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Theory-enriched practical knowledge in mathematics teacher education

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Theory-enriched practical knowledge

in

mathematics teacher education

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Theory-enriched practical knowledge

in

mathematics teacher education

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1 General introduction

1.1 Background and context

Since the 1980s, teacher training colleges have gradually come to realize that prescriptive transfer of theory is unsatisfactory. This was partly due to the fact that theory was insufficiently in step with reality and with the complexity of action in practice (e.g., Corporaal, 1988; Verloop, 2003). Furthermore, the observation was made that student teachers do encounter different types of 'theory' in their practice schools through the model function of the mentors. These theories are colored by various views (Zanting, 2001).

It is clear that the tension between theory and practice is an important factor in the practical training of student teachers. On the one hand both teacher educators and student teachers consider practical training as an effective way to acquire (practical) knowledge, on the other hand it is argued that the realization of teacher training goals is occasionally impeded by the conformist and conservative influence that practical training can have on student teachers (Zeichner, Tabachnick & Densmore, 1987).

Over the last few years, research on the relationship between theory and practice in teacher training has focused on the question of how student teachers can integrate theory and practice and what the relationship between the two components should be, or which of the two has to be the point of departure when designing the learning environment (Eraut, 1994a,b; Leinhardt, McCarthy Young & Merriman, 1995; Ruthven, 2001). There is no unambiguous conception of theory, nor of practice or the relationship between the two. Little is known of how student teachers construct professional knowledge; this is particularly true in relation to primary teacher education in the Netherlands.

With respect to primary mathematics teacher education in the Netherlands, in the 1990s new developments were initiated by a group of twelve expert educators. This resulted in a book that became a standard work for teacher educators (Goffree & Dolk, 1995). This publication was also a reason for developing the Multimedia Interactive Learning Environment (MILE) for primary mathematics teacher education, as a medium between theory in teacher training colleges (Pabos) and student teachers' training practice (Dolk, Faes, Goffree, Hermsen & Oonk, 1996). MILE is a digital representation of primary school practice for mathematics, which enables student teachers to intensively study authentic practice within the primary school (see chapter 3). Research relating to the new learning environment from the very beginning targeted student teachers' ways of constructing knowledge, with teaching practice as the starting point for the student teachers' learning process.

Research into student teachers' knowledge construction is of vital importance for the current and future curriculum development of primary mathematics teacher education.

Such research can be considered in the context of at least three current issues.
First, there are complaints from inspectors, managers, teacher educators as well as from student teachers about the level of the programs offered by teacher training colleges (e.g., Inspectie van het Onderwijs, 1998; Onderwijsraad, 2005). Beyond organizational conditions (such as no time for developing deeper understanding; overloaded programs), there are 'content-dependent' reasons for this superficial level of programs. One is the nature and the content of the learning environment for student teachers, which often lacks a well thought-out strategy for linking theory and practice. Another reason is the problem of how to gauge student teachers' level of reflecting on practice, particularly in relation to their use of theoretical knowledge.
Second, student teachers do not automatically appreciate theory (Lampert & Loewenberg Ball, 1998). They often have their doubts about the point of (formal) theory (e.g., Clark & Peterson, 1986).
Third, the age-old 'gap' between theory and practice exists in different forms and on different levels. Although Freudenthal contended already in 1987 (p. 14) that "a gap is not necessary", recent researchers and teacher educators still refer – directly or indirectly – to the existence of that phenomenon (e.g., Ball & Cohen, 1999; Jaworski, 2006; Van Zanten & Van Gool, 2007; see section 2.4).

