



Universiteit  
Leiden  
The Netherlands

## **Explanatory latent variable modeling of mathematical ability in primary school : crossing the border between psychometrics and psychology**

Hickendorff, M.

### **Citation**

Hickendorff, M. (2011, October 25). *Explanatory latent variable modeling of mathematical ability in primary school : crossing the border between psychometrics and psychology*. Retrieved from <https://hdl.handle.net/1887/17979>

Version: Not Applicable (or Unknown)

License: [Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden](#)

Downloaded from: <https://hdl.handle.net/1887/17979>

**Note:** To cite this publication please use the final published version (if applicable).

# **Explanatory latent variable modeling of mathematical ability in primary school**

*Crossing the border between  
psychometrics and psychology*

Hickendorff, Marian

Explanatory latent variable modeling of mathematical ability in primary school:  
Crossing the border between psychometrics and psychology.

Copyright ©2011 by Marian Hickendorff

Cover design by Moon grafisch ontwerp

Printed by Proefschriftmaken.nl, Oisterwijk

All rights reserved. No part of this book may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronically, mechanically, by photocopy, by recording, or otherwise, without prior written permission from the author.

ISBN 978-90-8891-326-6

# **Explanatory latent variable modeling of mathematical ability in primary school**

*Crossing the border between  
psychometrics and psychology*

PROEFSCHRIFT

ter verkrijging van de graad van Doctor aan de Universiteit Leiden,  
op gezag van de Rector Magnificus prof. mr. P. F. van der Heijden,  
volgens besluit van het College voor Promoties  
te verdedigen op dinsdag 25 oktober 2011  
klokke 16.15 uur

door Marian Hickendorff  
geboren te Leiden in 1981

## PROMOTIECOMMISSIE

Promotor prof. dr. W. J. Heiser

Copromotores dr. C. M. van Putten (Universiteit Leiden)  
prof. dr. N. D. Verhelst (Cito Instituut voor Toetsontwikkeling)

Overige leden	dr. A. A. Béguin	(Cito Instituut voor Toetsontwikkeling)
	prof. dr. P. A. L. de Boeck	(Universiteit van Amsterdam)
	dr. E. H. Kroesbergen	(Universiteit Utrecht)
	prof. dr. L. Verschaffel	(K.U. Leuven, België)

# Contents

## Contents *v*

### **Introduction *xiii***

Outline *xvi*

### **1 Performance outcomes of primary school mathematics programs in the Netherlands: A research synthesis *1***

- 1.1 Introduction *2*
- 1.2 Method of the current review *11*
- 1.3 Intervention studies *15*
- 1.4 Curriculum studies *27*
- 1.5 Summary, conclusions, and implications *32*
- 1.A Study characteristics of intervention studies *35*
- 1.B Study characteristics of curriculum studies *43*

### **2 Solution strategies and achievement in Dutch complex arithmetic: Latent variable modeling of change *45***

- 2.1 Introduction *46*
- 2.2 Method *50*
- 2.3 Results *59*
- 2.4 Discussion *69*

### **3 Complex multiplication and division in Dutch educational assessments: What can solution strategies tell us? *75***

- 3.1 Introduction *76*

3.2	Part I: Changes in strategy choice and strategy accuracy in multiplication	85
3.3	Part II: Effect of teachers' strategy instruction on students' strategy choice	99
3.4	General discussion	104
<b>4</b>	<b>Individual differences in strategy use on division problems: Mental versus written computation</b>	<b>111</b>
4.1	Introduction	112
4.2	Method	120
4.3	Results	124
4.4	Discussion	137
4.A	Item Set	142
<b>5</b>	<b>Solution strategies and adaptivity in complex division: A choice/no-choice study</b>	<b>143</b>
5.1	Introduction	144
5.2	Method	152
5.3	Results	154
5.4	Discussion	162
5.A	Complete item set	168
<b>6</b>	<b>The language factor in assessing elementary mathematics ability: Computational skills and applied problem solving in a multidimensional IRT framework</b>	<b>169</b>
6.1	Introduction	170
6.2	Method	174
6.3	Results	181
6.4	Discussion	187
6.A	Sample problems (problem texts translated from Dutch)	193
<b>7</b>	<b>The effects of presenting multidigit mathematics problems in a realistic context on sixth graders' problem solving</b>	<b>195</b>
7.1	Introduction	196
7.2	Method	203

7.3	Data analysis and results	210
7.4	Discussion	218
7.A	The 8 problem pairs in test form A, texts translated from Dutch	224
7.B	Examples of solution strategy categories of Table 7.1	226
<b>8</b>	<b>General discussion</b>	<b>227</b>
8.1	Substantive findings	229
8.2	Contributions to psychometrics	240
<b>References</b> 247		
<b>Author Index</b> 265		
<b>Summary in Dutch (Samenvatting)</b> 271		
<b>Curriculum vitae</b> 283		



# List of Figures

- 2.1 Examples of the traditional long division algorithm and a realistic strategy of schematized repeated subtraction for the problem  $432 \div 12$ . 49
- 2.2 Design of the assessments. 51
- 2.3 Conditional probabilities of the 4-class LC-model. 61
- 2.4 Item-specific effect parameters of each strategy, from model M2. 66
- 2.5 Interaction effects of strategy use with year of assessment (left panel) and with general mathematics level (right panel) from model M3b. 68
  
