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Explanatory latent variable modeling of mathematical ability in primary school : crossing the border between psychometrics and psychology

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References

- Abedi, J., & Hejri, F. (2004). Accommodations for students with limited English proficiency in the National Assessment of Educational Progress. *Applied Measurement in Education*, 17, 371-392.
- Abedi, J., & Lord, C. (2001). The language factor in mathematics tests. *Applied Measurement in Education*, 14, 219-234.
- Adams, R. J., Wilson, M., & Wang, W.-C. (1997). The multidimensional random coefficients multinomial logit model. *Applied Psychological Measurement*, 21, 1-23.
- Adams, R. J., & Wu, M. L. (2000). *PISA 2000 technical report*. Paris: OECD.
- Agresti, A. (2002). *Categorical data analysis* (2nd ed.). New York: Wiley.
- Albert, J. H. (1992). A Bayesian estimation of normal ogive item response curves using Gibbs sampling. *Journal of Educational Statistics*, 17, 251-269.
- Ambrose, R., Baek, J.-M., & Carpenter, T. P. (2003). Children's invention of multidigit multiplication and division algorithms. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (p. 305-336). Mahwah, NJ: Lawrence Erlbaum Associates.
- Anghileri, J. (1989). An investigation of young children's understanding of multiplication. *Educational Studies in Mathematics*, 20, 367-385.
- Anghileri, J., Beishuizen, M., & Van Putten, C. M. (2002). From informal strategies to structured procedures: Mind the gap! *Educational Studies in Mathematics*, 49, 149-170.
- Baroody, A. J., & Dowker, A. (Eds.). (2003). *The development of arithmetic concepts and skills: Constructing adaptive expertise*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- Bates, D., & Maechler, M. (2010). *lme4: Linear mixed modeling using S4 classes*.

- (Computer program and manual). Available from <http://cran.r-project.org/web/packages/lme4/index.html>.
- Béguin, A. A., & Glas, C. A. W. (2001). MCMC estimation and some model-fit analysis of multidimensional IRT models. *Psychometrika*, 66, 541-562.
- Beishuizen, M. (1993). Mental strategies and materials or models for addition and subtraction up to 100 in Dutch second grades. *Journal for Research in Mathematics Education*, 24, 294-323.
- Beishuizen, M., Van Putten, C. M., & Van Mulken, F. (1997). Mental arithmetic and strategy use with indirect number problems up to one hundred. *Learning and Instruction*, 7, 87-106.
- Bembenutty, H. (2009). Academic delay of gratification, self-regulation of learning, gender differences, and expectancy-value. *Personality and Individual Differences*, 46, 347-352.
- Berends, I. E., & Van Lieshout, E. C. D. M. (2009). The effect of illustrations in arithmetic problem-solving: Effects of increased cognitive load. *Learning and Instruction*, 19, 345-353.
- Blöte, A., Van der Burg, E., & Klein, A. S. (2001). Students' flexibility in solving two-digit addition and subtraction problems: Instruction effects. *Journal of Educational Psychology*, 93, 627-638.
- Borsboom, D. (2006). The attack of the psychometricians. *Psychometrika*, 71, 425-440.
- Bouwmeester, S., Sijtsma, K., & Vermunt, J. K. (2004). Latent class regression analysis to describe cognitive developmental phenomena: An application to transitive reasoning. *European Journal of Developmental Psychology*, 1, 67-86.
- Buijs, C. (2008). *Leren vermenigvuldigen met meercijferige getallen* [Learning to multiply with multidigit numbers]. Utrecht, The Netherlands: Freudenthal Institute for Science and Mathematics Education.
- Byrnes, J. P., Miller, D. C., & Shafer, W. D. (1999). Gender differences in risk taking: A meta-analysis. *Psychological Bulletin*, 125, 367-383.
- Campbell, J. I. D., Fuchs-Lacelle, S., & Phenix, T. L. (2006). Identical elements model of arithmetic memory: Extensions to addition and subtraction. *Memory & Cognition*, 34, 633-647.
- Carr, M., & Davis, H. (2001). Gender differences in arithmetic strategy use: A function of skill and preference. *Contemporary Educational Psychology*, 26, 330-347.
- Carr, M., & Jessup, D. L. (1997). Gender differences in first-grade mathematics strategy

