Sixth-grade students' mathematical problem-solving behavior. Motivational variables and gender differences

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SIXTH-GRADE STUDENTS' MATHEMATICAL PROBLEM-SOLVING BEHAVIOR: MOTIVATIONAL VARIABLES AND GENDER DIFFERENCES

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"What's one and one and one and one and one and one and one and one and one and one? "
"I don't know", said Alice. "I lost count".
"She can't do Addition", the Red Queen interrupted.

LEWIS CARROLL

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CONTENTS

1 INTRODUCTION

Success in mathematics depends not only on cognitive variables, such as sufficient knowledge about facts and procedures, but also on motivational variables, including for instance beliefs about one's capacity and interest in the subject. In this thesis motivational variables are studied in relation to mathematical problem solving. Emphasis is placed on students' displayed confidence when working on two types of mathematics tasks.

The reasons for setting up this research were twofold. Firstly, this research was aimed at gaining better insights into different aspects of motivational variables and achievement in mathematics, especially at the task-specific level. The study that is described here draws on research that has been directed at students' motivation in concrete learning situations (e.g., Boekaerts, 1991; Seegers & Boekaerts, 1993). Secondly, the research was set up to further explore gender differences in mathematics.

This chapter serves as an introduction to the research. We first focus on gender differences in mathematics performance (section 1.1). Here we restrict ourselves to a short description of the gender differences in mathematics achievement that have consistently been reported in the literature, and to possible causes of these differences. A more detailed outline of gender differences in relation to motivational variables and mathematics will be presented in chapter 3. In section 1.2 we provide a short description of the role of different types of problems within realistic mathematics education in the Netherlands. The specific aim of this study is described in section 1.3, and in section 1.4 the structure of the thesis is outlined.

1.1 Gender differences in mathematics performance

Gender differences in mathematics performance have been the subject of research for many years. A consistent finding in research has been that boys generally outperform girls in mathematics, although lately some authors have stated that in the American context these differences have tended to decrease (Hyde, Fennema, & Lamon, 1990), or even disappear (Frost, Hyde, & Fennema, 1994). These conclusions were based on meta-analyses that were performed on studies of the last decades. However, with respect to the Dutch situation, this tendency has not been confirmed. In two studies that were executed five years apart by the National Institute for Educational Measurement (CITO), clear differences in performance between boys and girls in the final year of primary school were reported (Wijnstra, 1988; Bokhove, Van der Schoot, & Eggen, 1996).

Boys appear to have an advantage which is present from an early age, and this advantage seems to increase with age (Fennema & Carpenter, 1981; Hall & Hoff, 1988). There is no consensus, however, about the age at which these differences appear. Beller and Gafni (1996), for instance, did not find gender differences in the performances of 9-year old students. Van der Heijden (1993) reported that boys outperformed girls at the age of 8 years. In any case, in most studies there is agreement that gender differences are apparent by the time students reach secondary school age (about 12 years) (e.g., Beller & Gafni, 1996).

Possible causes

Over the years a wide range of explanations about the causes of gender differences in mathematics achievement have been offered. Biological, sociological, psychological, and educational factors have been considered as possible causes. Various models have been presented (Fennema, 1985; Ethington, 1992). A distinction can be made between research that examines the effect of environmental variables - such as the influences of parents, teachers, peer group, and the wider society - and research which explores person-related variables, including cognitive and affective variables (Leder, 1992). There is evidence that some environmental variables exert a positive influence on the choices and behavior of males. For example, parental beliefs are a critical factor in determining students' attitudes toward mathematics, and it is believed that parents are often more encouraging of their sons' than their daughters' mathematical studies (Fennema & Sherman, 1977). Differences in patterns of teacher interactions with boys and girls also seem to affect mathematics learning. For example, males tend to receive more encouragement and are more frequently praised for correct answers than females (Hart, 1989; Koehler, 1990; Leder, 1987). It is difficult, however, to estimate the effects of environmental variables on the mathematics performance of boys and girls. Although we acknowledge the importance of this type of research, we restrict ourselves here to the effects of person-related variables on gender differences in mathematics, without making statements about the causes of these differences. Our starting point is that gender differences in mathematics performance are the outcome of complex interaction effects, in which both cognitive and motivational variables play a role.

Content areas

Whether or not gender differences are found may be influenced by the way a test is administered (e.g., open questions versus multiple choice items) and by the content area. In several studies on gender differences in mathematics a distinction was made between different content areas. Marshall (1984) reported that sixth-grade girls performed better in computations than boys, whereas boys performed better than girls when story problems (application problems) were involved. It has often been found that boys score higher than girls on tests that entail problem solving (Ecdes et al., 1985; Kimball, 1989). In a meta-analysis performed by Frost et al. (1994), the effect of the cognitive level of a test was included with their results. They found a slight female superiority in computation, no gender difference in the understanding of mathematical concepts, and a slight male superiority in problem solving. As the complexity of the problems increased, the differences between boys' and girls' achievements also increased, with the boys scoring higher than the girls. Marshall and Smith (1987) found that in grade 6 girls surpassed boys on computations, as they did in the third grade, but that boys had a clear advantage in solving word problems (application problems) and geometry/measurement items.

In the Netherlands, Wijnstra (1988) reported that at the end of primary school boys outperformed girls on almost every subscale, except for computations. This pattern was confirmed in a second study that was executed five years later (Bokhove et al., 1996). In an item-specific analysis that was based on these studies, Van den Heuvel-Panhuizen (1996b) selected items on which differences in favor of boys were the most and the least evident. This analysis revealed, among other things, that boys were better at the subscales measurement, ratios, percentages, and estimation. Girls, on the contrary, performed better on assignments in which detailed and precise calculations were required, and on routine assignments with standard procedures. Seegers and Boekaerts (1996) found that sixth-grade boys scored better than girls on the mathematical topics fractions, percent problems, ratios, and measurement. They found that the differences were more pronounced when the difficulty of the items increased.

A general pattern that can be derived from these studies is that boys perform better than girls when it comes to more complex applied problem solving, but that no differences, or even slight differences in favor of girls, can be found when exact computations are involved. These conclusions are worrisome, all the more because with the renewal of mathematics education, solving application problems is becoming a major part of the mathematics curriculum in the Netherlands. This renewal has been inspired by the educational theory of realistic mathematics education. In the next section of this chapter, we will briefly sketch the main characteristics of this theory of mathematics education. Our objective is to illustrate the importance of applied problem solving within realistic mathematics education.

For a more detailed overview, we refer to Gravemeijer (1994), and Van den Heuvel-Panhuizen (1996a).

1.2 The role of problems within realistic mathematics education

Realistic mathematics education in the Netherlands has its roots in the seventies, when Freudenthal's ideas on mathematics education inspired educators to gradually change the mathematics curriculum. Freudenthal (1973, 1991) emphasized the idea of mathematics as a human activity, in contrast to the idea of mathematics as a closed system of formal rules, algorithms, and definitions. Mathematics, according to Freudenthal, must be connected to reality and can best be learned by doing. According to this view, students should be given the opportunity to develop all sorts of mathematical skills and insights themselves, starting from concrete problems in realistic settings. Thus, the students' own contributions, instead of formal rules, are the starting point from which learning takes place. In this context, Freudenthal used the term *guided reinvention principle.* According to this principle, students should be given the opportunity to experience a process similar to the process by which mathematics was invented. However, a certain degree of guiding within this process is inevitable. Freudenthal (1991) admitted that this is a far from easy task for educators, by stating that "guiding reinvention means striking a subtle balance between the freedom of inventing and the force of guiding, between allowing the learner to please himself and asking him to please the teacher" (p. 48). The implications of this theory for educational practices are still subject of study (e.g., Gravemeijer, 1994; Treffers & De Moor, 1990).

Freudenthal ideas' on mathematics education have mainly influenced mathematics education in primary schools (4-12 year-old students). Until now, these influences are especially evident in curriculum development and textbooks for mathematics education. Nowadays more than 75% of all primary schools in the Netherlands use a modern method for mathematics education that is based on the principles of realistic mathematics education (Treffers, 1991a). These modern textbooks differ from more traditional textbooks in regard to the learning material, the composition of the learning strands, and the place, nature, and extent of applications or context problems (Treffers, 1991b). Applied mathematics problems are considered to be an important part of the curriculum. A distinction is made between traditional application problems and context problems (Treffers & De Moor, 1990). Traditional application problems fulfill the function of applications afterwards, that is, after having learned the procedures within the formal system.

According to these authors the applicability of these problems is rather limited. The problems are routine, the solution schema is ready-made, and the nature of these problems does not allow reasoning from the context in which the problem is stated. Nevertheless, these problems do have a function within mathematics education, but mainly as practice assignments.

According to Treffers and De Moor (1990), realistic mathematics education should be aimed, to a large extent, at problems that are stated within a context. As such, the problem itself is central, and mathematical knowledge serves as a tool to solve this problem. An important feature of context problems is that there is always a variable element, which can lead to real considerations when a solution is verified within the context of the problem. Furthermore, context problems are characterized by the associations a problem calls up. Students are allowed to bring in their own knowledge about the situation; in fact they rely on this common knowledge for problem-solving. Consider, for example, the following context problem:

> *A bottle contains* 75 *centiliter of wine.*
> *How many 20 centiliter glasses can be filled from this bottle?*

The context allows more solutions to the problem: Some students will state that 3 glasses can be filled; others will state that 3¾ glasses can be filled; and others will give the solution "almost 4". It is also imaginable that these types of problems evoke a lot of questions, such as, for instance, "How full are you supposed to fill the glasses? If you put 15 centiliter in each glass, you can fill 5 glasses". However, if the computation 75 : 20 = ? had been given without any context, only one solution would have been possible.

To summarize, we stated that an adequate explanation of gender differences in mathematics performance should be based upon the complex interactions between cognitive and motivational variables. We described that in the research literature no gender differences (or slight differences in favor of girls) have been reported with respect to exact computations. However, it has often been found that boys perform better than girls when it comes to solving application problems. With the renewal of mathematics education, it is exactly this kind of problems that has become an important part of the mathematics curriculum in the Netherlands.

1.3 Object of this study

In this study, elements from research within cognitive psychology on mathematical problem solving on the one hand (e.g., Schoenfeld, 1983, 1985, 1992), and elements from research that is directed at a task-specific approach of motivation and learning on the other hand (e.g., Boekaerts, 1987, 1991, 1992), are integrated. Theory regarding these two approaches will be discussed in chapters 2 and 3, respectively.

Our first purpose was to investigate gender differences and intraindividual differences in both cognitive and motivational variables in relation to two types of mathematics tasks. Our focus was on students' actual behavior while they were solving problems. Drawing on studies in which gender differences have been reported involving content-specific areas of performance, a distinction was made between two types of mathematics problems: computation problems and application problems. In the research literature computation problems are also referred to as algorithms, bare problems or numerical expressions; application problems are also known as word problems, story problems, verbal problems or context problems. A computation problem is characterized by the fact that a precise, systematic and detailed plan should be executed. If this plan is carried out completely and in the right order, it will lead to the right solution with a hundred percent certainty. An application problem may also include a sequence of steps, but this sequence is less complete and less systematic than within a computation problem. Characteristic of these problems is that one or more translations have to made from a text version to one or more mathematical operations. Especially with context problems, an extra difficulty is involved because students should evaluate their solution within the context of the problem. As will be stated in chapter 2, solving application problems can be considered to be a form of mathematical problem solving.

Our next purpose was to investigate relations between cognitive and motivational variables. As described in section 1.1, an adequate explanation of gender differences in mathematics performance should be based upon the complex interactions between cognitive and motivational variables. We examined to what extent students' self-referenced cognitions influence their behavior while solving the two types of mathematics problems, and whether gender differences exist in this context. Here, the model of adaptable learning as developed by Boekaerts was taken as a starting point (see chapter 3).

1.4 Structure of this thesis

Chapters 2 and 3 provide the theoretical background for this thesis. In chapter 2, a selected review is presented of relevant cognitive, metacognitive, and affective variables that contribute to mathematical problem solving. We defend the view that it is important to include, besides cognitive issues, metacognitive and affective issues in the study of mathematical problem-solving behavior. Chapter 3 focuses on students' self-referenced cognitions in relation to mathematics, and on gender differences in this context. We describe Boekaerts' model of adaptable learning, in which cognitive and affective variables are integrated. Chapters 4, 5, and 6 describe the empirical research. Chapter 4 outlines the method of our research, whereas the results are described in chapters 5 and 6. Chapter 5 provides results on descriptive statistics, as well as gender differences in relation to the variables in our research and intra-individual differences across the two types of tasks. Chapter 6 describes the results on relations between cognitive and motivational variables. Chapter 7 presents the conclusions and discussion.

2 MATHEMATICAL PROBLEM SOLVING: COGNITIVE, METACOGNITIVE AND AFFECTIVE VARIABLES

As described in chapter 1, an important aspect of realistic mathematics education is the focus on students' own contributions and solution strategies, instead of on formal rules and algorithms. In this context, application problems comprise an important part of the mathematics curriculum. This chapter focuses on relevant skills that students need to possess in order to solve these problems successfully. We argue that those skills are not only cognitive by nature, and stress the importance of including metacognitive and affective variables in the study of mathematical problem solving. We focus on the interplay between these variables in consideration of an adequate explanation of individual differences (especially gender differences) in mathematics performance. We restrict ourselves to a descriptive analysis of mathematical problem-solving behavior; instructional issues fall beyond the scope of this thesis.

After an introduction to relevant concepts in section 2.1, the following factors that may help or hinder successful mathematical problem-solving are discussed: prior knowledge, heuristics, metacognition, beliefs, attitudes, and emotions (2.2). These factors can be distinguished according to the cognitive loading they possess, and according to the impact these factors may have in the different phases of the solution process. The importance of affective issues will be briefly discussed in this chapter. A more extensive theoretical background will be outlined in chapter 3. In section 2.3 we address the interplay between (meta)cognitive and affective variables, trying to explain individual differences in problem-solving behavior. Finally, the research perspective of this thesis is described (2.4).

2.1 Conceptual framework

Problem solving

Resnick and Glaser (1976) stated that "psychologists agree that the term 'problem' refers to a situation in which an individual is called upon to perform a task not previously encountered and for which externally provided instructions do not specify completely the mode of solution. The particular task, in other words, is new for the individual, although processes or knowledge already available can be called upon for solution" (p. 209). With this definition Resnick and Glaser apply the term problem solving only to situations that are new to a student. In this way, the ability to solve problems can be seen as equivalent to intelligence.

Other authors proposed a broader definition of problem solving. Frijda and Elshout (1976) defined problem solving as "that cognitive activity (that is, that information-processing activity), at which the subject (person or animal) tries to find an answer to a problem" (p. 414). According to these authors, a problem can be defined as a situation in which (1) the subject is confronted with a task, assignment or difficulty, and (2) he has no immediate answer available, and he can not find the answer by means of an automated series of actions. Mayer's (1985) definition is comparable. He stated that "a problem occurs when you are confronted with a given situation - let's call that the *given state* - and you want another situation - let's call that the *goal state* - but there is no obvious way of accomplishing your goal" (p. 123). In Mayer's definition, problem solving refers to the process of moving from the given state to the goal state of a problem. This involves a series of mental operations that are directed toward that goal.

We adopted the definitions given by Frijda and Elshout (1976) and Mayer (1985), which are broad enough to be applied to problems ranging from geometry (e.g., Polya, 1957) to chess (e.g., De Groot, 1965). However, as a consequence of these definitions, it is impossible to objectively define a situation as a problem. Whether or not something is experienced as a problem depends on the subject in a certain situation. For example, for most sixth-grade students, the question: "How much is 18 : 6 ?" is not a problem because they have the fact memorized, but it is a problem for younger children. It is also possible that a certain task can become a problem for a person, although it had not been a problem before. Van Streun (1989) reported that for some 13-14 year old students the assignment "- 6 - 4 = ?" was considered a problem, although it had not been a problem at the time when instruction on the addition and subtraction of negative numbers was given half a year earlier.

Mathematical problem solving

Mathematical problems can be defined as problems in which one or more numerical relations are presented. Mathematical assignments can be presented as algorithms or computation problems, or as application problems (see chapter 1). When solving computation problems, students may or may not perform an automatized series of actions. For example, when sixth-grade students are asked to add $\frac{1}{3} + \frac{1}{8}$, some students will remember the rule for adding fractions, and will begin making calculations immediately. Others, who have forgotten how to add fractions, may nevertheless attempt to solve this problem by, for instance, making a drawing.

When solving applied mathematical problems, students are confronted with problems to which they have no immediate answer. For application problems a distinction is often made between non-algorithmic problems (for which students have no ready-made solution method), and routine problems (which require the application of a familiar procedure). In this way, routine problems are usually considered not to apply to mathematical problem solving. In our view, however, this distinction is rather artificial and certainly does not hold for realistic mathematics education (see chapter 1) in which much importance is ascribed to taking account of the context in which a problem is stated. As such, solving application problems requires more than an automated series of actions or the application of rules. For application problems the student first has to understand the problem, then, if necessary, transform it into a problem for which a solution method is available, execute the solution method, and finally verify the answer in the context of the problem given. In all these stages of the solution process possible stumbling blocks to successful problem solving may be encountered by students. The translation of a problem into a representation is not an automatized process, but a process which involves understanding and reflection. And after the problem has been represented, it is not a matter of blindly applying rules. Consider again the following problem:

> *A bottle contains 75 centiliter of wine.*
> *How many 20 centiliter glasses can be filled from this bottle?*

For some students this will be a routine exercise, because they immediately see which algorithm should be applied; others will need more time in order to explore the problem and translate it into a mental representation. Suppose a student immediately sees what algorithm should be applied. According to the definition of Frijda and Elshout (1976), that particular student is not solving a problem, but performing a routine exercise. However, after having done the routine exercise, the problem is not yet solved, because the solution must be verified within the concrete context of the problem. This is not an automated process, and therefore we can call this process problem solving.

These illustrations show that the distinction between mathematical problem solving and the mere applications of rules is not always clear. As already stated before, it depends on the subject in a certain situation whether or not something is experienced as a problem.

Phases within the problem-solving process

In the study of problem solving, Polya's (1957) book "How to solve it" has been influential. Polya distinguished four phases that a competent problem solver should pass through. They are: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) looking back. When solving mathematical problems the verbal statement of the problem must first be understood. The principal parts of the problem have to be identified, namely the unknown, the data, and the condition. Then a plan should be devised. According to Polya, we have a plan when we know, at least in the form of an outline, which calculations, computations, or constructions we have to perform in order to obtain the unknown. Carrying out the plan is much easier then formulating this plan. According to Polya: "To devise a plan, to conceive the idea of the solution is not easy. It takes so much to succeed; formerly acquired knowledge, good mental habits, concentration upon the purpose, and one more thing; good luck. To carry out the plan is much easier; what we need is mainly patience" (1957, p. 12). The last phase of the solution process involves examining the solution that was obtained. Relevant questions are, for instance: Can you check the result? Can you derive the result differently?

Polya's four phases have served as a framework for many researchers investigating a multitude of processes that may foster successful problem solving. Garofalo and Lester (1985) presented a framework for studying a wide range of mathematical tasks, not only those classified as problems. This framework comprises four categories of activities involved in performing a mathematical task: orientation, organization, execution, and verification. The four categories are related to, but are more broadly defined than, Polya's four phases. Orientation refers to strategic behavior to assess and understand a problem. This category includes for instance comprehension strategies, analysis of information and conditions, and assessment of level of difficulty and chances of success. Organization includes planning of behavior and choice of actions. Identification of goals and subgoals are important features within this category. Execution refers to the regulation of behavior conformable to plans. This regulation includes performance of local actions, monitoring of progress of specific and general plans, and trade-off decisions. Verification, finally, implies evaluation of decisions made and of outcomes of executed plans. In this phase a distinction is made between evaluation of orientation and organization, and evaluation of execution, respectively.

Of course, for different problems, a particular phase in the solution process needs more attention than the others. When solving non-routine application problems, it is evident that the student has to put a lot of effort into representing the problem (orientation phase). Based on the outcome of the orientation phase, students will plan and execute solution steps to solve the problem. In addition, it is clear that for many context problems, the verification phase will be crucial. These phases sometimes overlap. For instance, students will already make plans while reading the problem, especially when solving routine application problems. So executing the plan and making (new) plans may overlap, and, especially when difficulties are encountered, students will be inclined to adjust their plans.

2.2 Factors contributing to mathematical problem solving

Substantial progress has been made in characterizing cognitive processes that are important to success in mathematical problem solving. Within the cognitive approach (also called the information processing approach) research has been directed at developing models that describe how people store information in memory, and how they activate this information in problem-solving situations. Central questions within this approach have been: What information relevant to the problem does the problem solver possess? And how is this information accessed and used? Emphasis has been placed on how different types of knowledge contribute to problem solving, which is seen as the central issue in problem solving. It is assumed that students' performances differ because of differences in information processing systems and in amounts, as well as types of knowledge.

However, in the last decade, prominent researchers in the field have come to agree that problem solving is a process in which various factors interact (e.g., Schoenfeld, 1985; Lester, 1985). Schoenfeld (1985) outlined a framework for the analysis of mathematical behavior which distinguishes four categories of knowledge and behavior that contribute to successful problem solving: resources, heuristics, control, and belief systems. Resources refer to mathematical knowledge, such as factual knowledge and knowledge of algorithmic procedures. Heuristics are rules of thumb for effective problem solving, including, for instance, drawing figures, and exploiting related problems. Control refers to general decisions regarding the selection and implementation of resources and strategies, such as planning, monitoring, and decision-making. In short, these are decisions about what to do in order to reach a solution. As such, control can be considered a metacognitive aspect of problem solving, referring to the regulation of cognition. Belief systems, finally, refer to "one's mathematical world view" about self, the environment, the topic, and mathematics. In Schoenfeld's view, belief systems shape cognition, even when one is not consciously aware of holding those beliefs. According to Lester (1994), problem-solving performance seems to be a function of several interdependent categories of factors (e.g. knowledge acquisition and utilization, control, beliefs, affects, and sociocultural contexts). These categories overlap and interact in a variety of ways. There is general agreement on the importance of knowledge, heuristics, metacognitive issues, beliefs, attitudes, and emotions for successful mathematical problem solving (see, e.g., De Corte, Gréer, & Verschaffel, 1996). However, the terminologies which are used may differ. Some authors, for instance, consider beliefs as affective variables (e.g., McLeod, 1992); whereas others classify beliefs as metacognitive issues (e.g., Garofalo & Lester, 1985).

Prior knowledge

In the literature, a distinction is often made between declarative and procedural knowledge. The former kind of knowledge refers to *knowing that*, while the latter refers to *knowing how*. Mayer, Larkin, and Kadane (1984) made a more detailed distinction between four types of knowledge that are central in mathematical problem solving: (1) linguistic and factual knowledge, (2) schematic knowledge, (3) strategic knowledge, and (4) algorithmic knowledge. Linguistic and factual knowledge are necessary in order to translate the words of the problem into an internal representation. Schematic knowledge refers to knowledge about different types of problems - knowledge which is needed in order to understand the problem. A scheme refers to a structure that clarifies the relations among variables in the problem, which allows a student to fit the variables of the problems into a structure that is already familiar. Strategic knowledge refers to the problem solver's knowledge concerning how to establish and monitor plans for goals. Algorithmic knowledge refers to knowledge about how to carry out some procedure that is needed for problem execution. For example, consider the following application problem:

> *A plumber earns f54, - an hour.*
> *He needs 1 hour and 20 minutes to finish the job.*
> *How much money should he get?*

A student first has to transform the problem into a mental representation. In the understanding phase, the bits of information in the problem must be integrated into a coherent whole. In order to do this, the student needs to know that there are 60 minutes in an hour. Furthermore, students should be aware that this is a time-money problem, in which there is a proportional relation between the amount of money earned and the amount of time needed to do the work. In the planning phase, a more concrete plan has to be developed in order to solve the problem. Different plans are possible. For example, students may translate 1 hour and 20 minutes into 80 minutes and try to figure out what x is in the proportion: 54 : 60 as x: 80. Another plan would be to figure out how many times 20 minutes fit into one hour, divide the amount by the right number, and add the amount to *f* 54.- . In the last phase of the solution process, students must know how to carry out certain procedures, such as *f* 54.- : 3 =.

It may be evident that it depends on the type of mathematics problem what knowledge is of crucial importance, and what knowledge is not. When solving application problems, all types of knowledge may add to the solution. However, when solving computation problems, knowledge about how to carry out a specific procedure may be sufficient.

Heuristics or problem solving strategies

Heuristic strategies are rules of thumb for successful problem solving or general suggestions that help an individual to understand a problem better or to make progress towards its solution. Heuristics are especially important when it comes to solving non-routine problems. When students are confronted with problems for which they can not retrieve an answer from memory, or for which they have no ready-made solution method, problem analysis is required. Heuristic procedures can be helpful in the problem analysis. The aim of using heuristics is to transform the problem into a familiar task for which a solution procedure is already known.

Polya (1957) stressed the importance of heuristics for effective mathematical problem solving. Some examples of heuristic methods are: dissecting the problem into subproblems, finding an easier or related problem, or visualizing the problem using a diagram or a drawing. For example, consider the following problem (Van Essen, 1991):

> *There are 9 apple trees in a line.*
> *The distance between two trees is 3.4 meters.*
> *What's the distance between the first and the last tree?*

In this situation, making a drawing can be very helpful. In the plumber problem given earlier, making a proportion table may be a useful heuristic. A heuristic strategy can be general or specific. General heuristic strategies can be applied to a variety of problems because they do not require specific domain-specific knowledge. An example of a general heuristic is making an inventory of the problem.

Problem analysis should be made answering questions like "What is the precise nature of the question?"; "What are the given?". Specific heuristics can only be applied to a limited range of problems. An example would be making the problem easier by making the number in the problem smaller and seeing what will happen.

Several studies have identified a relationship between the use of heuristic procedures and mathematical problem-solving behavior (e.g., Kantowski, 1977). Based on these findings, researchers have concentrated on training students to use relevant problem-solving heuristics. However, these programs did not have as much effect as was expected.

The importance of metacognitive and affective factors in problem solving

In the last few years, researchers have become aware of the shortcomings of models in which only knowledge and heuristics are considered relevant for problem solving. Research has indicated that relevant knowledge and procedures may be available, but may simply be ignored in "real-world" contexts. Lester (1983) and Schoenfeld (1983) believe that the failure of most efforts to improve students' problem-solving performance is largely due to the fact that instruction has overemphasized the development of heuristic skills and has ignored the managerial skills necessary to regulate one's thinking.

Lately, the importance of metacognitive and affective factors in problem solving has been stressed. For example, doing mathematics requires not only knowledge of rules, facts and principles, but also an understanding of when and how to use that knowledge. In general, information-processing theories have not placed much emphasis on metacognitive and affective issues. Nevertheless, these issues are important in mathematical problem solving in school situations.

Polya already acknowledged the fact that solving problems is not purely an "intellectual affair". He claimed that "teaching to solve problems is education of the will" (1957, p. 94). Polya identified several characteristic behaviors of students in the different phases of the solution process that may impair successful problem solving. First he pointed out that there was often an incomplete understanding of the problem, owing to lack of concentration. With respect to devising a plan, he distinguished two opposite behaviors: Some students rush into calculations and constructions without any plan or general idea, whereas other students wait passively for some idea to come and do nothing to accelerate the generation of ideas. When carrying out a plan, the most frequent concern is carelessness, or a lack of patience in checking each step. Finally, students often fail to check the result at all.

The student is glad to get an answer, throws down his or her pencil, and is not alerted by the most unlikely results.

Metacognitive issues

Metacognition has two separate but related aspects. According to Flavell (1976), metacognition includes both knowledge about cognition and the regulation and control of cognitive actions. In relation to the first meaning, a distinction can be made between knowledge of cognitions related to the person, task, or strategy (Flavell & Wellman, 1977). In the context of mathematics, metacognitive knowledge consists of how one views oneself and others as cognitive beings. Within the task category, metacognitive knowledge includes knowledge about the scope and requirements of tasks, as well as knowledge about the factors and conditions that make some tasks more difficult than others. Lester and Garofalo (1982) found that many third and fifth graders believe that the size and the number in a verbal problem (application problem) are important indicators of difficulty, and that verbal problems are harder than computation problems. They also found that students believe that verbal problems can be solved by a direct application of one or more arithmetic operations, and that the operations which should be used can be determined merely by identifying the key words. Metacognitive knowledge about strategies includes knowing when certain strategies can be used and knowing when and how to apply them.

The regulation and control of cognition is concerned with a variety of decisions and strategic activities. Examples are selecting appropriate strategies to carry out plans, monitoring execution activities, and abandoning non-productive strategies. Consider, for instance, the plumber problem: When students decide to solve the problem by calculating how much the plumber earns in 20 minutes and add this amount to *f* 54.- , they should be aware that the problem is not solved after having calculated the first step. Monitoring of progress is an important aspect of this problem.

