Fermions, criticality and superconductivity

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SUMMARY

The many-body systems around us, created by nature or by humans, consist of electrons. Electrons are fermions. The fermion wavefunction changes sign when any two fermions are interchanged. So when we do a quantum-mechanical summation over all the many-particle states, we are adding terms with both positive and negative signs, and the result will be highly oscillatory. This leads to the infamous fermion sign problem, which is as hard as it can be, called non-deterministic polynomial-time (NP) hard in mathematical terms. The sign problem generally impedes a mathematically exact solution of the many-body systems.

Surprisingly, several strongly interacting fermionic systems, e.g. normal metals and ^{3}He at low temperatures, can be satisfactorily described by a phenomenological theory of Landau based on the assumption of adiabatic connectivity to the free Fermi gas. However more and more experimental evidences are accumulating in the last few decades, pointing towards the breakdown of the Landau paradigm and release of fermion signs from hell. These exotic materials usually lie on the verge of a zero temperature phase transition to another stable phase. Associated with such a singular point, anomalous scaling behaviors are constantly observed in a finite region of the phase diagram. This motivates the idea of quantum criticality, trying to describe the strange finite temperature properties of the system in terms of the low energy degrees of freedom of the ground state at zero temperature.

We explore the signful fermionic world in this thesis. In Chapter 2 and 3, we study the fermion sign problem in the worldline path integral formalism, in which the sign problem is most transparent. In Chapter 4 through 6, we explore the idea of quantum criticality. We focus on the instabilities of the quantum critical points (QCPs) at low temperatures, where exotic new phases appears.

In Chapter 2, we present a simple exercise of the signful worldline path integrals. We explain in this formalism the mechanism behind the Anderson-Higgs effect for a gas of charged bosons in a background magnetic field, and then use the method to prove the absence of the effect for a gas of fermions. In this formalism, the fermionic statistics are encoded via the inclusion of additional Grassmann coordinates in a manner that leads to a manifest worldline supersymmetry. This extra symmetry is key in demonstrating the absence of the effect for charged fermions.

In Chapter 3, we start to tackle the fermion sign problem in the worldline formalism. The insightful work of Ceperley in constructing fermionic path integrals in terms of constrained worldlines is reviewed. In this representation, the minus signs associated with Fermi-Dirac statistics are self consistently translated into a geometrical constraint structure, the nodal hypersurface, acting on an effective bosonic dynamics. Working with the path integral in momentum space, we then show that the Fermi gas can be understood by analogy to a Mott insulator in a harmonic trap.

In Chapter 4, we explore the instabilities of QCPs arising from the competition between the bosonic order parameters. The phases near QCPs are assumed to be either classical or quantum and assumed to repulsively interact via quadratic-quadratic interactions. We find that for any dynamical exponents and for any dimensionality strong enough interaction renders QCPs unstable, and drives transitions to become first order. We propose that this instability and the onset of first-order transitions lead to spatially inhomogeneous states in practical materials near putative QCPs.

In Chapter 5, we explore the instability of the fermionic degrees of freedom near QCPs. In particular, we study the instability in the particle-particle channel, and present a simple phenomenological scaling theory for superconductivity in the quantum critical metals. Asserting that the normal state is a strongly interacting quantum critical state of fermions, we propose that the pairing susceptibility becomes relevant, instead of the BCS marginal form, which has the effect that the pairing instability becomes much stronger. Even with a weak attractive interaction, we can get a high transition temperature comparable to what is found in real materials. We also discuss the behavior of the orbital-limited upper critical magnetic field as a function of the zero-temperature coupling constant. Compared to the variation in the transition temperature, the critical field might show a much stronger variation pending the value of the dynamical critical exponent.

In Chapter 6, we propose to use the second order Josephson effect as a direct probe of the Cooper channel of quantum critical metals, to shed light on the problem of unconventional superconductivity in such systems. To provide templates for experimentalists, we calculate the pair susceptibility for several different scenarios. The evolution of the peak structure in the imaginary part of the susceptibility is investigated in detail. We find that models assuming the electrons are in a critical normal state differ substantially from the Fermi liquid BCS model and its modern extensions.