-
- use: Social and metacognitive influences. *Journal of Educational Psychology*, 89, 318-328.
- CITO. (2005a). *Leerling- en onderwijsvolgsysteem, rekenen-wiskunde groep 3* [Monitoring and evaluation system for primary pupils – Arithmetic and Mathematics, grade 1]. Arnhem, The Netherlands: CITO.
- CITO. (2005b). *Leerling- en onderwijsvolgsysteem, rekenen-wiskunde groep 4* [Monitoring and evaluation system for primary pupils – Arithmetic and Mathematics, grade 2]. Arnhem, The Netherlands: CITO.
- CITO. (2006). *Leerling- en onderwijsvolgsysteem, rekenen-wiskunde groep 5* [Monitoring and evaluation system for primary pupils – Arithmetic and Mathematics, grade 3]. Arnhem, The Netherlands: CITO.
- CITO. (2007). *Terugblik en resultaten 2007. Eindtoets basisonderwijs* [Retrospect and results of the End of Primary School Test 2007]. Arnhem, The Netherlands: CITO.
- CITO. (2009). *Terugblik en resultaten 2009. Eindtoets basisonderwijs* [Retrospect and results of the End of Primary School Test 2009]. Arnhem, The Netherlands: CITO.
- Cohen, J. (1960). A coefficient of agreement for nominal scales. *Educational and Psychological Measurement*, 20, 37-46.
- Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: Lawrence Erlbaum Associates.
- Cummins, D. D., Kintsch, W., Reusser, K., & Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive Psychology*, 20, 405-438.
- De Boeck, P. (2008). Random item models. *Psychometrika*, 73, 533-559.
- De Boeck, P., Bakker, M., Zwitser, R., Nivard, M., Hofman, A., Tuerlinckx, F., et al. (2011). The estimation of item response models with the `lmer` function from the `lme4` package in R. *Journal for Statistical Software*, 39(12), 1-28.
- De Boeck, P., & Wilson, M. (Eds.). (2004). *Explanatory item response models: A generalized linear and nonlinear approach*. New York, NY: Springer.
- De Brauwier, J., & Fias, W. (2009). A longitudinal study of children's performance on simple multiplication and division problems. *Developmental Psychology*, 45, 1480-1496.
- De Corte, E., Verschaffel, L., & De Win, L. (1985). Influence of rewording verbal problems on children's problem representations and solutions. *Journal of Educational Psychology*, 77, 460-470.
- De la Torre, J., & Patz, R. J. (2005). Making the most of what we have: A practical application of multidimensional item response theory in test scoring. *Journal of*

- Educational and Behavioral Statistics*, 30, 295-311.
- De Smedt, B., Torbeyns, J., Stassens, N., Ghesquière, P., & Verschaffel, L. (2010). Frequency, efficiency and flexibility of indirect addition in two learning environments. *Learning and Instruction*, 20, 205-215.
- Dias, J. G., & Vermunt, J. K. (2006). Bootstrap methods for measuring classification uncertainty in latent class analysis. *COMPSTAT 2006 - Proceedings in Computational Statistics, part I*, 31-41.
- Dutch Inspectorate of Education. (2008). *Basisvaardigheden rekenen-wiskunde*. [Basic mathematics abilities.] Den Haag, The Netherlands: Inspectie van het Onderwijs.
- Dutch Ministry of Education, Culture, and Sciences. (1998). *Kerndoelen basisonderwijs 1998* [Educational standards for Dutch primary education]. Den Haag, The Netherlands: Ministerie van OCW.
- Dutch Ministry of Education, Culture, and Sciences. (2006). *Kerndoelen basisonderwijs* [Educational standards for Dutch primary education]. Den Haag, The Netherlands: Ministerie van OCW.
- Ellis, S. (1997). Strategy choice in sociocultural context. *Developmental Review*, 17, 490-524.
- Embretson, S. E., & Reise, S. P. (2000). *Item response theory for psychologists*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Ericsson, K. A., & Simon, H. A. (1993). *Protocol analysis: Verbal reports as data* (rev. ed.). Cambridge, MA: MIT Press.
- Expertgroep Doorlopende leerlijnen Taal en Rekenen. (2008). *Over de drempels met rekenen. Consolideren, onderhouden, gebruiken en verdiepen*. [Crossing the thresholds with mathematics. Strengthen, maintain, use, and deepen.] Enschede, The Netherlands: Expertgroep Doorlopende leerlijnen Taal en Rekenen.
- Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (p. 147-164). New York: MacMillan.
- Fischer, G. H. (1987). Linear logistic test model as an instrument in educational research. *Acta Psychologica*, 37, 359-374.
- Foxman, D., & Beishuizen, M. (2003). Mental calculation methods used by 11-year-olds in different attainment bands: A reanalysis of data from the 1987 APU survey in the UK. *Educational Studies in Mathematics*, 51, 41-69.
- Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht, The Netherlands:

Reidel.

- Freudenthal, H. (1991). *Revisiting mathematics education. China lectures*. Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Fuchs, L. S., Fuchs, D., Compton, D. L., Powell, S. R., Seethaler, P. M., Capizzi, A. M., et al. (2006). The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology*, 98, 29-43.
- Fuchs, L. S., Fuchs, D., Stuebing, K., Fletcher, J. M., Hamlett, C. L., & Lambert, W. (2008). Problem solving and computational skill: Are they shared or distinct aspects of mathematical cognition? *Journal of Educational Psychology*, 100, 30-47.
- Fuson, K. C., Wearne, D., Hiebert, J. L., Murray, H. G., Human, P. G., Olivier, A. I., et al. (1997). Children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 28, 130-162.
- Gallagher, A. M., De Lisi, R., Holst, P. C., McGillicuddy-De Lisi, A. V., Morely, M., & Cahalan, C. (2000). Gender differences in advanced mathematical problem solving. *Journal of Experimental Child Psychology*, 75, 165-190.
- Geary, D. C. (2003). Arithmetical development: Commentary on chapters 9 through 15 and future directions. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (p. 453-464). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research*, 79, 1202-1242.
- Gilmore, C. K., & Bryant, P. (2006). Individual differences in children's understanding of inversion and arithmetical skill. *British Journal of Educational Psychology*, 76, 309-331.
- Goodman, L. A. (1974). Exploratory latent structure analysis using both identifiable and unidentifiable models. *Biometrika*, 61, 215-231.
- Gravemeijer, K. (1997a). Commentary. Solving word problems: A case of modelling? *Learning and Instruction*, 7, 389-397.
- Gravemeijer, K. (1997b). Instructional design for reform in mathematics education. In M. Beishuizen, K. Gravemeijer, & E. C. D. M. Van Lieshout (Eds.), *The role of contexts and models in the development of mathematical strategies and procedures* (p. 13-34). Utrecht, The Netherlands: Freudenthal Institute.