1.2 Purpose and relevance

The importance of integrating theory and practice by (future) teachers is acknowledged everywhere. Very little is known even now about the character of that process of integration. The complexity of behaviour in practice and a lack of clarity about the concepts of theory, practice and the relationship between the two, complicates the discussion about the subject. This study intends to contribute to that discussion and to the development of theory regarding the relationship between theory and practice.
The purpose of the present study was to gain insight in the student teachers' process of integrating theory and practice, and particularly to find out *how* they relate theory and practice and *to what extent* they are competent to use theoretical knowledge in multimedia educational situations.
It demands a huge effort of (future) teachers and their educators to become familiar with the idiosyncratic and complex reality of teaching. In an elaborated model of teaching practice, Lampert designed an image of the 'Complicated Terrain of Teaching' (Lampert, 2001). Practice and theory as well as the relationship between the two are a part of this complicated terrain (Eraut, 1994b; Leinhardt et al., 1995; Jaworski, 1999).
The developments over the last thirty years in the area of Dutch primary mathematics teacher education (Pabo), led to an approach of integrating subject matter, pedagogical content matter and school practice. However, such an approach does not in itself lead

automatically to student teachers' integration of theory and practice. Acquiring a teacher's professional knowledge base in primary mathematics teacher education, the area in which this study takes place, requires a constructive commitment and much effort to become 'owner' of the specific insights and procedures. Further research should show how student teachers link theory and practice in an adequate – for example multimedia – learning environment and should also express the quality of these activities.

The major *scientific relevance* of this research lies in its contribution to gaining an insight in the student teachers' process of integrating theory and practice and to find out to what extent they are competent in relating the two. Insight in that relationship can lead to a better understanding of the complexity of acting in practice.

The *societal relevance* of this research is twofold. First, (future) teachers' use of theory is part of the 'linking process' between theory and practice, particularly in the way that theory supports observing and analyzing practice, and can therefore lead to improving (future) teachers' practice. Theories can provide an instrument for teachers to recognize more quickly and adequately all kinds of aspects of the teaching-learning process. Teachers that can handle such an instrument are able to see more in the same situation and therefore can think, speak and act more effectively (Fenstermacher & Richardson, 1993). Second, establishing a knowledge base that underlies teachers' practice is a condition for improving the status of teaching as a profession (Booth, Hargreaves, Bradley & Southworth, 1995). Theoretical knowledge as part of practical knowledge (see section 2.3) is considered to form a part of the professional knowledge base of teachers (Verloop, Van Driel & Meijer, 2001). Prospective teachers should provide the experience that using theory is interesting and will gain a profit for one's practice as a professional teacher.

The present research might be a contribution *to avoiding the gap* between theory and practice.

1.3 Research questions

The research questions described below are related to the consecutive research phases of this thesis.

The first exploratory study (section 3.5) focused on knowledge construction and on investigation processes experienced by student teachers in the Multimedia Learning Environment MILE. In total, 15 meetings with two student teachers were held, eight of which were two-hour sessions with participation of the researcher. The culmination of the student teachers' investigations consisted of an oral exam, a written report, and a presentation. Audio recordings during the discourse, e-mail communications, and written reflections documented the collaboration and the individuals' learning and thinking processes.

The underlying research question was:
What is the character of the investigation process of student teachers in MILE and what is the output of their learning process in terms of knowledge construction?
The second exploratory study (section 3.8) was designed to find out how prospective teachers made connections between theory and practice in MILE. Ten two-hour meetings were held in two classes of 25 student teachers. Four pairs of student teachers were observed and interviewed, and a participating study of the group work was conducted with two student teachers. A list of possible signals of theory in action ('Signals of use of theory') was generated to support and analyze the observations of student teachers at work (cf. appendix 1).
The research question in this context was:
Which signals of utilizing theory do student teachers show in their reflections on studied practices of MILE?
The small scale study (chapter 4) targeted student teachers' use of theory in a more structured and 'theory-enriched' learning environment. The research procedure consisted of eight components for triangulation. Five meetings were held in Amsterdam (6 student teachers) and Alkmaar (8 student teachers). All meetings were videotaped by the researcher. In the first meeting student teachers were given an initial assessment and a written numeracy test. After the third meeting, a 45-minute stimulated recall interview was held with each student teacher. In the fourth meeting student teachers made a concept map. In the last meeting there was a final assessment and a (anonymous) questionnaire. Shortly after the course each student teacher was interviewed.
The research question for this phase was:
In what way and to what extent do student teachers use theoretical knowledge when they describe practical situations, after spending a period in a learning environment that invites the use of theory?
A sub-question to this question in the small scale study was:
To what extent is there a relationship between student teachers' use of theory and their level of numeracy?
The main goal of the large scale study (chapter 5) can be described globally as gaining insight into the phenomenon of 'theory use' by students in teacher training colleges for primary school teachers. Two dimensions are distinguished. On the one hand theory use manifests itself in the way students describe situations with the aid of theory; this is called the *nature* of theory use. This may occur for example in a factual description of a teaching situation or by responding to a situation. On the other hand, theory use is expressed by the degree to which the students use the theoretical concepts meaningfully, the *level* of theory use.