Lester and Garofalo (1982) found that elementary students often do not analyze problem information, monitor progress, or evaluate results. Schoenfeld (1985) reported similar results for college students. He found that they did not adequately monitor and control their solution behavior when solving mathematical problems. He developed a method which is useful for studying the control behaviors of problem solvers by examining problem-solving protocols. The protocols are divided into major episodes, or periods of time when the problem solver is engaged in certain actions: reading, analyzing, exploring, planning, implementing, or verifying. According to Schoenfeld, it is precisely during the transitions between these episodes that students make managerial decisions. The method is designed to help the researcher locate those places where problem solvers either should be, or are likely to be, engaging in metacognitive behaviors.

Beließ, attitudes, and emotions

The term *affect* is usually referred to as a wide range of feelings and moods that are generally regarded as something different from pure cognition. According to the Encyclopedia of Psychology, affect refers to "a wide range of concepts and phenomena including feelings, emotions, moods, motivation, and certain drives and instincts" (Corsini, 1984, p. 36). The distinction between metacognition and affect is not always clear. For example, Schoenfeld (1983) and Garofalo and Lester (1985) consider belief systems and motivation metacognitive components, whereas others (e.g., McLeod, 1989) consider those aspects affective components.

In the framework proposed by McLeod (1992), beliefs, attitudes, and emotions reflect the range of affective reactions involved in mathematics learning. These three types of affective reactions are not only distinct with respect to stability, but also with respect to their degree of cognitive loading. Beliefs have a very strong cognitive component; this cognitive loading decreases as one progresses from beliefs to attitudes to emotions. Beliefs and attitudes can be considered to be rather stable concepts, whereas emotions are more situation dependent. Here, we adopt Mandler's view: It is possible to differentiate between concepts which have a higher or lower degree of cognitive loading. A strict distinction, however, between variables that refer only to cognition or only to affect, is not possible.

One's beliefs and attitudes in relation to mathematics can determine how one chooses to approach a problem, which techniques will be used or avoided, how long and how hard one will work on it, and so on. According to Lester, Garofalo and Kroll (1989) an individual's beliefs (about self, mathematics, and problem solving) play a dominant, often overpowering, role in his or her problem-solving behavior. Research on self-concept, attributions and related areas tend to focus on beliefs about the self. We consider those aspects of beliefs to be important variables in our research, and therefore we will discuss them in detail in the next chapter. In this chapter, we will only discuss beliefs related to mathematics as a subject-matter.

Research on students' beliefs about mathematics has received considerable attention in the last 15 years. Lester and Garofalo (1982) reported that third and fifth graders believe that mathematical problems can always be solved by using basic operations and can always be solved in only a few minutes. Schoenfeld (1985), for example, found that many students believe that problems can be solved quickly or not at all.

The role of emotions in mathematical problem solving has not yet been the subject of systematic research, although many authors within the field of cognitive research stress the importance of emotional issues (e.g., Norman, 1981; Mandler, 1989). Burton (1984) described how affective responses may occur in the problem-solving process: As problem solvers engage in a problem, their curiosity is aroused. This entry phase can be followed by embarking on the problem (by those who have sufficient confidence) or withdrawing from it (by those who do not). Buxton (1981) reported that some adults described their emotional reaction to mathematics as panic. Their reports of panic were accompanied by a high degree of physiological arousal; this arousal was so difficult to control that they found it disrupted their ability to concentrate on the task. According to Mandler (1989), an important reason for the appearance of emotions during mathematical problem solving is the interruption of plans. These interruptions of planned sequences of thought or actions are called blockages, or discrepancies between what was expected and what is experienced. Thus, the blocks that inevitably interrupt problem-solving activities may lead to intense emotions. We agree with Carver and Scheier (1988) that the existence of certain emotions is less important than the way persons respond to these emotions. These authors argue that, in spite of feeling frustrated, people believe that they will be successful in attaining a desired goal, they will continue striving, will use resources effectively, and, in the end, there will be little or no impairment of their performance. Even when frustrated, people who are confident will continue to try. However, if a student is doubtful about the possibility of a good outcome, he or she may experience an impulse to disengage from the task, and this may cause a deterioration in performance.

Interactions between cognitive, metacognitive, and affective variables

In mathematical problem solving, the variables that were described above may interact in various ways. According to McLeod (1990), knowledge of one's own cognitive processing is closely related to notions of self-concept and confidence, and the regulation or executive control of cognitive processes is intimately connected to one's reactions to the frustrations of working on nonroutine problems and the willingness to persist in mathematical tasks.

Garofalo and Lester (1985) proposed a cognitive-metacognitive framework in which key points are specified where metacognitive decisions are likely to influence cognitive actions. Based on Flavell and Wellman's (1977) variables, a distinction should be made between person variables (e.g., motivation, perseverance), task variables (e.g., task content), and strategy variables (e.g., individual's awareness of the usefulness of a strategy). The interactions of person, task, and strategy knowledge have an influence on the decision to regulate one's activity (Garofalo & Lester, 1985). For example, if students believe that all problems can be solved by merely applying the operations suggested by the key-words in the problem and have previously experienced success in solving word problems with this approach, then students are likely to continue to use this approach.

When studying metacognitive and affective variables during problem solving, one must be aware that they are closely tied to one another. Bandura (1986) stated that "Moods can affect self-referent thinking which, in turn, affects how well people execute what they know. A comprehensive approach to problem solving must therefore consider how self-referent thinking impinges on problem-solving thinking as people experience successes, setbacks, and failures in the search for adequate solutions" (p. 465).

According to Schoenfeld (1992), metacognition in the sense of self-regulatory procedures, including monitoring and "on-line" decision-making, is closely tied to affective phenomena. For example, in trying to solve a problem, a student must make decisions regarding which strategy to apply and how long to keep on trying before attempting a new strategy. Schoenfeld (1983) has argued that the decisions that have to be made during problem solving can be influenced by all sorts of affective factors, such as expectations regarding success and failure, confidence in one's mathematical ability, and the capacity to persist in the face of difficulties.

These affective influences on problem solving will vary depending on the heuristic strategies being used (McLeod, 1989). Consider, for example, a student who attempts to solve every problem through trial and error. A succession of errors may undermine confidence and pleasure in doing the task. If this student had more heuristic strategies at his or her disposal, the affective response might have been different.

In summary, factors that contribute to mathematical problem solving should be considered in relation to each other. Both cognitive and affective variables may help or hinder successful mathematical problem solving. In the next part of this chapter, the consequences of this approach for studying individual differences in problem solving will be discussed.

2.3 Individual differences in mathematical problem-solving behavior

Studying individual differences in problem-solving behavior can be done from two different perspectives, namely from the task perspective and from the person perspective. Two questions can be raised: (1) What makes a problem difficult for students, and (2) How are successful and unsuccessful problem solvers different? When research on problem solving first began, researchers often studied task variables, such as syntax variables and types of problems. Recently, researchers have become interested in problem solver characteristics and in the interaction between task- and problem solver characteristics. With respect to the latter, a lot of research has concentrated on comparing novices and experts, in particularly concerning the development and organization of knowledge.

Expert-novice comparisons

The rationale behind comparing experts and novices is that it may provide theoretical insights into the nature of effective problem-solving performance as well as into the kinds of difficulties inexperienced problem solvers may encounter. Major conclusions with respect to expert-novice differences are summarized by VanLehn (1989). In general, researchers have found that experts not only have more quantitative knowledge, but also have a qualitatively different organization of this knowledge, as compared to novices. An important finding is that experts are better at monitoring the progress of their problem solving and directing their efforts appropriately. Schoenfeld (1985) found that experts have superior self-monitoring abilities. A related finding is that experts are able to estimate the difficulty level of a task with higher accuracy than novices.

Schoenfeld (1983, 1985, 1987) has done a lot of research comparing experts and novices with respect to the monitoring of their solution behaviors. His research suggests that good problem solvers can be distinguished from poor problem solvers with respect to the following aspects: (1) Good problem solvers know more than poor problem solvers and their knowledge is well-organized - it is well connected and composed of rich schemata, (2) Good problem solvers tend to focus their attention on structural features of problems, compared to poor problems solvers who concentrate on surface features, (3) Good problem solvers are more aware of their strengths and weaknesses in problem solving, and (4) Good problem solvers are better at monitoring and regulating their problem-solving efforts. The first conclusion refers to the knowledge base of the problem solver, both quantitative and qualitative. The second conclusion refers to the strategies that problem solvers use, whereas the third and fourth conclusion are related to metacognitive issues.

Possible stumbling blocks during mathematical problem solving

There are several reasons why a problem-solving attempt can go wrong. Failures in mathematical problem solving can not solely be traced back to inadequate knowledge or strategies. During problem solving, all kinds of decisions have to be made about when (and when not) to apply certain knowledge and strategies. Metacognitive and affective issues, such as monitoring one's progress and being persistent despite difficulties, are also essential factors in problem solving. However, less is known about the influence of metacognitive and affective factors on problem-solving behavior. In the following part of this chapter, a number of possible stumbling blocks to mathematical problem solving will be discussed, tracing the different phases of the solution process. Here we adapt the categories of activities that were distinguished by Garofalo and Lester (1985), namely orientation, organization, execution, and verification. In our view there is a lot of overlap between the second and the third categories, so they are considered here as one category.

Orientation

According to Silver and Marshall (1990), there is considerable evidence suggesting that failures to solve problems can often be attributed to failures to understand the problem adequately: that is, failures to construct adequate initial problem representations. De Corte and Somers (1982) found that 78% of the wrong answers on a word problem (application problems) test administered to sixth graders reflected interpretation errors. Understanding the problem involves both analyzing the grammatical and semantic structure of the text, and developing a representation of the problem. Students may lack adequate linguistic or factual knowledge in order to understand the problem, or knowledge may be present in the wrong way. The latter case we refer to as misconceptions.

Translation from words to equation(s) appears to be difficult, particularly when students are confronted with problems in which relational propositions are stated. Loftus and Suppes (1972), for instance, found that sixth graders who were asked to solve a number of word problems had the most difficulties with the problem:

> *Mary is twice as old as Betty was 2 years ago.*
> *Mary is 40 years old. How old is Betty?*

Other possible sources of failure are a lack of knowledge about particular types of problems and a lack of knowledge of appropriate strategies. Errors may occur when students miscategorize a problem and use an inappropriate schema, and as a consequence inappropriate strategies. It was found that students often analyze problems superficially and decide to apply a certain strategy on the base of key words in the problem. Lester and Garofalo (1982) asked third and fifth graders to solve the following problem:

> *Tom and Sue visited a farm and noticed there were chickens and pigs.*
> *Tom said, 'There are 18 animals.'*
> *Sue said, 'Yes, and they have 52 legs in all.'*
> *How many of each kind of animal were there?*

They found that almost all third graders added 18 and 52, while most of the fifth graders tried to solve the problem by dividing 52 by 18. This "number crunching" without reflection, also known as "blind calculating", is often mentioned in the literature.

It is hypothesized that students' misconceptions contribute to their difficulties with application problems. Research has revealed that many students believe that multiplication always makes numbers bigger and division always makes them smaller (e.g., Bell, Fischbein & Gréer, 1984; De Corte, Verschaffel & Van Coillie, 1988).

There are reasons to believe that some students give up at the beginning of the problem solving process even though adequate knowledge and strategies are available. There can be many reasons for giving up, many of which may be connected with students' expectations of success, or confidence. When a problem is conceived of as too difficult, some students will not even try to analyze the problem. Beliefs with regard to one's ability in mathematics play a significant role. For instance, students who believe that they are poor in solving mathematics problems where percentages are involved, will express doubts about their ability to be able to solve the problem as soon as they see a percent sign. Although the problem might not be difficult, doubts in the orientation phase may have a significant impact on problem-solving behavior.

Organization and execution

When carrying out the solution plan, errors may occur in the calculations or procedures. These errors can be either temporary or consistent. A lot of research has been done on diagnosing and classifying computational errors, in particular so called *bugs,* which are described as consistently incorrect actions based on misunderstandings (e.g., Brown & Burton, 1978; Brown & VanLehn, 1980).

Lack of monitoring and control in problem solving is an issue that is getting more attention nowadays. Schoenfeld (1985) described disastrous decisions at the planning stage, and failing to monitor and evaluate problem-solving activities, as being causes for unsuccessful problem solving. He analyzed protocols of students who failed to solve a geometry problem because of poor executive control. These students, who possessed the adequate knowledge for solving the problem, appeared to explore inadequate approaches to the problem without assessing whether progress was being made. Kroll (1988) observed college-age students solving mathematical problems and found a tendency for some students to go in the wrong direction for a long time. She noted that these students had less success in problem solving than students who changed plan whenever necessary. Based on these and other findings, Lester *et* al. (1989) concluded that persistence is not necessarily a virtue in problem solving.

According to Schoenfeld (1985), students' beliefs about mathematics may weaken their ability to solve non-routine problems. If students believe that mathematical problems should always be completed in five minutes or less, then they may be unwilling to persist in trying to solve problems that may take substantially longer.

As mentioned above, Mandler (1989) viewed the interruption of plans as an important reason for the appearance of emotions during problem solving. These interruptions of planned sequences of thought or actions are called blockages, or discrepancies between what was expected and what is experienced. The lack of a systematic plan may result in frequent interruptions, especially during mathematical problem solving. There are indeed many reasons why an anticipated sequence of actions might not be completed as planned, and the individual's knowledge and beliefs about the mathematical problem-solving process play a significant role in the interpretation of these interruptions. For instance, students who believe that all mathematical problems can be solved by applying specific rules may feel stuck after having tried in vain to apply one particular rule. When they think that no other heuristic is available to solve a specific problem, or that the allotted time has almost passed, they may doubt that they can solve the problem, which may in turn lead them to experience anxiety or to give up easily. In the context of Mandler's theory, metacognition plays a crucial role. If a students's initial plan for solving a problem is interrupted and further progress is blocked, the student has to deal with two metacognitive issues. Firstly, the student must become aware of the blockage, rather than blindly plugging away at meaningless computations. Secondly, the student needs to make a decision about what new strategy to try.

An important question is whether or not students are aware that something is going wrong in the solution process. Decisions about what to do next can only be made when students are aware of the fact that something is wrong. When difficulties arise, decisions about whether to persevere along a possible solution path may be influenced by students' expectations of successful goal attainment.

Verification

Lack of control is an important source of failure within the verification phase. In the literature, two categories of verification activities are often mentioned (e.g., Garofalo & Lester, 1985). The first category involves activities that are directed towards checking the understanding of the problem and the appropriateness of the plan that was executed. The second category concerns evaluating the execution of the solution method - in other words, checking whether the steps have been executed correctly. Lester and Garofalo (1982) found that primary school students rarely verify the correctness of their answers.

In our view, another category should be added, namely the verification of the adequateness of the answer within the context of the problem. Especially when solving context problems, this phase needs special attention. When solving the computation problems 1128 : 36 = , a different answer should be given than for the context problem (Treffers, 1991b):

> *1128 soldiers are transported on buses that have 36 seats.*
> *How many buses are needed?*

Students may fail to give a correct answer to the latter problem in spite of having understood the problem, and despite having executed an effective solution plan without computational errors. Students may give an incorrect answer if they fail to take notice of the context in which the problem is stated. Verschaffel, De Corte, and Lasure (1994), for instance, demonstrated students' strong tendency to exclude real-world knowledge and realistic considerations from their solution. These authors found that only 17% of the students' solutions could be considered as realistic, either because the students wrote a realistic answer, or because they made an additional realistic comment.

Studying problem-solving behavior: relations between variables

A problem-solving approach in which cognitive, metacognitive, and affective variables are considered to be of crucial importance has major implications for the study of individual differences. When trying to explain individual differences (especially gender differences) in problem-solving behavior, attention should be paid to both cognitive and affective variables, and to the interactions between these variables. In this approach, knowledge is seen as necessary, but not sufficient by itself for successful problem-solving performance. Good problem solvers are not only characterized by sufficient domain-specific knowledge, but also by a variety of metacognitive and motivational strategies that are helpful for successful performance. On the other hand, bad problem solvers may have sufficient domain-specific knowledge, but may not know when and how to use that knowledge.

An approach in educational research in which cognitive, metacognitive, and affective variables are integrated is directed at the development of self-regulatory skills. According to Zimmerman (1989), the systematic use of metacognitive, motivational, and/or behavioral strategies is a key feature of most definitions of self-regulated learning. In terms of metacognitive processes, self-regulated learners are aware of when they know a fact or possess a skill and when they do not. They plan, set goals, organize, self-monitor, and self-evaluate at various points during the learning process. Boekaerts (1996) stresses the importance of motivational self-regulation, consisting of a knowledge component and a skill component. The former component refers to self-referenced cognitions; the skill component refers to motivational strategies and self-defined goals. Self-referenced cognitions can be divided into two sets, including (1) beliefs, judgments, and values related to curricular tasks and subject-matter areas, and (2) beliefs, judgments and values related to one's capacity in relation to a domain of study. Motivational regulatory strategies refer to the capacity to regulate motivational, emotional, and social processes before, during and after learning activities.

In terms of mathematical problem solving, skillful regulation of cognitive processes is very important. Schoenfeld (1992) stated that monitoring and assessing progress "on-line" and responding to the assessments of on-line progress are the core components of self-regulation. According to Schoenfeld, these monitoring skills can be learned as a result of explicit instruction that focuses on metacognitive aspects of mathematical thinking. In many studies a positive relationship was found between control processes and mathematics performance. Span and Overtoom (1986) compared the executive control processes of intellectually gifted students and average students when solving mathematical problems. They found that gifted students spent more time analyzing the problems, worked more systematically, verified their answers more often, and were better able to reflect on their problem-solving strategies.

In sum, we described relevant (meta)cognitive and affective variables that contribute to students' mathematical problem solving. In addition, we argued that these variables should be studied in relation to each other when studying individual differences. Below, we will sketch our research perspective based on the reflections and arguments made in this chapter. At the end of chapter 3, we will formulate the research questions.

2.4 Research perspective

As described in chapter 1, this project was set up to further explore gender differences in mathematics, especially in relation to applied problem solving. Until now, research concerning students' problem-solving behavior has mainly focused on cognitive variables. In this project the interaction of cognitive, metacognitive and affective variables during mathematical problem solving was addressed. Emphasis was put on students' expectations concerning successful goal attainment while they were working on mathematics tasks, and their reactions to failure, when it occurred. We hypothesized that students will make an estimation of the extent to which they can (still) succeed on the task when difficulties are anticipated or encountered. The *confidence* or *doubt* that results in the different phases of the problem-solving process is considered to be an important variable. Hence, the main research question in this project concerned students' capability to assess their progress "on-line". We wanted to know whether their perceived confidence is congruent with their actual performance. In cognitive psychology this is referred to as a calibration of confidence. According to Lundeberg, Fox, and Punćochař (1994) the calibration of confidence is an important aspect of metacognition.

As stated in chapter 1, we made a distinction between solving computation problems and solving applied mathematics problems. We examined the expressed confidence in relation to these two types of tasks. Furthermore, the influence of failure experiences on problem-solving behavior (persistence) were studied. Our expectation was, that this influence would have a more deteriorating effect on behavior when it comes to solving application problems, compared to solving computation problems. In particular, when students experience difficulties while solving application problems, we hypothesized that students can ascribe failure to many different causes: They may have chosen an inadequate solution strategy or there may have been a slip in the execution of the solution. Because there is more uncertainty involved, this may lead to the students (further) doubting their ability to solve the problem, and therefore they may reduce their effort. We especially expected this behavior in girls.

Of course, individual differences are of major importance here, both in expressed confidence and in persistence. Some students will quit instead of looking for another approach when confronted with failure, while other students will put in more effort when they decide to try to solve the problem again. We therefore examined the influence of students' self-referenced cognitions on problem-solving behavior. The next chapter describes the theoretical background and the perspective from which these variables were studied.

3 SELF-REFERENCED COGNITIONS IN RELATION TO MATHEMATICS

The preceding chapter focused on (meta)cognitive and affective variables that contribute to students' problem-solving behavior. We briefly mentioned the important influence of students' beliefs about themselves on their problem-solving behavior. This chapter will further outline the theoretical background of our research, focusing on motivational issues and students' beliefs about the self in relation to mathematics. Beliefs about the self and motivational issues, here called self-referenced cognitions, are discussed at both the domain-specific and task-specific levels (see Boekaerts, 1995; Seegers & Boekaerts, 1993).

In section 3.1 a conceptual framework is outlined. In section 3.2 the focus is on Boekaerts' model of adaptable learning (Boekaerts, 1991, 1992, 1995), in which cognitive and affective variables at both the domain-specific and task-specific levels are integrated. Section 3.3 consists of a description of three motivational beliefs that have proven to be relevant to the study of mathematics, namely self-concept of mathematics ability, goal orientation, and attributions. In section 3.4 gender differences with respect to these motivational beliefs are discussed. Functional and dysfunctional motivational patterns for learning are highlighted in section 3.5. Finally, it is argued that it is crucial to study self-referenced cognitions at the task-specific level (section 3.6). At the end of this chapter, the variables within our study and the research questions will be outlined (section 3.7).

3.1 Conceptual framework

Self-referenced cognitions

Students' beliefs about themselves and their motivation are receiving more attention nowadays in research on mathematics learning. Many researchers who are interested in studying individual differences in mathematics achievement agree that both cognitive and motivational components contribute to those differences (e.g., Pintrich & De Groot, 1990). It is often found that differences in mathematics achievement, especially gender related differences, can not solely be traced back to differences in cognitive abilities. It is assumed that beliefs that students develop about their abilities, and about their motivations are both important aspects in mathematics learning, especially when it comes to higher order thinking processes, such as mathematical problem solving.

The amount of literature on achievement motivation and related variables is overwhelming. One factor that makes this area so complicated, is the use of different concepts for the same phenomena, and vice versa. In addition, it is confusing that variables overlap, both conceptually and in the way they are operationalized. In this thesis, we restrict ourselves to beliefs that students have about themselves as mathematics students and to their motivation related to mathematics. Researchers generally agree that beliefs about the self include capacity-related beliefs and control-related beliefs, whereas motivational issues involve students' goals and interests. As such, beliefs have a more cognitive component, whereas motivation has a more affective loading. Boekaerts (1995) encompasses all these variables under the construct of self-referenced cognitions; others refer to these variables as motivational beliefs (e.g., Pintrich, Wolters, & De Groot, 1995).

Domain-specific versus task-specific level of measurement

Following Cantor (1981), Boekaerts (1995) distinguishes between self-referenced cognitions measured at the superordinate, the middle, and the subordinate levels. Self-referenced cognitions measured at the superordinate level refer to the general motivation to learn. At the middle level, self-referenced cognitions reflect students' beliefs towards specific academic subjects, whereas at the subordinate level these variables are measured in relation to specific learning situations. When studying self-referenced cognitions in school settings, this distinction is similar to the distinction between general, subject-matter specific (domain-specific) and task-specific variables.

In the past, most research on achievement motivation concentrated on the general level: Motivational and related variables were seen as stable personality traits which could be measured by the use of questionnaires. Nowadays most theorists in the field of motivation assume that motivation is partly context dependent and situation specific, and should be measured as such. According to Boekaerts (1987, 1991, 1992, 1996), general measures of motivation do not provide insights into the interactions of personality traits with the learning process itself. She reasoned that confronting students with a task will trigger personality variables at the general and domain-specific level, and that this subjective information will affect task-specific cognitions and affects. Following Lazarus (Lazarus, 1991; Lazarus & Folkman, 1984), she referred to these task-specific cognitions and affects as *appraisals*. The term appraisals was adopted in order to make an explicit distinction between self-referenced cognitions that are measured at the domain-specific level and self-referenced cognitions that are measured at the task-specific level. In this thesis, we will reserve the term appraisals for the task-specific cognitions that are measured before students start working on a specific task. Self-referenced cognitions in relation to mathematics will be referred to as motivational beliefs. In the next part of this chapter, a model of learning - one in which motivational beliefs and task-specific appraisals are integrated - will be described.

3.2 The model of adaptable learning

The model of adaptable learning as developed by Boekaerts (1991, 1992, 1995) integrates cognitive and affective variables of the learning context, at both the domain-specific and task-specific levels. It specifies that students, when confronted with a task, will use information from three main sources. The first source of information is the perception of the task and the context in which it is embedded. The second source of information is activated domain specific knowledge and skills relevant to the task, including cognitive strategies and metacognitive knowledge relevant to the task. The third source consists of motivational beliefs (including self-concept of mathematics ability, goal orientation, and attributions). Information from these three main sources is used to dynamically appraise mathematics tasks at the beginning, during, and at the end of the task. These appraisals have a central position in the model.

It is assumed that when students are confronted with a learning situation (task onset), they may note a discrepancy between perceived task demands and perceived resources to meet these demands. Such appraisals may be predominantly favorable or unfavorable at task onset, and will, as such, elicit dominantly positive or negative emotions. Intense emotions may influence upcoming and ongoing cognitive processes, not only because they draw the learner's attention away from the task, but also because toning down emotions may have a negative effect on processing capacity (Bower, 1981). Both unfavorable and favorable appraisals and negative and positive emotions may be experienced upon confrontation with a mathematics task, or they may develop while working on the task.

It is theorized in the model of adaptable learning that when the learner interprets learning situations or tasks as having a negative impact on well-being, unfavorable appraisals and negative emotions (e.g., doubt) may be dominant. The student's primary goal will then be to initiate activity in the "coping mode" in order to restore well-being. On the other hand, when learning situations or tasks are seen as leading to gains in competence for reasonable costs, favorable appraisals and positive emotions (e.g., confidence) will be dominant, leading to a learning intention and to activity in the "mastery mode".

For example, sometimes students have high expectations of success when starting to solve a mathematics problem, and a high learning intention. However, during the problem-solving process difficulties may be experienced. When that occurs, students have to make decisions about whether or not to put in more effort. This decision is influenced by subjective appraisals, such as expectations of success (or failure) and the importance students ascribe to imagined success or failure. For students who believe that making mistakes during problem solving is an inevitable part of the solution process, the chances are higher that they will decide to try to solve the problem again. However, for students who are afraid of making mistakes and have low estimates for their chances of getting the right answer, it is likely that they will withdraw from the mathematics problem.

It is stressed in the model of adaptable learning, that both motivational beliefs (in relation to mathematics) and task-specific appraisals (in relation to a specific task or assignment) influence the learning process. Motivational beliefs refer to issues such as self-concept of mathematical ability, causal attributions for successes and failures in mathematics, and goal orientation. The first two constructs refer to capacity-related beliefs and control-beliefs; goal orientation is a motivational issue. Individual differences in the motivational beliefs related to mathematics learning have been reported frequently, especially with respect to gender differences. Many of the models that have been used to explain gender related differences in mathematics highlight the contribution of belief variables (e.g., Eccles et al. 1985; Fennema & Peterson, 1985; Ethington, 1992). In section 3.3 an overview will be given of three motivational beliefs that have proven to be of importance for mathematics learning.

3.3 Motivational beliefs

Self-concept of mathematics ability

An important aspect that is linked to students' motivation, is self-concept of ability in relation to a specific domain. Related constructs that are found in the literature are: self-confidence in the ability to learn mathematics, beliefs about one's competence in mathematics, perceived competence, and self-efficacy. The conceptu-al differences between the constructs are not always clear. In our view, the constructs are essentially the same and can be used interchangeably, except for self-efficacy. Bandura (1982) stated that perceived self-efficacy concerns "judgments of how well one can execute courses of actions required to deal with prospective situations" (p. 122). Most authors agree that self-efficacy differs from the other constructs mentioned above because of its content-specific character, and most authors distinguish between self-efficacy and self-concept of ability (e.g., Norwich, 1987; Pajares & Miller, 1994). According to Pajares and Miller (1994), self-concept differs from self-efficacy in the sense that the latter is a context-specific assessment of competence to perform a specific task: a judgment of one's capabilities to execute specific behaviors in specific situations. Self-concept, on the other hand, is not measured at that level of specificity, and includes beliefs of self-worth associated with one's perceived competence. According to these authors, self-concept judgments are more general and less context dependent. The question "Are you a good mathematics student?" taps different cognitive and affective processes than the self-efficacy question "Do you have the skills to solve this specific problem?". According to Bandura (1986), judgments of self-efficacy are task-specific and must be measured as closely as possible in time to the task.

However, many researchers do not measure self-efficacy in this way, which causes confusion when interpreting research findings. In our research we will consider the self-concept of mathematics ability as a domain-specific measure of perceived competence that can be measured independent of a specific situation. We will reserve the construct self-efficacy for only those situations in which perceived competence is measured in relation to specific tasks.

The relationship between self-concept of ability and mathematics achievement has been the subject of considerable research. A positive relation between self-concept of mathematics ability and achievement in mathematics has been demonstrated by many authors. The relationship is generally moderate, with correlation coefficients of around .40 (Reyes, 1984). In addition, it has often been found that self-confidence has a stronger correlation with achievement than do other affective variables (e.g., Kloosterman, 1988). A reciprocal relationship is often assumed: It seems likely that performance in mathematics will influence students' beliefs in their capacities, which, in turn, will influence how confidently students will approach mathematics tasks. Helmke (1989, 1990) conducted a longitudinal study in which the relation between self-concept and achievement in mathematics in grades 5 and 6 was a central issue. It was found that in grade 5 self-concept was influenced by achievement, but that self-concept did not influence achievement. In grade 6 this pattern changed. A reciprocal relation was found between self-concept and achievement in mathematics.