- Gravemeijer, K. (2007). Reken-wiskundeonderwijs anno 2007 - tussen oude waarden en nieuwe uitdagingen [Mathematics education in the year 2007 - between old values and new challenges]. *Reken-wiskundeonderwijs: onderzoek, ontwikkeling, praktijk*, 26(4), 3-10.
- Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics*, 39, 111-128.
- Gravemeijer, K., Van den Heuvel-Panhuizen, M., Van Donselaar, G., Ruesink, N., Streefland, L., Vermeulen, W., et al. (1993). *Methoden in het reken-wiskundeonderwijs, een rijke context voor vergelijkend onderzoek*. [Methods in mathematics education, a rich context for comparative research.] Utrecht, The Netherlands: Freudenthal Instituut.
- Greer, B. (1997). Modelling reality in mathematics classrooms: The case of word problems. *Learning and Instruction*, 7, 293-307.
- Harskamp, E. G. (1988). *Rekenmethoden op de proef gesteld*. [Mathematics textbooks put to the test]. Doctoral dissertation, Groningen University, Groningen, The Netherlands.
- Harskamp, E. G., & Suhre, C. J. M. (1995). *Hoofdrekenen in het speciaal onderwijs*. [Mental computation in special education]. Groningen, The Netherlands: GION.
- Harskamp, E. G., Suhre, C. J. M., & Willemsen, T. F. W. P. (1993). *Remediële rekenprogramma's voor het basisonderwijs beproefd*. [Remedial mathematics programs for primary education put to the test]. Groningen, The Netherlands: GION.
- Hartig, J., & Höhler, J. (2008). Representation of competencies in multidimensional IRT models with within-item and between-item multidimensionality. *Zeitschrift für Psychologie / Journal of Psychology*, 216, 89-101.
- Hartzel, J., Agresti, A., & Caffo, B. (2001). Multinomial logit random effects models. *Statistical Modelling*, 1, 81-102.
- Hecht, S. A. (2006). Group differences in adult simple arithmetic: Good retrievers, not-so-good retrievers, and perfectionists. *Memory & Cognition*, 31, 207-216.
- Hedeker, D., & Gibbons, R. D. (2006). *Longitudinal data analysis*. Hoboken, NJ: John Wiley & Sons, Inc.
- Heinze, A., Marschick, F., & Lipowsky, F. (2009). Addition and subtraction of three-digit numbers: adaptive strategy use and the influence of instruction in german third grade. *ZDM Mathematics Education*, 41, 591-604.
- Heinze, A., Star, J. R., & Verschaffel, L. (2009). Flexible and adaptive use of strategies

-
- and representations in mathematics education. *ZDM Mathematics Education*, 41, 535-540.
- Hickendorff, M. (2010a, October). *Beyond measurement: Applications of explanatory IRT models to primary school mathematics tests*. Paper presented on the 2010 RCEC workshop on IRT and Educational Measurement, University of Twente, The Netherlands.
- Hickendorff, M. (2010b). The language factor in elementary mathematics assessments: computational skills and applied problem solving in a multidimensional IRT framework. *Manuscript submitted for publication*.
- Hickendorff, M. (2010c, September). *Subtraction by addition and compensation: Results from a study into shortcut strategy use by Dutch sixth graders*. Paper presented on the Advanced Study Colloquium on Mathematical Inversion, Leuven, Belgium.
- Hickendorff, M., Heiser, W. J., Van Putten, C. M., & Verhelst, N. D. (2008). Clustering nominal data with equivalent categories. *Behaviormetrika*, 35, 35-54.
- Hickendorff, M., Heiser, W. J., Van Putten, C. M., & Verhelst, N. D. (2009a). How to measure and explain achievement change in large-scale assessments: A rejoinder. *Psychometrika*, 74, 367-374.
- Hickendorff, M., Heiser, W. J., Van Putten, C. M., & Verhelst, N. D. (2009b). Solution strategies and achievement in Dutch complex arithmetic: Latent variable modeling of change. *Psychometrika*, 74, 331-350.
- Hickendorff, M., & Janssen, J. (2009). De invloed van contexten in rekenopgaven op de prestaties van basisschoolleerlingen [The effect of contexts in mathematics items on primary-school pupils' performance]. *Reken-wiskundeonderwijs: onderzoek, ontwikkeling, praktijk*, 28(4), 3-11.
- Hickendorff, M., & Van Putten, C. M. (2010). Complex multiplication and division in Dutch educational assessments: What can solution strategies tell us? *Manuscript submitted for publication*.
- Hickendorff, M., Van Putten, C. M., Verhelst, N. D., & Heiser, W. J. (2010). Individual differences in strategy use on division problems: Mental versus written computation. *Journal of Educational Psychology*, 102, 439-452.
- Hiebert, J., & Grouws, D. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (p. 371-404). Charlotte, CT: Information Age Publishing.
- Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers' mathematical

