the training). The independent variable consisted of two levels: non-/weak-analogical reasoners ($N=11$) and variable analogical reasoners ($N=5$). The third group (strong analogical reasoners) was not included in this analysis since these children could not improve much more. A preliminary analysis evaluating the homogeneity-of-slopes assumption indicated that the relationship between the covariate and the dependent variable did not differ significantly as a function of the independent variable, $F(1, 12)=.02$, $MSE=26.40$, $p=.89$, partial $\eta^2=.00$. The results of the ANCOVA were non-significant, indicating that the short training procedure did not have a greater effect on children showing variable, inconsistent analogical reasoning over trials than on children not showing this kind of behavior.

Duration of the effects

The next question related to whether the improvement in the use of analogical strategies observed in the microgenetic study lasted over a longer period of time. For the purpose of this issue all children – not trained and trained – were seen in a follow-up test session 3

month after test session 4. A one within (Session: test sessions 4 and follow-up test session) and one between (Condition: not trained and trained) repeated-measures ANOVA, with the dependent variable being the sum of correct analogical – explicit and implicit – responses for each session, was conducted. The results of the analysis associated with the within-subject effects showed a statistically significant effect for session, Wilks' $\lambda=.82$, $F(1, 34)=7.39$, $p=.01$, partial $\eta^2=.18$, but no interaction effect. The means of both the trained and the not-trained condition were higher on the follow-up test session ($M=13.06$, $SD=6.10$ and $M=6.06$, $SD=6.81$, respectively) than on test session 4 ($M=12.00$, $SD=6.22$ and $M=5.06$, $SD=6.70$, respectively), indicating an increase in the use of analogical strategies over a period of 3 months for both conditions. Since there was no interaction effect, it can be concluded that children in both conditions improved their analogical performance in a similar manner. The results of the analysis associated with the between subject effects showed statistically significant differences in the use of analogical strategies among the two conditions, $F(1, 34)=10.80$, $p=.002$, partial $\eta^2=.24$. Observation of the means revealed that the trained children ($M=12.53$, Std. Error=1.5) gave considerably more analogical responses than the not-trained children ($M=5.56$, Std. Error=1.5) on the two sessions combined.

Relation between variables

The next question concerned whether children's analogical reasoning performance was related to their memory or inductive reasoning scores. We computed correlational coefficients among the analogical reasoning measure (represented by the number of correct analogical explicit and implicit solutions for the first—two test sessions before the training—and second—two test sessions after the training—half of sessions) and the inductive reasoning and memory (concrete and abstract) measures. The correlational coefficients were computed for the two conditions separately to eliminate conditions from the relations, allowing us to make comparisons across conditions. The results showed for the two conditions non-significant correlations among the inductive reasoning and analogical reasoning scores on both halves of sessions. The correlations among the concrete memory and analogical reasoning scores on the two halves of sessions were also non-significant for the two conditions. However, for the relation between the abstract memory and analogical reasoning scores, we found statistically significant correlations for the trained children on the second half of sessions ($r_{\text{pearson}}=.678$, $p=.02$). These results suggest that the trained children who were scoring high on the abstract memory measure also tended to score high on the analogical reasoning measure during the two test sessions after the short training procedure.

2.4 Discussion

The main purpose of this study was to gain insight into the nature of young children's analogical reasoning ability by investigating whether the analogical performance on geometric tasks of 6–8 year-old first-grade children changed due to repeated practice alone, without explicit prompting, and whether a short training procedure that provided children with some explicit modeling and feedback added to this effect. As stated in the introduction, several studies on children's analogical reasoning ability have been conducted in the past, but only one study (Tunteler & Resing, 2007a) has focused on a comparison of the path of natural, unprompted changes in analogical reasoning performance with that of changes induced by a short training procedure. The present study differs from the Tunteler and Resing (2007a)

study in that it focused on geometric tasks, rather than problem analogy tasks. In addition, the short training procedure in our study was given between test session 2 and 3, which allowed us to investigate changes in children's analogical performances both before and after the short training procedure.

