



Universiteit  
Leiden  
The Netherlands

## **Explanatory latent variable modeling of mathematical ability in primary school : crossing the border between psychometrics and psychology**

Hickendorff, M.

### **Citation**

Hickendorff, M. (2011, October 25). *Explanatory latent variable modeling of mathematical ability in primary school : crossing the border between psychometrics and psychology*. Retrieved from <https://hdl.handle.net/1887/17979>

Version: Not Applicable (or Unknown)

License: [Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden](#)

Downloaded from: <https://hdl.handle.net/1887/17979>

**Note:** To cite this publication please use the final published version (if applicable).

# Introduction

Children's mathematical ability is a hotly debated topic in many countries, including the Netherlands. One point of discussion is mathematics *education*. A reform movement of international scope has taken place, which can roughly be described as a shift away from teachers directly instructing arithmetic skills that children have to drill, towards an approach that considers children's existing pre-knowledge as the basis on which to build mathematical knowledge, attempting to attain not only procedural expertise but also mathematical insight, flexibility, and creativity. Another point of discussion is the the mathematics *performance* level of students in primary school and in secondary school. Results from large-scale national and international assessments of students' mathematical ability, reporting trends over time, international comparisons, and deviations from educational standards that hold within a country, usually form the starting point of this discussion.

In this thesis – the result of a collaborative research project of the Institute of Psychology of Leiden University and CITO, the Dutch National Institute for Educational Measurement – the focus is on primary school students' mathematical ability in the Netherlands. The findings of the most recent national mathematics assessment at the end of primary school (sixth grade; 12-year-olds) carried out by CITO in 2004 (called PPON [*Periodieke Peiling van het OnderwijsNiveau*]; J. Janssen, Van der Schoot, & Hemker, 2005; see also Van der Schoot, 2008) were the starting point. The 2004-assessment was the fourth cycle, with earlier assessments carried out in 1987, 1992, and 1997; the fifth cycle is planned in 2011. Trends over the time period from 1987 to 2004 showed diverse patterns: in some mathematics domains students' performance increased, while in other domains it decreased. Moreover, in general students' performance lagged behind the educational standards, in some domains more than in others. In the newspapers and other platforms

of the public debate people expressed their opinions on these developments. One returning element is the didactical theory of Realistic Mathematics Education (RME; e.g., Freudenthal, 1973, 1991; Treffers, 1993) that has become the dominant theory in primary school mathematics education in the 80s and 90s of the previous century, which evokes strong feelings. In the public debate, however, commonsense beliefs and personal sentiments with anecdotal foundations usually prevail over robust insights based on empirical study of what students know and can do in mathematics and of what are the performance outcomes of different mathematics programs. The purpose of the current thesis is to provide these empirically-based insights.

First, to give an overview of what is known empirically – and what is *not* known – about performance outcomes of different mathematics programs or curricula, a research synthesis of empirical studies that address this question for primary school students in the Netherlands is presented. Next, with respect to what primary school students know and can do in mathematics, CITO's mathematics assessments are a rich source of information on students' performance level compared to the educational standards, as well as on differences between students (e.g., boys and girls) and on trends over time. However, these assessments are surveys and therefore limited to descriptive analyses. Explanations for apparent differences or trends require further study. That is exactly what has been done in the current research project and what is reported on in six empirical studies in this thesis.

These empirical studies, addressing determinants of students' ability in the domain of arithmetic (addition, subtraction, multiplication, and division), cross the border between the academic fields of substantive educational and cognitive psychology on the one hand and psychometrics on the other hand. Substantively, *solution strategies* are a key element of all but one of the empirical studies. Strategies were deemed relevant both from an educational psychology perspective, because they are a spearhead in mathematics education reform, as well as from a cognitive psychology perspective where mechanisms of strategy choice and concepts as strategic competence are important research topics. In the current studies, solution strategies were considered both as outcome measures in analyses of determinants of strategy choice, and as explanatory variables in analyses of determinants of mathematics performance. Related recurring elements are individual differences in strategy choice and in performance, and differences between groups of students such as boys and girls.

Psychometrically, the data in the empirical studies reported are complex, requiring

---

advanced statistical modeling. In the substantive fields of educational and cognitive psychology, such techniques are not very common. That is why the current thesis can be said to be an attempt to integrate psychometrics and psychology, such as advocated by Borsboom (2006). One salient complicating aspect of the data in the current studies is that they involve repeated observations within subjects (i.e., each student responds to several mathematics problems), leading to a correlated or dependent data structure. To take these dependencies into account, it is argued that *latent variable models* are appropriate: one or more latent (unobserved) variables reflect individual differences between students, and the dependent responses within each student are mapped onto these variables. Latent variables can be either categorical, modeling qualitative individual differences between students, or continuous, modeling quantitative individual differences. Furthermore, the influence of explanatory variables such as assessment cycle or students' gender can be addressed by analyzing the effect on the latent variable.

In the studies reported, the responses on each trial (when a student is confronted with an item) are of categorical measurement level. That is, two types of responses are dealt with: the strategy used to solve the problem (several unordered categories) and the accuracy of the answer given to the problem (dichotomous: correct/incorrect). To analyze individual differences in strategy choice, latent class analysis (LCA; e.g. Goodman, 1974; Lazarsfeld & Henry, 1968) was used. In latent class models, it is assumed that there are unobserved subgroups (classes or clusters) of students that are characterized by a specific response profile, in this case, a specific strategy choice profile. In order to address the influence of student characteristics on latent class membership, latent class models with covariates (Vermunt & Magidson, 2002) were used. To analyze individual differences in performance, item response theory (IRT; e.g., Embretson & Reise, 2000; Van der Linden & Hambleton, 1997) models were used, in which the probability of giving the correct answer is determined by one or more continuous latent (ability) dimensions. In particular, measurement IRT models were extended with an explanatory part, in which predictors at the person level, at the item level, or at the person-by-item level can be incorporated (De Boeck & Wilson, 2004; Rijmen, Tuerlinckx, De Boeck, & Kuppens, 2003). One innovating application of these explanatory IRT models was to use the strategy used on an item as a person-by-item predictor, thereby modeling strategy accuracy (the probability to obtain a correct answer with a certain strategy) while statistically accounting for individual differences in overall ability and for differences in difficulty level between problems, something that had not been accomplished before in psychological

research into solution strategies.