- knowledge: What knowledge matters and what evidence counts? In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (p. 111-155). Charlotte, CT: Information Age Publishing.
- Imbo, I., & LeFevre, J.-A. (2009). Cultural differences in complex addition: Efficient Chinese versus adaptive Belgians and Canadians. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 35, 1465-1476.
- Imbo, I., & Vandierendonck, A. (2007). Do multiplication and division strategies rely on executive and phonological working memory resources? *Memory & Cognition*, 35, 1759-1771.
- Janssen, J., & Hickendorff, M. (2009). Categorieënanalyse bij de LOVS toetsen rekenen-wiskunde. [Profile analysis in the LOVS mathematics tests]. In M. Van Zanten (Ed.), *Leren van evalueren: De lerende in beeld bij reken-wiskundeonderwijs*. (p. 49-60). Utrecht, The Netherlands: Flsmc.
- Janssen, J., Van der Schoot, F., & Hemker, B. (2005). *Balans van het reken-wiskundeonderwijs aan het einde van de basisschool 4* [Fourth assessment of mathematics education at the end of primary school]. Arnhem, The Netherlands: CITO.
- Janssen, J., Van der Schoot, F., Hemker, B., & Verhelst, N. D. (1999). *Balans van het reken-wiskundeonderwijs aan het einde van de basisschool 3* [Third assessment of mathematics education at the end of primary school]. Arnhem, The Netherlands: CITO.
- Janssen, R., Schepers, J., & Peres, D. (2004). Models with item and item group predictors. In P. D. Boeck & M. Wilson (Eds.), *Explanatory item response models: A generalized linear and nonlinear approach* (p. 189-212). New York: Springer.
- Kagan, J. (1966). Reflection-impulsivity: The generality and dynamics of conceptual tempo. *Journal of Abnormal Psychology*, 71, 17-24.
- Keijzer, R. (2003). *Teaching formal mathematics in primary education*. Doctoral dissertation, Freudenthal Instituut, Utrecht, The Netherlands.
- Keijzer, R., & Terwel, J. (2003). Learning for mathematical insight: a longitudinal comparative study on modelling. *Learning and Instruction*, 13, 285-304.
- Kerkman, D. D., & Siegler, R. S. (1997). Measuring individual differences in children's addition strategy choices. *Learning and Individual Differences*, 9, 1-18.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up. Helping children learn mathematics*. Washington, D.C.: National Academy Press.
- Klein, A. S. (1998). *Flexibilization of mental arithmetic strategies on a different knowledge*

-
- base: the empty number line in a realistic versus gradual program design. Doctoral dissertation, Leiden University, Leiden, The Netherlands.
- Klein, A. S., Beishuizen, M., & Treffers, A. (1998). The empty number line in Dutch second grades: Realistic versus gradual program design. *Journal for Research in Mathematics Education*, 29, 443-464.
- Klein Entink, R. H., Fox, J.-P., & Van der Linden, W. J. (2009). A multivariate multilevel approach to the modeling of accuracy and speed of test takers. *Psychometrika*, 74, 21-48.
- KNAW. (2009). *Rekenonderwijs op de basisschool. Analyse en sleutels tot verbetering* [Mathematics education in primary school. Analysis and recommendations for improvement]. Amsterdam, The Netherlands: KNAW.
- Kraemer, J.-M., Janssen, J., Van der Schoot, F., & Hemker, B. (2005). *Balans van het reken-wiskundeonderwijs halvewege de basisschool 4* [Fourth assessment of mathematics education halfway primary school]. Arnhem, The Netherlands: CITO.
- Kroesbergen, E. H., & Van Luit, J. E. H. (2002). Teaching multiplication to low math performers: Guided versus structured instruction. *Instructional Science*, 30, 361-378.
- Kroesbergen, E. H., & Van Luit, J. E. H. (2003). Mathematics interventions for children with special educational needs. *Remedial and Special Education*, 24, 97-114.
- Kroesbergen, E. H., & Van Luit, J. E. H. (2005). Effects of different forms of instruction on acquisition and use of multiplication strategies by children with math learning difficulties. In P. Ghesquière & A. Ruijsenaars (Eds.), *Learning disabilities: A challenge to teaching and instruction* (p. 161-181). Leuven, Belgium: Leuven University Press.
- Kroesbergen, E. H., Van Luit, J. E. H., & Maas, C. J. M. (2004). Effectiveness of explicit and constructivist mathematics instruction for low-achieving students in the Netherlands. *Elementary School Journal*, 104, 233-251.
- Kuss, O., & McLerran, D. (2007). A note on the estimation of the multinomial logistic model with correlated responses. *Computer Methods and Programs in Biomedicine*, 87, 262-269.
- Lazarsfeld, P. F., & Henry, N. W. (1968). *Latent structure analysis*. New York: Houghton-Mifflin.
- Lee, J. (2007). Making sense of the traditional long division algorithm. *Journal of Mathematical Behavior*, 26, 48-59.
- Leighton, J. P., & Gierl, M. J. (Eds.). (2007). *Cognitive diagnostic assessment for education: Theory and application*. New York: Cambridge University Press.