The large scale study was performed on 269 students from 11 different teacher training colleges. The learning environment of the student teachers was a more sophisticated version of the learning environment from the small scale study. The research procedure consisted of four components: the initial assessment, the final assessment, followed by an anonymous questionnaire and a written numeracy test after the course for the participating student teachers. The emphasis of the data-analysis was on the student teachers' reflective note in the final assessment.
The large scale study focused on three main questions, with the third question split into two sub-questions:
1. In what way do student teachers use theoretical knowledge when they describe practical situations, after spending a period in a learning environment that invites the use of theory?
2. What is the theoretical quality of statements made by the student teachers when they describe practical situations?
3a. Is there a meaningful relationship between the nature and the level of theory use? If so, how is that relationship expressed in the various components of theory use and in various groups of students?
3b. To what extent is there a relationship between the nature or the level of the student teachers' use of theory and their level of numeracy?

1.4 Nature of the study

The research approach of the whole study – comprising the four sub-studies – can be considered as an amalgam of exploratory research, qualitative research (Denzin & Lincoln, 2000), empirical research (Richardson, 2001) and design research (Gravemeijer, 1994). The design research emerges especially in the way in which the teacher education curriculum – in particular the student teachers' learning environment – for this research was constructed, both in the way that it was formed and in its use by pre-service teachers. The learning environment – necessary to develop for elaborating and answering the research questions – was gradually refined in accordance with the developmental research or educational design research approach (Gravemeijer & Cobb, 2006; section 2.7.2).

1.5 Outline of the thesis

In chapter two the theoretical foundation for this thesis is worked out. First, in the context of the discussion about relating practical and propositional knowledge, it is necessary to know in which way teacher education programs are different. Section 2.2 distinguishes ways in which teacher training colleges attempt to come to a balanced view of both theory and practice and, as a consequence, to a view of the relationship between those two components of the knowledge base of teaching. After discussing the concepts of theory and practice in teacher education (2.3), we focus on that relationship

(2.4) finally pointed specifically to the situation in primary mathematics teacher education in the Netherlands (2.5). Section 2.6 aims at gaining insight in characteristics of a domain-specific instructional theory, which insight will lead us to qualifying focal points for theory in teacher education, derived from the characteristics that have been found. These points of interest are important for designing the learning environment (2.7) for the student teachers involved in this research.
Chapter three continues with a description of the (making of the) multimedia learning environment MILE. Next, we address the question – in the first exploratory study (3.5) – of how student teachers do their investigations in MILE and which knowledge they acquire from their particular way of studying and learning. In section 3.8, on the second exploratory study, the main question is how prospective teachers make connections between theory and practice. The focus is particularly on signals of utilizing theory that student teachers show in their reflections on studied practices in MILE. The results of the second exploratory research provided us with tentative evidence that the intended learning by student teachers in the digitized learning environment could not be realized without theoretical enrichment. The study raised questions concerning quality, namely the depth of learning from practice. Among other things it was established that good practice of primary mathematics teaching in MILE – indicated as practice that is theoretically founded and observable in classrooms – does not naturally lead to good practice for primary mathematics teacher education.
Chapter four describes the small scale study in the adapted 'theory-enriched' learning environment for the student teachers involved. We address the question of how, and to what extent, student teachers use theory when they describe practical situations after spending a period in this learning environment. A sub-question focuses on the relationship between the use of theory and student teachers' level of numeracy. The small scale research provided insight into the thinking and learning processes of student teachers – particularly their reasoning – and also provided elements for refining the design of the learning environment in the large scale study.
The fifth chapter, on the large scale research, addresses the issues of the *nature* and the *level* of use of theory. The questions in this research were about the way that prospective teachers used theory when they described practical situations, about the quality of their reflections on the (multimedia) practice situations, and the possible link between the nature and the level of use of theory. A reflection-analysis instrument was developed for analyzing the data of the student teachers' reflections of the final assessment. As in the small scale study, one question focused on the relationship between the use of theory and student teachers' level of numeracy. The instrument for the last question was a refined version of the one that was used in the small scale study.
Chapter six concludes this thesis with a general conclusion and discussion.