The results of our microgenetic study revealed new insights into the nature of changes in analogical reasoning on geometric analogical tasks in grade 1 level children. One of our findings was that the analogical performances of children of this grade level changed spontaneously as a result of practice alone, without any helpful instruction to the task or prompting. According to Siegler (2006), this type of change may be considered as natural because it does not arise from explicit interventions. The finding that practice alone was sufficient to activate the use of analogical strategies on geometric tasks then suggests that analogical reasoning skills must have been already present, though in a more rudimentary form, in the repertoire of children of this age and that the opportunity to practice accelerated its spontaneous use. This result is consistent with earlier findings of Tunteler and Resing (2002, 2007a,b), who demonstrated that 4- to 8-year-old children spontaneously improved their use of analogical strategies to solve problem analogy tasks after practice experiences. Similar results have also emerged from studies that have examined young children's knowledge of geometric transformations in the plane; Logo experiences have been shown to increase young children's awareness of geometric figures and their ability to communicate about spatial ideas (see Clements & Battista, 1992; Clements, Battista, Sarama & Swaminathan, 1997).

Another finding of the study is that spontaneous, unprompted changes in analogical reasoning performance was most prominent between test sessions 1 and 2; the analogical performance of the not-trained children remained rather stable during test sessions 3 and 4. In a study by Alexander et al. (1989), similar changes in geometric analogical reasoning were found in preschoolers and kindergartners. The authors suggested that this improvement was due to familiarity with the test-items, but they gave the children in their study extensive instructions to the tasks. As instructions may be seen as a form of training, one may object that the increase in analogical performance observed in the Alexander et al. study may also be attributed to the instructions repeatedly given to the children. In our study, the geometrical analogy tasks were given with no instruction other than that the tasks were like a 'puzzle'; neither was any feedback given. Therefore, our results clearly demonstrate that young children can indeed change their knowledge and strategy for analogy by experience alone, confirming that it is well within their developmental range of growth to grasp analogical ideas related to geometric shapes.

Our results were interesting in the light of other microgenetic studies, which tended to reveal that changes in a cognitive competency is a result of a changing distribution of various strategies of varying adequacy (Kuhn, 1995; Siegler, 2006). In this study, changes in the distribution of the various response categories were analyzed at both the group and the individual level. Analyses at the group level indicated that the natural, unprompted increase in analogical performance observed between test sessions 1 and 2 was greatly due to an increase in implicit correct analogical solutions. More in-depth analysis of the patterns of change in the use of analogical strategies within individuals of the not-trained condition added to this finding that approximately half of the not-trained children did not progress during the study period, while another half of the children did. The distributions of the response categories of this latter subgroup showed considerable variability in task performance within individuals, referring to diverse and variable strategy use within as well as across trials. This finding is consistent with findings of earlier studies with problem analogy tasks (Tunteler & Resing,

2002, 2007a,b). The children who relatively often showed incomplete analogical responses during the first test session, made the most improvement during test session 2, suggesting that the observed increase in analogical performance was mostly due to a progression from incomplete to complete – either implicit or (to a less extent) explicit – analogical solutions. Although not the focus of the study, this finding seems to provide evidence for the propositions that children make unconscious discoveries of analogical strategies, which subsequently becomes conscious analogical reasoning. This observation confirms research from Siegler and Stern (1998) who found similar results with children in a study on strategy discovery on an arithmetic task. However, it should be noted that the not-trained children in our study were not given any instruction or feedback with respect to their solutions. They may therefore have executed an analogical strategy at a conscious level, but did not report it because they did not know that it was better to do so. Therefore, further research is recommended to investigate this issue in more detail.

With respect to the paths of change of the trained children, the results showed that the short training procedure caused an increase in the use of analogical strategies on geometric tasks that could not be explained by practice alone. Examination of the performance of this group of children generated by each strategy separately revealed that the short training procedure particularly had an effect on children's use of explicit correct analogical strategies and, although to a lesser extent, on their use of incomplete analogical strategies. Our findings suggest that children after the short training procedure mastered their correct responses mostly explicitly and when they were not up to a complete analogical solution, they solved the item partly analogically without entirely reverting back to an associative response. These changes in the use of analogical strategies observed in the trained group tended to go together with a rather rapid decrease in the use of non-analogical, associative responses, while a rather continuous decrease in the use of this non-analogical strategy was observed in the not-trained group.