- Lemaire, P. (2010). Executive functions and strategic aspects of arithmetic performance: The case of adults' and children's arithmetic. *Psychologica Belgica*, 50, 335-352.
- Lemaire, P., & Callies, S. (2009). Children's strategies in complex arithmetic. *Journal of Experimental Child Psychology*, 103, 49-65.
- Lemaire, P., & Lecacheur, M. (2002). Children's strategies in computational estimation. *Journal of Experimental Child Psychology*, 82, 281-304.
- Lemaire, P., & Siegler, R. S. (1995). Four aspects of strategic change: Contributions to children's learning of multiplication. *Journal of Experimental Psychology: General*, 124, 83-97.
- Lesaffre, E., & Spiessens, B. (2001). On the effect of the number of quadrature points in a logistic random-effects model: an example. *Journal of the Royal Statistical Society Series C - Applied Statistics*, 50, 325-335.
- Linzer, D., & Lewis, J. (2010). *poLCA: Polytomous variable latent class analysis. R package version 1.2* (Computer program and manual). Available from <http://userwww.service.emory.edu/~dlinzer/poLCA>.
- Linzer, D., & Lewis, J. (2011). *poLCA: An R package for polytomous variable latent class analysis. Journal of Statistical Software*, 42(10), 1-29.
- Lipsey, M. W., & Wilson, D. (2001). *Practical meta-analysis*. Thousand Oaks, CA: Sage.
- Luwel, K., Onghena, P., Torbeyns, J., Schillemans, V., & Verschaffel, L. (2009). Strengths and weaknesses of the choice/no-choice method in research on strategy choice and strategy change. *European Psychologist*, 14, 351-362.
- Luwel, K., Verschaffel, L., Onghena, P., & De Corte, E. (2003). Analysing the adaptiveness of strategy choices using the choice/no-choice method: The case of numerosity judgement. *European Journal of Cognitive Psychology*, 15, 511-537.
- Mabbott, D. J., & Bisanz, J. (2003). Developmental change and individual differences in children's multiplication. *Child Development*, 74, 1091-1107.
- Mauro, D. G., LeFevre, J.-A., & Morris, J. (2003). Effects of problem format on division and multiplication performance: division facts are mediated via multiplication-based representations. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 29, 163-170.
- McCulloch, C. E., & Searle, S. R. (2001). *Generalized, linear, and mixed models*. New York: Wiley.
- Meelissen, M. R. M., & Drent, M. (2008). *TIMSS-2007 Nederland. Trends in leerprestaties in exacte vakken van het basisonderwijs*. [TIMSS 2007 the Netherlands. Trends in

-
- achievement in mathematics and science in primary education]. Enschede, The Netherlands: Twente University.
- Menne, J. (2001). *Met sprongen vooruit. een productief oefenprogramma voor zwakke rekenaars in het getallen gebied tot 100 – een onderwijsexperiment*. [A productive training program for low mathematics achievers in the number domain up to 100 – a design experiment]. Doctoral dissertation, Freudenthal Institute, Utrecht, The Netherlands.
- Milo, B. F., & Ruijsenaars, A. J. J. M. (2005). Strategy use and math instruction for students with special needs. In P. Ghesquière & A. Ruijsenaars (Eds.), *Learning disabilities: A challenge to teaching and instruction* (p. 183-200). Leuven, Belgium: Leuven University Press.
- Milo, B. F., Ruijsenaars, A. J. J. M., & Seegers, G. (2005). Math instruction for students with special educational needs: Effects of guiding versus directing instruction. *Educational & Child Psychology*, 22, 70-80.
- Milo, B. F., Seegers, G., Ruijsenaars, A. J. J. M., & Vermeer, H. (2004). Affective consequences of mathematics instruction for students with special needs. *European Journal of Special Needs Education*, 19, 49-68.
- Molenberghs, G., & Verbeke, G. (2004). An introduction to (generalized) (non)linear mixed models. In P. De Boeck & M. Wilson (Eds.), *Explanatory item response models: A generalized linear and nonlinear approach* (p. 111-153). New York, NY: Springer.
- Mulligan, J. T., & Mitchelmore, M. C. (1997). Young children's intuitive models of multiplication and division. *Journal for Research in Mathematics Education*, 28, 309-330.
- Mullis, I. V. S., Martin, M. O., & Foy, P. (2008). *TIMSS 2007 international mathematics report. Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eighth grades*. Boston: Boston College, TIMSS & PIRLS International Study Center.
- Mullis, I. V. S., Martin, M. O., Kennedy, A. M., & Foy, P. (2007). *PIRLS 2006 international report. IEA's Progress in International Reading Literacy Study in primary school in 40 countries*. Chestnut Hill, MA: Boston College, TIMSS & PIRLS International Study Center.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school*