2 Theory and practice in teacher education

2.1 Introduction

Over the last decades, the problem of theory versus practice in teacher education has increasingly become of interest. Before, the topic was highlighted in particular by Dewey (1933), who distinguished 'reflective action' and 'routine action.' In the 1980s, there was renewed interest for this topic through the work of Donald Schön (1983). His ideas and conceptions – not primarily concerned with teachers – are among those that have contributed to researchers and teacher educators becoming aware that professionals rarely simply 'apply' theory in their practice. A teacher decides on the basis of all kinds of situation-related components. Theoretical knowledge and insight do play a part, but they do not unambiguously determine the behavior of the teacher (Schön 1983, 1987).

Schön mentions the 'reflective practitioner' as someone who is able to consider his practice reflectively, not only before and after, but also *during* the performance of that practice (reflection in action). There is an extensive literature relevant to Schön's ideas, gradually also followed by critical response (e.g., Gilroy, 1993; Eraut, 1995a). Other shifts of accents in the last few years have influenced the theory versus practice discussion. The focus on the (prospective) teacher's thinking process and beliefs characterizes the changes in research on teaching. This focus originates from the idea that the behavior of the teacher can only be understood well, if the cognitions and conceptions that guide this behavior are also taken into consideration. Along with content knowledge, pedagogical content knowledge and general pedagogical knowledge, practical knowledge is seen as an important component of the knowledge base that underlies all actions by teachers (Elbaz, 1983; Carter, 1990; Verloop, 1992).

Teacher training colleges have come to realize that prescriptive transfer of theory is not enough (Brouwer, 1989). At the same time it has become clear that the content itself failed to meet expectations; theory was insufficiently in step with reality and with the complexity of action in practice (Cohen, 1998; Coonen, 1987, p. 243; Corporaal, 1988, p.13; Drever & Cope, 1999; Verloop, 2003, p. 203). Furthermore, student teachers are confronted with different types of 'theory' in their practice schools – through their supervisors' exemplary role (Zanting, 2001). The extent to which the activities of students match the goals of training will partly depend on the level and type of cooperation between training institute and practice school (Emans, 1983; Watts, 1987; Wubbels, Korthagen & Brekelmans, 1997).

It is clear that practical training of student teachers is a factor in the tension between theory and practice. On the one hand both teacher educators and student teachers consider practical training to be an effective way to acquire (practical) knowledge, on

the other hand it is claimed that the realization of teacher education goals – also in terms of integrating theory and practice – is occasionally impeded by the conformist and conservative influence that practical training can have on student teachers (Zeichner et al., 1987). That influence can be a disadvantage for strongly practice-oriented teacher training. There is still another disadvantage to the practice-directed approach. The one-sided focus on school practice leads to insufficient depth in the reflective competence of student teachers (Coonen, 1987).