Goal orientation

Goals that individuals pursue in achievement situations are central in theories on achievement motivation. Several sets of two contrasting achievement goals have been proposed to explain differences in students' behavior: mastery versus performance (Ames & Archer, 1988); learning versus performance (Dweck & Elliot, 1983); task- versus ego orientation (Nicholls, 1984a). These sets of contrasting goals are similar. In general, individuals who are task-oriented and pursue mastery or learning goals are focused on developing new skills and on trying to understand what they are doing. On the other hand, students who are ego-oriented, or pursue performance goals, are concerned with performing better than others. These students are likely to make attempts to learn only when they expect to show superior ability.

In this thesis, we adopted Nicholls' (1984a, 1984b) theory on task versus ego orientation. This distinction has given us additional insights into processes that underlie student motivation in the classroom, especially with regard to mathematics learning. Although ego and task orientation have been described as representing two forms of "approach tendencies" (Nicholls, Patasnick, Chung Cheung, Thorkildsen, & Lauer, 1989), characteristics of the learning situation itself are also likely to influence students' goals (e.g., Ames 1992). We restricted ourselves to studying students' perceptions at the individual level and were not interested in task versus ego orientation at the classroom level.

Nicholls' theory focuses on the standard of comparison that is used by students. When students are task-oriented, they compare their achievement with their own prior achievement: Feelings of competence result if these students gain new insights or improve their performance. Ego-oriented students, on the other hand, compare their achievement with that of other students: Feelings of competence result if these students perform equal or better than other students. As a consequence, a willingness to invest effort is reached differently by the two groups of students. Students who are task-oriented will invest effort as long as they are interested in the task and consider further competence within reach. Ego-oriented students however, are only likely to make effective attempts to learn when they believe that their attempts will show that they are better at the task than other students. Nicholls (1983, 1984a) related the distinction between ego and task orientation to differences in conceptions of ability that students develop. For young children, ability is judged with reference to one's previous level of performance. Hence, for young children, ability can be improved by investing effort. However, according to Nicholls, for adolescents and adults to be judged able, one must learn more than others with equivalent effort or achieve an equivalent level of performance with less effort than others. In this differentiated conception ability is considered equivalent to capacity.

Task orientation and ego orientation have been distinguished as two individual difference dimensions both in high school students (e.g., Nolen, 1988) and in primary school students (e.g., Nicholls, Cobb, Wood, Yackel, & Patasnick, 1990). These authors stated that task and ego orientation are independent of one another: Students' status on one dimension has proved not to be a reliable predictor of their status on the other dimension. Meece and Holt (1993) identified three clusters of students with different achievement profiles in science: (1) students who were high on both ego and task orientation, (2) students who were low on both ego and task orientation, and (3) students who exhibited a pattern in which task orientation was stronger than ego orientation. They found that the latter group showed the most positive achievement profile, whereas students who were low on both goals showed the most negative achievement profile.

Attributions

Attributions or perceived causes of successes and failures play a central role in research on motivation in achievement situations. According to Weiner (1985), attributions of results can be classified along three dimensions: (1) locus, (2) stability, and (3) controllability. Locus refers to the location of a cause, which can be internal (e.g., ability) or external to the person (e.g., task difficulty). Stability refers to the temporal nature of a cause, and controllability is related to the degree of influence that can be exerted over a cause. Effort for instance can be seen as an internal, unstable cause which is controllable; Whereas ability is usually seen as an internal, stable cause which is uncontrollable.

In general, motivation is poor when uncontrollable causes are blamed for failure because students feel that there is little they can do to increase their chances of success (Weiner, 1979; 1985). For example, if students blame their failure in mathematics on a lack of ability, then they will not expect to do better next time, because they consider ability as an internal, stable cause beyond their control. Considerable research has been done on effort and ability as important causes for successes and failures. It is generally assumed that attributing success to ability and failure to lack of effort has less deteriorating effects on the learning of mathematics than vice versa (e.g., Ames & Archer, 1988).

Attributions in relation to mathematics have often been studied in relation to other motivational beliefs. The causes that students give for their successes and failures in mathematics, are closely related to their self-concept of mathematics ability. Students who possess a low self-concept of ability are not likely to attribute a success to high ability. Kloosterman (1988) found that students who were high in confidence in mathematics were also likely to attribute success to ability and failure to lack of effort.

Furthermore, perceived causes of success or failure in mathematics are also related to the goals that students pursue. Task-oriented and ego-oriented individuals will interpret their performance differently. For task-oriented students, failure is likely to elicit questions like: "What must I do differently to succeed?". Task orientation often goes together with a belief that effort will lead to success or a sense of mastery. For ego-oriented students, however, failure is more likely to elicit the question: "What can I do to avoid looking stupid?". When trying hard does not lead to success, ego-oriented students are inclined to experience the expenditure of effort as threatening their self-concept of ability. Covington and Omelich (1979) referred to effort in this context as a "double-edged sword". Jagacinski and Nicholls (1987, 1990) argued that when students are more ego-oriented, negative results have a stronger negative effect on self-efficacy and on the willingness to invest effort in similar tasks in the future. When ego-oriented students are repeatedly confronted with failure, they will be more inclined to judge their capacity as insufficient.

3.4 Gender differences in motivational beliefs

Self-concept in mathematics ability is one of the variables that has been proven to be of major importance when studying gender related differences in mathematics learning. Eccles and Jacobs (1986) identified students' self-concept of their mathematics ability as one of three significant factors that affected students' achievement in mathematics as well as their intention to continue studying mathematics. Many authors report that girls have lower perceptions of competence and lower performance expectations than boys in mathematics, even when girls have equal or better results (Boekaerts, Seegers, & Vermeer, 1995; Eccles et al., 1985; Hyde et al., 1990). These differences in expectations seem to be present at an early age: Entwisle and Baker (1983) found that boys in the first, second and third grades had developed higher expectations for their own performance in mathematics than their female classmates, although their arithmetic marks and/or general aptitude did not exceed that of the girls.

Research findings on gender related differences in goal orientation are not consistent. Results from experimental research have shown that girls are more often characterized as ego-oriented (e.g., Licht & Dweck, 1984). However, there is also evidence that boys are more inclined toward interpersonal competition (Fennema, 1985). Seegers and Boekaerts (1996) found that 11-12 year old boys had a higher level of ego orientation in relation to mathematics, implying a more competitive attitude.

Furthermore, gender differences in perceptions of the causes of success and failure in mathematics have been frequently reported. Several authors reported that female students are less likely than male students to attribute mathematics success to their own ability and are more likely to attribute failure in mathematics to low ability (e.g., Fennema, 1985). Boys stress the role of effort in their explanations of failure more than girls do. This implies that girls are less likely than boys to believe that success in mathematics can be achieved through hard work (Spence & Helmreich, 1983; Fennema, 1985). Seegers and Boekaerts (1996) found that girls tended to attribute failure to lack of capacity more often than boys.

The motivational beliefs described above were mostly measured in group settings using questionnaires. Through this research, we have learned more about the existence of rather stable beliefs and motivational orientation in relation to mathematics, and about gender related differences in this area. However, important questions remain unanswered, namely: To what extent are these motivational beliefs related to actual behavior during a task? And also, which behaviors are beneficial (or detrimental) to mathematics performance? In section 3.5, research that has revealed functional and dysfunctional motivational patterns will be discussed.

3.5 Functional and dysfunctional motivational patterns

In general, a distinction can be made between two types of research that dealt with functional and dysfunctional motivational patterns: (1) experimental research in which one or more variables have been manipulated, and (2) correlational studies in which students' self-reports have been central.

The first type of research has often concentrated on students' reactions following failure in an experimental setting. Most research in this context has been done within a social-cognitive framework. Diener and Dweck (1978, 1980) investigated fifth-grade students' performance following failure on a discrimination learning task, for which each student was given extensive training prior to the test problems. Students were requested to talk aloud while performing the task. They found that ego-oriented students tended to show challenge avoidance and low persistence in the face of difficulties, when they had a low self-concept of ability and attributed failure to lack of ability. Researchers found that these "helpless" students felt that success was beyond their control and that effort was useless because it probably would not lead to success. They reacted to failure by abandoning problem-solving strategies where intensification or modification of strategic behavior would have been more appropriate. This failure-avoidance is a "dysfunctional motivational pattern". It is referred to as "learned helplessness", and has been studied by several researchers.

Learned helplessness has been found more often in girls than in boys (e.g., Diener & Dweck, 1978, 1980; Licht & Dweck, 1984). Research findings suggest that task orientation is often associated with a motivational pattern that is likely to maintain involvement in learning (e.g., Covington, 1983; Dweck, 1986). This "functional motivational pattern" is characterized by challenge seeking, and high, effective persistence in the face of difficulties. Students who attribute their success to ability and their failure to effort are more likely to persevere on tasks which are not solved immediately (e.g., Elliot & Dweck, 1988).

The second type of research that dealt with functional and dysfunctional motivational patterns in relation to school situations used mainly self-reports. A considerable number of studies focused on describing how different goal orientations elicit qualitatively different learning behaviors. It was revealed that task orientation is associated with a number of motivation related variables, such as attributional beliefs that effort leads to success, preferences for challenging work and risk-taking, and persistence in the face of difficulties (Ames, 1992). Furthermore, there is evidence that different motivational patterns are related to the use of different learning strategies. Some researchers have concluded that task orientation fosters the use of effortful and effective learning strategies, whereas ego orientation is associated with the use of superficial or ineffective learning strategies (e.g., Ames & Archer, 1988; Meece, Blumenfeld, & Hoyle, 1988; Pintrich & De Groot, 1990).

To summarize, the first type of research has provided evidence for the existence of different motivational patterns that elicit different behaviors in students. It often concerns laboratory and experimental studies, which means that the behavior studied is not representative of students' behavior in classroom settings. In addition, the tasks that were assigned in experimental settings differed from those which are relevant in school learning. Although the second type of research provided more ecologically valid results across different domains, the question remains whether the self-reports that have been administered in group settings give an adequate picture of students' individual behavior during school activities. Furthermore, because mainly domain-specific measures were included in the research (middle level of measurement), the results from these studies do not give insights into processes that actually occur when students start working on a specific task. Therefore, students' perceptions of specific learning situations should be included in research.

3.6 Task-specific appraisals

Variables that are measured at the task-specific level have not been extensively used in research on motivational issues in mathematics. A distinction can be made between variables that are measured (1) before starting with a specific task, (2) during the task, and (3) when the task is finished. Most research on task-specific appraisals has emphasized students' cognitions before starting with a task, in particular self-efficacy beliefs. Self-efficacy has usually been measured using students' estimates of their chances of success after they were told what type of task they were going to do (e.g., Schunk, 1981). In general it has been found that self-efficacy is a more predictive measure of mathematics achievement than self-concept of mathematics ability (e.g., Pajares & Miller, 1994).

Boekaerts (1987, 1988) stressed the importance of studying student motivation and behavior in specific contexts. She argued that in order to predict student motivation in specific learning situations, the unique ways in which students experience every-day curricular activities should be addressed. To capture students' motivation at the momentary level, their task-specific cognitions, affects, and learning intention should be registered in relation to the specific learning situation at hand. Boekaerts (1988) developed the On-line Motivation Questionnaire to measure these situation-specific variables. This questionnaire consists of two parts. The first part is administered just before students start working on a task, whereas the second part is administered after students have completed a task. The first part taps the values of a number of task-specific variables which include: (1) subjective competence - including self-efficacy, success expectation and perceived level of difficulty - (e.g., "How good are you at doing these types of tasks?"), (2) task attraction (e.g, "How much do you like these types of tasks?"), (3) the value ascribed to the task or personal relevance (e.g, "How useful do you consider this task?").

Items of the second part of the questionnaire concern among other things attributions of perceived positive or negative results.

An important issue of research has been how these appraisals influence willingness to invest effort in doing a task (learning intention) and performance. In one study (Seegers & Boekaerts, 1993) these task-specific variables were combined with motivational beliefs (self-concept of mathematics ability, attributional style, goal orientation) in order to test the assumption underlying the model of adaptable learning that attitudes and beliefs concerning a specific subject area (e.g., mathematics) influence task specific variables (subjective competence, pleasure in doing the task, and personal relevance). It was found that willingness to invest effort, emotional state, and achievement were relatively independent outcomes of the appraisals. Task orientation was found to have a direct positive effect on estimated personal relevance and on pleasure in doing the task, and an indirect effect on willingness to invest effort. A tendency to attribute failure to lack of capacity had a negative influence on subjective competence, while subjective competence had an effect on both emotional state and task performance.

The importance of a task-specific approach when studying gender differences

An approach in which students' motivation is studied at a more concrete level, seems valuable when studying gender differences in mathematics. Because gender differences in performance have usually been found to be dependent on the content of the problems (see chapter 1), it is of special interest to examine students' motivation (and actual behavior) in relation to different content areas. Moreover, such an approach also makes it possible to examine intraindividual differences across different content areas.

Lundeberg et al. (1994) investigated gender differences in item-specific confidence judgments. Their subjects were psychology students. They found that gender differences in confidence were dependent on the content of questions asked. In the mathematics items, males appeared to be more confident than females, even when performance was equal.

Attributions for success or failure on a specific task have often been reported in relation to gender differences. Stipek and Gralinski (1991) for instance reported that third-grade girls (between 8 and 9 years old) not only rated their math ability lower than did boys, but they were also less likely than boys to attribute success to capacity and failure to luck. Moreover, they reported less pride following successful math performance, showed a stronger desire to hide their paper after failure, and were less convinced that success could be achieved through effort. Kloosterman

(1990) examined the relationship between attributions, performance following failure, and mathematics achievement for seventh-grade students. To assess the extent to which failure in mathematics would result in reduced performance on mathematical word problems (application problems), he designed the Performance Following Failure instrument. Using this scale, the procedure for inducing and measuring reactions to failure was as follows: At first, students were given several moderately difficult word problems, and were then assigned a "pre-failure score" based on the results. Students were given several difficult word problems in order to induce failure. Finally, in order to measure performance following failure, the students were asked to complete several more word problems similar to those on the pre-failure part of the instrument, and were subsequently given a "post-failure" score. Performance following failure was measured by comparing a post-failure score to a pre-failure score for each student. He found that girls' performance declined more strongly following failure than boys' performance.

Seegers and Boekaerts (1996) found that boys and girls took different starting positions when confronted with mathematics problems: Girls displayed less self-efficacy and had less favorable beliefs about their mathematics ability. These results were also found by Crombach, Voeten, and Boekaerts (1994).

In summary, students' motivational beliefs in relation to mathematics are considered to be important aspects of mathematics learning. Gender related differences have consistently been found. However, the impact of these beliefs on students' behavior during the execution of school-related tasks and on performance is still not clear. In the field of educational psychology, research has recently been directed at specific learning situations. Research directed at students' cognitions and motivation in relation to mathematics tasks has confirmed gender related differences at the task-specific level.

Cognitions that are measured before and after task execution do not provide insight into the processes that actually occur while students are solving mathematics problems. For instance, we do no know much of students' confidence while solving problems, nor of their actual persistence when confronted with failure. Another important question is how motivational beliefs and task-specific appraisals affect actual problem-solving behavior. In our research we began to investigate these processes.

3.7 Variables and research questions

In Table 3.1 all the variables in our study are outlined.

Table 3.1
Variables within our study

	GENERAL LEVEL Abstract reasoning ability	
	DOMAIN-SPECIFIC LEVEL *Motivational beliefs:* Goal orientation Attributions Self-concept of mathematics ability	
	TASK-SPECIFIC LEVEL [1]	
Before the task:	**During the task:** [2]	**After the task:**
	Task performance Solution strategy use Solution time	
Task-specific appraisals: Subjective competence Perceived task attraction Personal relevance Learning intention	Perceived confidence Persistence following failure	Task-specific attributions

[1]) These variables were examined for both task conditions.
[2]) These behavior-related variables were examined for each of the six problems of both task conditions.

Our research was directed at sixth-grade students' mathematical problem-solving behavior in actual situations. We further explored task-specific behavior by integrating variables that are measured while students are working on mathematics problems. By measuring cognitive and motivational variables during task execution, we expected to gain more insight into individual differences in task-specific behavior, in this case mathematical problem-solving behavior. Central to this research were the students' confidence and doubt judgments during mathematical problem solving and their reactions to failure (see chapter 2).

Both intra- and interindividual differences were examined, that is differences in cognitive and affective variables across tasks (computation problems versus application problems), as well as gender related differences. It was hypothesized that students will generally show more confidence while solving computation problems, because this relates more to the execution of a precise and systematic plan than applied problem-solving. We also expected students to give up earlier after failure when it comes to solving application problems compared to solving computation problems (see chapter 2).

With respect to gender differences, we did not expect to find differences in boys' and girls' confidence while solving computations. However, we expected girls to have lower confidence than boys while solving application problems, and hence to give up more easily after failure experiences. We also expected girls to attribute failure to lack of capacity more often than boys do. Finally, the influences of motivational beliefs and task-specific appraisals on problem-solving behavior were addressed. We examined relations between these variables and task performance, perceived confidence and persistence following failure, respectively.

Our research questions were:

- *Do gender differences exist with respect to the cognitive and motivational variables in our study? (chapter 5)*
- *Do intraindividual differences in cognitive and motivational variables exist across the two types of task, and what is the influence of gender? (chapter 5)*
- *How are students' task performance, perceived confidence, and persistence related and what are the influences of gender and type of task? (chapter 6)*
- *How are students' objective competence, task-specific appraisals, and motivational beliefs related to their task performance? (chapter 6)*
- *How are students' objective competence, task-specific appraisals, and motivational beliefs related to their perceived confidence? (chapter 6)*
- *How are students' objective competence, task-specific appraisals, and motivational beliefs related to their persistence following failure? (chapter 6)*

4 METHOD

4.1 Subjects

Schools which used the same mathematics method were invited to participate in our project. The method we chose to use was "De Wereld in Getallen" ("The World in Numbers"), which is a realistic method for mathematics education in primary schools (see chapter 1). At the time of the research about 20% of the primary schools in the Netherlands were using this method in the sixth grade (Bokhove et al., 1996). Sixth-grade teachers from forty schools situated in the urban regions of Den Haag, Leiden and Utrecht were asked to participate in our research. Twelve schools agreed to take part. These schools had a total of 276 students (129 boys and 147 girls) in the sixth grade (11-12 years). These students formed the *total sample* that participated in the group sessions of our research.

From the total sample, 160 students were selected to participate in the *individual part* of the research. The selection was based on a three-step process. First, students who scored within the highest or lowest decile of two subtests for non-verbal intelligence were excluded from our research. This decision was made because we expected that the mathematics tasks would be too easy (or respectively too difficult) for these groups of students. Secondly, we asked all teachers whether they expected students to have problems with the reading of the questionnaires and the tasks. Students with an inadequate knowledge of the Dutch language were not included in our research. Thirdly, stratified random sampling according to school and gender was employed. This was done by randomly selecting 80 boys and 80 girls from the twelve schools in which the total number of students per school and the distribution of boys/girls per school was taken into account. Two students dropped out of the study - because they changed schools during the research period - which resulted in a *selected sample* of 158 students (79 boys and 79 girls). The number of students per school ranged from 8 to 42 in the total sample, and from 3 to 16 in the selected sample.

4.2 Measures

Abstract reasoning ability

Two subscales of the revised Snijders-Oomen Non-verbal intelligence test (SON-R) (Laros & Tellegen, 1991) were used to measure non-verbal intelligence, namely the scales of *analogies* and *categories.* Both subscales are tests for abstract reasoning. The subscale categories requires the classification of objects into categories, whereas geometrical figures analogue to a given pair must be formed in the analogies subscale. The latter subscale consists of 27 items and the former subscale consists of 24 items. Example items are given in Figures 4.1 and 4.2. Both subscales can be measured in a group setting. Laros and Tellegen (1991) found reliability coefficients (Cronbach's alphas) in this age group of .79 and .75 for the analogies and categories subscales, respectively. The Pearson correlation coefficient between the two subscales was .47 in this age group.

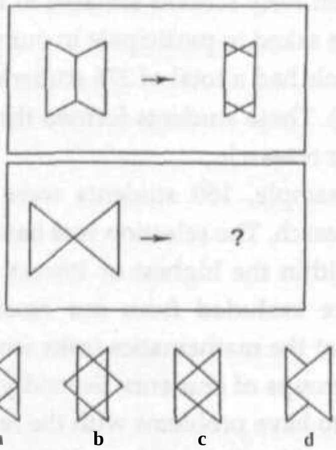

Figure 4.1. An example of the analogies subscale.

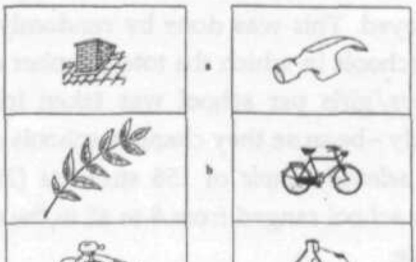

Figure 4.2. An example of the categories subscale.

Goal Orientation Questionnaire

In our study, we used a revised version of the Goal Orientation Questionnaire, as developed by Seegers and Boekaerts (1993), and based on Nicholls' theory. The original version of this questionnaire included 32 items that refer to situations and behavior in relation to mathematics. All items were answered on a 4-point scale, ranging from 1 *(never)* to 4 *(always)*. These authors reported that four factors could be identified that represent ego or task orientation in relation to success and failure experiences respectively. The first factor, which they labeled *task orientation,* implies a positive attitude towards and interest in mathematics tasks (e.g., "I feel satisfied when I learn something interesting in math"). The second factor included items that express the idea that failure indicates a personally experienced lack of capacity (e.g., "I think I am stupid when I make a mistake"). This factor was labeled *fear of making mistakes.* The third factor *ego orientation* referred to ego orientation in situations where capacity can be demonstrated (e.g., " I feel good when I know the answer first"). The fourth factor, which was labeled *hiding mistakes,* included situations where students are afraid that others will notice their mistakes (e.g., "I'm afraid that other students will notice when I make a mistake in math"). The contrast between hiding mistakes and fear of making mistakes runs parallel to the contrast between ego- and task orientation. Hiding mistakes refers to situations in which students are preoccupied by avoiding being negatively judged by others. In contrast, fear of making mistakes is related to situations in which students blame themselves for their mistakes. However, in our view the distinction between the labels fear of making mistakes and hiding mistakes is not clear at first sight, and does not do justice to the distinction between ego and task orientation. We will follow the terminology of Skaalvik (1997), who proposed to label the two dimensions of ego orientation as *self-enhancing ego orientation* versus *self-defeating ego orientation.* Therefore, the factor hiding mistakes will be replaced by the term self-defeating ego orientation, whereas ego orientation will be labeled self-enhancing ego orientation. In addition, the term fear of making mistakes will be replaced by the term *error frustration.* The term task orientation remains unchanged.

The revised version of the Goal Orientation Questionnaire included 24 items that were answered on a 4-point scale ranging from 1 *(almost never)* to 4 *(almost always)*. This revised questionnaire was administered to the 276 sixth-grade students in our research. A Principal Components Analysis confirmed the existence of the four subscales, as described by Seegers and Boekaerts (1993). These subscales varied from five to six items. Internal consistencies in term of Cronbach's alpha were .78 for task orientation, .76 for error frustration, .85 for self-enhancing ego orientation, and

.76 for self-defeating ego orientation. Pearson correlation coefficients between the four subscales are given in Table 4.1.

Table 4.1
Pearson correlation coefficients between the subscales of the Goal Orientation Questionnaire

Subscale	1	2	3	4
1. Task orientation	–	.26	.28	.05
2. Error frustration		–	.46	.50
3. Self-enhancing ego orientation			–	.38
4. Self-defeating ego orientation				–

Attribution Questionnaire

The Attribution Questionnaire (Seegers & Boekaerts, 1993) was developed to measure the extent to which students think that (lack of) ability or (lack of) effort are responsible for their successes or failures in mathematics. This questionnaire included 20 items, referring to success and failure experiences in mathematics, plus 10 filler items. The items were answered on a 4-point scale, ranging from 1 *(completely true)* to 4 *(definitely not true)*. Two factors were found to explain the underlying data structure most adequately. The first factor, which was labeled *failure:-capacity,* refers to a tendency to ascribe failure to a lack of capacity (e.g., "Math is too difficult for me"). The second factor was labeled *success:+effort*. This factor expresses the important role that students ascribe to effort in cases of success (e.g., "If I work hard enough, I will get a good mark in math").

We used a revised version of this questionnaire in our study. The revised version of the Attribution Questionnaire contains 20 items in total, of which 12 items refer to effort and capacity attributions in mathematics, and 8 are filler items. Answers are on a 4-point scale ranging from 1 *(almost never)* to 4 *(almost always)*. This questionnaire was administered to the 276 students in our study. A Principal Components Analysis confirmed the existence of two subscales, each consisting of five items. In the two subscales both success and failure experiences were clustered together. Therefore, the first factor is referred to as *capacity* (Cronbach's alpha= .83), whereas the second factor is referred to as *effort* (Cronbach's alpha= .67). The Pearson correlation coefficient between the two subscales was .11.

Self-concept of Mathematics Ability Questionnaire

This questionnaire was developed for our study and measures the students' estimated ability in relation to mathematics. The items covered eight aspects of the mathematics curriculum, namely: mental arithmetic, computations, application problems, fractions, percentages, graphs and tables, decimal numbers, and measurement. Students indicated how adequate they estimated their ability in relation to each of these topics on a 5-point scale with answers ranging from 1 *(not good at all)* to 5 *(very good).* Furthermore, questions concerning perceived importance of these topics were included in the questionnaire. Students had to indicate on a 4-point scale how important they considered it to be good at the eight topics of the mathematics curriculum. Answers ranged from 1 *(unimportant)* to 4 *(very important).* This questionnaire consisted of 16 items.

The questionnaire was administered to the 276 students in our study. A Principal Component Analysis that was applied to the data confirmed that estimated ability and perceived importance were two latent variables. The first factor includes 8 items that refer to self-concept of mathematics ability and was called *self-concept* (e.g., "How good are you at mental arithmetic?"). The second factor includes 8 items that refer to the importance students ascribe to being good at mathematics (e.g., "How important is it for you to be good at fractions?"). This factor defines a subscale that will be referred to as *importance.* Internal consistencies in terms of Cronbach's alpha were .86 for the subscale self-concept and .85 for the subscale importance. The Pearson correlation coefficient between the two subscales was .30.

On-line Motivation Questionnaire

The On-line Motivation Questionnaire (OMQ) was developed by Boekaerts (1987) to obtain students' perceptions about relevant aspects of the learning situation during actual learning tasks. The OMQ consists of two parts. In both parts, subjects rate their position on 4-point scales. An important feature of this questionnaire is that it is administered when students are confronted with specific tasks.

The first part is administered prior to the task and includes 24 items. Students fill out this part of the questionnaire after they are told what specific task they are going to do. Analyses of data from different samples of students, ranging in age from 10 to 14 years, served to distinguish clusters of items reflecting the student's appraisals, emotions and learning intention. Appraisals covered the following topics: *subjective competence* - including success expectation, self-efficacy judgment, and perception of difficulty - (e.g., "How good are you at doing this type of task?"),

task attraction (e.g., "How much do you like this type of tasks?"), *personal relevance* (e.g., "How useful do you consider this task?"), and *learning intention* (e.g., How much effort are you going to invest in this task?"). The subscale subjective competence differs from self-concept of mathematics ability that was described earlier. Subjective competence is a task-specific variable; it refers to perceived competence in relation to a specific task or assignment that students are going to perform. Self-concept of mathematics ability is measured at the domain-specific level. Students judge their competence in a domain on the basis of activated or hypothetical information.

The second part, which is filled in after the task, consists of 14 items measuring among other things *attributions of results.* First, students indicated on a 4-point-scale how well they thought they had done the task (ranging from *not well at all* to *very well).* Students who indicated that they had done the task well or very well were asked to answer the questions formulated as success attributions, whereas students who indicated that they had done the task not well or not well at all, were asked to answer the questions formulated as failure attributions. Then they were asked to indicate on a 4-point-scale (1= *strongly agree;* 4= *strongly disagree)* to what extent they ascribed their (good or bad) result to a number of causes. These causes included *capacity, pleasure, luck, effort,* and *difficulty level* (e.g., "I did very well on this task because I did my best").

In a study that was conducted by Seegers and Boekaerts (1993) the OMQ was administered to 162 sixth-grade students just before they took a mathematics test in the classroom. Internal consistencies (Cronbach's alphas) of these subscales were satisfactory to good (ranging from .72 to .86).

Confidence and Doubt Questionnaire

The Confidence and Doubt Questionnaire (CDQ) was developed for this research (see also Boekaerts, 1994; Boekaerts, Seegers, & Vermeer, 1995). This questionnaire is an instrument for registering confidence on-line during mathematical problem solving. In order to investigate these processes on-line, a special notation system was developed. In the left margin of every work sheet on which a problem was written, five faces were drawn ranging from very sad to very happy in their expression. They symbolized the degree of doubt or confidence a student had while working on the problem (see Figure 4.3).