- mathematics*. Reston, VA: NCTM.
- National Mathematics Advisory Panel. (2008). *Foundations for success. The Final Report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.
- Neuman, D. (1999). Early learning and awareness of division: A phenomenographic approach. *Educational Studies in Mathematics*, 40, 101-128.
- Nye, B., Konstantopoulos, S., & Hedges, L. V. (2004). How large are teacher effects? *Educational Evaluation and Policy Analysis*, 26, 237-257.
- OECD. (2004). *Learning for tomorrow's world. First results from PISA 2003*. Paris: OECD.
- OECD. (2009). *PISA 2006 technical report*. Paris: OECD.
- OECD. (2010). *PISA 2009 results: What students know and can do. Student performance in reading, mathematics and science (volume I)*. Paris: OECD.
- Phillips, L. H., & Rabbitt, P. M. A. (1995). Impulsivity and speed-accuracy strategies in intelligence-test performance. *Intelligence*, 21, 13-29.
- Poland, M. (2007). *The treasures of schematising. The effects of schematising in early childhood on the learning processes and outcomes in later mathematical understanding*. Doctoral dissertation, Free University, Amsterdam, The Netherlands.
- Poland, M., & Van Oers, B. (2007). Effects of schematising on mathematical development. *European Early Childhood Education Research Journal*, 15, 269-293.
- Porter, A. C. (2006). Curriculum assessment. In J. L. Green, G. Camilli, & P. B. Elmore (Eds.), *Handbook of complementary methods in education research* (p. 141-160). Mahwah, NJ: Lawrence Erlbaum Associates.
- Prenger, J. (2005). *Taal telt! Een onderzoek naar de rol van taalvaardigheid en tekstbegrip in het realistische rekenonderwijs*. [Language counts! A study into the role of linguistic skill and text comprehension in realistic mathematics education]. PhD thesis, University of Groningen, The Netherlands.
- R Development Core Team. (2009). R: A language and environment for statistical computing [Computer software manual]. Vienna, Austria. Available from <http://www.R-project.org>
- Reckase, M. D. (2009). *Multidimensional item response theory*. New York, NY: Springer.
- Reys, R. E. (1984). Mental computation and estimation: Past, present, and future. *The Elementary School Journal*, 84, 546-557.
- Rijmen, F., Tuerlinckx, F., De Boeck, P., & Kuppens, P. (2003). A nonlinear mixed model framework for item response theory. *Psychological Methods*, 8, 185-205.

-
- Robinson, K. M. (2001). The validity of verbal reports in children's subtraction. *Journal of Educational Psychology*, 93, 211-222.
- Robinson, K. M., Arbuthnott, K. D., Rose, D., McCarron, M. C., Globa, C. A., & Phonexay, S. D. (2006). Stability and change in children's division strategies. *Journal of Experimental Child Psychology*, 93, 224-238.
- Russo, J. E., Johnson, E. J., & Stephens, D. L. (1989). The validity of verbal reports. *Memory & Cognition*, 17, 759-769.
- Ruthven, K. (1998). The use of mental, written and calculator strategies of numerical computation by upper primary pupils within a 'calculator-aware' number curriculum. *British Educational Research Journal*, 24, 21 - 42.
- SAS Institute. (2002). *SAS online doc (version 9)*. Cary, NC: SAS Institute Inc.
- Schopman, E. A. M., & Van Luit, J. E. H. (1996). Learning transfer of preparatory arithmetic strategies among young children with a developmental lag. *Journal of Cognitive Education*, 5, 117-131.
- Selter, C. (2001). Addition and subtraction of three-digit numbers: German elementary children's succes, methods and strategies. *Educational Studies in Mathematics*, 47, 145-173.
- Sherin, B., & Fuson, K. (2005). Multiplication strategies and the appropriation of computational resources. *Journal for Research in Mathematics Education*, 36, 347-395.
- Sheu, C.-F., Chen, C.-T., Su, Y.-H., & Wang, W.-C. (2005). Using SAS PROC NL MIXED to fit item response theory models. *Behavior Research Methods*, 37, 202-218.
- Shrager, J., & Siegler, R. S. (1998). SCADS: A model of children's strategy choices and strategy discoveries. *Psychological Science*, 9, 405-410.
- Siegler, R. S. (1988a). Individual differences in strategy choices: good students, not-so-good students, and perfectionists. *Child Development*, 59, 833-851.
- Siegler, R. S. (1988b). Strategy choice procedures and the development of multiplication skill. *Journal of Experimental Psychology: General*, 117, 258-275.
- Siegler, R. S., & Lemaire, P. (1997). Older and younger adults' strategy choices in multiplication: Testing predictions of ASCM using the choice/no-choice method. *Journal of Experimental Psychology: General*, 126, 71-92.
- Siegler, R. S., & Shipley, C. (1995). Variation, selection, and cognitive change. In G. Halford & T. Simon (Eds.), *Developing cognitive competence: New approaches to process modeling* (p. 31-76). Hillsdale, NJ: Erlbaum.
- Silverman, I. W. (2003). Gender differences in delay of gratification: A meta-analysis. *Sex*