More in-depth analysis of the patterns of changes within individuals of the trained condition revealed a varied picture. Within this condition, there were 2 children who remained consistent non-analogical, regardless of the short training procedure, and 3 children who spontaneously improved their analogical reasoning through practice alone. Another 4 children improved through practice and also through the short training procedure, after which they particularly became more explicitly. These children showed a gradual change in their distribution of correct analogical responses over time, confirming the microgenetic picture of learning proposed by Kuhn (1995). The remaining 9 children made a relatively larger change towards analogical reasoning after the short training procedure. Surprisingly, in this subgroup, it were not the children with variation in strategies that showed the most improvement, but the children exhibiting only non-analogical strategies during the first two practice sessions. This latter group of children even went from completely associative reasoning to consistent analogical reasoning after the short training procedure, indicating a rather rapid change within one time point. These results provide evidence for Siegler's observation (2006) that microgenetic studies tend to show a relatively large number of children going through a gradual change in their rate of discovery and generalization of a cognitive strategy, while a smaller number shows a more rapid change in this respect. They also challenge the position that analogical reasoning on geometric tasks is an age related competence which can not be induced by training in children only showing non-analogical, associative reasoning (Hosenfeld, Van der Maas, et al., 1997b). Apparently analogical reasoning was already present in the cognitive processing abilities of these young children, but needed some prompting.

Taken together, consistent with results of Tunteler and Resing (2007a) with problem analogy tasks, our results showed considerable inter and intra-individual variability in the use of analogical strategies in both not-trained and trained first-grade children. According to Siegler (2006) such intra-individual variability in strategy use has been shown to predict later learning substantially. The findings described above provide evidence for this position for the not-trained group. Within this group, a natural increase in analogical reasoning was evidenced in children showing variable, diverse strategies on the first test session, whereas children practically only showing non-analogical, associative reasoning did not change their performance over time. However, no conclusive evidence was found for the trained group. The short training procedure did not only induce change in the analogical performances of children initially showing variable analogical reasoning, but also in children only showing non-analogical, associative reasoning during the test session prior to the training session. Moreover, analysis at the group level showed that the short training procedure did not have a greater effect on children who displayed variable analogical reasoning, defined by having 25%–75% correct analogical solutions on the test session prior to the training session, than on children not showing this kind of behavior. It should however be noted that this outcome should be interpreted with some caution, since the groups in this analysis were rather small and of unequal sizes and the age range of the participants was a bit wide. Replication of these results is therefore needed to verify and extend our results.

Another result of the study is that both the trained and not-trained children improved their analogical performances in a follow-up test session that was conducted after a retention period of 3 months after test session 4. It is not clear whether this improvement occurred as a result of our experimental procedure or because of another variable. More important however, this finding indicates that the observed progress in analogical performance lasted over a more extended period of time, over both conditions in the study. Apparently, changes in analogical reasoning obtained through experience or a short training procedure do not only persist over a period of time during which children are repeatedly given practice opportunities, as Tunteler and Resing (2007a,b) showed earlier with problem analogy tasks, but also last over a longer period of time during which children are not given these experiences. This observation then suggests that these young children had incorporated the analogical strategy within their existing set of strategies for solving geometric analogy tasks.

Finally, in an attempt to investigate the role memory capacity and inductive reasoning skills may play in the development of analogical reasoning, we investigated whether children's analogical reasoning performance was related to their memory (abstract and concrete) or inductive reasoning scores. Our results only revealed a relation between children's analogical reasoning scores and their scores on the abstract memory test for the trained children during the test sessions after the training session. The analogical items used in our studies consisted of geometric shapes of various difficulty levels. The difficult items contained more information than the easier ones, but they could be solved in small steps by reducing them to smaller sets of information. The short training procedure consisted of a standardized step by step procedure, which prompted the child to explain the reasoning behind the experimenters' correct analogical solution. We suggest that the short training procedure made children more aware of the fact that they could solve the analogical tasks in small steps, reducing the amount of information that had to be processed in parallel, allowing them to solve analogical items of increasing levels of difficulty. The finding that this relation was only found for the trained children during the test sessions after the training provides evidence for this position.