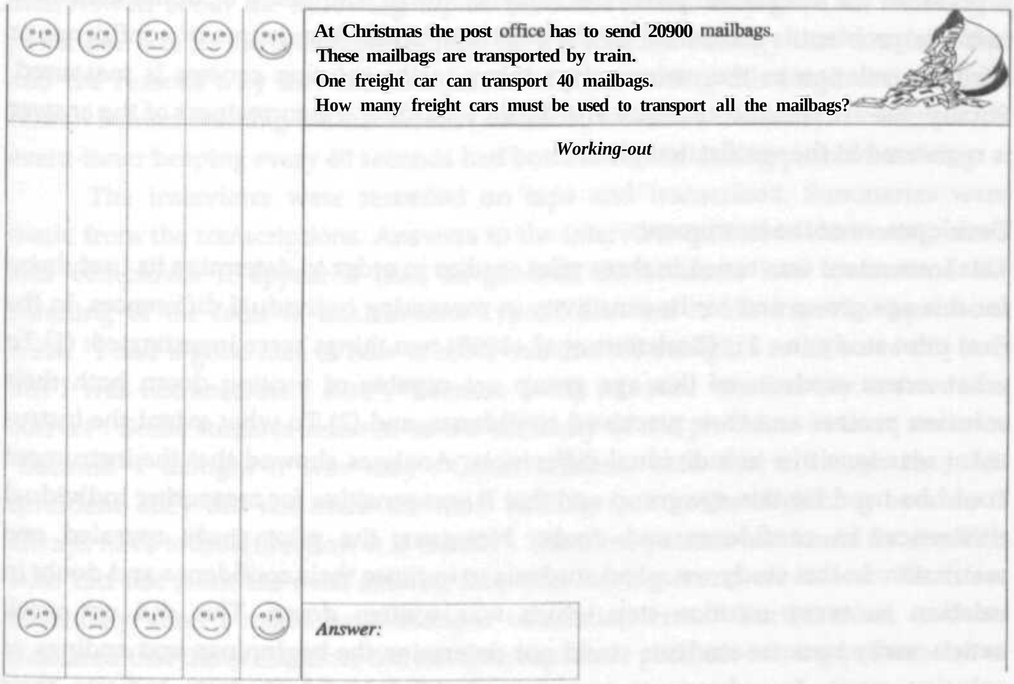

Figure 4.3. An example of the Confidence and Doubt Questionnaire

While working on the task, students were asked to indicate to what extent they thought that their strategy would lead to the right solution. The students were asked to put a mark under one of the faces (1) after having read the problem (the orientation phase), (2) at 40-seconds intervals during the solution process (the execution phase), and (3) after having found an answer (the verification phase). For this purpose, an event-timer or "beeper" was used, which beeped every 40 seconds during the solution process from the moment students had put their first mark under one of the five faces.

Marked faces can be translated to scores ranging from 1 *(very doubtful)* to 5 *(very confident)*. This notation system provides at least three scores of confidence and doubt. It first measures an initial indication of confidence and doubt (smile 1). Next an indication of the confidence and doubt experienced during the solution process (smiles 2a, 2b, 2c,..., 2n). And finally, the test indicates the students' confidence and doubt in relation to the solution (smile 3). This instrument can be used to measure individual differences in (1) the use of solution strategies and (2) the degree of confidence and doubt displayed during the different phases of the problem-solving process. In the orientation phase the students' initial confidence or doubt after they read the problem is measured. In the execution phase, the degree of confidence or doubt in relation to the various steps taken in the solution process is measured. Finally, the estimated confidence and doubt regarding the correctness of the answer is registered in the verification phase.

Development of the instrument

This instrument was tested in three pilot studies in order to determine its usefulness for this age group and for its sensitivity in measuring individual differences. In the first pilot study (n= 31) (Boekaerts et al., 1993) two things were investigated: (1) To what extent students of this age group are capable of writing down both their solution process and their perceived confidence, and (2) To what extent the instrument was sensitive to individual differences. Analyses showed that the instrument could be used for this age group and that it was sensitive for measuring individual differences in confidence and doubt. However, the pilot study revealed one restriction: In this study we asked students to indicate their confidence and doubt in relation to every solution step which was written down. This did not work satisfactorily because students could not determine the beginnings and endings of solution steps. In subsequent tests, we decided to let students indicate their confidence at 40-seconds intervals. In addition, problems that were found to be unsuitable were replaced by other problems.

In a second pilot study (Vermeer, Seegers, & Boekaerts, 1994) the revised instrument was administered to 51 students. It appeared from this study that the students could handle the instrument very well. Analyses that were performed on the data revealed that the CDQ was sensitive to measuring gender differences in perceived confidence. The problems that were used (2 computation problems and 2 application problems) were suitable. Individual differences were found in the use of solution strategies. Because of these findings, we decided to extend the CDQ by adding more problems.

In a third pilot study we measured the construct validity of the CDQ. Construct validity refers to the extent to which a test measures a particular theoretical construct (Neale & Liebert, 1980). In this study, we examined whether the CDQ can measure the degree of confidence that students experience while working on the problems. In this context, we registered what reasons students gave for marking faces in the different phases of the solution process. A study was set up with 48 sixth-grade students. These students worked on three applied mathematics problems in combination with the CDQ. After the test, the students were interviewed about the following topics: (1) The reasons they gave for marking a particular face in the orientation phase, (2) the solution strategy that was followed and the reasons why they marked particular faces during this stage, and (3) their reason behind marking the last face. In addition, we asked the students whether the event-timer beeping every 40 seconds had bothered them during problem solving.

The interviews were recorded on tape and transcribed. Summaries were made from the transcriptions. Answers to the interview questions were categorized and compared. It appeared that, in general, the students had understood the meaning of the faces in this context. Typical answers to the interview questions were: "I had a good idea of how to solve this one"; "I thought I could solve this one, but I was not absolutely sure"; "Because I was not sure whether my answer was correct". Some students referred to the difficulty of the problems in their answers: "Because I thought it was easy". Other students said that although they felt confident, they did not mark the most smiling face in the beginning because "I always have to look first how it is exactly". The same pattern was found for students who did not mark the most smiling face after having found an answer: "I am not absolutely sure, I might have made a calculation error." Furthermore, students indicated that the event-timer did not interrupt their problem-solving process.

The mathematics tasks

Two types of mathematics tasks were constructed for this research, including six computation problems, and six application problems. The contents of the individual problems were chosen in such a way, that the application problems could be solved by applying the same computations as in the other task, only using different numbers. In this way, domain-specific knowledge that the students need to activate with respect to procedures and algorithms was the same in both tasks, and therefore comparisons between their confidence judgments during the two tasks could be made more easily. The problems were constructed in pairs. Each pair consisted of problems with the same computational structure. As a consequence, a particular type of non-routine problems, such as estimation problems, was not suitable for our study.

Because we are especially interested in students' reactions following failure, we included two problems that were more difficult than the other problems. The difficulty of the problems was manipulated in the following way: The first two problems could be solved by about 50 to 75% of the students. These problems were followed by two difficult items that could be solved by less than half of the students. Next came two items that could again be solved by 50 to 75% of the students. Some problems were chosen from the national assessment study that had been conducted in the Netherlands in 1988, but most problems were taken from the pilot studies. The problems of both tasks were chosen such that different types of computations were needed to solve the problems.

In addition, the following criteria for the application problems were met: (1) The problems should be described in clear and simple language, (2) The students should be able to solve the problems using a variety of solution strategies, (3) The problems should refer to at least one authentic context, and the correctness of the answer should be determined by reference to that context (4) The context of the problems should be interesting for both boys and girls. This resulted in the following problems:

Computation problems	Application problems
14820 : 38 =	*At Christmas the post office has to send 20900 mailbags. These mailbags are transported by train. One freight car can transport 40 mailbags. How many freight cars must be used to transport all the mailbags?*
68.2 - 4.73 =	*When Bas left home, the mileage indicator of his bike was on 021.4. When he came back later the indicator was on 086.2. How far did he bike that day?*
4¾% off 1816.- =	*Yvonne has f 3660.- in the bank. Her bank gives an interest rate of 5¼% over a year. How much interest does Yvonne receive after a year?*
0.825 : 0.01 =	*A pharmacist has a bottle which contains 0.75 liter of a specific liquid. With the contents of this bottle he fills some smaller medicine bottles. Every medicine bottle will hold 0.05 liter of this liquid. How many medicine bottles can be filled?*
5% of 46460 •-	*45450 tickets are available for a rock concert. The day before the concert 4% of the tickets have not yet been sold. How many tickets are still available?*
236 x 405 =	*A campground is 406 meters long and 235 meters wide. What is the surface area of Ms campground?*

4.3 Procedures

Data collection

Data were collected in both group and individual sessions. The group sessions preceded the individual sessions and took place during two separate sessions. During the first session, one subscale of a test for non-verbal intelligence was administered, as well as the Goal Orientation Questionnaire. In the second group session, the other subscale of the test for non-verbal intelligence was administered together with the Attribution Questionnaire and the Self-concept of Mathematics Ability Questionnaire. Both group sessions took about one hour. Individual testing took place in two separate sessions with an interval of about three months. Both individual sessions were the same except for the contents of the mathematics task. More specifically, the computation problems were given to the students in combination with the OMQ and the CDQ. After about three months, the students were invited to do the application problems, again in combination with the OMQ and the CDQ. The procedures for both sessions were the same. We did not counterbalance the order of administration of the two testing situations, because our main interest was not in comparing performances, but in comparing problem-solving behavior. Students' behavior during the two testing situations was investigated under identical conditions.

Procedure for the individual sessions

Students were observed individually while they worked on the mathematics tasks. They were in a separate room in their regular school surroundings with one observer present. Two observers participated in the research. Each observer saw half of the students during both sessions. The students were told that we are interested in the way they solve mathematical problems. At the beginning of every session, the students received detailed instruction about the testing procedure. They were told by the observers that we were not only interested in the way in which they solved the mathematical problems, but also in their feelings of confidence before, during, and after they solved problems. We showed them the five faces and explained what they represented in this context. Then we introduced the beeper and explained that a mark had to be put under one of the five faces, each time they heard a beep. Students were also instructed to write down their solution process and calculations in as detailed a manner as possible in the space provided. In general, students quickly understood what was expected of them. In order to familiarize the students with the special notation system of the CDQ, two problems were given prior to testing. After students confirmed that they had understood the instruction and had completed the two pretest problems, they were told they were going to do six comparable problems. Students were then shown a glimpse of these problems. When they knew what was expected of them, they were requested to fill out the first part of the OMQ.

The students filled out the CDQ while they worked on the problems. After they had finished a problem, they were told whether their solution was correct or not. If their solution was correct, they were asked to do the next problem. Students who had given up, and thus had no solution, were instructed to continue with the next problem. If their solution was incorrect, they were asked whether they wanted to try the problem again. Students who decided to retry a problem were given a new work sheet. This procedure was not always followed as we will explain in the next section under the heading "scoring procedures". After the students' second attempt, no feedback was given on the correctness or incorrectness of the solution. If students asked whether their solution was correct after their second attempt, they were told that this would be discussed at the end of the session. In this study, no help was given to the students, because we were interested in their problem-solving behavior, as well as in their judgments of confidence and doubt. Working time was recorded for each problem. The students filled out the second part of the OMQ after solving the computation problems and the application problems, respectively. Each individual session took about 40 minutes.

Scoring procedures

With respect to two application problems, namely problems 1 and 6, there were some difficulties in scoring, because the distinction between correct and incorrect solutions was not immediately obvious. We will therefore consider the scoring procedures of problems 1 and 6.

Problem 1

> *At Christmas the post office has to send 20900 mailbags.*
> *These mailbags are transported by train.*
> *One freight car can transport 40 mailbags.*
> *How many freight cars must be used to transport all the mailbags?*

After having solved this problem, students should interpret the numerical outcome of the solution process in the wider context of the problem. That is, after having made the correct calculations, students should round up the answer 522.5 to 523. However, many students did not do this. If students had 522.5 or 522 remainder 20 as an answer, we asked the question: "Does this make sense, half a freight car?". This question or hint enabled them to correct their first answer. The number of different solutions to problem 1 was as follows:

	Solution	Number of students
(a)	*Correct*	28
(b)	*Incorrect*	67
(c)	*Rounded up after a hint*	51
(d)	*Rounded down after a hint*	4
(e)	*No change after a hint*	8
	Total	158

Only 28 students out of 158 (about 18%) immediately considered the context of the problem and rounded their answer up to 223. After a hint, another 51 students realized that their solutions should be adjusted. We decided to use a flexible scoring method and considered the answers in category (c) as correct, and the categories (d) and (e) as incorrect. Please note that category (b) consisted of all the other solutions. The causes for these incorrect solutions were diverse: Some students only made small calculation errors, whereas other students had not understood the problem at all. These students were not given a hint, but were asked whether they wanted to try the problem again. Students who had 222.5 as an answer and either did not change their answer after a hint or rounded down their answer after a hint, were not asked whether they wanted to try the problem again.

Problem 6

A campground is 406 meters long and 235 meters wide.
*What is the surface area of this campground?*__________

After having solved this problem, not all students reported a unit of measure. If students only mentioned the number 95410, we asked: "Shouldn't there be something after that number?". This question enabled them to complete their answer. The number of different solutions was as follows:

	Solution	Number of students
(a)	*Correct*	43
(b)	*Incorrect*	71
(c)	*Correct unit after a hint*	25
(d)	*Incorrect unit or no unit after a hint*	19
	Total	158

Forty-three students (about 27%) put the correct unity (m^2) after their answer. After questioning, another 25 students solved the problem correctly. We decided to consider answers in category (c) as correct. Category (b) consisted of all the other incorrect solutions. As in the first problem, the causes of the incorrect solutions were diverse. Students who had 95410 as an answer and either did not change their answer after a hint or put an incorrect unit after their answer after a hint, were not asked whether they wanted to try the problem again.

Solution strategies

Solution strategies for each application problem were listed, based on the students' written work and on the observer's notes made during the individual sessions. This was done for the application problems only, because the computation problems elicited less variety in students' use of solution strategies. This resulted in different lists of solution strategies for each application problem. In order to compare solution strategies across problems, we further categorized them as one of the three general solution strategies: *ineffective, conventional,* and *unconventional.* A solution strategy was called ineffective, when students either, (1) showed no attempt to solve the problem at all, (2) used the wrong computations or combination of computations, or (3) did not complete all the necessary steps in order to solve the problem. The solution strategies that were effective were further analyzed. A solution strategy was called conventional if one or more standard computational strategies were applied, such as the execution of long division. Furthermore, a solution method was labeled unconventional if it was mainly non-routine, for example primarily based on students' insight and logical reasoning, such as estimation or mental computation. The distinction between conventional and unconventional was primarily based on our own observations; the specific instruction that the students had received in the classroom setting was not included here. Within this classification, ineffective solution strategies lead to incorrect solutions by definition, but effective solution strategies (conventional or unconventional) may result in either correct or incorrect solutions. For instance, students might make calculation errors even though they apply the correct computational strategies. In Figures 4.4 and 4.5 two students' solutions to the first application problem are displayed. The solution strategy used by the student in Figure 4.4 is scored as conventional, whereas the solution strategy reported by the student in Figure 4.5 is scored as unconventional.

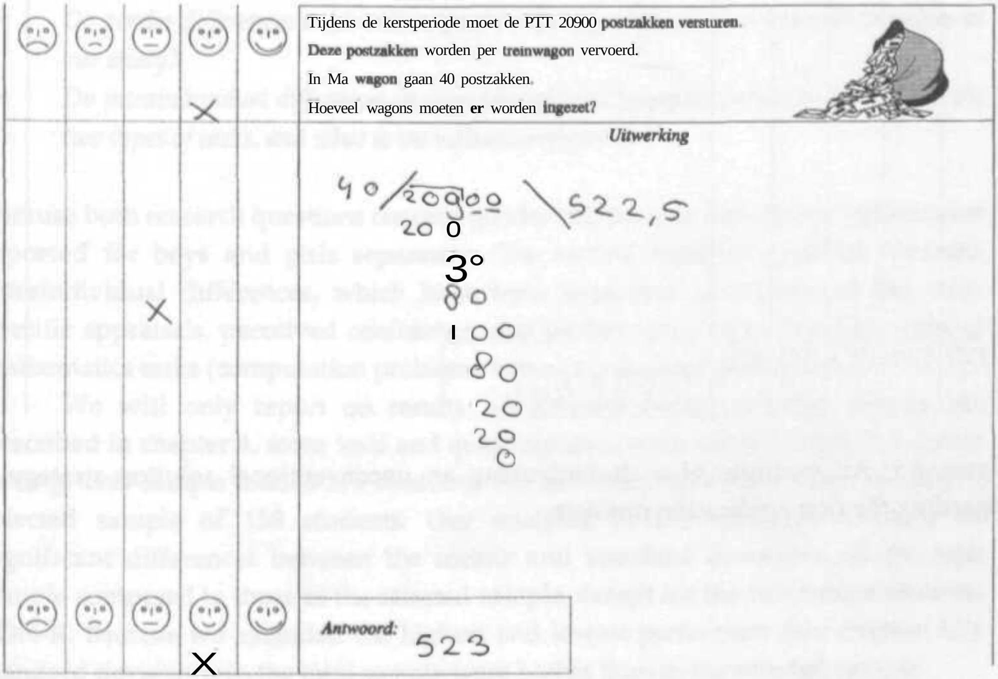

Figure 4.4. An example of a student using a conventional solution strategy regarding the first application problem.

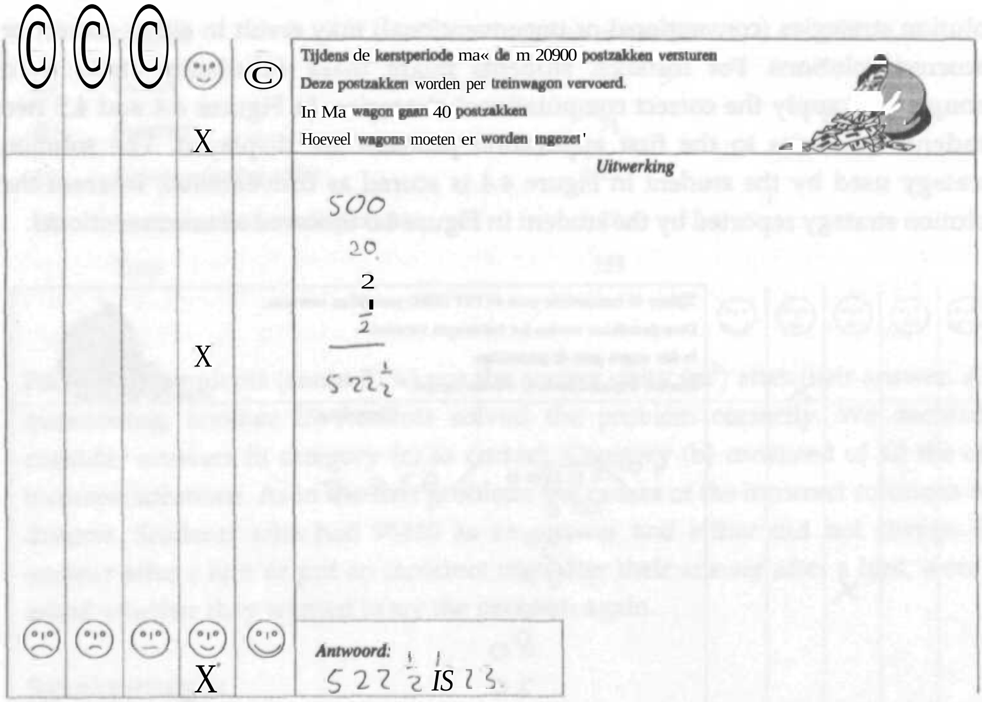

Figure 4.5. An example of a student using an unconventional solution strategy regarding the first application problem.

5 GENDER DIFFERENCES AND INTRAINDIVIDUAL DIFFERENCES

In this chapter we will discuss the initial analyses that were performed on the variables in our study. Descriptive statistics are presented for all the variables, that is for general, domain-specific, task-specific, and behavior-related measures. Furthermore, the following research questions are answered:

- *Do gender differences exist with respect to the cognitive and motivational variables in our study?*
- *Do intraindividual differences in cognitive and motivational variables exist across the two types of tasks, and what is the influence ofgender?*

Because both research questions concern gender differences, descriptive statistics are reported for boys and girls separately. The second research question concerns intraindividual differences, which have been examined in relation to the task-specific appraisals, perceived confidence, and performance across the two types of mathematics tasks (computation problems versus application problems).

We will only report on results which refer to the selected sample. As described in chapter 4, some tests and questionnaires were administered in a group setting. This sample totaled 279 students. We have data on all the variables from a selected sample of 158 students. Our analyses of the variables revealed no significant differences between the means and standard deviations of the total sample compared to those of the selected sample, except for the two subscales of the SON-R. Because we excluded the highest and lowest performers (see chapter 4.1), standard deviations in the total sample were higher than in the selected sample.

5.1 Abstract reasoning ability

Subscales ofthe SON-R

Descriptive statistics for the raw scores of the two subscales of the SON-R are presented in Table 5.1. The range of scores for the selected sample was from 8-24 for the analogies subscale (4-27 for the total sample) and from 9-20 for the categories subscale (1-24 for the total sample). Internal consistencies in terms of Cronbach's alpha of both subscales were .73 for analogies and .75 for categories. The results revealed no significant differences between boys' and girls' scores, neither on the subscale analogies ($t(156)= .13$, $p= .89$), nor on the subscale categories ($t(156)= -.15$, p= .88). This is consistent with what has usually been reported in the literature (e.g., Laros & Tellegen, 1991; Seegers & Boekaerts, 1996).

Table 5.1
Descriptive statistics for the raw scores of the SON-R

		Selected sample (n=158)		Boys (n= 79)		Girls (n- 79)	
Subscales	Range	M	SD	M	SD	M	SD
Analogies	8-24	17.63	2.96	17.66	2.95	17.59	2.99
Categories	9-20	15.79	2.69	15.76	2.66	15.82	2.74

5.2 Motivational beliefs

In Table 5.2 descriptive statistics are presented for the questionnaires on motivational beliefs.

Table 5.2
Descriptive statistics for the questionnaires on motivational beliefs

		Selected sample (n=158)		Boys (n= 79)		Girls (n=79)	
Measure	a	M	SD	M	SD	M	SD
Goal orientation							
Task orientation (5)	.78	2.89	.62	2.99*	.60	2.79	.64
Error frustration (5)	.76	2.00	.55	2.14"	.63	1.87	.43
Self-enhancing ego orientation (6)	.84	2.04	.70	2.21"	.75	1.86	.60
Self-defeating ego orientation (6)	.75	1.51	.47	1.61"	.53	1.42	.37
Attributions							
Capacity (6)	.78	2.52	.52	2.56	.53	2.48	.52
Effort (6)	.62	2.91	.48	2.99	.49	2.84	.46
Self-concept of mathematics ability							
Self-concept (8)	.87	3.52	.59	3.62*	.55	3.42	.61
Importance (8)	.86	3.22	.43	3.25	.43	3.18	.42

Note. The number of items for every subscale are given in parentheses.
*p < .05. "p < .01.

Goal orientation

As can be seen in Table 5.2, means and standard deviations were computed for the four different subscales of the Goal Orientation Questionnaire: task orientation, error frustration, self-enhancing ego orientation, and self-defeating ego orientation. Scores ranged from 1 to 4. A high score on the subscale self-enhancing ego orientation indicates a tendency to be competitive in mathematics, whereas a high score on the subscale self-defeating ego orientation indicates a tendency to be afraid that others will notice one's mistakes in mathematics. Furthermore, a high score on task orientation refers to a positive attitude towards and interest in mathematics, whereas a high score on the subscale error frustration refers to a tendency to experience failure as a self-referenced shortcoming.

To test for gender differences, a multivariate analysis of variance (MANOVA) was applied to the data with the students' scores on the four subscales of the Goal Orientation Questionnaire as the dependent variables, and gender as the between-subjects variable. Results revealed a main gender effect (Pillais, $F[l,156]= 4.00$, $p< .01$). Univariate tests revealed gender differences on all the subscales: task orientation ($F[l,156]= 4.36$, $p < .05$), error frustration ($F[l,156]= 9.71$, $p < .01$), self-enhancing ego orientation ($F[l,156]= 10.39$, $p < .01$), and self-defeating ego orientation ($F[l,156]= 6.62$, $p < .01$). Inspection of the means (see Table 5.2) revealed that boys scored higher than girls on each subscale.

Attributions

Means and standard deviations were computed for the two different subscales of the Attribution Questionnaire: capacity and effort (see Table 5.2). Scores ranged from 1-4. A high score on the subscale capacity indicates a tendency to ascribe success in mathematics to capacity, whereas a high score on effort indicates a tendency to ascribe success in mathematics to effort.

A MANOVA was applied to the data with the students' scores on the two attributions as the dependent variables and gender as the between-subjects variable. No main gender effect was found (Pillais, $F[2,155]= 2.14$, $p= .12$).

Self-concept ofmathematics ability

Means and standard deviations were computed for the two subscales of the Self-concept of Mathematics Ability Questionnaire: self-concept and importance (see Table 5.2). Scores ranged from 1-5 for the subscale self-concept, and from 1-4 for the subscale importance. A high score on the subscale self-concept refers to a high self-concept of mathematics ability, whereas a high score on the subscale importance indicates that a student ascribes a lot of importance to being good at mathematics.

A MANOVA that was applied to the data with the students' scores on the subscales self-concept and importance as the dependent variables and gender as the independent variable showed no main gender effect (Pillais, F[l,156]= 2.39, p= .10). Univariate results, however, showed that boys' self-concept of mathematics ability was significantly higher than girls' (F[l,156]= 4.59, p < .05).

Correlations between subscales of the questionnaires on motivational beliefs

Pearson correlation coefficients were computed between the scores on the various subscales that measure motivational beliefs. A high positive correlation was found between students' scores on the subscales self-concept and capacity attributions (r= .74). This indicates that students with a high self-concept of mathematics ability are inclined to perceive their capacity as an important cause for success in mathematics. A moderate positive association was found between task orientation and effort attributions (r= .44), implying that a positive attitude towards and interest in mathematics are related with high effort attributions. In addition, task orientation had modest to moderate positive associations with capacity attributions (r= .30), self-concept (r=.29) and importance (r= .23). A moderate positive correlation was found between effort attributions and importance (r= .37). Also, a moderate positive association was found between effort attributions and self-enhancing ego orientation (r= .21), whereas a modest negative association was found between capacity attributions and self-defeating ego orientation (r= -.21). The other subscales were not correlated significantly.

To test whether these relations were the same for boys and girls, Pearson correlation coefficients were computed for boys and girls separately. The results are displayed in Table 5.3. This table shows that the patterns of associations were generally the same for boys and girls. However, a significant negative correlation between scores on the subscales self-defeating ego orientation and capacity (r= -.29), was found only for boys. This indicates that for boys the fear of others noticing their mistakes in mathematics was related to low capacity attributions. In addition, a significant positive correlation between scores on the subscales task orientation and importance was found only for boys (r= .24). These findings are partly consistent with previous research findings. Seegers and Boekaerts (1996) reported on relations between the different types of goal orientations and attributions for failure (lack of capacity) and success (high effort). They also found a positive association between task orientation and effort attributions for both boys and girls. However, in our study an association between self-defeating ego orientation and capacity attributions was found only for boys, whereas in Seegers' and Boekaerts' study this relation was found only for girls. This may have been due to the fact that in our study capacity attributions for success were measured, whereas Seegers and Boekaerts investigated to what extent students ascribed failure to lack of capacity.

Table 5.3

Pearson correlation coefficients between subscales of the questionnaires on motivational beliefs for boys (n= 79) and girls (n= 79) separately

Measure	1	2	3	4	5	6	7	8
Coal orientation								
1. Task orientation	–				.37**	.43**	.31**	.24*
2. Errorfrustration		–			-.03	.10	.10	.17
3. Self-enhancing ego orientation			''		.08	.21	.15	.20
4. Self-defeating ego orientation				–	-.29*	.06	-.25	.18
Attributions								
5. Capacity	.23*	-.07	.10	-.16	–		.75**	.10
6. Effort	.42"	.14	.14	-.14		–	.18	.33-
Self-concept of mathematics ability								
7. Self-concept	.25*	-.15	.25*	-.15	.73**	.09	–	
8. Importance	.06	.05	.20	.22	.09	.41**		–

Note. For boys, the correlations are printed above the main diagonal, whereas for girls, the correlations are printed below the main diagonal.
* $p < .05$. ** $p < .01$.

5.3 Task-specific appraisals and attributions

Task-specific appraisals

Table 5.4 shows means and standard deviations of the students' scores on the four subscales of the On-line Motivation Questionnaire (OMQ) that was administered before working on the task. The results from the two task conditions are displayed separately. Scores ranged from 1-4 for every subscale.