In the course of the next sections we go from a more general analysis of the concepts of theory and practice in teacher education to a more specific focus on these concepts within the context of mathematics teacher education.

2.2 Orientations in teacher education programs

Over the last few years, research into the relationship between theory and practice in teacher training has focused on the question of how student teachers can integrate theory and practice and in which sense the design of the learning environment can contribute to that integration. However, no unambiguous conception of theory exists, nor of practice or the relationship between the two. In the context of the discussion about relating practical and propositional knowledge, Thiessen (2000) distinguishes three orientations that have been emphasized in teacher education over the last 40 years:

- 'impactful behaviors,' leading to the training of prospective teachers in behaviors that appeared to be effective in process-product research;
- 'reflective practices' and,
- 'development of professional knowledge.'

The three orientations should not be seen as mutually exclusive, all are more or less recognizable in current programs. The 'impactful behaviors' orientation dominated in the 1970s. Particularly according to the initial teaching preparation programs, this orientation appeared to be unsuccessful in linking student teachers' theoretical and practical knowledge. Gradually the awareness grew that in order to understand the behavior of the teacher, cognitions have to be considered as well (Clark & Peterson, 1986). The 'reflective practices' orientation emerged in the 1980s, after increasing criticism on the empirical base underlying the 'impactful behaviors' orientation. According to Thiessen, the reflective practices orientation concentrates on skills which help beginning teachers think through what they have done, are doing or are about to do (Thiessen, 2000, p. 520). In his view, while there are numerous published reports on program innovations in support of the reflective practices orientation, the conceptual rigor and empirical foundation of this work are uneven and less developed. Zeichner (1994) presented an analysis of the different conceptions within this orientation. He distinguished five traditions of reflective practice for teaching and teacher education: the Academic, the Social Efficiency, the

Developmental, the Social Reconstructionist, and the Generic Reflection Tradition. Though intended for the U.S., Zeichner recognizes his framework of traditions of reflective practice in other countries as well. Reflective practice is still an important orientation in many teacher education programs, although this approach is also criticized by different authors. For example, Eraut (1995a) posits that a (prospective) teacher is often faced with lack of time to reflect in action, because of the necessity to react immediately (cf. Dolk, 1997). Furthermore, a danger is that reflections remain superficial through lack of – subtly 'fed' – adequate theoretical knowledge (Kennedy, 1992; Oonk, 2001). Another problem is the (tacit) interpretation of the different concepts. Terms such as reflective practice and reflection in action encompass some notion of reflection in the process of professional development, but at the same time disguise conceptual variations that have implications for the design and organization for teacher education courses (Calderhead, 1989; Boerst & Oonk, 2005).

The third orientation – the 'development of professional knowledge' – that Thiessen (2000) mentioned, is the most recent one. He claims that this orientation is the most promising for teacher education. In his view – considering the image of teaching as 'knowledge work' – the emphasis on concurrent use of practical and propositional knowledge distinguishes this orientation from the impactful behaviors and the reflective practices orientations. He argues that student teachers should experience "the concurrent use of knowledge in each pedagogical phase and context – on campus through strategies which focus on practically relevant propositional knowledge and in schools through strategies which focus on purposeful, defensible practice (i.e. propositionally interpreted practical knowledge)" (Thiessen, 2000, p. 529). What he contends in this way about relating theory and practice, is to some extent in accordance with ideas of Eraut (1995b) and Leinhardt et al. (1995). Verloop et al. argue that, although the importance of integrating formal theoretical knowledge and teacher knowledge is evident, it is necessary to come to a balanced view of both theory and practice before the relationship between those two components of the knowledge base of teaching can be studied adequately (Verloop et al., 2001, p. 445).

In fact, the central question here is which training method will prevent a gap arising between theory and practice. Another, related question, focusing on the development of student teachers, is how integrating several elements of the knowledge base of (prospective) teachers can be realized and how this integration can be stimulated.