- Roles*, 49, 451-463.
- Sinharay, S., Puhan, G., & Haberman, S. J. (2010). Reporting diagnostic scores in educational testing: Temptations, pitfalls, and some solutions. *Multivariate Behavioral Research*, 45, 553-573.
- Slavin, R. E. (2008). What works? Issues in synthesizing educational program evaluation. *Educational Researcher*, 37, 5-14.
- Slavin, R. E., & Lake, C. (2008). Effective programs in elementary mathematics: A best-evidence syntheses. *Review of Educational Research*, 78, 427-515.
- Snijders, T. A. B., & Bosker, R. J. (1999). *Multilevel analysis. An introduction to basic and advanced multilevel modeling*. London: Sage Publications.
- Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education*, 36, 404-411.
- Star, J. R., & Newton, K. J. (2009). The nature and development of experts' strategy flexibility for solving equations. *ZDM Mathematics Education*, 41, 557-561.
- Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (p. 319-370). Charlotte, CT: Information Age Publishing.
- Stone, C. A., Ye, F., Zhu, X., & Lane, S. (2010). Providing subscale scores for diagnostic information: A case study when the test is essentially unidimensional. *Applied Measurement in Education*, 23, 63-86.
- Swanson, H. L., & Carson, C. (1996). A selective synthesis of intervention research for students with learning disabilities. *School Psychology Review*, 25, 370-392.
- Swanson, H. L., & Hoskyn, M. (1998). Experimental intervention research on students with learning disabilities: A meta-analysis of treatment outcomes. *Review of Educational Research*, 68, 227-331.
- Terwel, J., Van Oers, B., Van Dijk, I. M. A. W., & Van Eeden, P. (2009). Are representations to be provided or generated in primary mathematics education? Effects on transfer. *Educational Research and Evaluation*, 15, 25-44.
- Timmermans, R. E., & Van Lieshout, E. C. D. M. (2003). Influence of instruction in mathematics for low performing students on strategy use. *European Journal of Special Needs Education*, 8, 5-16.
- Timmermans, R. E., Van Lieshout, E. C. D. M., & Verhoeven, L. (2007). Gender related effects of contemporary math instruction for low performers on problem solving behavior. *Learning and Instruction*, 17, 42-54.

-
- Torbeyns, J., De Smedt, B., Ghesquière, P., & Verschaffel, L. (2009a). Acquisition and use of shortcut strategies by traditionally schooled children. *Educational Studies in Mathematics*, 71, 1-17.
- Torbeyns, J., De Smedt, B., Ghesquière, P., & Verschaffel, L. (2009b). Jump or compensate? Strategy flexibility in the number domain up to 100. *ZDM Mathematics Education*, 41, 581-590.
- Torbeyns, J., Ghesquière, P., & Verschaffel, L. (2009). Efficiency and flexibility of indirect addition in the domain of multi-digit subtraction. *Learning and Instruction*, 19, 1-12.
- Torbeyns, J., Verschaffel, L., & Ghesquière, P. (2002). Strategic competence: Applying Siegler's theoretical and methodological framework to the domain of simple addition. *European Journal of Psychology of Education*, 27, 275-291.
- Torbeyns, J., Verschaffel, L., & Ghesquière, P. (2004a). Strategic aspects of simple addition and subtraction: the influence of mathematical ability. *Learning and Instruction*, 14, 177-195.
- Torbeyns, J., Verschaffel, L., & Ghesquière, P. (2004b). Strategy development in children with mathematical disabilities: Insights from the choice/no-choice method and the chronological-age/ability-level-match design. *Journal of Learning Disabilities*, 37, 119-131.
- Torbeyns, J., Verschaffel, L., & Ghesquière, P. (2005). Simple addition strategies in a first-grade class with multiple strategy instruction. *Cognition and Instruction*, 23, 1-21.
- Torbeyns, J., Verschaffel, L., & Ghesquière, P. (2006). The development of children's adaptive expertise in the number domain 20 to 100. *Cognition and Instruction*, 24, 439-465.
- Treffers, A. (1987). Integrated column arithmetic according to progressive schematisation. *Educational Studies in Mathematics*, 18, 125-145.
- Treffers, A. (1993). Wiscobas and Freudenthal: Realistic mathematics education. *Educational Studies in Mathematics*, 25, 89-108.
- Van den Boer, C. (2003). *Als je begrijpt wat ik bedoel. Een zoektocht naar verklaringen van achterblijvende prestaties van allochtone leerlingen in het wiskundeonderwijs* [If you get what I mean. A search for explanations of lagging achievement of non-native students in mathematics education]. Utrecht, The Netherlands: CD- β press.
- Van den Heuvel-Panhuizen, M. (Ed.). (2008). *Children learn mathematics*. Rotterdam, the Netherlands: Sense Publishers.