Table 5.4
Descriptive statistics for the subscales of the OMQ before working on the computation problems and application problems

		Selected sample (n=158)		Boys (n= 79)		Girls (n= 79)	
Measure	a	M	SD	M	SD	M	SD
Computation problems							
Subjective competence (8)	.85	2.72	.38	2.71	.,41	2.73	.35
Task attraction (3)	.77	2.93	.50	2.93	.48	2.92	.53
Personal relevance (2)	.69	2.89	.50	2.88	.48	2.89	.52
Learning intention (4)	.66	3.22	.42	3.19	.45	3.25	.40
Application problems							
Subjective competence (8)	.91	2.60	.47	2.68*	.46	2.53	.48
Task attraction (3)	.82	2.83	.55	2.87	.56	2.79	.54
Personal relevance (2)	.77	2.75	.56	2.64	.55	2.85**	.56
Learning intention (4)	.71	3.16	.44	3.13	.45	3.18	.43

Note. The number of items for every subscale are given in parentheses.
* $p < .05$. ** $p < .01$.

Intraindividual differences in task-specific appraisals across both types of tasks were examined using the BMDP4V statistical program (Davidson & Toporek, 1983). A multivariate design of repeated measures was used, also known as profile analysis (Tabachnick & Fidell, 1989). This design is a between-within-subjects design with multiple dependent variables that are repeated over time. In this study the four task-specific appraisals in the two task conditions (computation problems versus application problems) were considered as the dependent variables. Gender was included as the between-subjects variable, and task as the within-subjects variable. Results of this analysis are displayed in Table 5.5.

Results demonstrated overall effects for the within-subjects variable task ($F[4,152]= 6.27$, $p < .01$), and for the interaction gender by task ($F[4,152]= 4.83$, $p < .01$). Task effects were found for all the appraisals (see Table 5.5). As Table 5.4 shows, students' scores on the four appraisals were higher for the computation problems than for the application problems. The gender by task interaction effect indicates that boys' and girls' scores on the appraisals differed dependent on the type of task.

Table 5.5

Results of analysis of variance with the task-specific appraisals as the multiple dependent variables repeated over the two types of tasks (computation problems versus application problems)

Source		df	SS	F-statistic	Sign.
Between-subjects					
Gender:	Overall	4,152	5.65	1.39	n.s.
	Subjective competence	1,155	.31	1.02	n.s.
	Task attraction	1,155	.12	.28	n.s.
	Personal relevance	1,155	1.07	2.55	n.s.
	Learning intention	1,155	.20	.64	n.s.
Within-subjects					
Task:	Overall	4,152	25.57	6.27	.01
	Subjective competence	1,155	1.12	17.68	.01
	Task attraction	1,155	.79	7.01	.01
	Personal relevance	1,155	1.56	10.90	.01
	Learning intention	1,155	.33	4.82	.05
Gender x Task:	Overall	4,152	19.69	4.83	.01
	Subjective competence	1,155	.58	9.04	.01
	Task attraction	1,155	.09	.83	.36
	Personal relevance	1,155	.83	5.81	.05
	Learning intention	1,155	.00	.00	.97

For the different appraisals, significant interaction effects were found for subjective competence ($F[1,155]= 9.04$, $p < .01$), and personal relevance ($F[1,155]= 5.81$, $p < .05$). Inspection of the means (see Table 5.4) revealed gender differences only during the application problems: Boys' subjective competence was significantly higher than girls', and girls perceived higher personal relevance than boys. This pattern was not found for the computations.

Task-specific attributions

The post-task part of the OMQ included task-specific attributions. For the computation problems, questions formulated as success attributions were answered by 91 students (49 boys and 42 girls), whereas questions formulated as failure attributions were answered by 65 students (29 boys and 36 girls). Two students forgot to answer these questions. For the application problems, success attributions were filled out by 96 students (50 boys and 46 girls), and failure attributions were filled out by 59 students (28 boys and 31 girls). Three students forgot to answer these questions.

We investigated whether boys and girls differed in ascribing their perceived success or failure to different causes, that is capacity, pleasure, luck, effort, and difficulty level. Mann-Whitney tests were used, in which the difference in ranking between boys and girls for ascribing their perceived success (or failure) to the different causes was examined. These tests revealed no differences in boys' and girls' attributions regarding the computation problems. With respect to the application problems, however, we found that girls attributed a bad result more often to a lack of capacity ($Z= 1.99$, $p < .05$), and to the difficulty level of the task ($Z= 2.95$, $p < .01$) than boys did.

Correlations between task-specific appraisals

In Table 5.6 Pearson correlation coefficients between the subscales of the OMQ (the pretask part) are displayed. It appeared that almost all the subscales were correlated moderately. This pattern was also found by Seegers and Boekaerts (1993, 1996). There was, however, no significant association between the subscales subjective competence and personal relevance. For the computation problems, this correlation was significant only for boys ($r= .37$, $p < .01$).

Table 5.6

Pearson correlation coefficients between the subscales of the OMQ administered before the computation problems (before the slash) and the application problems (behind the slash)

Subscale	1	2	3	4
1. Subjective competence	–	.59** / .66**	.37** / .19	.44** / .44**
2. Task attraction	.48** / .66**	–	.41** / .44**	.44** / .62**
3. Personal relevance	.00 / .22	.43** / .45**	–	.54** / .42**
4. Learning intention	.22* / .32**	.52** / .63**	.38** / .68**	–

Note. For boys (n=79), the correlations are printed above the main diagonal, whereas for girls (n=79), the correlations are printed below the main diagonal.

* $p < .05$. ** $p < .01$.

5.4 Behavior-related variables

Perceived confidence

Students' confidence during the orientation, execution, and verification phases of the solution process was measured using the Confidence and Doubt Questionnaire (see chapter 4.2). Confidence ratings ranged from 1 *(not confident at all)* to 5 *(very confident).* Because students indicated their confidence during the execution phase at time intervals of 40 seconds, the total number of confidence scores during this phase was different for every student. Therefore, for all students the mean confidence during the execution phase was calculated by dividing the sum of all the confidence ratings by the number of confidence ratings that were registered. In this way, we obtained one confidence rating for each student for the execution phase of each individual problem.

As a first step, we investigated to what extent students' confidence fluctuated across the three phases within the solution process. Tables 5.7 and 5.8 display mean confidence scores during the three phases of the solution process for the computation problems and the application problems, respectively.

Table 5.7

Students' mean confidence scores and standard deviations during the three phases of the computation problems (n=158)

	Orientation		Execution		Verification	
Problems	M	SD	M	SD	M	SD
1. 14820:38=	3.70	.82	3.75	.87	4.23	.91
2. 68.2 - 4.73=	4.11	.78	4.31	.69	4.44	.67
3. 4¾% of f 1816.-=	2.97	1.10	3.11	1.14	3.20	1.22
4. 0.825 : 0.01=	3.54	.97	3.48	1.16	3.68	1.18
5. 5% of 46460=	3.78	1.03	3.76	1.10	3.78	1.13
6. 236x405-	4.32	.79	4.35	.77	4.29	.80

Table 5.8

Students' mean confidence scores and standard deviations during the three phases of the application problems (n=158)

	Orientation		Execution		Verification	
Problems	M	SD	M	SD	M	SD
1. Mailbags	3.59	.82	3.65	.87	3.61	1.02
2. Bicycle	3.91	.93	3.98	.90	4.05	.95
3. Interest	3.40	1.01	3.50	1.03	3.60	1.06
4. Pharmacist	3.89	.83	3.95	.87	4.06	.92
5. Rock concert	3.56	1.00	3.52	1.12	3.55	1.06
6. Campground	3.72	1.00	3.71	1.06	3.60	1.04

Pearson correlation coefficients were computed between students' mean confidence ratings during the orientation, execution, and verification phase. These correlation coefficients are displayed in Appendix A. Because these correlations were rather high, we calculated a mean confidence score across the three phases of the solution process for every student on every problem. If a student had given up on a problem before the verification phase, and therefore had no confidence score for this phase, the mean confidence score was calculated on the basis of the first two phases. If students did not have a confidence score during the execution phase because they had solved the problem within 40 seconds, the mean confidence was calculated from the first and last phase. In Tables 5.9 and 5.10 students' mean confidence is displayed separately for the computation problems and application problems.

As a next step, we examined intraindividual differences in perceived confidence across both types of task. Confidence scores of the problems within both tasks can be considered as repeated measures, because these scores are available with respect to analogue problems during the two task conditions (computation problems versus application problems). A multivariate design of repeated measures was performed on the data using the BMDP4V program. Gender was included as the between-subjects variable, task as the within-subjects variable, and the confidence ratings for the different problems repeated over the two types of tasks as the dependent variables. Results of the analysis are displayed in Table 5.11.

Table 5.9

Boys' and girls' mean confidence scores and standard deviations for the computation problems

	Total (n= 158)		Boys (n= 79)		Girls (n= 79)	
Problems	Mean	SD	Mean	SD	Mean	SD
1. 14820 : 38=	3.84	.74	3.86	.75	3.82	.72
2. 68.2 - 4.73=	4.29	.63	4.26	.66	4.31	.59
3. 4¾% of ƒ 1816.-=	3.02	1.04	3.18	.99	2.87	1.07
4. 0.825 : 0.01=	3.53	.99	3.63	1.03	3.42	.94
5. 5% of 46460=	3.75	1.04	3.89	.98	3.60	1.08
6. 236 x 405=	4.32	.72	4.39	.69	4.25	.75
Total mean	3.79	.68	3.87	.67	3.71	.69

* p < .05. ** p < .01.

Table 5.10

Boys' and girls' mean confidence scores and standard deviations for the application problems

	Total (n=158)		Boys (n= 79)		Girls (n=79)	
Problems	Mean	SD	Mean	SD	Mean	SD
1. Mailbags	3.61	.77	3.81**	.69	3.41	.80
2. Bicycle	3.97	.87	4.16**	.81	3.79	.90
3. Interest	3.46	.95	3.56	.96	3.36	.93
4. Pharmacist	3.97	.82	4.14	.75	3.79	.86
5. Rock concert	3.52	.98	3.65	.99	3.39	.96
6. Campground	3.65	.97	3.79	.96	3.52	.96
Total mean	3.70	.74	3.85**	.70	3.54	.76

* p < .05. ** p < .01.

Table 5.11

Results of analysis of variance with perceived confidence ratings as the multiple dependent variables repeated over the two types of tasks (computations versus applications)

Source		df	SS	F-statistic	Sign.
Between-subjects					
Gender	Overall	6,148	5.83	.94	as.
	Problem 1	1,153	3.39	3.95	.05
	Problem 2	1,153	1.49	1.91	as.
	Problem 3	1,153	4.16	2.64	as.
	Problem 4	1,153	5.32	4.54	.05
	Problem 5	1,153	6.19	3.83	as.
	Problem 6	1,153	3.08	3.19	as.
Within-subjects					
Task:	Overall	6,148	197.09	31.78	.01
	Problem 1	1,153	3.72	15.83	.01
	Problem 2	1,153	7.38	23.11	.01
	Problem 3	1,153	15.35	39.75	.01
	Problem 4	1,153	14.31	34.12	.01
	Problem 5	1,153	3.86	9.65	.01
	Problem 6	1,153	31.44	68.94	.01
Gender x Task:	Overall	6,148	21.55	3.47	.01
	Problem 1	1,153	1.85	7.87	.01
	Problem 2	1,153	3.20	10.01	.01
	Problem 3	1,153	.28	.73	as.
	Problem 4	1,153	.18	.44	as.
	Problem 5	1,153	.03	.09	as.
	Problem 6	1,153	.17	.36	n.s.

The results demonstrated overall effects for the variables task ($F[6,148]= 31.78$, $p < .01$), and the interaction gender by task ($F[6,148]= 3.47$, $p < .01$). As Table 5.11 shows, task effects were significant for all problems. Students' confidence ratings were higher for the computation problems 1, 2, 5, and 6 than for the equivalent application problems (see Tables 5.9 and 5.10). For the third and fourth problem, however, the pattern was reverse. The gender by task interaction indicates that boys' and girls' perceived confidence differed dependent on the type of task. This interaction effect was significant for problem 1 ($F[1,153]= 7.87$, $p < .01$), and problem 2 ($F[1,153]= 10.01$, $p < .01$). Inspection of the means (see Tables 5.9 and 5.10) revealed gender differences, but only during the application problems: Boys' perceived confidence was significantly higher than girls' during these problems.

Task performance

The number of correct and incorrect responses was analyzed for each problem in both task conditions (see chapter 4.3 for scoring procedures). The mean number of correct solutions was 3.23 (SD= 1.36) for the computation problems, and 3.35 (SD= 1.58) for the application problems. Differences between boys' and girls' performances on the application problems were displayed, where the boys' mean score (M= 3.65; SD= 1.49) was significantly higher than the girls' mean score (M= 3.06; SD= 1.63), $t(156)= 2.34$, $p < .05$. As can be viewed from Table 5.12, there were significant differences between boys' and girls' performances on the third computation problem: More girls than boys solved this problem correctly, $\chi^2(1, n=158)= 4.11$, $p < .05$. Furthermore, the boys solved all the application problems as well as or better than the girls. The boys' scores were significantly higher than the girls' scores for problem 4, $\chi^2(1, n=158)= 11.11$, $p < .01$, and problem 5, $\chi^2(1, n=158)= 3.75$, $p < .05$.

Next, intraindividual differences in performance across the two types of tasks were examined. Sixty-five students (41%) performed better on the application problems than on the computation problems, 55 students (35%) performed better on the computation problems than on the application problems, and 38 students (24%) performed equally well on both types of tasks. Chi-square testing revealed gender differences in performance across the two types of tasks, $x^2(2, n=158)= 9.57$, $p < .01$. Inspection of the data revealed that more girls than boys performed better on the computation problems than on the application problems (21 boys versus 34 girls), whereas more boys than girls performed better on the application problems than on the computation problems (42 boys versus 23 girls).

Table 5.12

Number of correct solutions on the computation problems and the application problems

Problems	Total correct (n=158)	Boys (n= 79)	Girls (n= 79)
Computation problems			
1. 14820 : 38 =	95 (60%)	46 (58%)	49 (62%)
2. 68.2 - 4.73 =	121 (77%)	64 (81%)	57 (72%)
3. 4¾ % of ƒ 1816.-=	13 (8%)	3 (4%)	10 (13%)*
4. 0.825 : 0.01 -	65 (41%)	35 (44%)	30 (38%)
5. 5% of 46460 =	99 (63%)	46 (58%)	53 (67%)
6. 236 x 405 =	119 (75%)	55 (70%)	64 (81%)
Application problems			
1. Mailbags	79 (50%)	40 (51%)	39 (49%)
2. Bicycle	121 (77%)	65 (82%)	56 (71%)
3. Interest	40 (25%)	20 (25%)	20 (25%)
4. Pharmacist	130 (82%)	73 (92%)**	57 (72%)
5. Rock concert	92 (58%)	52 (66%)*	40 (51%)
6. Campground	68 (43%)	38 (48%)	30 (38%)

* $p < .05$. ** $p < .01$.

Solution strategies

As explained in chapter 4, students' solution strategies were examined only with respect to the application problems. Students' solution strategies for each problem were listed, and were scored as conventional, unconventional or ineffective (see chapter 4.3). In Table 5.13 frequencies of solution strategies are displayed. This table shows that some problems elicited more unconventional strategies than others. About 59% of the students used unconventional strategies on the fourth problem, whereas no unconventional strategies were used at all on the sixth problem. More ineffective strategies were used for the third problem (47%) than for any of the other problems.

Table 5.13

Frequencies of solution strategies on the application problems

	Conventional			Unconventional			Ineffective		
Problems	Total	Boys	Girls	Total	Boys	Girls	Total	Boys	Girls
1. Mailbags	115 (73%)	50	65	27 (17%)	19*	8	16 (10%)	10	6
2. Bicycle	114 (72%)	53	61	28 (18%)	19	9	16 (10%)	7	9
3. Interest	72 (46%)	37	35	12 (8%)	7	5	74 (47%)	35	39
4. Pharmacist	48 (30%)	18	30	93 (59%)	59**	34	17 (11%)	2	15**
5. Rock concert	109 (69%)	61	48	17 (11%)	5	12	32 (20%)	13	19
6. Campground	109 (69%)	55	54	0 (0%)	–	–	49 (31%)	24	25

* $p < .05$. ** $p < .01$.

Strategy scores were summed, and the number of solutions that were conventional, unconventional and ineffective were computed for each student for all the application problems. For the entire sample, the mean numbers of conventional, unconventional, and ineffective solution strategies were 3.59 (SD= 1.47), 1.12 (SD= 1.02), and 1.29 (SD= 1.21), respectively. Gender differences were found only with respect to the use of unconventional strategies, $t(156)=3.30$, $p < .01$: Boys (M= 1.38, SD= 1.04) used more unconventional strategies than girls (M= .86, SD= .93). Next, we examined gender differences in strategy use for each problem and found gender differences in strategy use on the first problem, χ^2 (2, n=158)= 7.44, $p < .05$, and on the fourth problem, χ^2 (2, n=158)= 19.66, $p < .01$. Inspection of the data revealed that more boys than girls applied unconventional strategies to solve these problems. Also, more girls than boys used ineffective strategies to solve the fourth problem.

Solution time

The mean solution time was calculated for each problem of both types of tasks. Students who gave up immediately after having read a problem were not included in this analysis. The mean solution time for the computations varied from 46.51 seconds (SD= 33.75) for problem 2 to 173.96 seconds (SD= 98.08) for problem 1. For the application problems the mean solution time varied from 49.08 seconds (SD= 39.10) for problem 4 to 126.46 seconds (SD= 85.40) for problem 1. The logarithms of the mean solution time for each problem were computed, and gender differences were examined. For the computations, we found that boys (M= 52.40, SD= 43.59) needed more time than girls (M= 40.61, SD= 17.97) to solve the second problem, $t(156)= 2.78$, $p < .01$. As concerns the application problems, we found that girls (M= 58.85, SD= 43.12) worked longer on the fourth problem than boys (M= 39.30, SD= 31.99), $t(156)= -3.15$, $p < .01$. However, no differences were found in boys' and girls' solution time on the total set of problems, neither for the computation problems, nor for the application problems.

Persistence following failure

When students had given an incorrect solution to a problem, they were asked whether they wanted to try to solve the problem again. However, students who already gave up while solving the problem, and thus had no solution at all, were not asked whether they wanted to try to solve the problem again. The number of students who gave up while working on the computation problems varied from 1 (the second computation problem) to 32 (the third computation problem). For the application problems, this ranged from 3 (the fourth application problem) to 26 (the sixth application problem).

Reactions following failure were scored as a dichotomous variable with the reactions *try again* and *quit.* In Tables 5.14 and 5.15 frequencies of students who tried again or quit are given for the computations and application problems, respectively. The mean percentage of students who retried a problem was 54% for the computations and 59% for the application problems. We found gender differences in reactions following failure only on the application problems. When comparing boys' and girls' reactions on the application problems it appeared that girls were more inclined than boys to try again: When their solution was incorrect, 64% of the girls tried again, compared to 51% of the boys, x^2 (1, n=344)= 6.01, $p < .01$. An inspection of the individual application problems showed the greatest differences between boys' and girls' willingness to try again on application problem 3, χ^2= (1, n=102)= 3.92, $p < .05$.

Table 5.14

Frequencies of reactions following failure on the computation problems. Relative percentages are given in parentheses

	Incorrect			Quit			Try again		
Problems	Total	Boys	Girls	Total	Boys	Girls	Total	Boys	Girls
1. 14820 : 38=	56	31	25	19 (34%)	9	10	37 (66%)	22	15
2. 68.2 - 4.73=	36	15	21	8 (22%)	3	5	28 (78%)	12	16
3. 4¾% of ƒ1816.-=	114	60	54	63 (55%)	32	31	51 (45%)	28	23
4. 0.825 : 0.01=	75	39	36	48 (64%)	28	20	27 (36%)	11	16
5. 5% of 46460«	50	31	19	22 (44%)	18	4	28 (56%)	13	15
6. 236 x 405=	38	23	15	10 (26%)	6	4	28 (74%)	17	11
Total	369	199	170	170 (46%)	96	74	199 (54%)	103	96

Note. Students who gave up before having a solution were not included here, because they were not asked whether they wanted to try to solve a problem again.

* $p < .05$. ** $p < .01$.

Table 5.15

Frequencies of reactions following failure on the application problems. Relative percentages are given in parentheses

	Incorrect			Quit			Try again		
Problems	Total	Boys	Girls	Total	Boys	Girls	Total	Boys	Girls
1. Mailbags	65	31	34	22 (34%)	8	14	43 (66%)	23	20
2. Bicycle	32	11	21	11 (34%)	5	6	21 (66%)	6	15
3. Interest	102	50	52	41 (40%)	25	16	61 (60%)	25	36*
4. Pharmacist	25	6	19	11 (44%)	4	7	14 (56%)	2	12
5. Rock concert	56	22	34	24 (43%)	12	12	32 (57%)	10	22
6. Campground	64	30	34	33 (52%)	19	14	31 (48%)	11	20
Total	344	150	194	142 (41%)	73	69	202 (59%)	77	125"

Note. Students who gave up before having a solution were not included here, because they were not asked whether they wanted to try to solve a problem again.
* $p < .05$. ** $p < .01$.

5.5 Conclusions

Analyses that were described in this chapter revealed gender differences at the domain-specific, task-specific, and behavior-related levels of measurement. No gender differences were found in abstract reasoning ability. At the domain-specific level, we found that boys' self-concept was higher than girls', and that boys and girls displayed different goal orientations towards mathematics. Boys appeared to score higher than girls on both ego orientation and task orientation in relation to success and failure experiences. These results are partly consistent with results that have been found earlier. Seegers and Boekaerts (1996) reported higher scores for boys, only on self-enhancing ego orientation. No gender differences were found in students' attributions in relation to mathematics.

At the task-specific level, gender differences were especially apparent with respect to the application problems. Before starting the application problems, boys displayed higher subjective competence than girls, whereas girls attached more importance to being good at these kinds of tasks. Seegers and Boekaerts (1996) also reported a higher subjective competence for boys than for girls before starting with a mathematics task. However, they also found that girls reported a higher learning intention than boys, which was not confirmed in this study. In addition, intraindividual analyses revealed that students' scores on the appraisals were higher for the computations than for the application problems. With respect to task-specific attributions, we found that more girls than boys attributed their perceived failure on the application problems to a lack of capacity and to the difficulty level of the task.

Both affective and cognitive variables registered *during* the task (behavior-related measures) revealed gender differences. As was expected, boys perceived higher confidence than girls, but only while working on the application problems. At the problem-specific level, boys' higher confidence was apparent with respect to the first and second application problem. This pattern was not found for the computation problems. Consistent with expectations, boys outperformed girls only on the application problems. At the problem-specific level, boys performed better than girls on two of the problems. Intraindividual analyses revealed that more girls performed better on the computation problems than on the application problems, and that more boys performed better on the application problems than on the computations. Gender differences were also apparent in the use of solution strategies: Boys used more unconventional solution strategies than girls for the application problems. This pattern was apparent in two of the problems. No significant gender differences were found when comparing students' total solution time on the computations and the application problems. An unexpected finding was that girls showed higher persistence than boys on the application problems after failure experiences. Especially on the third problem, more girls than boys tried again. These findings will be further discussed in chapter 7.

6 RELATIONS BETWEEN COGNITIVE AND MOTIVATIONAL VARIABLES

In the previous chapter gender differences and intraindividual differences in the cognitive and motivational variables were described. In this chapter, the relations between these variables are examined. As discussed in chapter 3, an important purpose of this study was to examine the interplay between cognitive and motivational variables in relation to mathematical problem-solving behavior. By doing this, we expect to gain more insight into individual differences (especially gender differences) in problem-solving behavior. Central to this are the behavior-related variables perceived confidence, task performance, and persistence following failure. These variables were measured during the two types of tasks (computation problems versus application problems). The behavior-related variables solution strategies and solution time are not the subject of the multivariate analyses in this chapter. Solution strategies were not included, because task performance and solution strategy use are highly correlated - that is, ineffective solution strategies inherently lead to incorrect solutions. Moreover, solution strategies were examined in relation to the application problems only. Finally, solution time was only considered as a descriptive variable.

Our first purpose was to examine how these behavior-related variables are interconnected. A relevant question in this context is to what extent task performance and perceived confidence are related and whether this relation is different for boys and girls and for the two task conditions. A second purpose of the analyses in this chapter was to study relations between variables measured at the general, domain-specific, task-specific, and behavior-related levels. We first investigated relations between the three behavior-related variables task performance, perceived confidence, and persistence. The first research question was:

→ *How are students ' task performance, perceived confidence, and persistence related and what are the influences ofgender and type of task?*

Next, we examined to what extent task-specific appraisals and motivational beliefs influence these behavior-related measures, when an objective measure of competence has been accounted for. Research questions were:

→ *How are students' objective competence, task-specific appraisals, and motivational beließ related to their task performance?*

- → *How are students' objective competence, task-specific appraisals, and motivational beließ related to their perceived confidence?*
- → *How are students' objective competence, task-specific appraisals, and motivational beliefs related to their persistence following failure?*

To answer these questions, the assumptions from the model of adaptable learning were taken into account. In this model, motivational beliefs are assumed to influence behavior through task-specific appraisals (see chapter 3.2). In contrast to the analyses that were performed in other studies (Seegers & Boekaerts, 1993, 1996), learning intention was considered a criterion variable in this study. This decision was made because learning intention is assumed to have an influence on the behavior-related variables task performance, perceived confidence, and persistence. The general measure, objective competence, is assumed to affect all the variables. In Figure 6.1 the hypothesized relations are displayed.

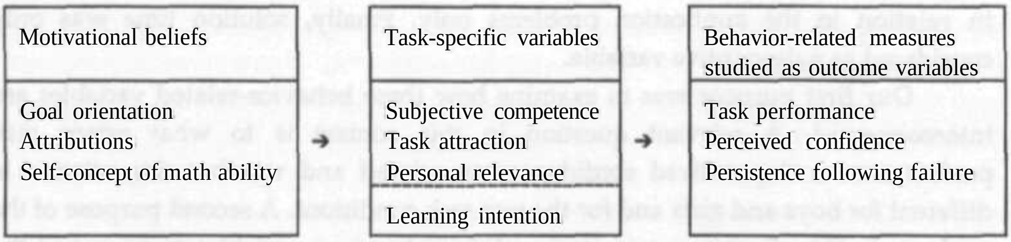

Figure 6.1. Relations between domain-specific, task-specific and behavior-related variables.

Because our main goal was to investigate the unique contributions of motivational influences on the behavior-related measures, we first accounted for the influence of objective competence. Therefore, in all the regression analyses objective competence was entered first. Following the hierarchical structure of the model of adaptable learning, the task-specific appraisals were entered in the regression as a next step, and, finally, the motivational beliefs. All these analyses were done for the two task conditions, and for boys and girls separately.

6.1 Relations among behavior-related variables

Our first research question addressed the relations between the three behavior-related variables, task performance, perceived confidence, and persistence. The latter variable needs special attention, because of its specific character. As described in chapter 5.4, students' reactions following failure (quit versus try again) were listed in relation to each problem. The variable *persistence* refers to students' reactions following failure in relation to each type of task (computation problems versus application problems). We dichotomisized the variable, persistence, into the categories *high* and *low*, by taking into account the total number of incorrect solutions students had on each type of task, and the number of times students decided to try to solve a problem again. Students who tried again less than half of the problems they had solved incorrectly, had a low persistence score, whereas students who retried at least half of the problems they had solved incorrectly had a high persistence score. Students who solved all the problems correctly were excluded from the analysis (six boys and three girls were excluded from the analysis of the application problems, and nobody was excluded from the analysis of the computation problems). We expected positive relations between all behavior-related measures. Pearson correlation coefficients between task performance, perceived confidence and persistence are displayed in Table 6.1.

Table 6.1
Pearson correlation coefficients between persistence, perceived confidence, and task performance

Measure	1	2	3
Computation problems			
1. Task performance	–	.53"	.29**
2. Perceived confidence	.41"	–	.36"
3. Persistence	.18	.16	–
Application problems			
1. Task performance	–	.52"	.31"
2. Perceived confidence	.51"	–	.17
3. Persistence	.32"	.24"	–

Note. For boys (n= 79), the correlations are printed above the main diagonal, whereas for girls (n= 79), the correlations are printed below the main diagonal.
* p < .05. ** p < .01.

As was expected, task performance was strongly related to perceived confidence for boys and girls in relation to both types of tasks (the correlations varied from .41 to .53). However, the associations between persistence and perceived confidence were smaller than was expected. Persistence during the computation problems was moderately related to perceived confidence for boys (r= .36), but this correlation was not significant for girls (r= .16). With respect to the application problems, persistence was significantly related to perceived confidence for girls (r= .24), but not for boys (r= .17). These results imply that perceived confidence is not consistently related to persistence. In addition, persistence showed moderately positive correlations with task performance. Persistence during the computation problems was significantly related to task performance for boys (r= .29), but not for girls (r= .18). Furthermore, the correlations between persistence during the application problems and task performance were significant for boys (r= .31) and for girls (r= .32). These results suggest that high performers are more inclined to persist than low performers. The bivariate relations between the three measures are graphically displayed in Figures 6.2 to 6.4.

