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Explanatory latent variable modeling of mathematical ability in primary school : crossing the border between psychometrics and psychology

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Hickendorff, Marian

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Explanatory latent variable modeling of mathematical ability in primary school

*Crossing the border between
psychometrics and psychology*

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