- Van den Heuvel-Panhuizen, M., Buys, K., & Treffers, A. (Eds.). (2001). *Kinderen leren rekenen. Tussendoelen annex leerlijnen: Hele getallen bovenbouw basisschool*. [Children learn mathematics]. Groningen, the Netherlands: Wolters-Noordhoff.
- Van den Heuvel-Panhuizen, M., Robitzsch, A., Treffers, A., & Köller, O. (2009). Large-scale assessments of change in student achievement: Dutch primary school students' results on written division in 1997 and 2004 as an example. *Psychometrika*, 74, 351-365.
- Van der Craats, J. (2007). Waarom Daan en Sanne niet kunnen rekenen [Why Daan and Sanne can't add]. *Nieuw Archief voor de Wiskunde*, 8(2), 132-136.
- Van de Rijt, B. A. M., & Van Luit, J. E. H. (1998). Effectiveness of the Additional Early Mathematics program for teaching children early mathematics. *Instructional Science*, 26, 337-358.
- Van der Linden, W. J., & Hambleton, R. K. (1997). *Handbook of modern item response theory*. New York, NY: Springer-Verlag.
- Van der Schoot, F. (2008). *Onderwijs op peil? Een samenvattend overzicht van 20 jaar PPOON* [A summary overview of 20 years of national assessments of the level of education]. Arnhem, The Netherlands: CITO.
- Van Dijk, I. M. A. W., Van Oers, B., Terwel, J., & Van Eeden, P. (2003). Strategic learning in primary mathematics education: Effects of an experimental program in modelling. *Educational Research and Evaluation*, 9, 161-187.
- Van Luit, J. E. H. (1994). The effectiveness of structural and realistic arithmetic curricula in children with special educational needs. *European Journal of Special Needs Education*, 9, 16-26.
- Van Luit, J. E. H., & Naglieri, J. A. (1999). Effectiveness of the MASTER program for teaching special children multiplication and division. *Journal of learning disabilities*, 32, 98-107.
- Van Luit, J. E. H., & Schopman, E. A. M. (2000). Improving early numeracy of young children with special educational needs. *Remedial and special education*, 21, 27-40.
- Van Putten, C. M., & Hickendorff, M. (2009). Peilstokken voor Plasterk: Evaluatie van rekenvaardigheid in groep 8. [Assessments for Plasterk: Evaluation of mathematical ability in sixth grade]. *Tijdschrift voor Orthopedagogiek*, 48, 183-194.
- Van Putten, C. M., Van den Brom-Snijders, P. A., & Beishuizen, M. (2005). Progressive mathematization of long division strategies in Dutch primary schools. *Journal for Research in Mathematics Education*, 36, 44-73.
- Van Schilt-Mol, T. M. M. L. (2007). *Differential item functioning en itembias in de Cito-*

-
- Eindtoets Basisonderwijs* [Differential item functioning and item bias in CITO's End of Primary School Test]. PhD thesis, Tilburg University, The Netherlands.
- Varol, F., & Farran, D. (2007). Elementary school students' mental computation proficiencies. *Early Childhood Educational Journal*, 35, 89-94.
- Verhelst, N. D. (2007). *Profielanalyse met Item Respons Theorie* [Profile analysis with Item Response Theory]. Arnhem, The Netherlands: CITO.
- Verhelst, N. D. (in press). Profile analysis: A closer look at the PISA 2000 Reading data. *Scandinavian Journal of Educational Research*.
- Verhelst, N. D., & Glas, C. A. W. (1995). The one-parameter logistic model. In G. H. Fischer & I. Molenaar (Eds.), *Rasch models. Foundations, recent developments and applications* (p. 215-237). New York: Springer.
- Verhelst, N. D., & Verstralen, H. H. F. M. (2002). *Structural analysis of a univariate latent variable (SAUL)* (computer program and manual). Arnhem, The Netherlands: CITO.
- Vermeer, H. J., Boekaerts, M., & Seegers, G. (2000). Motivational and gender differences: Sixth-grade students' mathematical problem solving behavior. *Journal of Educational Psychology*, 92, 308-315.
- Vermunt, J. K. (1997). *LEM 1.0: A general program for the analysis of categorical data*. Tilburg, The Netherlands: Tilburg University.
- Vermunt, J. K., & Magidson, J. (2002). Latent class cluster analysis. In J. A. Hagenaars & A. L. McCutcheon (Eds.), *Applied latent class analysis* (p. 89-106). Cambridge, England: Cambridge University Press.
- Verschaffel, L. (2009). 'Over het muurtje kijken': achtergrond, inhoud en receptie van het Final Report van het 'National Mathematics Advisory Panel' in de U.S. [Background, content, and reception of the Final Report of the National Mathematics Advisory Panel in the U.S.] *Reken-wiskundeonderwijs: Onderzoek, ontwikkeling en praktijk*, 28(1), 3-20.
- Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems*. Lisse, The Netherlands: Swets and Zeitlinger.
- Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning*. Charlotte, CT: Information Age Publishing.
- Verschaffel, L., Luwel, K., Torbeyns, J., & Van Dooren, W. (2009). Conceptualizing, investigating, and enhancing adaptive expertise in elementary mathematics education. *European Journal of Psychology of Education*, 24, 335-359.

- Vicente, S., Orrantia, J., & Verschaffel, L. (2007). Influence of situational and conceptual rewording on word problem solving. *British Journal of Educational Psychology*, 77, 829-848.
- Von Davier, M., & Sinharay, S. (2009). *Stochastic approximation methods for latent regression item response models* (ETS-RR-09-09). Princeton, NJ: Educational Testing Service.
- Willemsen, T. F. W. P. (1994). *Remediële rekenprogramma's voor de basisschool* [Remedial mathematics programs for primary education]. Doctoral dissertation, GION, Groningen, The Netherlands.
- Wilson, M., & De Boeck, P. (2004). Descriptive and explanatory item response models. In P. De Boeck & M. Wilson (Eds.), *Explanatory item response models: A generalized linear and nonlinear approach* (p. 43-74). New York, NY: Springer.
- Wu, M. L., & Adams, R. J. (2006). Modelling mathematics problem solving item responses using a multidimensional IRT model. *Mathematics Education Research Journal*, 18, 93-113.
- Xin, Y. P., & Jitendra, A. K. (1999). The effects of instruction in solving mathematical word problems for students with learning problems: A meta-analysis. *The Journal of Special Education*, 32, 207-225.