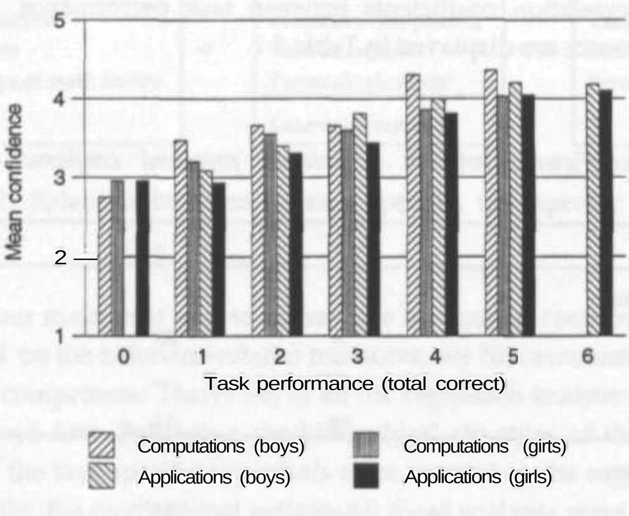

Figure 6.2. Relations between task performance and mean confidence. None of the students solved six computation problems correctly; none of the boys solved six application problems incorrectly.

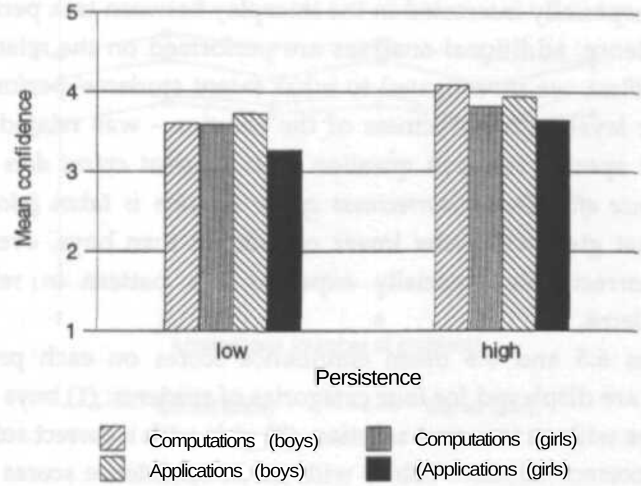

Figure 6.3. Relations between persistence and mean confidence.

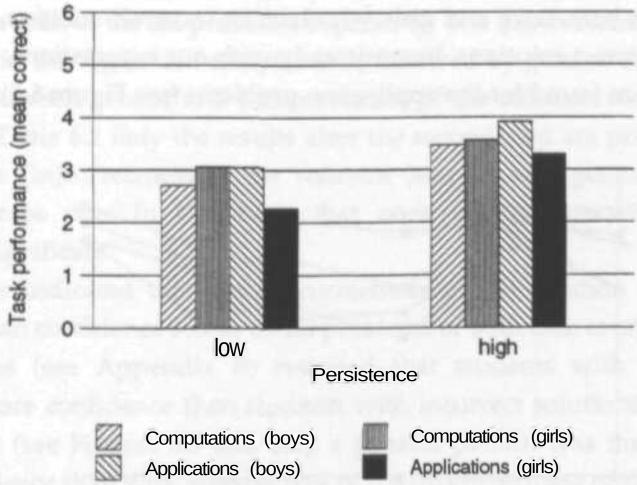

Figure 6.4. Relations between persistence and task performance.

Additional analyses: Relations between task performance and perceived confidence

Because we are especially interested in the interplay between task performance and perceived confidence, additional analyses are performed on the relations between these variables. First, we investigated to what extent students' performance at the problem-specific level - (in)correctness of the solution - was related to perceived confidence. Our specific research question was: *To what extent does gender add to perceived confidence after the (in)correctness of the solution is taken into account?* We hypothesized that girls will show lower confidence than boys, even when their solutions are correct. We especially expected this pattern in relation to the application problems.

In Figures 6.5 and 6.6 mean confidence scores on each problem of the respective tasks are displayed for four categories of students: (1) boys with a correct solution, (2) boys with an incorrect solution, (3) girls with a correct solution, and (4) girls with an incorrect solution. Tables with mean confidence scores and standard deviations are printed in Appendix B. As Figures 6.5 and 6.6 show, boys perceived higher confidence than girls, both for the correct and incorrect solutions. In addition, there was a decline in students' perceived confidence during the most difficult computation problems, namely 3 and 4 (see Figure 6.5). This decline in confidence was apparent for both boys and girls who had an incorrect solution, and for girls who displayed correct solutions. Inconsistently with our expectations, this decline in confidence was not found for the application problems (see Figure 6.6).

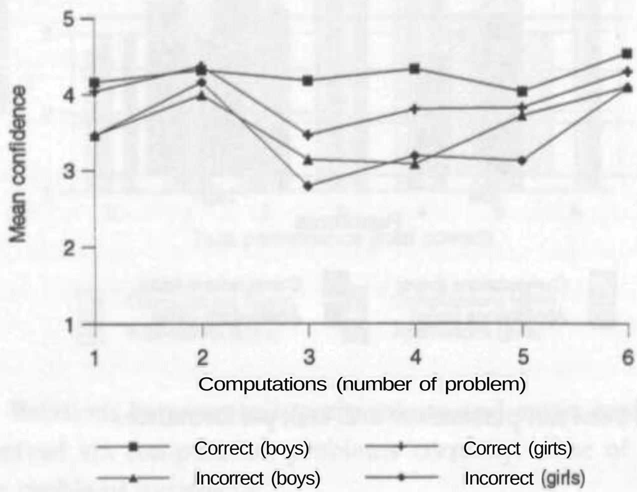

Figure 6.5. Students' mean confidence scores on the computation problems.

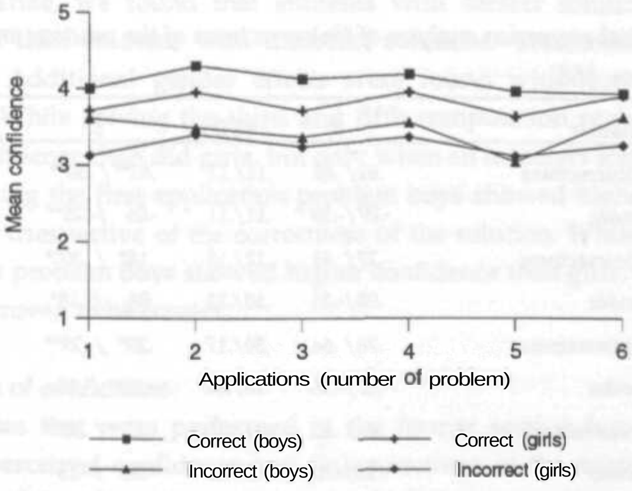

Figure 6.6. Students' mean confidence scores on the application problems.

To assess the influence of gender on perceived confidence after the (in)correctness of the solution had been accounted for, hierarchical regression analyses were conducted on each of the six problems separately. The (in)correctness of the solution was entered in the regression analysis first, followed by gender. As a final step, the interaction between gender and (in)correctness of the solution was included in the analysis. In Table 6.2 only the results after the second step are printed, that is after including the (in)correctness of the solution (step 1) and gender (step 2) in the analyses. Please note in this table that none of the interaction effects were statistically significant.

Results indicated that the (in)correctness of the solution was significantly related to mean confidence scores on all problems of both task conditions. Inspection of the means (see Appendix B) revealed that students with correct solutions displayed more confidence than students with incorrect solutions on all problems. At first sight (see Figures 6.5 and 6.6), a general pattern was that boys displayed higher confidence than girls, irrespective of the (in)correctness of the solution. These results reached statistical significance for some problems only. When (in)correctness of the solution had been included in the analysis as a first variable, additional significant effects were found with respect to the third (R^2 change= .03, $p < .05$) and fifth (R^2 change= .02, $p < .05$) computation problem, and with respect to the first (R^2 change= .06, $p < .01$) and second (R^2 change= .03, $p < .05$) application problem.

Table 6.2
Summary of hierarchical regression analyses of (in)correctness of the solution and gender on perceived confidence (n= 158)

Problems	Variables	B	SE B	ß	R^2 change
Problem 1	(In)correctness	.62/ .46	.11/.11	.42**/ .30"	.17**/.09**
	Gender	-.07/-.39	.11/.11	-.05 /-.25**	.01 /.06**
Problem 2	(In)correctness	.27/ .61	.12/.15	.18* / .30**	.03* /.10**
	Gender	.08/-.31	.10/.13	.06 /-.18*	.00 /.03*
Problem 3	(In)correctness	.76/ .64	.30/.17	.20* / .29**	.03* /.09**
	Gender	-.37/-.19	.16/.14	-.18* /-.10	.03* /.01
Problem 4	(In)correctness	.94/ .62	.14/.16	.47**/ .30**	.23**/.11**
	Gender	-.15/-.22	.14/.13	-.08 /-.13	.00 /.02
Problem 5	(In)correctness	.50/ .70	.17/.15	.23**/ .35**	.05**/.13**
	Gender	-.34/-.16	.16/.15	-.16* /-.08	.02* /.01
Problem 6	(In)correctness	.35/ .48	.13/.15	.21**/ .25**	.04**/.07**
	Gender	-.18/-.23	.11/.15	-.13 /-.12	.01 /.01

Note. Results on the computation problems are printed before the slash, and results on the application problems are printed after the slash.
* $p < .05$. ** $p < .01$.

In order to compare boys' and girls' confidence scores on these problems, post-hoc t-tests were computed separately for the correct and incorrect solutions. Boys only displayed a statistically significant higher confidence than girls for some of the incorrect solutions, more specifically while solving the third and fifth computation problem. On the third computation problem, boys' mean confidence score was 3.14 (SD= .99) for the incorrect solutions, whereas girls' mean confidence score was 2.79 (SD= 1.08), t(141)= 2.02, p < .05. On the fifth computation problem, boys' mean confidence score was 3.71 (SD= 1.01) for the incorrect solutions, whereas girls' mean confidence score was 3.12 (SD= 1.21), t(55)= 1.98, p < .05.

With respect to the first application problem, boys displayed higher confidence scores (M= 3.61, SD= .74) than girls (M= 3.12, SD= .81) when they had achieved a correct solution, t(77)= 2.00, p < .05. Boys' confidence score was also higher (M= 4.00, SD= .60) than that of girls (M= 3.71, SD= .68) when an incorrect solution was produced, t(77)= 2.84, p < .01. On the second application problem, boys had more confidence (M= 4.30, SD= .72) than girls (M= 3.95, SD= .85) only with respect to a correctly solved problem, t(119)= 2.50, p < .01.

To summarize, we found that students with correct solutions displayed more confidence than students with incorrect solutions. This conclusion holds for all problems. Additional gender effects were found with respect to some of the problems: While solving the third and fifth computation problem boys displayed higher confidence than did girls, but only when an incorrect solution was produced. While solving the first application problem boys showed higher confidence scores than girls, irrespective of the correctness of the solution. While solving the second application problem boys showed higher confidence than girls, but only when their solutions proved to be correct.

Calibration of confidence

The analyses that were performed in the former section focused on the relation between perceived confidence and (in)correctness of the solution at the problem-specific level. In this section, we examined to what extent there was correspondence between students' confidence ratings and their performance at the task-specific level. We investigated to what extent students express high confidence in problems they solve correctly and low confidence in problems they solve incorrectly. In the literature this is known as *calibration of confidence* (e.g., Lundeberg et al., 1994). Our specific research question concerned the extent to which students show calibration of confidence and how this is influenced by type of task and gender.

A statistical tool to measure calibration of confidence was introduced by Shaughnessey (1979). It is called the Confidence-judgment Accuracy Quotient (CAQ). This measure is a ratio, the numerator of which consists of the difference between the mean confidence judgment assigned to problems the students answered correctly and the mean confidence judgment assigned to problems answered incorrectly. The denominator of the ratio is the standard deviation of all confidence judgments across both correct and incorrect answers. In formula:

$$CAQ = \frac{MCC - MCW}{SD(MC)}$$

In which: MCC= mean confidence correct; MCW= mean confidence wrong; MC= mean confidence for all problems.

A CAQ of zero denotes an individual who displays the same level of confidence irrespective of the (in)correctness of the solution. Positive CAQ scores indicate higher confidence when correct than when incorrect; negative CAQ scores denote a higher confidence when incorrect than when correct. CAQ scores exclude from the analysis students who give all correct or all incorrect answers. For our sample this meant that with respect to the computation problems, 7 students with only incorrect solutions were excluded. With respect to the application problems, 16 students were excluded (7 students had only incorrect solutions and 9 students had only correct solutions). Table 6.3 presents students' mean confidence scores and CAQ scores for both types of tasks. The CAQ scores ranged from -1.09 to 2.28 for the computation problems and from -2.06 to 2.45 for the application problems.

Table 6.3

Students' mean confidence scores and calibration of confidence scores

		Confidence correct		Confidence incorrect		CAQ	
Problems		M	SD	M	SD	M	SD
	Computation problems						
Total	(n=151)	4.15	.62	3.46	.81	.92	.77
Boys	(n= 77)	4.25*	.61	3.58	.76	.92	.72
Girls	(n= 74)	4.05	.61	3.34	.85	.93	.82
	Application problems						
Total	(n=142)	3.86	.74	3.54	.77	.47	.91
Boys	(n= 73)	4.01*	.64	3.63	.75	.55	.89
Girls	(n= 69)	3.71	.80	3.45	.78	.39	.92

Note. CAQ means Confidence Accuracy Quotient.
* $p < .05$. ** $p < .01$.

As can be concluded from this table, boys displayed higher confidence than girls for correctly solved computation problems, $t(149)= 2.02$, $p < .05$, and application problems, $t(149)= 2.54$, $p < .05$. Students' mean CAQ scores for both tasks were positive, indicating that students generally express higher confidence in problems they solve correctly than in problems they solve incorrectly. Although boys' CAQ scores were higher than girls' for the application problems, these results did not reach statistical significance.

Intraindividual differences in CAQ scores across tasks were also examined. A multivariate analysis of variance with repeated measures was performed on the data with gender as the between-subjects variable, task as the within-subjects variable, and mean CAQ score as the dependent variable. This analysis revealed a task effect ($F= 31.36$, $p < .01$) only. The interaction of gender by type of task did not reach statistical significance ($F= 2.26$, $p= .14$). Inspection of the means (see Table 6.3) revealed that both boys and girls had higher CAQ scores in relation to the computation problems than the application problems, indicating that they were more capable of calibrating their confidence while solving the computation problems, compared to the application problems.

We may conclude that, when considering students' mean confidence scores on each type of task, boys displayed significantly higher confidence than girls for the correct solutions. This pattern was found in both task conditions. In addition, results indicated that students were more capable of calibrating their confidence during the computation problems than during the application problems. However, no statistically significant gender differences were found in students' calibration of confidence.

6.2 Variables contributing to task performance

Our next research question concerned how students' objective competence, task-specific appraisals and motivational beliefs are related to task performance. More specifically, we investigated to what extent task-specific appraisals and motivational beliefs contribute to task performance after the influence of an objective measure of competence was accounted for. Objective competence (abstract reasoning ability) was measured by the subscales analogies and categories of the SON-R (see chapter 4). Pearson correlation coefficients between the variables objective competence, motivational beliefs, and task-specific appraisals are displayed in Appendix C. Table 6.4 displays the Pearson correlation coefficients between task performance and the variables objective competence, task-specific appraisals, and motivational beliefs.

As Table 6.4 shows, performance on both types of tasks was positively related with students' scores on both the analogies and categories subscales (the correlations varied from .26 to .33). Positive associations were found between task performance and subjective competence (from .40 to .50), capacity attributions (from .23 to .55), and self-concept of mathematics ability (from .34 to .55).

Table 6.4

Pearson correlation coefficients between objective competence, task-specific appraisals, motivational beliefs, and task performance

Measure	Task performance (computation problems) Boys (n= 79)	Task performance (computation problems) Girls (n= 79)	Task performance (application problems) Boys (n= 79)	Task performance (application problems) Girls (n= 79)
Objective competence				
Analogies	.26**	.33**	.30**	.31**
Categories	.27*	.37**	.26**	.42**
Task-specific appraisals				
Subjective competence	.41**	.40**	.44**	.50**
Task attraction	.21	.01	.28*	.23*
Personal relevance	.16	-.01	.09	.13
Learning intention	.30**	.00	.15	.21
Motivational beliefs				
Goal orientation				
Self-enhancing ego orientation	-.06	-.06	-.18	.05
Self-defeating ego orientation	-.25*	-.11	-.32**	-.25*
Task orientation	.10	-.03	.03	.09
Error frustration	.07	-.12	-.05	-.07
Attributions				
Capacity	.41**	.23*	.38**	.55**
Effort	.05	.00	-.07	.22
Self-concept of mathematics ability				
Self-concept	.45**	.55**	.34**	.49**
Importance	.09	.19	-.16	.05

* $p < .05$. ** $p < .01$.

Low positive correlations were found between task attraction and performance for the application problems only (r= .28 for boys and r= .23 for girls). For boys, a positive correlation was found between learning intention and performance on the computation problems (r= .30). For girls, these variables were not correlated. In addition, negative correlations were found between self-defeating ego orientation and task performance (correlations varied from -.11 to -.32). In other words, high fear that others will notice one's mistakes in mathematics was related with low task performance. However, this correlation was not significant for girls in relation to the computation problems.

Students' scores on the subscales analogies and categories were not significantly correlated (r= .04 for boys and r= .28 for girls). Therefore, both measures of abstract reasoning ability were included in the regression analyses. Hierachical regression analyses on task performance were performed to assess the contribution of task-specific appraisals and motivational beliefs after the influence of abstract reasoning ability had been accounted for. First, students' scores on the subscales analogies and categories were entered in the regression analysis together. As a second step, task-specific appraisals were entered in a stepwise procedure, meaning that the variables were left free to enter when reaching the significance criterion of .05. As a third step, the motivational beliefs were entered, also in a stepwise procedure. In each step, the residuals of the former step were saved and served as dependent variable in the next step. These analyses were done for the two task conditions and for boys and girls, separately. The results of the regression analyses are presented in Tables 6.5 and 6.6. Only those results are presented, that appeared to be significant in the stepwise procedures ($p < .05$). In Table 6.5 the results after the third step (motivational beliefs) are omitted, because these appeared to be not statistically significant. Furthermore, when a variable appeared to add significantly to the variance explained for one group of students (boys or girls), the same variable was forced into the regression for the other group of students. This was done in order to make comparisons between the two groups of students more easily.

Table 6.5

Summary of hierarchical regression analyses for variables predicting performance on the computation problems

Variable	B	SE B	β	R^2 change
Step 1: Objective competence				.13**/.19**
Analogies	.11 / .12	.05/.05	.25* /.25*	
Categories	.13 / .15	.05/.06	.26»/.30**	
Step 2: Task-specific appraisals				.11**/.08*
Subjective competence	1.00/1.00	.32/.40	.34**/.28*	
			Total R^2=	.24 /.27

Note. Results for boys (n= 79) are printed before the slash, whereas results for girls (n= 79) are printed after the slash.

* $p < .05$. - $p < .01$.

Table 6.6

Summary of hierarchical regression analyses for variables predicting performance on the application problems

Variable		B	SE B	β	R^2 change
Step 1:	Objective competence				.15** / .22**
	Analogies	.15 / .11	.05 / .06	.29** / .21*	
	Categories	.14 / .22	.06 / .06	.25* / .36**	
Step 2:	Task-specific appraisals				.10** / .12**
	Subjective competence	.98 / 1.03	.33 / .32	.32** / .34**	
Step 3:	Motivational beliefs				.00 / .10
	Capacity attributions	.10 / .82	.28 / .28	.04 / .32**	
				Total R^2=	.25 / .44

Note. Results for boys (n= 79) are printed before the slash, whereas results for girls (n= 79) are printed after the slash.
* $p < .05$. ** $p < .01$.

As was expected, abstract reasoning ability was a significant predictor of task performance. However, subjective competence contributed significantly to task performance, after abstract reasoning ability had been accounted for. These effects were found for boys and girls in both task conditions. For boys, abstract reasoning ability accounted for 13% of the total variance in performance on the computation problems (see Table 6.5). When the variance due to abstract reasoning ability was accounted for, subjective competence explained an additional 11% of the variance in performance. For girls, these percentages were 19% and 8%, respectively. No additional effects of motivational beliefs on task performance were found, after objective competence and task-specific appraisals were entered in the analysis.

For the application problems, abstract reasoning ability accounted for 15% of the variance in performance for boys. Subjective competence explained an additional 10% of the variance in performance. For girls, these percentages were 22% and 12%, respectively. In addition, capacity attributions explained another 10% of the variance in performance for girls. For boys, no additional effects of motivational beliefs on task performance were found. These results indicate that task-specific appraisals and motivational beliefs contributed more to the variance in applied problem solving performance for girls (22%) than for boys (10%).

In sum, results showed that subjective competence expressed before working on the problems significantly contributed to task performance, after the influences of objective measures of competence were taken into account. This was found for both boys and girls under both task conditions. Additional effects of capacity attributions on task performance (application problems) were found for girls only.

Additional analyses on applied problem-solving performance

We consider performance on the computation problems an important predictor of successful performance on the application problems. Namely, the contents of the application problems were chosen in such a way, that they could be solved by applying the same computations used in the computation problems. Therefore, we investigated to what extent task-specific appraisals and motivational beliefs had additional influences on students' performance on the application problems, when both abstract reasoning ability and performance on the computation problems were taken into account. The Pearson correlation coefficient between performance on the computation problems and performance on the application problems was .53 for boys and .48 for girls.

Hierarchical regression analyses on performance (application problems) were conducted, in which the scores on the subscales analogies and categories were entered first. In the second step, performance on the computation problems was entered. In the third step, the task-specific appraisals were included in a stepwise procedure, and in the fourth step, the motivational beliefs were entered. Results of the analyses are displayed in Table 6.7. Again, only significant results are displayed.

As can be seen from Table 6.7, all the variables together explained 46% of the variance in task performance for boys, and 68% of the variance in task performance for girls. For the objective measures of competence (abstract reasoning ability and performance on the computation problems together) these percentages were 43% for boys, and 45% for girls. These results show that performance on the computation problems was a significant predictor of performance on the application problems, after the influence of abstract reasoning ability was accounted for. However, performance on the computation problems explained more of the variance in performance on the application problems for boys (28%) than for girls (23%). In addition, subjective competence and capacity attributions significantly contributed to task performance for girls, after both abstract reasoning ability and performance on the computation problems had been accounted for. For boys, there were no additional effects of task-specific appraisals nor of motivational beliefs on task performance.

Table 6.7
Summary of hierarchical regression analyses for variables predicting task performance on the application problems with performance on the computation problems included as predictor variable

Variable		B	SE B	ß	R^2 change
Step 1:	**Objective competence**				.15**/.22**
	Analogies	.15/.11	.05/.06	.29**/.21*	
	Categories	.14/.22	.06/.06	.25* /.36**	
Step2:	**Performance on the** computation problems	.51/.39	.11/.12	.45**/.34**	.28**/.23**
Step 3:	**Task-specific** appraisals				.03 /.11**
	Subjective competence	.51/.99	.31/.32	.18 /.33**	
Step 4:	Motivational beliefs				.00 /.12**
	Capacity attributions	.17/.90	.27/.28	.07 /.35**	
				Total R^2=	.46 /.68

Note. Results for boys (n= 79) are printed before the slash, whereas results for girls (n= 79) are printed after the slash.
* $p < .05$. ** $p < .01$.

6.3 Variables contributing to perceived confidence

Our next research question addressed the contributions of task-specific appraisals and motivational beliefs to students' perceived confidence while working on the problems. Again, the assumptions of the model of adaptable learning were taken into account. We investigated to what extent perceived confidence was influenced by task-specific appraisals and motivational beliefs, when the influence of objective competence was accounted for. Pearson correlation coefficients between objective competence, task-specific appraisals, motivational beliefs, and perceived confidence are displayed in Table 6.8.

Table 6.8
Pearson correlation coefficients between objective competence, task-specific appraisals, motivational beliefs, and perceived confidence

Measure	Perceived confidence (computation problems)		Perceived confidence (application problems)	
	Boys (n= 79)	Girls (n= 79)	Boys (n= 79)	Girls (n= 79)
Objective competence				
Analogies	.14	.08	.13	.20
Categories	.43**	.35**	.31**	.35**
Task-specific appraisals				
Subjective competence	.50**	.59**	.56-	.71-
Task attraction	.23*	.36**	.33-	.54-
Personal relevance	.11	.10	.00	.26*
Learning intention	.38**	.26*	.20	.41-
Motivational beliefs				
Goal orientation				
Self-enhancing ego orientation	.10	.03	-.02	.01
Self-defeating ego orientation	-.26*	-.14	-.34-	-.09
Task orientation	.16	.14	.17	.11
Error frustration	.08	-.06	-.11	-.18
Attributions				
Capacity	.39-	.37**	.47**	.40**
Effort	.21	.08	.14	.07
Self-concept of mathematics ability				
Self-concept	.47**	.55-	.50**	.56-
Importance	.06	.19	-.04	.12

* $p < .05$ - $p < .01$.

As can be viewed from this table, the subscale categories was positively related with perceived confidence. Surprisingly, the subscale analogies showed no significant correlation with perceived confidence. As was expected, subjective competence before starting the task and the domain-specific self-concept of mathematics ability were positively related with perceived confidence. This was found for boys and girls in both task conditions. The correlations were somewhat higher for subjective competence (from .50 to .71) than for self-concept of mathematics ability (from .47 to

.56). Moderate associations were found between perceived confidence and the task-specific appraisals task attraction (from .23 to .54) and learning intention (from .20 to .41). Personal relevance had a low correlation with perceived confidence (r= .26), but only for girls in relation to the application problems. In addition, moderate correlations were found between capacity attributions and perceived confidence (from .37 to .47). As for goal orientation, modest negative associations were found between self-defeating ego orientation and perceived confidence (r= -.26 and r= -.34) for boys only. This implies that boys, who fear that others will notice their mistakes in mathematics, report low perceived confidence. This association was not evident in girls.

Hierarchical regression analyses on perceived confidence were conducted to examine the unique contributions of task-specific appraisals and motivational beliefs. These analyses were done separately for boys and girls and for the two task conditions. Again, we first controlled for the influence of objective competence. In the first step, the subscales analogies and categories were entered, and in the second step the task-specific appraisals were left free to enter in a stepwise procedure. In the third step, the motivational beliefs were entered in a stepwise procedure. The results of the analyses are displayed in Tables 6.9 and 6.10. These tables show only effects that proved to be statistically significant in the stepwise procedures (in steps 2 and 3).

Table 6.9

Summary of hierarchical regression analyses for variables predicting students' perceived confidence while solving the computation problems

Variable	B	SE B	[illegible]	R^2 change
Step 1: Objective competence				.20**/.13**
Analogies	.03/.00	.02/.03	.12 / -.03	
Categories	.11/4»	.03/.03	.42**/ .36"	
Step 2: Task-specific appraisals				.20**/.27**
Subjective competence	.66/.96	.15/.18	.45**/.52**	
			Total R^2=	.40 /.40

Note. Results for boys (n= 79) are printed before the slash, whereas results for girls (n= 79) are printed after the slash.
* p < .05. ** p < .01.

Table 6.10

Summary of hierarchical regression analyses for variables predicting students' perceived confidence while solving the application problems.

Variable		B	SE B	ß	R^2 change
Step 1:	Objective competence				.11* / .13**
	Analogies	.03 / .03	.03 / .03	.12 / .11	
	Categories	.03 / .09	.02 / .03	.13 / .32**	
Step 2:	Task-specific appraisals				.24** / .43**
	Subjective competence	.75/76	.16 / .14	.52** / .51**	
	Learning intention	-.11 *IM*	.16 / .15	-.07 / .28**	
				Total R^2=	.35 / .57

Note. Results for boys (n= 79) are printed before the slash, whereas results for girls (n= 79) are printed after the slash.
* p < .05. ** p < .01.

As was expected, subjective competence before starting to work on the problems had an additional influence on perceived confidence, after the effect of objective competence had been accounted for. This effect was found for boys and girls in both task conditions. As Table 6.9 shows, subjective competence before working on the computation problems explained an additional 20% in variance of perceived confidence for boys. For girls, this percentage was 27%.

For girls, more of the variance in perceived confidence while solving the application problems could be explained by task-specific appraisals than for boys (see table 6.10). For girls, subjective competence and learning intention explained an additional 43% of the variance in perceived confidence, after the influence of objective competence was accounted for. For boys, there was an effect of subjective competence only (R^2 change= .24, p < .01). There were no additional effects of motivational beliefs on perceived confidence, neither for boys, nor for girls.

To summarize, boys' and girls' subjective competence before working on the problems significantly contributed to their perceived confidence while working on the problems, after the effect of objective competence had been accounted for. This effect was found for both task conditions, and was consistent with our expectations. A statistically significant effect of learning intention on perceived confidence while solving the application problems, was found for girls only. This implies that for girls, intended effort expenditure contributed positively to their perceived confidence while solving the application problems. When the task-specific appraisals were entered in the regression analyses, no additional effects were found of motivational beliefs on perceived confidence.