- 3.1 Largest trends over time from Dutch national assessments (PPONs) of mathematics education at the end of primary school (Van der Schoot, 2008, p. 22), in effect sizes (standardized mean difference) with 1987 as baseline level. Effects statistically corrected for students' gender, number of school years, and socio-economical background, socio-economical composition of school, and mathematics textbook used. 78
- 3.2 Example strategies for multidigit multiplication for the problem  $18 \times 24$ . 82
- 3.3 Distribution of multiplication items over test booklets, in the 1997 and in the 2004 assessment cycles. Symbol  $\times$  indicates item was administered. 88
- 3.4 Conditional probabilities of strategy choice on multiplication problems of the 4 latent classes model, 1997 and 2004 data. 94
- 3.5 Graphical display of interaction effect between strategy used and student's general mathematics level on IRT ability scale, based on multiplication problems in 1997 and 2004 cycles. 98
- 3.6 Fourth grade, fifth grade, and sixth grade teachers' approach to complex multiplication and division problem solving, as reported in J. Janssen et al. (2005, p. 44). 101

4.1 Examples of solution strategies for the problem  $736 \div 32$ . 117

4.2 Probability of applying mental calculation in 3 latent classes. 131

4.3 Hypothesized group means on logistic latent ability scale for one item pair. 134

4.4 Estimated probabilities to solve items 1 to 9 correctly for students at the mean level of mathematics achievement. Left plot: items administered in Choice as well as No-Choice condition, per item students who used mental calculation on that item in the Choice condition are separated from those who used a written procedure. Right plot: items only administered in Choice condition. 136

5.1 Examples of solution strategies for the problem  $306 \div 17$ . 149

6.1 Graphical representation of between-item two-dimensional IRT model. 179

6.2 Graphical display of home language effects (left plots) and reading comprehension level effects (right plots) for the two ability dimensions, grade 1 (upper part), grade 2 (middle part), and grade 3 (bottom part). 185

7.1 Design of experimental task forms. A = Addition, S = Subtraction, M = Multiplication, and D = Division. Problem indices 1 (small numbers) and 2 (large numbers) denote the specific pair within each operation, indices  $a$  and  $b$  denote the two parallel versions within each problem pair. Problems in unshaded cells present numerical problems, problems in cells shaded gray are the contextual problems. 205

7.2 Graphical representation of between-item two-dimensional IRT model. 212

7.3 Strategy choice proportion of recoded solution strategies on numerical (num) and contextual (context) problems, per operation. 216

7.4 Estimated mean accuracy of the three strategies, by operation. 219

# List of Tables

- 1.1 Dutch mathematics assessments results, from Van der Schoot (2008, p. 20-22). 5
- 1.2 Synthesis of results from six studies comparing guided instruction (GI) and direct instruction (DI) in low mathematics performers. 18
- 2.1 Specifications of the items. 52
- 2.2 Part of the data set. 54
- 2.3 Part of the data set in long matrix format. 58
- 2.4 Strategy use in proportions. 59
- 2.5 Latent class models. 60
- 2.6 Class sizes in 1997 and 2004. 62
- 2.7 Relevant proportions of Year, Gender, GML and PBE crossed with class membership. 63
- 2.8 Explanatory IRT models. 64
- 3.1 Specifications of the multiplication problems. 87
- 3.2 Strategy use on multiplication problems in proportions, based on 1997 and 2004 data. 92
- 3.3 Cross-tabulations of the student background variables general mathematics level, gender, and SES with latent strategy class membership (in proportions); multiplication problems, 1997 and 2004 data. 95
- 3.4 Strategy use on multiplication and division problems, split by teacher's instructional approach, based on 2004 data. 102
- 4.1 Descriptive statistics of strategy use and strategy accuracy. 125

4.2 Distributions of written strategies in the No-Choice condition, separate for students who solved that item with a mental (m) or written (w) strategy in the Choice condition. 127

4.3 Distribution of mental computation strategies on items in the Choice condition. 128

4.4 Estimated class probabilities, conditional on gender and GML. Standard errors (SEs) between brackets. 132

5.1 Distribution of type of strategies used in choice condition. 156

5.2 Strategy performance in the choice condition, by gender and general mathematics level. 157

5.3 Strategy performance in the no-choice conditions, by gender and general mathematics level. 158

5.4 Number characteristics of the items. 168

6.1 Pupil background information: distribution of home language and reading comprehension level. 175

6.2 For both subscales, the number of problems per operation, descriptive statistics of the proportion correct scores  $P$  (correct), and Cronbach's  $\alpha$ . 176

6.3 Correlations between total number correct scores, latent correlations between computational skills and contextual problem solving, and Likelihood Ratio (LR) test results comparing fit of the one-dimensional (1D) versus the two-dimensional (2D) IRT models. 181

7.1 Categories solution strategies. 207

7.2 Descriptive statistics of performance (proportion correct) on numerical and contextual problems, by operation, gender, and home language. 210

7.3 Distribution in proportions of solution strategy categories of numerical (num) problems and contextual (con) problems, per operation. Strategy categories refer to Table 7.1. 215

7.4 Strategy choice distribution (in proportions), by gender and language achievement level. 217