Although individual differences and variability in the acquisition of novel strategies are not unique for the area of analogical reasoning (e.g., Kuhn, 1995; Siegler, 1995, 2006), it is important to consider these issues with respect to the acquisition of analogical strategies on geometric tasks more closely in future studies. At this moment it is not clear why some of the children in our study increased their use of analogical strategies through practice, some others through training, again some others through a combination of the two, while still some others did not progress even despite of the short training procedure. One objection may be that the age range of the participants was a bit wide, and that these children therefore may have been in a different developmental range of growth with respect to analogical reasoning. Yet, it should be noted that no age differences were found among the various subgroups with different patterns of responding over time. Explanations in terms of a lack of inductive reasoning skills or memory capacity are not sufficient either, because our results showed no differences in this respect among the various subgroups.

It is also not clear why the children who only gave associative responses prior to the short training procedure, improved their analogical reasoning performance more during the unprompted test sessions after the short training procedure, than their peers who already showed some analogical reasoning prior to the short training procedure. Apparently, the absence of any partially formed strategy among these children allowed them to adopt strategies they had learned about as they tried to explain the steps taken by an interviewer as she solved the tasks. A possible explanation for this finding could therefore be that children who already solved some analogical items correctly may have improved less, because learning a completely new strategy may be easier than integrating a more advanced strategy with an old, less advanced one or replacing a less advanced strategy with a more advanced one. This observation might then indicate that correctly instructing young children from the beginning, before giving them the opportunity to practice, might prove a better way to accelerate the process of acquisition and generalization of analogical strategies in some children, because correctly instructing them from the beginning might prevent interference of less adequate or deficient analogical strategies. This result has important implications for geometry education as it clarifies how 6-8 year old children from first grade can address logical operations on spatial objects through analogies. Yet, the data here came from one experiment and the subgroups consisted of relatively small numbers of children; any conclusions drawn from these data must be treated with the necessary caution. We therefore recommend further research into young children's use of analogies on tasks involving geometric transformations.

It was clear from our data that the short training procedure had an additional effect above practice alone on a considerable number of children of this age. One may then question what factors could have led to this effect. In essence, our short training procedure consisted of modeling and a step by step procedure of prompting children to self-explain all the steps of correct analogical reasoning proposed by Sternberg and Rifkin (1979) and Resing (1990). As discussed above, the increase in analogical performance due to the short training procedure observed in this study was mostly due to an increase in explicit correct analogical responses, meaning that children not only solved the analogical item correctly, but also verbally explained correct analogical reasoning for their solutions without explicitly being told to do so. Additionally, besides the increase in complete analogical responses, there was also an increase in incomplete analogical responses after the short training, while completely associative responses were nearly extinguished. These findings certainly reveal conceptual mastery. We therefore suggest that our results confirm the prediction made by Rittle-Johnson (2006) "... that direct instruction on a correct procedure and conceptual explanation for the

procedure would lead to the greatest learning and transfer if students were also prompted to self-explain” (p. 13). Yet, in this study we did not compare the effects of various types of training on analogical reasoning at this age. Therefore, future research needs to be conducted in which instruction procedures are varied in order to give a conclusive answer to the question cited above. Moreover, it would be advisable to investigate whether similar results can be obtained while instructing children of other ages and also with different types of analogical tasks.

In sum, we can conclude from our results that although repeated practice has a beneficial effect on the natural, unprompted use of analogical strategies to solve geometric tasks by first-grade children, the provision of a short training procedure that provides children with some explicit modeling and feedback adds to this effect. The short training procedure used in our study induced in 9 of the 18 children a continuation of a gradual process of change in the acquisition of analogical strategies, while in 4 others it induced a rather rapid change in analogical reasoning. Since this observation may have important implications for analogical learning in educational settings, it would be good to investigate whether teachers could obtain similar results with their children (Alexander et al., 1987) and whether children will extend their learning to other (analogical) tasks as well. Moreover, it would be interesting to extend this research to research on dynamic assessment in educational settings (Elliott, 2003). It could, for example, be investigated exactly how much instruction children need to improve their analogical reasoning and what this amount of help subsequently means for their readiness to learn by analogy.

Given that the analogical ability is generally assumed to be related to a variety of mathematical competencies besides geometry (e.g., numeric representations, calculation, understanding fractions), our results imply that children’s potential growth on tasks with analogical reasoning may contribute to mathematical development besides geometry. This is an important implication for mathematics education. However, in this study we did not assess transfer to other mathematical competencies. We therefore recommend evaluating this issue in future research.