6.4 Variables contributing to persistence

Our last research question concerned the relations between students' objective competence, task-specific appraisals, motivational beliefs, and persistence following failure. We hypothesized that students with high levels of task orientation, high effort attributions, and high learning intention would show higher persistence than students who score low on these variables. Pearson correlation coefficients are printed in Table 6.11. This table shows that no significant correlations were found between objective competence and persistence, except for one: The subscale analogies was positively correlated with persistence in relation to the application problems for boys only (r= .35). Persistence was moderately associated with subjective competence before starting the computation problems, for boys (r= .39), and with subjective competence before starting the application problems, for girls (r= .24). Low positive correlations were found between task attraction and persistence during the application problems for girls (r= .27), and between learning intention and persistence during the computation problems for boys (r =.23). These associations were lower than expected.

Moderate positive correlations were found between capacity attributions and persistence on both types of tasks (correlations varied from .26 to .40). For girls, however, these correlations were lower than for boys. Inconsistent to our expectations, the associations between effort attributions and persistence were rather low. A statistically significant positive association was found between effort attributions and persistence during the application problems, but only for girls (r= .25). In addition, moderate positive associations were found between self-concept of mathematics ability and persistence on both types of tasks (varying from .22 to .38). The goal orientation subscales appeared to have no significant associations with persistence. Only the subscale self-enhancing ego orientation was moderately correlated with boys' persistence during applied problem solving (r= .28). This implies that boys who showed high competitiveness were more inclined to persist.

Table 6.11
Pearson correlation coefficients between objective competence, task-specific appraisals, motivational beliefs, and persistence following failure

	Persistence (computation problems)		Persistence (application problems)	
Measure	Boys (n= 79)	Girls (n= 79)	Boys[a] (n= 73)	Girls[b] (n= 76)
Objective competence				
Analogies	.17	.07	.35-	.06
Categories	.17	.14	.04	.12
Task-specific appraisals				
Subjective competence	.39"	.16	.17	.24*
Task attraction	.17	.13	.27»	.06
Personal relevance	.13	.06	.09	.09
Learning intention	.23*	.07	.17	.18
Motivational beliefs				
Goal orientation				
Self-enhancing ego orientation	.06	.01	.28»	.00
Self-defeating ego orientation	-.15	-.10	-.06	-.21
Task orientation	.13	.02	.04	.22
Errorfrustration	.09	.02	.16	-.04
Attributions				
Capacity	.40**	.27*	.37**	.26*
Effort	-.08	.05	-.20	.25*
Self-concept of mathematics ability				
Self-concept	.38**	.35**	.35-	.22
Importance	.09	.12	-.06	.05

* Six boys were excluded from the analysis, because they solved all the problems correctly.
[b] Three girls were excluded from the analysis, because they solved all the problems correctly.
* $p < .05$. - $p < .01$.

As a next step, logistic regression analyses were performed to examine whether task-specific appraisals and motivational beliefs showed unique contributions to persistence. These analyses were done separately for boys and girls and for the two task conditions. Logistic regression analyses were used because persistence is a dichotomous criterion variable. This implies that we could not analyze the residuals, and that, consequently, the contribution of each variable in the regression changed after each step. Four logistic regression analyses were performed with the dichotomous variable persistence (high versus low) as the criterion variable, and objective competence, task-specific appraisals, and motivational beliefs as the predictor variables. First, the subscales analogies and categories were entered together. In the second step, the task-specific appraisals were entered in a forward stepwise procedure, in which only those variables that were statistically significant were added to the regression. Finally, motivational beliefs were entered, also in a forward stepwise procedure. The results of the analyses are displayed in Tables 6.12 and 6.13. Only those results are printed that appeared to be statistically significant in the forward stepwise procedures.

Table 6.12 shows that objective competence (the subscales analogies and categories) did not contribute significantly to persistence in relation to the computation problems. We further note that subjective competence had a positive influence on persistence during the computation problems for both boys and girls. Higher levels of subjective competence before starting the computation problems resulted in higher persistence for both boys and girls. For girls, this effect was of more importance than for boys. Moreover, there was an additional effect of capacity attributions on persistence in relation to the computation problems, but for boys only. This result indicates that boys who have the tendency to attribute success in mathematics to capacity are inclined to persist. For girls no additional influences of motivational beliefs on persistence were found during the computation problems.

Table 6.12

Summary of hierarchical logistic regression analyses for variables predicting persistence in relation to the computation problems

Variables	B	Wald	Exp(B)
Step 1			
Analogies	.12 / .02	2.16 / .07	1.13 / 1.02
Categories	.13 / .10	2.16 / 1.17	1.14 / 1.10
Step 2			
Analogies	.08 / -.03	.76 / .10	1.09 / .97
Categories	.11 / .00	1.21 / .00	1.11 / 1.00
Subjective competence	2.13 / 3.04	8.56** / 10.48**	8.44 / 20.84
Step 3			
Analogies	.03 / -.04	.11 / .14	1.03 / .96
Categories	.07 / .01	.55 / .01	1.08 / 1.01
Subjective competence	1.65 / 2.91	4.46* / 6.97**	5.19 / 18.33
Capacity attributions	1.19 / .14	3.87* / .05	3.30 / 1.15

Note. Results for boys (n= 79) are printed before the slash, and results for girls (n= 79) are printed after the slash.

* p < .05. ** p < .01.

With respect to the application problems, the effects of objective competence on persistence were also of minor importance: The subscale analogies positively influenced persistence for boys only (see Table 6.13). After including the task-specific appraisals in the regression analyses, there was an effect of subjective competence on persistence for boys only. However, this effect disappeared when the motivational beliefs were entered as a third step. For boys, effects were found of the subscales, self-enhancing ego orientation, capacity and effort attributions. However, the latter effect was a negative one, which was inconsistent with our expectations. These results imply that boys with high competitiveness, and with a tendency to attribute success in mathematics to capacity and not to effort, showed high persistence during applied problem solving. For girls, we found a positive effect of effort attributions only.

Table 6.13

Summary of hierarchical logistic regression analyses for variables predicting persistence in relation to the application problems

Variables	B	Wald	Exp(B)
Step 1			
Analogies	.27 / .02	7.82** / .05	1.31 / 1.02
Categories	.02 / .09	.06 / .88	1.02 / 1.09
Step 2			
Analogies	.24 / -.02	5.75* / .04	1.27 / .98
Categories	.00 / .05	.01 / .25	.99 / 1.05
Subjective competence	1.31 / .87	4.18* / 2.24	3.72 / 2.38
Step 3			
Analogies	.29 / -.08	5.80* / .74	1.34 / .92
Categories	.08 / .06	.36 / .30	1.08 / 1.06
Subjective competence	.46 / .50	.25 / .56	1.58 / 1.65
Self-enhancing ego orientation	1.28 / -.24	7.75** / .24	3.61 / .79
Capacity attributions	1.74 / 1.06	4.03* / 2.73	5.70 / 2.88
Effort attributions	-2.00 / 1.21	7.39** / 3.93*	.14 / 3.36

Note. Results for boys (n= 73) are printed before the slash, and results for girls (n= 76) are printed after the slash. Students who solved all the problems correctly, were not included in this analysis.
* $p < .05$. ** $p < .01$.

To summarize, subjective competence before working on the computation problems positively influenced boys' and girls' persistence during the computation problems. No effects were found of objective competence. For boys, a tendency to attribute success in mathematics to capacity also positively influenced their persistence. With respect to the application problems, the effects of motivational beliefs were of more importance than the task-specific appraisals. It was found that boys with a high level of ego orientation and high capacity attributions showed high persistence. An unexpected finding was that effort attributions were negatively related to persistence for boys, whereas for girls, this relation was a positive one.

6.5 Conclusions

In this chapter relations between cognitive and motivational variables were investigated. The first research question addressed the relations between the behavior-related variables task performance, perceived confidence, and persistence following failure. High positive associations were found between task performance and perceived confidence, and moderate associations were found between task performance and persistence. These relations held for both boys and girls in relation to both types of problems. The associations between persistence and perceived confidence, however, were lower than expected. Results showed that persistence was only marginally related to perceived confidence.

In additional analyses, the relations between task performance and perceived confidence were further examined. Problem-specific analyses revealed that for all the problems in both task conditions students with correct solutions displayed more confidence than students with incorrect solutions. Furthermore, boys were inclined to display higher confidence than girls, irrespective of the (in)correctness of a solution. When (in)correctness of the solution had been accounted for, we found an additional significant influence of gender on perceived confidence in relation to some problems. During two of the computation problems boys showed higher confidence than girls when the solution was incorrect, whereas during two of the application problems boys showed higher confidence than girls when the solution was correct. One of these problems also elicited higher confidence in boys than girls, when the solution was incorrect. We have no adequate explanation for the fact that differences in boys' and girls' confidence were more pronounced in relation to some of the problems. We confine ourselves to the observation that this pattern shows that boys were inclined to overestimate their performance, while girls were inclined to underestimate their performance. In addition, students' calibration of confidence was examined. Although we found that boys showed higher confidence than girls when comparing the correct solutions on both types of tasks, no gender differences in calibration of confidence scores were found. These results are not surprising, because boys were inclined to overestimate their performance, whereas girls were inclined to underestimate their performance. However, the CAQ measures to what extent students' confidence ratings are realistic. When comparing students' calibration of confidence across tasks it was noted that both boys and girls were better able to calibrate their confidence during the computation problems than during the application problems.

The last three research questions addressed in this chapter focused on the relations between variables at different levels of measurement. We examined to what extent the task-specific appraisals and motivational beliefs contributed to the behavior-related measures task performance, perceived confidence, and persistence, respectively. In these analyses, we controlled for the influence of objective competence (abstract reasoning ability). Analyses revealed that, consistent with earlier research findings, subjective competence before working on the problems significantly contributed to task performance. This was found for boys and girls in both task conditions. We also noted that motivational variables contributed more to the variance explained in applied problem-solving performance for girls than for boys.

The same analyses were done with perceived confidence as the dependent variable. Again, boys' and girls' subjective competence significantly contributed to their perceived confidence while working on the two types of problems. This result was expected, because both variables assess students' estimated chances of success. For girls, their intended effort expenditure also contributed positively to their perceived confidence while solving the application problems. No additional effects were found of motivational beliefs on perceived confidence.

Our last research question concerned relations between students' objective competence, task-specific appraisals, and persistence following failure. Objective competence showed no significant relations with persistence. For both boys and girls, subjective competence before working on the computation problems positively influenced their persistence while solving these computation problems. Furthermore, boys with a high tendency to ascribe success in mathematics to capacity showed high persistence in relation to the computation problems. With respect to the application problems, motivational beliefs were found to contribute more to persistence than the task-specific appraisals. This pattern was especially evident in boys. High tendencies to ascribe success in mathematics to capacity resulted in high persistence for boys. Furthermore, boys with a high level of ego orientation were inclined to persist longer on the application problems. For girls, high effort attributions resulted in high persistence. Surprisingly, this relation was negative for boys. These findings will be further discussed in the following chapter.

7 DISCUSSION

In this chapter we will discuss the findings of our study. In section 7.1 the general conclusions of our study will be summarized and discussed in light of the research questions and the theoretical framework that were outlined in the first three chapters. At the end of this section, limitations to these findings will be considered. In section 7.2 the implications of our study for research and education will be discussed.

7.1 General conclusions

The goals of this study were: (1) to investigate gender differences and intraindividual differences in motivational and cognitive variables in relation to different types of mathematics problems, and (2) to investigate relations between motivational and cognitive variables. An important feature of our study is the centrality of different content areas within mathematics. Drawing on studies in which gender differences have been reported involving content-specific areas of performance (see chapter 1), a distinction was made between two types of mathematics problems: computation problems and application problems. Our main focus was on students' appraisals and actual behavior in relation to these two types of problems. In consideration of an adequate explanation of individual differences in mathematics, we stressed the importance of investigating the interplay between cognitive and affective variables (see chapter 2). In addition, we examined students' self-referenced cognitions in relation to mathematics, which are considered important aspects of mathematics learning (see chapter 3). Following Boekaerts' model of adaptable learning (see chapter 3), motivational beliefs were related to students' task-specific appraisals and their actual behavior while solving problems.

Our findings provide evidence for the idea that variations in students' cognitions and motivation depend not only on the domain of learning (in this study: mathematics), but also on the content areas within a domain. The results of the present study demonstrate that 11-12 year-old boys and girls differed not only in the motivational beliefs they displayed with respect to mathematics, but also in their appraisals when confronted with mathematics problems, and in their actual behavior when solving those problems. As hypothesized, these gender differences were especially evident during applied problem solving (see chapter 5). Another important finding to emerge was that cognitive and affective variables were differently related for boys and girls and for the two task conditions (see chapter 6). In the following, conclusions with respect to each of the research questions will be summarized and discussed. Findings concerning the two research questions that were answered in chapter 5, will be discussed jointly.

Gender differences and intra-individual differences in motivational and cognitive variables
Gender differences in *motivational beliefs* in relation to mathematics were partly confirmed. Consistent with earlier research findings the boys in our sample showed higher self-concept than girls for mathematics in general, and displayed a more competitive attitude than girls. However, in contrast with previous research, boys also scored higher than girls on the other subscales of the goal orientation questionnaire. This implies that the boys in our sample displayed not only a higher level of ego orientation than the girls, but also a higher level of task orientation. It is not likely that boys' higher scores on all the subscales are caused by differences in self-representation, because half of the items that were answered positively by boys referred to situations in which failure occurred (e.g., " I feel unpleasant when other children see that I make mistakes in math"). Our results support the view that students may adopt diverse achievement goals. Meece and Holt (1993) also found that 40% of the students in their sample scored relatively high on both mastery (task) and ego goals.

No significant gender differences were found in attributional style. However, as the following section will show, our study did reveal gender differences in attributions that were measured in relation to specific mathematics tasks.

At the *task-specific level*, gender differences in appraisals were more pronounced in relation to applied problem solving. As hypothesized, girls started working on the application problems with lower subjective competence than did boys. In general, students' scores on the four appraisals were higher for the computations than for the applications. A plausible explanation for this finding is that students may feel more comfortable when confronted with computation problems, because this type of problem appeals more to the execution of a precise and systematic solution plan than do application problems (see chapter 1).

Causal attributions for failure on the task also appeared to be dependent on the content of the task and on gender. Although gender differences in attributions for success and failure were not confirmed, when measured at the domain-specific level, we found that with respect to applied problem solving, girls attributed bad results more often than boys to lack of capacity and to the difficulty of the task. This finding is consistent with earlier research findings (e.g., Fennema, 1985; Seegers & Boekaerts, 1996).

Both cognitive and motivational variables investigated *during the mathematics tasks* revealed gender differences. These differences in problem-solving behavior were found to be dependent not only on gender, but also on the contents of the mathematics tasks. Analyses revealed that gender differences were more pronounced when applied problem solving was involved.

With regard to boys' and girls' performances on the two types of tasks, the pattern described in chapter 1 was confirmed: In general, boys do better than girls at application problems. Analyses across tasks revealed that girls performed worse than boys on the applications, and worse on the applications than on the computations. What is even more interesting is that for boys the pattern was the reverse: They performed better on problems that were embedded in a context than on pure computations. These differences are noteworthy, considering that the contents of the two types of tasks were comparable. In other words, the application problems could be solved by applying the same computations as in the other type of task, only using different numbers. Girls appeared to have sufficient knowledge (as their results on the first task indicated), but had more difficulty knowing when and how to use their knowledge. Marshall and Smith (1987) found similar results in their study, and suggested that automatized rules for computation problems may, in fact, be a hindrance rather than a help in understanding mathematics. However, the explanation these authors give for the observed gender differences is directed only at cognitive variables, with particular attention to how mathematical knowledge is acquired and stored and to the role of automaticity in problem solving. As we discussed in chapter 2, failures in mathematical problem solving cannot be solely traced back to inadequate knowledge or strategies. All kinds of decisions have to be made about when to apply certain knowledge and strategies. It seems that girls have difficulty applying their knowledge, which, in our view, can be partly traced back to their lower levels of confidence.

As for solution strategies, we found that boys were more inclined than girls to use unconventional strategies when solving application problems. Girls were more inclined than boys to use the same computations as they has used in the computation problems, probably because they considered this a safer strategy. This suggests that more girls than boys believe that doing mathematics is based on applying a set of rules. However, we can only speculate on this, because these variables were not included in our research.

In this study gender differences in self-confidence were not only confirmed at the domain-specific level (self-concept of mathematics ability) and at the task-specific level (subjective competence), but also while solving problems (perceived confidence). In line with our expectations, we found that girls perceived lower confidence than boys only while working on application problems. Intraindividual analyses revealed that girls perceived lower confidence than boys while solving the application problems, and also perceived lower confidence in solving the application problems than the computation problems.

Contrary to our hypothesis, girls showed higher persistence than boys during applied problem solving. As explained in chapter 2, we expected that girls would give up more easily after failure experiences than boys. Possible explanations for these unexpected findings are offered in the next sections, when we discuss relations between persistence and other variables.

Relations among behavior-related variables

The first research question addressed in chapter 6 focused on relations between the behavior-related measures task performance, perceived confidence, and persistence following failure. The expected positive associations between these variables were confirmed for both boys and girls in relation to both types of problems. However, expectations concerning relations between perceived confidence and persistence following failure were only partly confirmed in our study. This result was unexpected, because confidence in doing a task is usually considered a critical factor for motivation and persistence. Because we expected girls' confidence to be lower than boys' during applied problem solving, we hypothesized that girls would give up more easily after failure. Although the finding that girls display lower confidence than boys during applied problem solving was confirmed, the hypothesized relation between perceived confidence and persistence was not. In other words, learned helplessness behavior or a dysfunctional motivational pattern (see chapter 3) that has often been ascribed to girls was not confirmed in our sample. Apparently, the relations between self-confidence and persistence are more complex. Girls' higher effort expenditure, although not following failure experiences, has been reported in the literature. Seegers and Boekaerts (1996) found that girls indicated that they were more prepared to invest effort in a mathematics task than did boys. These authors stated that high perceived competence in itself is not enough to guarantee that students will put in effort. Zimmerman and Martinez-Pons (1990) also found that girls were more actively engaged in a task and reported greater use of self-regulated learning strategies, even though they judged themselves as less capable than boys did. Several authors have stressed that the quality of the effort expenditure is of great importance. As Lester et al. (1989) already stated, persistence is not necessarily a virtue in problem solving. In this context, Helmke (1989) distinguished between qualitative and quantitative effort. He found that students with a high self-concept of mathematics ability invested more qualitative effort during instruction, by expending more mental effort in order to understand and remember the material. On the contrary, students with a low self-concept of mathematics ability expended more quantitative effort, in terms of more preparation time for homework and exams. Qualitative effort appeared to have a positive influence on achievement, whereas quantitative effort had a negative effect on achievement. Another explanation is offered by Kloosterman (1990). He found that those students who gave up most easily on mathematics problems were the highest achievers. He explains that this might be because good problem solvers know when to quit.

Additional analyses that were performed on the relations between task performance and perceived confidence partly confirmed our expectations. As was expected, students with correct solutions generally displayed more confidence than students with incorrect solutions. However, our data suggest that performance alone does not account for differences in confidence. We found that when performances were equal, boys perceived higher confidence than girls. This was found for one third of the computation problems when comparing boys' and girls' confidence for the incorrect solutions, and for one third of the application problems when comparing boys' and girls' confidence for the correct solutions. These patterns suggest overconfidence for boys (especially in relation to computations) and underconfidence in girls (especially in relation to the applications). However, we must take into account that this over- and underconfidence is relative, namely when comparing boys' and girls' confidence. When comparing students' confidence in relation to an absolute standard, that is, their positions on the five-point scale, we found that students generally display a high degree of confidence: Mean confidence scores were above the midpoint on this scale (see Figures 6.5 and 6.6). Students' inclination to overestimate themselves has been reported in the literature by Pajares and Miller (1994). They found that 57% of the university students in their sample overestimated their performance on a mathematics task and that 20% underestimated it. Lundeberg et al. (1994) also reported that most of the psychology students in their sample, when asked to indicate their confidence after having answered items on a multiple-choice test, overestimated their performance. These authors noted more overconfidence for males than for females on a mathematics test.

In the literature, underconfidence is usually seen as detrimental to motivation and performance, but less is known about the influences of overconfidence. According to Schunk (1981) accurate appraisal is important, since misjudgments in either direction can have negative consequences. He stated that persons who overestimate their abilities are apt to become demoralized through repeated task failure, whereas those who underestimate their capabilities may avoid challenges, thereby precluding opportunities for skill development. We agree, however, with Bandura (1986) who stated that some degree of overconfidence may be useful. Underconfidence, on the contrary, is less desirable.

Other than the findings reported above, we did not find gender differences in students' calibration of confidence. As already stated in chapter 5, these results are not surprising, given that boys were inclined to overestimate their performance, whereas girls were inclined to underestimate their performance. Calibration of confidence concerns the extent to which students' estimates are realistic. It is interesting that both boys and girls estimated their performance more accurately on the computation problems than on the application problems, indicating that the perceived confidence students expressed while solving computation problems was more realistic than their perceived confidence solving application problems.

Variables contributing to task performance

Results showed that subjective competence expressed before working on the problems significantly contributed to task performance, after the influences of objective measures of competence were taken into account. This was found for boys and girls in both task conditions. Additional effects of capacity attributions on task performance (application problems) were found for girls only. These results are consistent with earlier research findings. Seegers and Boekaerts (1996) reported that performance on mathematics tasks was mostly influenced by subjective competence and intelligence. We also noted that motivational variables contributed more to the variance explained in applied problem-solving performance for girls than for boys. This was also found when performance on the computations was entered as an additional predictor variable. Additional effects of capacity attributions on applied problem-solving performance were found for girls only. These findings suggest that students' beliefs about their capabilities are important mediators of their performance. Variations in subjective competence and capacity attributions may have a strong impact on task performance, especially for girls. It should be noted that minor influences were found of motivational beliefs on task performance, because these variables were entered in the regression after the task-specific appraisals had been included. As explained in chapter 6, this was done because we followed the assumptions from the model of adaptable learning, which is hierarchically structured. It prescribes that variables measured at the domain-specific level exert an influence on behavior-related measures through task-specific appraisals.

Variables contributing to perceived confidence

The results of multiple regressions indicated that subjective competence, which was measured just before students started with a task, accounted for most of the variance in students' perceived confidence while working on the tasks. Students who started working on the tasks with positive appraisals about their ability to do these specific tasks displayed high perceived confidence while doing the task. This result could be expected, because both subjective competence and perceived confidence measure students' estimated chances of success. We also reported that more of the variance in perceived confidence, while solving the application problems, could be explained for girls than for boys by task-specific measures, including subjective competence and learning intention. After the direct influences of these task-specific appraisals on perceived confidence had been accounted for, no effects were found of motivational beliefs. Inspection of the Pearson correlation coefficients showed that a significant association between fear that others will notice one's mistakes in mathematics and perceived confidence. This was evident in boys only. However, this variable did not add significantly to the variance explained, because of its high association with subjective competence. The same conclusion holds for the variables capacity attributions, and self-concept of mathematics ability.

Variables contributing to persistence

In contrast to the findings reported above, persistence during applied problem solving was more strongly related to motivational beliefs than to objective competence and task-specific appraisals. This pattern was especially apparent in boys. We found that for the boys in our sample, a tendency to have a competitive attitude in mathematics, as well as a tendency to attribute success in mathematics to capacity had positive influences on persistence. On the other hand, a tendency to attribute success to effort had a deteriorating effect on persistence for boys. This was unexpected, because in the literature effort attributions are usually considered to be beneficial for motivation and performance (see chapter 3). For girls however, the expected relations between effort attributions and persistence were confirmed. We may conclude that the boys in our sample were only likely to persist when they thought they could show how capable they were, and not when they thought they had to put in a lot of effort in order to succeed. A possible explanation is that these students reduce their effort in order to protect their perceived ability. Various researchers (e.g., Jacacinski & Nicholls, 1990) have suggested that when students expect a failure that will indicate their incompetence, they will reduce their effort so that failure can be attributed to low effort, rather than to low ability. Covington and Omelich (1979) referred to effort in this context as a "double-edged sword", by which they meant that students must find a compromise between expending high effort or no effort at all. According to Covington (1983) a combination of high effort with failure may lead to feelings of incompetence. However, one cause of failure (e.g., low ability) is discounted when not trying at all. The finding, in our study, that for boys, high competitiveness in mathematics resulted in higher persistence, provides further support for this explanation.

Limitations to these findings

The findings from this study must be put into perspective regarding several factors. These factors concern the specific testing situation, the design of the study, and the content of the mathematical problems. First of all, our study was not carried out in an actual classroom environment, which may have influenced students' motivation (Pintrich, Marx, & Boyle, 1993). Students were in a test situation in which an observer was present, and in which feedback was given after incorrect solutions. This was done because we were interested in students' reactions following failure just after they heard that their solution was incorrect. This specific situation may have influenced students' behavior. In addition, for some students their problem-solving may have been interrupted by the event-timer. Although we did not experience this in the pilot studies, it is possible that, especially for very anxious students, this caused a deterioration in performance and/or confidence.

Secondly, the within-subjects comparisons or intraindividual differences that were described in this thesis must be put into perspective. Because the administration of the computation problems preceded the administration of the application problems, test effects or carryover effects may have been produced. Students' experiences during the first individual session may have influenced their behavior during the second individual session. However, counterbalancing the order in which the students solved computation or application problems, did not seem an adequate alternative for our study. Because we considered the applications more difficult than the computations, counterbalancing the order of administration would have introduced a number of complex side effects.

Thirdly, the conclusions from this study are restricted to a limited set of problems. The problems were mainly routine; no real higher order thinking was involved in this study. We did not use context problems that were completely new for the students, or problems that demanded complex solution steps. The reasons for not choosing these problems were twofold. First of all, the contents of the tasks were chosen in such a way that the same computations could be applied in both types of tasks. This enabled us to make within-subjects comparisons. Another reason was that students in this age group are not used to solve complex problems individually. Usually these problems are solved in classroom situations in discussion with peers and teacher.

Finally, generalizibility is restricted to this age group (11-12 years). It is plausible, however, that gender differences in cognitive and motivational variables related to mathematics learning will persevere in older students, and will negatively influence the development of their mathematics attitude and achievement.

7.2 General implications

Implications for research

The present findings support the view that in research on individual differences in motivation and related variables two approaches are particularly useful: (1) a task-specific or content-specific approach, and (2) an approach in which cognitive and motivational variables are investigated in relation to each other. A task-specific approach not only makes it possible to investigate cognitions and observe behavior in concrete situations, but also allows one to detect intra-individual differences across tasks or contents within one domain. In addition, when studying cognitive and motivational variables in relation to each other, conclusions can be drawn concerning over- and underperformance of students. Consistent with what has been reported by other researchers (e.g., Pajares & Miller, 1994), we found that confidence that was measured in relation to a specific task was more closely related to actual performances than was a more general measure of confidence. With respect to gender differences in mathematics, our results underscore that students' confidence is the variable that needs more attention in research.

The findings reported here suggest the need for further research. Firstly, it is important to know how and when students develop inaccurate beliefs about themselves. Longitudinal research studying the relations between cognitive and motivational variables could provide more insights into these processes. Although the results of our study revealed inter- and intraindividual differences in perceived confidence, the design of the study did not allow us to infer causality. However, our data suggest that low confidence results in underperformance. Secondly, future research should be aimed at gender differences in both metacognitive knowledge about and the use of solution strategies, and at students' beliefs about mathematics as a subject. In our study we found that boys were more inclined than girls to use unconventional strategies during applied problem solving. However, we did not collect data on students' reasons for choosing a specific solution strategy, neither did we ask them about their beliefs concerning how mathematics should be learned. Finally, future research should also be aimed at differences in boys' and girls' motivational orientations and self-confidence in different classroom settings.

Educational implications

Finally, the findings presented in this thesis have important implications for mathematics education. A central aim of mathematics education is to optimize both boys' and girls' engagement in learning. Therefore, educators should take into account that boys and girls display different motivational orientations towards mathematics in general and towards applied problem solving in particular. Especially, attention should be paid to gender differences in self-confidence. Assessing students' confidence can provide teachers with additional insights. Namely, students who perform well in the classroom may not necessarily have developed high confidence in their abilities. Although confidence is strongly related to performance, it is important for teachers to know for which students this is a negative relation, implying underconfidence. In the area of realistic mathematics education, where students can depend on rules to a lesser extent (see chapter 1), confidence in one's ability may have a strong impact on achievement. Therefore, it is important for teachers to know which students are inclined to underestimate themselves in relation to mathematics in general, and even in relation to specific contents within mathematics. Teachers should be aware, particularly in relation to applied problem solving, that students (especially girls) may be inclined to underestimate their abilities. Although not found in this study, it is not unlikely that underestimation of abilities in the long run will lead to reduced effort expenditure.

