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## Crossed product algebras associated with topological dynamical systems

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THOMAS STIELTJES INSTITUTE  
FOR MATHEMATICS



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# Preface

This thesis consists of an introduction, acknowledgements, a summary (in Dutch), a curriculum vitae (in Dutch) and the following five papers.

- Svensson, C., Silvestrov, S., de Jeu, M., *Dynamical systems and commutants in crossed products*, Internat. J. Math. **18** (2007), 455-471.
- Svensson, C., Silvestrov S., de Jeu M., *Connections between dynamical systems and crossed products of Banach algebras by  $\mathbb{Z}$* , in “Methods of Spectral Analysis in Mathematical Physics”, Janas, J., Kurasov, P., Laptev, A., Naboko, S., Stolz, G. (Eds.), Operator Theory: Advances and Applications **186**, Birkhäuser, Basel, 2009, 391-401.
- Svensson, C., Silvestrov, S., de Jeu, M., *Dynamical systems associated with crossed products*, to appear in Acta Applicandae Mathematicae (Preprints in Mathematical Sciences 2007:22, LUTFMA-5088-2007; Leiden Mathematical Institute report 2007-30; arXiv:0707.1881).
- Svensson, C., Tomiyama, J., *On the commutant of  $C(X)$  in  $C^*$ -crossed products by  $\mathbb{Z}$  and their representations*, to appear in Journal of Functional Analysis (Leiden Mathematical Institute report 2008-13; arXiv:0807.2940).
- Svensson, C., Tomiyama, J., *On the Banach  $*$ -algebra crossed product associated with a topological dynamical system*, submitted (Leiden Mathematical Institute report 2009-03; arXiv:0902.0690).