Appendix 2A

Examples of geometric analogy items of various levels of difficulty (Figure 6).

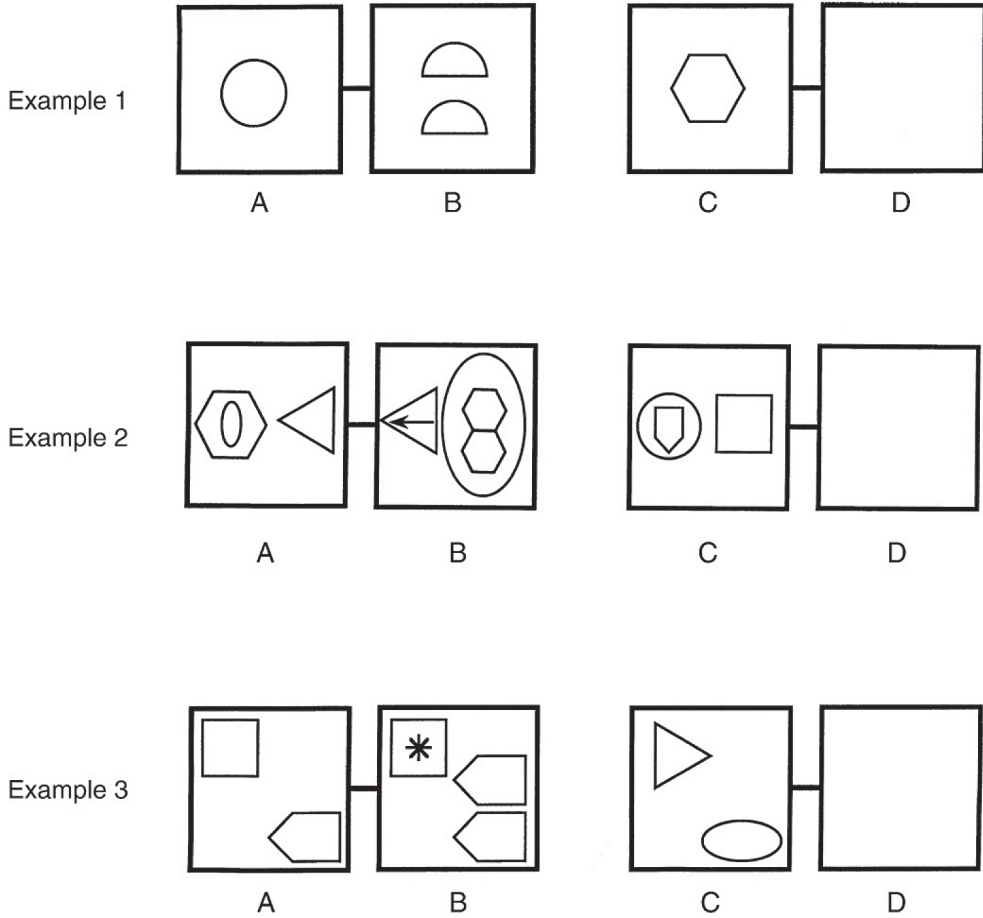


Figure 6.

Appendix 2B

Step by step procedure during the short training.

Step 1. [Encoding]

Question: What did I start with, do you think?

Answer: I first took a few good looks at all the figures in all of these boxes (point to A, B and C).

Step 2. [Inference]

Question: What did I do after that, do you think?

Answer: I then saw that this box (point to A) and this box (point to B) belong together'

Step 3. [Inference]

Question: 'How come that I thought of that, do you think?'

Answer: Experimenter explained for each geometric shape in A separately which transformation was applied to it in B.

Step 4. [Mapping]

Question: Which idea did I come up with next?

Answer: I then thought: "This box (point to A) and this box (point to C) look like each other, because..." (show/explain for all geometrical shapes separately why A looks like C).

Step 5. [Application]

Question: After this, what did I draw in this box (point to D) and why did I do that?

Answer: Well, I thought that I should draw something in this box (point to D) that would make these two boxes (point to C and D) belong to together, just like these two boxes (point to A and B) belong to together.

The experimenter then asked for each geometrical shape in A and B which transformations were applied and explained that the same transformations needed to be applied to the geometrical shapes in C in order to come to the solution drawn in D).