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APPENDIX A

Table A-1

Pearson correlation coefficients between students' mean confidence scores during the different phases of the solution process

Problems	Intercorrelations: Orientation phase / execution phase	Orientation phase / verification phase	Execution phase / verification phase
Computation problems			
1. 14820 : 38=	.64 (n= 158)	.38 (n= 133)	.48 (n= 133)
2. 68.2 - 4.73=	.79 (n= 158)	.48 (n= 155)	.74 (n= 155)
3. 4¾% of ƒ1816.-=	.72 (n= 158)	.56 (n= 115)	.72 (n= 115)
4. 0.825 : 0.01=	.75 (n= 158)	.51 (n= 137)	.73 (n= 137)
5. 5% of 46460=	.87 (n= 158)	.73 (n= 148)	.84 (n= 148)
6. 236 x 405=	.82 (n= 158)	.65 (n= 157)	.79 (n= 157)
Application problems			
1. Mailbags	.69 (n= 158)	.42 (n= 150)	.64 (n= 150)
2. Bicycle	.83 (n= 158)	.73 (n= 152)	.86 (n= 152)
3. Interest	.72 (n= 158)	.60 (n= 142)	.75 (n= 142)
4. Pharmacist	.89 (n= 158)	.71 (n= 154)	.86 (n= 154)
5. Rock concert	.78 (n= 158)	.63 (n= 146)	.80 (n= 146)
6. Campground	.86 (n= 158)	.70 (n=149)	.84 (n=149)

Note. The number of students are given in parentheses.

APPENDIX B

Table B-1

Mean confidence scores and standard deviations for four groups of students on the computation problems

	Correct solutions				Incorrect solutions			
	Boys		Girls		Boys		Girls	
Problems	M	SD	M	SD	M	SD	M	SD
1. 14820 : 38=	4.16 (46)	.59	4.04 (49)	.58	3.46 (33)	.77	3.46 (30)	.80
2. 68.2 - 4.73=	4.32 (64)	.62	4.37 (57)	.54	3.99 (15)	.79	4.16 (22)	.69
3. 4¾% van ƒ1816.-=	4.18 (3)	.22	3.46 (10)	.82	3.14 (75)	.99	2.79 (68)	1.08
4. 0.825 : 0.01=	4.33 (35)	.62	3.80 (30)	.74	3.08 (44)	.94	3.19 (49)	.99
5. 5% of 46460=	4.03 (46)	.94	3.82 (53)	.96	3.71 (33)	1.01	3.12 (24)	1.21
6. 236 x 405=	4.53 (55)	.57	4.29 (64)	.75	4.09 (24)	.83	4.08 (15)	.72

Note. The number of students are given in parentheses.

Table B-2

Mean confidence scores and standard deviations for four groups of students on the application problems

	Correct solutions				Incorrect solutions			
	Boys		Girls		Boys		Girls	
Problems	M	SD	M	SD	M	SD	M	SD
1. Mailbags	4.00 (40)	.60	3.71 (39)	.68	3.61 (39)	.74	3.12 (40)	.81
2. Bicycle	4.30 (65)	.72	3.95 (56)	.85	3.49 (14)	.88	3.40 (23)	.92
3. Interest	4.13 (20)	.68	3.71 (20)	.88	3.37 (59)	.97	3.24 (59)	.92
4. Pharmacist	4.19 (73)	.73	3.96 (57)	.79	3.56 (6)	.83	3.37 (22)	.90
5. Rock concert	3.97 (52)	.76	3.64 (40)	.88	3.05 (27)	1.11	3.12 (39)	.96
6. Campground	3.95 (38)	.91	3.92 (30)	.86	3.64 (41)	1.00	3.27 (49)	.94

Note. The number of students are given in parentheses.

APPENDIX C

Table C-1

Pearson correlation coefficients between the variables objective competence, motivational beliefs, and task-specific appraisals (n= 158)

	Objective competence		Task-specific appraisals			
Measure	ANA	CAT	SC	TA	PR	U
Objective competence						
Analogies			.22** / .31**	.02 / .16*	-.08 / .03	-.05 / .04
Categories			.23** / .28**	.12 / .13	.02 / .00	.09 / .07
Motivational beliefs						
Goal orientation						
Self-enhancing ego orientation	-.10	-.05	.16* / .14	.22** / .25**	.11 / .06	.22** / .25**
Self-defeating ego orientation	-.23**	-.20*	-.21** / -.16*	.04 / .04	.11 / .05	.08 / .06
Task orientation	.10	.04	.19* / .16*	.21** / .23**	.21** / .25"	.25** / .31"
Error frustration	.01	-.08	.00 / -.02	.12 / .11	.08 / .05	.16* / .09
Attributions						
Capacity	.34**	.18*	.54** / .58"	.22** / .38"	-.05 / .16*	.17* / .22"
Effort	-.03	.12	.16* / .10	.19* / .26**	.11 / .22**	.28** / .44**
Self-concept of mathematics ability						
Self-concept	.28**	.21"	.65** / .68**	.37** / .49**	.11 / .16*	.30** / .29**
Importance	-.08	-.09	.16* / .14	.27** / .31**	.33** / .38**	.42** / .46"

Note. Results on the computation problems are printed before the slash, and results on the application problems are printed after the slash. ANA=: Analogies; CAT= Categories; SC= Subjective competence; TA= Task attraction; PR= Personal relevance; LI= Learning intention.

* $p < .05$. ** $p < .01$.

SUMMARY

In this thesis motivational variables are studied in relation to mathematical problem solving. The reasons for setting up this study were twofold. Firstly, the research described draws on studies in which relations between non-cognitive variables and achievement in mathematics are investigated. Secondly, the research was set up to further explore gender differences in mathematics. Emphasis is placed on students' actual problem-solving behavior when working on two types of mathematics tasks: computation problems versus application problems.

Chapter 1 serves as an introduction to the study. A short description is provided of gender differences in mathematics achievement. A consistent finding has been that boys perform better than girls when it comes to applied problem solving, but that no differences exist when exact computations are involved. Our starting point is that gender differences in mathematics performance are the outcome of complex interactions, in which both cognitive and motivational variables play a role. Furthermore, this chapter outlines the role of different types of problems within realistic mathematics education. With the renewal of mathematics education, application problems have become a major part of the mathematics curriculum in the Netherlands. At the end of this chapter the aims of our study are described. Our first objective is to investigate gender differences and intraindividual differences in both cognitive and motivational variables in relation to the two types of mathematics tasks. Our next goal is to investigate relations between cognitive and motivational variables.

The second chapter describes relevant cognitive, metacognitive, and affective variables that contribute to students' mathematical problem solving. An outline is given of how, within cognitive psychology, a change from studying cognitive processes only to acknowledging the importance of non-cognitive processes, is apparent. The following factors are discussed: prior knowledge, heuristics, metacognition, beliefs, attitudes, and emotions. We argue that these factors should be studied in relation to each other when considering individual differences. We discuss possible stumbling blocks to mathematical problem solving, tracing the different phases of the solution process. The chapter ends with our research perspective. Our study is directed at students' actual problem-solving behavior during the two types of mathematics tasks. Students' perceived confidence, performance, and persistence following failure are central issues. In particular, gender differences in problem-solving behavior are examined.

An important question is to what extent students' self-referenced cognitions influence their problem-solving behavior. In chapter 3 these issues are outlined. A distinction is made between students' motivational beliefs (domain-specific level), and task-specific appraisals (task-specific level). The model of adaptable learning is described: Cognitive and motivational variables at both the domain-specific and task-specific levels are integrated in this model. Three motivational beliefs are discussed: self-concept of mathematics ability, goal orientation, and attributional style. Self-concept of mathematics ability concerns students' perceived competence in mathematics. Within the theory of goal orientation a distinction is made between two types of goals students may pursue. The focus may primarily be on learning new skills and trying to understand what one is doing (task orientation), or on performing better than others and showing superior ability (ego orientation). Attributional style refers to students' perceived causes of success and failure. Relations between these motivational beliefs are discussed, as well as gender differences. In general it has been reported that girls have a lower self-concept of mathematics ability than boys, and are more inclined than boys to attribute failure in mathematics to their low ability. Furthermore, it has often been reported that boys are more ego-oriented than girls.

Research is discussed in which functional and dysfunctional motivational patterns have been revealed. These studies often concern self-reports of students in relation to domain-specific motivation. Lately however, the importance of students' perceptions of specific learning situations has been stressed. Central issues are, for instance, students' success expectations before starting with a task, and their task-specific attributions concerning success or failure. We argue that a task-specific approach seems valuable when studying gender differences in mathematics. This chapter concludes with an outline of the variables and research questions.

The method of research is described in chapter 4. Subjects were 158 sixth-grade students (79 boys and 79 girls) selected from 12 schools. Measures for non-verbal intelligence and questionnaires on students' self-concept of mathematics ability, goal orientation, and attributional style were administered in the group setting. Individual testing took place in two separate sessions with an interval of about three months. During these individual sessions the Confidence and Doubt Questionnaire was central. This instrument was developed for this study in order to investigate different aspects of students' problem-solving behavior (perceived confidence, solution strategies, solution time, performance, persistence following failure). In addition, questionnaires were administered concerning students' task-specific appraisals before starting with the task (subjective competence, task attraction, personal relevance, learning intention), and their task-specific attributions concerning success or failure after having finished the task.

The results are discussed in chapters 5 and 6. Descriptive statistics on all the variables are presented in chapter 5, as well as gender differences and intraindividual differences. Analyses revealed gender differences at the domain-specific, task-specific, and behavior-related levels of measurement. At the domain-specific level, we found that boys' self-concept was higher than girls', and that boys and girls displayed different goal orientations towards mathematics. Boys appeared not only to be more ego-oriented than girls, but also more task-oriented. At the task-specific level, gender differences were especially apparent with respect to the application problems. Before starting the application problems, boys displayed higher subjective competence than girls, whereas girls attached more importance to being good at these kinds of tasks. With respect to task-specific attributions, we found that more girls than boys attributed their perceived failure on the application problems to a lack of capacity and to the difficulty level of the task.

Both cognitive and affective variables registered *during* the task (behavior-related measures) revealed gender differences. Consistent with expectations, boys perceived higher confidence than girls and performed better than girls, but only while working on the application problems. Intraindividual analyses revealed that more girls than boys performed better on the computation problems than on the application problems, and that more boys than girls performed better on the applications than on the computations. Gender differences were also apparent in the use of solution strategies: During applied problem solving boys used more unconventional solution strategies than girls. An unexpected finding was that girls showed higher persistence after failure experiences than boys.

Statistical analyses that were performed on relations between variables (chapter 6) showed that task performance, perceived confidence, and persistence following failure were positively related for boys and girls in relation to both types of tasks. However, expectations concerning relations between perceived confidence and persistence following failure were only partly confirmed. Additional analyses on relations between task performance and perceived confidence revealed that for some problems boys perceived higher confidence than girls, irrespective of the correctness of the solution. Boys were inclined to overestimate their performance, whereas girls were inclined to underestimate their performance.

Furthermore, relations between variables at the domain-specific, task-specific and behavior-related levels of measurement were investigated. We examined to what extent task-specific appraisals and motivational beliefs contributed to the behavior-related measures task performance, perceived confidence, and persistence, respectively. In these analyses, we controlled for the influence of an objective measure of competence (abstract reasoning ability). Multiple regression analyses revealed that, consistent with earlier research findings, subjective competence before working on the tasks significantly contributed to task performance. This was true for boys and girls in both task conditions. We also noted that motivational variables contributed more to the variance explained in applied problem-solving performance for girls than for boys. The same analyses were done with perceived confidence as the dependent variable. Boys' and girls' subjective competence significantly contributed to their perceived confidence while working on the two types of problems. This result was expected, because both variables assess students' estimated chances of success. For girls, their intended effort expenditure (learning intention) also contributed positively to their perceived confidence while solving the application problems. No additional effects were found of motivational beliefs on perceived confidence.

Finally, logistic regression analyses were performed with the dichotomous variable persistence (high/low) as the dependent variable. For both boys and girls, subjective competence before working on the computation problems positively influenced their persistence while solving these problems. Furthermore, boys with a high tendency to ascribe success in mathematics to capacity showed high persistence in relation to the computation problems. With respect to the application problems, motivational beliefs were found to contribute more to persistence than the task-specific appraisals did. This pattern was especially evident in boys. High tendencies to ascribe success in mathematics to capacity resulted in high persistence for boys. In addition, boys with a high level of ego orientation were inclined to persist longer on the applied problems. For girls, high effort attributions resulted in high persistence. Surprisingly, this relation was negative for boys.

In the last chapter, the general conclusions of our study are discussed and implications for research and education are considered. Our findings provide evidence for the idea that variations in students' cognition and motivation depend not only on the domain of learning (in this study: mathematics), but also on the content within a domain. The results of the present study demonstrate that 11-12 year-old boys and girls differed not only in the motivational beliefs they displayed with respect to mathematics, but also in their appraisals when confronted with mathematics problems and in their actual behavior when solving those problems. As hypothesized, these gender differences were especially evident during applied problem solving. Another important finding is that cognitive and motivational variables were differently related for boys and girls and for the two task conditions.

We discuss findings concerning boys' and girls' inclination to respectively overestimate or underestimate their abilities and girls' higher persistence during applied problem solving. It is argued that in research on individual differences in mathematics achievement a task-specific approach, in which cognitive and motivational variables are studied in relation to each other, is very useful. With respect to gender differences, our results confirm that students' confidence is the variable that needs most attention in research. We suggest that further research is conducted longitudinally, in order to understand how and when students develop inaccurate beliefs about themselves. Concerning practical implications, educators should take into account that boys and girls display different motivational orientations towards mathematics in general and towards applied problem solving in particular. Attention should be paid especially to gender differences in self-confidence. Within realistic mathematics education, in which students can depend on rules to a lesser extent, confidence in one's ability may have a strong impact on achievement.

SAMENVATTING

Het oplossen van rekenopgaven door leerlingen van groep acht: Motivationele variabelen en sekseverschillen

In dit proefschrift wordt een onderzoek beschreven waarin de relatie tussen motivatie en rekenvaardigheid centraal staat. De aanleiding tot dit onderzoek was tweeledig. Ten eerste bouwt het onderzoek voort op studies waarin relaties tussen niet-cognitieve variabelen en rekenprestaties nader worden onderzocht. Ten tweede wordt getracht meer inzicht te krijgen in geconstateerde sekseverschillen binnen het reken-wiskundeonderwijs. De nadruk wordt gelegd op het concreet oplossingsgedrag van leerlingen tijdens het uitvoeren van twee soorten rekentaken: rekenkundige bewerkingen (cijferen) versus toepassingsgerichte opgaven.

Hoofdstuk l dient als introductie tot het onderzoek. Er wordt een kort overzicht gegeven van sekseverschillen in reken-wiskundeprestaties. Een consistente bevinding is dat jongens beter presteren dan meisjes, niet zozeer op het gebied van cijferen, maar wel ten aanzien van het oplossen van toepassingsgerichte opgaven. Ons uitgangspunt is dat sekseverschillen in reken-wiskundeprestaties het gevolg zijn van complexe interacties, waarin zowel cognitieve als motivationele variabelen een rol spelen. Ook wordt de rol van verschillende soorten opgaven binnen het realistisch reken-wiskundeonderwijs belicht. Met de invoering van het realistisch reken-wiskundeonderwijs, vormt het oplossen van toepassingen een steeds groter onderdeel van het onderwijs in Nederland. Aan het eind van dit hoofdstuk worden de doelstellingen van het onderzoek nader omschreven. Een eerste doelstelling bestaat uit het onderzoeken van sekseverschillen en intra-individuele verschillen in zowel cognitieve als motivationele variabelen met betrekking tot de twee soorten rekentaken. Een volgend doel is het onderzoeken van relaties tussen cognitieve en motivationele variabelen onderling.

In het tweede hoofdstuk wordt een overzicht gegeven van relevante cognitieve, metacognitieve en affectieve factoren die een rol spelen tijdens het wiskundig probleemoplossea Geschetst wordt hoe binnen de cognitieve psychologie een omslag zichtbaar is van louter aandacht voor cognitieve processen naar het onderkennen van het belang van niet-cognitieve factoren tijdens probleemoplossen. De volgende factoren worden besproken: voorkennis, heuristieken, metacognitie, opvattingen, attitudes en emoties. We beargumenteren dat, teneinde tot een betere verklaring te komen van individuele verschillen in rekenvaardigheid, deze factoren in samenhang moeten worden onderzocht. Tevens wordt aandacht besteed aan mogelijke struikelblokken tijdens de verschillende fasen van het probleemoplossen. Dit hoofdstuk wordt afgesloten met het gezichtspunt van waaruit deze studie is opgezet. Het onderzoek richt zich op het concreet oplossingsgedrag van leerlingen tijdens het uitvoeren van de twee soorten rekentaken. Centraal staan de ervaren zekerheid/twijfel van leerlingen, hun prestaties en persistentie na falen. Met name wordt aandacht besteed aan verschillen in oplossingsgedrag tussen jongens en meisjes.

Een belangrijke vraag is hoeverre de motivatie en cognities van leerlingen ("self-referenced cognitions") hun probleemoplossingsgedrag beïnvloeden. In hoofdstuk 3 worden deze aspecten nader uitgewerkt. Een onderscheid wordt gemaakt tussen domein-specifieke variabelen ("motivational beliefs") en taak-specifieke variabelen ("task-specific appraisals"). Een model van adaptief leren wordt besproken, waarin zowel cognitieve als motivationele variabelen, gemeten op beide niveau's, worden geïntegreerd. Met betrekking tot het domein-specifieke niveau worden drie belangrijke concepten besproken: zelfbeeld ten aanzien van rekenen-wiskunde, doel-oriëntatie en attributie-stijl. Zelfbeeld betreft het beeld dat leerlingen hebben van hun rekenvaardigheid. Binnen de theorie van doel-oriëntatie wordt een onderscheid gemaakt tussen twee soorten doelen die leerlingen na kunnen streven. De aandacht kan voornamelijk gericht zijn op het leren van nieuwe vaardigheden en het vergroten van inzicht (taak-oriëntatie), of op het leveren van betere prestaties dan anderen en het laten zien van superieure vaardigheden (ego-oriëntatie). Attributie-stijl verwijst naar de oorzaken waaraan leerlingen succes of falen toeschrijven. Relaties tussen deze variabelen worden beschreven, evenals sekseverschillen. In de onderzoeksliteratuur wordt vaak gerapporteerd dat meisjes een lager zelfbeeld hebben dan jongens en dat ze meer geneigd zijn dan jongens om falen in rekenen-wiskunde toe te schrijven aan gebrek aan aanleg. Verder wordt vaak gemeld dat jongens meer ego-georiënteerd of competitie-gericht zijn dan meisjes. Onderzoeken worden besproken waarin nagegaan wordt welke vormen van motivatie functioneel respectievelijk dysfunctioneel zijn voor het leren van rekenen-wiskunde. In deze onderzoeken is vooral uitgegaan van zelf-rapportages van leerlingen met betrekking tot domein-specifieke motivatie. De laatste jaren echter wordt het belang onderkend van taak-specifieke motivatie in concrete leersituaties. Centrale thema's hierbij zijn onder meer de succes-verwachtingen van leerlingen voordat ze aan een taak beginnen en taak-specifieke attributies omtrent hun slagen of falen. We beargumenteren dat een taak-specifieke benadering van belang is in het onderzoek naar sekseverschillen in rekenvaardigheid. Aan het eind van dit hoofdstuk worden de variabelen en onderzoeksvragen besproken.

Hoofdstuk 4 behandelt de methode en opzet van het onderzoek. Aan het onderzoek namen 158 leerlingen (79 jongens en 79 meisjes) deel, geselecteerd uit de groepen 8 van 12 basisscholen. De meetinstrumenten die klassikaal werden afgenomen bestonden uit een test voor niet-verbale intelligentie, alsmede een drietal vragenlijsten die het zelfbeeld, de doel-oriëntatie en de attributiestijl van de leerlingen in kaart brachten. Bij elke leerling werden twee individuele onderzoeken afgenomen met een interval van ongeveer drie maanden. Tijdens deze individuele afnames stond de "Confidence and Doubt Questionnaire" centraal. Dit instrument is ontwikkeld voor deze studie om verschillende aspecten van het probleemoplossingsgedrag (ervaren zekerheid/twijfel, oplossingsstrategieën, oplossingstijd, prestaties, persistentie na falen) in kaart te brengen. Tevens werden tijdens de individuele sessies vragenlijsten afgenomen met betrekking tot taak-specifieke motivatie voor aanvang van de taak (subjectieve competentie, taakplezier, persoonlijk belang, leerintentie) en attributies met betrekking tot succes of falen na afloop van de taak.

In de hoofdstukken 5 en 6 worden de resultaten beschreven. In hoofdstuk 5 wordt de beschrijvende statistiek weergegeven met betrekking tot alle variabelen in de studie. Tevens wordt gerapporteerd over sekseverschillen en intra-individuele verschillen. Uit analyses kwamen sekseverschillen naar voren ten aanzien van het domein-specifieke, taak-specifieke en gedrags-specifieke meetniveau. Ten aanzien van het domein-specifieke meetniveau vonden we dat jongens een hoger beeld hadden van hun bekwaamheid ten aanzien van rekenen dan meisjes. Ook bleken jongens en meisjes een verschillende doel-oriëntatie te hebben: Jongens waren niet alleen meer ego-georiënteerd dan meisjes, maar ook meer taak-georiënteerd. Op taakspecifiek niveau bleken sekse-verschillen vooral op te treden met betrekking tot de toepassingsgerichte opgaven. Voordat leerlingen aan deze opgaven begonnen, schatten jongens hun competentie voor dit soort taken hoger in dan meisjes. Meisjes daarentegen gaven aan meer belang te hechten aan dit soort taken. Na afloop van de taak waren meisjes meer geneigd dan jongens om een slecht resultaat toe te schrijven aan gebrek aan aanleg en moeilijkheidsgraad van de taak. Verder brachten intra-individuele analyses aan het licht dat de taak-specifieke motivatie van leerlingen positiever was ten aanzien van het cijferen dan van de toepassingsgerichte opgaven.

Zowel cognitieve als affectieve variabelen die gemeten waren *tijdens* het uitvoeren van de rekentaken (gedrags-specifiek meetniveau) lieten sekse-verschillen zien. In overeenstemming met onze verwachtingen vertoonden jongens meer zekerheid dan meisjes en presteerden zij beter dan meisjes, maar alleen tijdens het oplossen van de toepassingen. Intra-individuele analyses wezen uit dat meer meisjes dan jongens beter presteerden tijdens het cijferen dan tijdens het oplossen van de toepassingen, terwijl meer jongens dan meisjes beter presteerden tijdens de toepassingen dan tijdens het cijferen. Sekseverschillen waren ook aanwezig in de gehanteerde oplossingsstrategieën: Tijdens het toegepast probleemoplossen gebruikten jongens meer onconventionele strategieën dan meisjes. Een onverwachte bevinding was dat meisjes meer persistentie na falen vertoonden dan jongens.

Statistische analyses, die werden uitgevoerd op relaties tussen variabelen (hoofdstuk 6) gaven weer dat taakprestatie, ervaren zekerheid en persistentie na falen positief samenhingen voor zowel jongens als meisjes in relatie tot beide taken. Onze verwachtingen ten aanzien van relaties tussen ervaren zekerheid en persistentie na falen werden echter maar gedeeltelijk bevestigd. Aanvullende analyses naar relaties tussen prestatie en ervaren zekerheid brachten naar voren dat, onafhankelijk van de correctheid van de oplossing, voor sommige opgaven jongens een hogere mate van zekerheid aangaven dan meisjes. Jongens waren geneigd zichzelf te overschatten, terwijl meisjes geneigd waren zichzelf te onderschatten.

Verder zijn relaties onderzocht tussen variabelen op domein-specifiek, taak-specifiek en gedrags-specifiek meetniveau. We onderzochten in hoeverre domein-specifieke en taak-specifieke motivatie bijdroegen aan de gedragsmaten taakprestatie, ervaren zekerheid en persistentie. In deze analyses werd gecontroleerd voor de invloed van niet-verbale intelligentie (abstract denkvermogen). Multipele regressie-analyses lieten zien dat, in overeenstemming met eerdere onderzoeksbevindingen, subjectieve competentie vóór het werken aan de taak significant bijdroeg aan taakprestatie. Dit gold voor zowel jongens als meisjes in beide taakcondities. Tevens werd gevonden dat voor meisjes motivationele variabelen meer bijdroegen aan de verklaarde variantie in taakprestatie dan voor jongens. Dezelfde analyses werden uitgevoerd met ervaren zekerheid als afhankelijke variabele. Subjectieve competentie droeg significant bij aan de ervaren zekerheid van zowel jongens als meisjes tijdens het maken van de twee typen opgaven. Dit resultaat was te verwachten, omdat beide variabelen de ingeschatte kans op succes in kaart brengen. Voor meisjes werd ook een positief effect gevonden van de bereidheid om inzet te leveren (leerintentie) op hun ervaren zekerheid tijdens het oplossen van de toepassingen. Er werden geen extra effecten gevonden van domein-specifieke variabelen op ervaren zekerheid.

Als laatste werden logistische regressie-analyses uitgevoerd met de dichotome variabele persistentie (hoog/laag) als afhankelijke variabele. Voor zowel jongens als meisjes werd persistentie tijdens het cijferen positief beïnvloed door subjectieve competentie vóór het werken aan de taak. Met betrekking tot de toepassingen, vonden we dat domein-specifieke variabelen meer invloed hadden op persistentie dan taakspecifieke variabelen. Dit patroon was vooral duidelijk voor jongens. Jongens die geneigd waren om succes in rekenen toe te schrijven aan aanleg, vertoonden een hoge mate van persistentie na falen. Verder waren jongens met een hoge mate van egooriëntatie geneigd om langer door te zetten tijdens het probleemoplossen. Voor meisjes hing de neiging om succes toe te schrijven aan inzet samen met hoge persistentie. Tegen onze verwachting in was deze relatie negatief voor jongens.

In het laatste hoofdstuk worden de algemene conclusies van deze studie besproken, alsmede de implicaties voor onderzoek en onderwijs. Onze bevindingen tonen aan dat individuele verschillen in cognitieve en motivationele variabelen niet alleen afhangen van het domein (in deze studie: rekenen-wiskunde), maar ook van het leergebied binnen een domein. De resultaten van dit onderzoek laten zien dat 11- tot 12-jarige jongens en meisjes niet alleen verschilden in domein-specifieke motivatie, maar ook in hun taak-specifieke motivatie met betrekking tot concrete rekentaken en in hun oplossingsgedrag. Zoals verondersteld, werden deze sekseverschillen vooral geconstateerd tijdens het toepassingsgericht probleemoplossen. Een andere belangrijke bevinding is dat cognitieve en motivationele variabelen verschillend samenhingen voor jongens en meisjes en voor de twee taakcondities.

Bevindingen worden besproken aangaande de neiging van jongens en meisjes om hun mogelijkheden te overschatten respectievelijk onderschatten, evenals de hogere mate van persistentie van meisjes tijdens het probleemoplossen. We benadrukken, dat binnen onderzoek naar individuele verschillen in rekenvaardigheid, cognitieve en motivationele variabelen in samenhang moeten worden bestudeerd. Met betrekking tot sekseverschillen bevestigen onze bevindingen dat de variabele zelfvertrouwen de meeste aandacht verdient binnen onderzoek. We bevelen aan dat volgend onderzoek longitudinaal opgezet wordt, teneinde beter inzicht te krijgen in hoe en wanneer leerlingen een onrealistisch beeld over zichzelf ontwikkelen. Binnen de onderwijspraktijk zou men er rekening mee moeten houden dat jongens en meisjes verschillen in motivatie ten aanzien van rekenen-wiskunde in het algemeen en ten aanzien van toegepast probleemoplossen in het bijzonder. Speciale aandacht zou besteed moeten worden aan sekseverschillen in zelfvertrouwen. Binnen het realistisch rekenwiskundeonderwijs, waarin leerlingen in mindere mate houvast hebben aan vaste regels, kan zelfvertrouwen een belangrijke invloed hebben op prestaties.

Curriculum Vitae

Harriet Vermeer werd op 18 januari 1961 geboren te Den Haag. Zij voltooide in 1979 het gymnasium aan de Hugo de Groot scholengemeenschap in Den Haag. In 1981 behaalde zij haar kandidaatsexamen pedagogiek aan de Rijksuniversiteit Leiden. Het doctoraalexamen orthopedagogiek (specialisatie school- en leermoeilijkheden) werd behaald in 1985 aan de Universiteit van Amsterdam. Van 1986 tot 1990 was zij werkzaam als toegevoegd onderzoeker aan de Universiteit van Amsterdam, Vakgroep Orthopedagogiek, aan een project betreffende tweetalig onderwijs voor allochtone kleuters. In diezelfde periode volgde zij een avondstudie MO-A wiskunde. In het studiejaar 1990-1991 werkte zij als docente wiskunde aan de scholengemeenschap "Nieuwer Amstel" te Amstelveen. Van 1992 tot 1996 was zij werkzaam als Onderzoeker In Opleiding aan de Rijksuniversiteit Leiden, Vakgroep Onderwijsstudies. In deze periode werd het onderzoek uitgevoerd dat in dit proefschrift resulteerde. Aansluitend aan deze periode werd zij bij dezelfde vakgroep voor een jaar aangesteld als onderzoeker op een project betreffende "Verschillen in reken-wiskundeprestaties tussen meisjes en jongens op de basisschool". Dit onderzoek (gefinancierd door het Ministerie van Onderwijs, Cultuur en Wetenschappen) wordt uitgevoerd in samenwerking met het Freudenthal Instituut te Utrecht. Op dit moment bevindt bovengenoemd project zich in de afrondingsfase.