## OUTLINE

The thesis starts with Chapter 1 reporting a research synthesis of empirical studies that were carried out in the Netherlands into the relation between mathematics education and mathematics proficiency. This chapter is based on work that was done for the KNAW (*Royal Dutch Academy of Arts and Sciences*) Committee on Primary School Mathematics Teaching<sup>1</sup>, whose report came out in 2009. Starting with an overview of results of Dutch national assessments and the position of Dutch students in international assessments, the main body of the chapter is devoted to a systematic review of studies in which the relationship between instructional approach and students' performance outcomes was investigated. The main conclusion that could be drawn was that much is unknown about the relation between mathematics programs and performance outcomes, and that methodologically sound empirical studies comparing different instructional approaches are rare, which may be because they are very difficult to implement. In the remainder of this thesis, the focus is shifted to other determinants of students' mathematics ability related to contemporary mathematics education, such as the strategies students used to solve the problems and characteristics of the mathematics problems.

First, two studies are reported in which secondary analyses on the raw student material (test booklets) of the two most recent national mathematics assessments of 1997 and 2004 were carried out. They both focus on complex or multidigit arithmetic: a mathematics domain on which performance decreased most severely over time, as well as stayed furthest behind the educational standards. Furthermore, the RME approach has changed the instructional approach as to how to solve these problems, paying less attention to the traditional algorithms and instead focusing more on informal whole-number approaches (Van den Heuvel-Panhuizen, 2008). Therefore, both studies focus on *solution strategies* as explanatory variables of performance, a recurring issue in this thesis. Specifically, in Chapter 2, solution strategies that students used to solve complex or multidigit *division* problems were studied, aiming to give more insight in the performance decrease between 1997 and 2004. The complex nature of the data necessitated advanced psychometric modeling, and latent variable models – latent class

---

<sup>1</sup> I worked as an associate researcher supporting the Committee. In particular, the Committee requested me to carry out the systematic literature review that formed the basis of chapter 4 in the report. Chapter 1 in the current thesis is based on this work.

analysis (LCA) and item response theory (IRT) – with explanatory variables are introduced in this chapter. Subsequently, in Chapter 3 the domain of division is broadened to include complex or multidigit *multiplication* problems as well. Furthermore, the influence of teachers' instructional approach to solving multiplication and division problems on students' strategy choice is addressed.

The subsequent part of this thesis reports on two studies in which new data were collected to answer specific research questions that were raised based on the findings of the secondary analyses on the division problems data in Chapter 2. Specifically, one important conclusion was that students increasingly answered without any written working, and that this shift was unfortunate with respect to performance, since it was the least accurate strategy. In Chapter 4 individual differences in strategy use in complex division problems were studied in a systematic research design: a partial *choice/no choice* design (Siegler & Lemaire, 1997). Sixth graders solved division problems in two different conditions: in the choice condition, they were free to choose how they solved the problem (with a written or a mental strategy), while in the subsequent no-choice condition, they were forced to write down how they solved the problem. In addition, individual interviews with students using a non-written strategy in the choice condition were carried out to investigate how they had solved the problem without using paper and pencil. Next, Chapter 5 reports on a study in which a complete choice/no-choice design was implemented, in which there was an additional no-choice condition in which students were forced to use a mental strategy. In addition, solution times were recorded, so that two aspects of strategy performance – accuracy and speed – could be taken into account simultaneously. In this study, it was possible to address the issue of *strategy adaptivity* at the student level: the extent to which a student chooses the best strategy for him or her on a particular division problem.

The final part of the current thesis addresses another aspect of contemporary mathematics education: an increased focus on mathematics problems in a *realistic context* – including word problems – in instruction as well as in tests. These contexts usually consist of a verbal description of a mathematical problem situation, which may be accompanied by an illustration. Such problems serve a central role for several reasons (e.g., Verschaffel, Greer, & De Corte, 2000): they may have motivational potential, mathematical concepts and skills may be developed in a meaningful way, and children may develop knowledge of when and how to use mathematics in everyday-life situations. Little is known, however, on the differences between solving computational problems

(bare numerical problems) and solving such contextual problems. Therefore, this question is addressed in two studies in the domain of the four basic arithmetical operations: one focusing on children in the lower – first, second, and third – grades of primary school, reported in Chapter 6; the other focusing on students in grade six, reported in Chapter 7. In both studies, special attention is paid to the influence of students' language level, because students need to understand the problem text in order to be able to successfully solve the problem. Chapter 6 focuses on performance only, modeling students responses (correct/incorrect) to mathematics problems of both types in a multidimensional IRT framework. Chapter 7 extends this focus by investigating strategy use as well.

Chapter 8 concludes this thesis with a general discussion. Besides reflecting on the substantive findings regarding mathematical ability in Dutch primary school students, attention is also paid to the psychometric modeling techniques that are used.

Finally, note that because the seven main chapters of this thesis are separate research papers, a certain amount of overlap is inevitable.