As yet, little is known about how student teachers construct knowledge or of the way in which students link theoretical knowledge and practical situations, both vital components of learning to teach. In the next section we will elaborate further on the concepts of theory and practice as well as on the relationship between those concepts.

2.3 The concepts of theory and practice in teacher education

2.3.1 Theory in teacher education

Research literature shows a large variation of definitions and opinions concerning the meaning of the concept of theory. The roots of that concept date back to ancient times, in particular to the Greek philosopher Aristotle. Over the last decades, researchers have rediscovered and deepened his ideas within the context of recent developments in education (Fenstermacher, 1994; Korthagen & Kessels, 1999), in ethics (Nussbaum, 1986), in the theory of knowledge (Toulmin, 1990) and in social science (Van Beugen, 1988). Particularly Aristotle's opinions on the manifestations of knowledge are frequently cited.

Aristotle distinguishes philosophical-contemplative knowledge (the nous), knowledge that is related to the surrounding world (epistème), knowledge of practical-ethical action (the phronèsis) and 'practical' knowledge, skills (the technè). Aristotle considers the first two forms of knowledge superior to the last two. The relationship between the nous or the epistème and reality remain limited to a mental connection, by which, in Aristotelian terms, those two forms of knowing distinguish themselves sharply from knowledge that is aimed at practical action. In present terminology, for nous and epistème, and to a smaller degree phronèsis, we might speak of knowledge which originates from considering phenomena. Such a consideration involves reflection on reality by taking distance from that reality. According to Van Beugen (1988) such a reflective attitude emerges on three levels:

- the reflective attitude that one can adopt in contact with the surrounding reality as an expression of the human ability to know;
- knowledge that rests on generalized experiences;
- knowledge as a system of verifiable judgments according to epistemological rules (scientific theory).

Reflection can lead to 'theory,' according to our view meaning a coherent collection of underpinned judgments or predictions concerning a phenomenon. At the highest level – that of scientific theory – we then end up at the development of a theory that can be expressed in theoretical terms and laws (Koningsveld, 1992). Fenstermacher (1994) demands different requirements of theoretical (formal) knowledge, this is 'justified true belief' for formal knowledge in scientific settings and, 'objectively reasonable belief' as an acceptable form for formal knowledge that is used within the context of the educational practice. In section 2.6 we will elaborate further on the concept of theory by focusing on the characteristics of domain-specific instructional theory (Treffers, 1987).

Also in the field of research on teacher training, we find a jumble of almost equal, related or overlapping elaborations of the concept of theory. Thus, there is a distinction

between objective and subjective theory (Corporaal, 1988), public and personal theory (Eraut, 1995b) academic and reflective theory (Smith, 1992) or academic and practical theory (Even, 1999). Considered in extremes, the distinction concerns the difference between scientifically oriented conceptualization and personal, situational perception of educational phenomena. Between these extremes there exists a range of ideas and conceptions concerning the meaning of the concept of theory, for example characterized by the concepts of abstract or concrete, universal or specific, generalizable or situational, true or not proven, objective or subjective, formal or informal, justified or plausible. Eraut's description of what he defines as theory reflects the common (broad) interpretation of researchers: "Educational theory comprises concepts, frameworks, ideas, and principles that may be used to interpret, explain, or judge intentions, actions, and experiences in educational or educational-related settings" (Eraut, 1994a, p. 70).
However, in that plurality of conceptions a tendency can be observed. Many researchers who distinguish personal or subjective theory in their descriptions of theory honor the belief that each action of the teacher is also an expression of theory (Schön, 1987; Carr & Kemmis, 1986; Elliot, 1987; Griffiths, 1987). The source of that idea must be sought in Aristotle and Dewey (Van Beugen, 1988) and the recent tradition of critical theory (Griffiths & Tann, 1992).
Little is known as yet of how student teachers construct theoretical knowledge and how that process of acquiring knowledge is influenced by their experiences and beliefs (Branger, 1973; Cooney, 2001a; Eraut, 1994a,b; Kagan, 1992; Corporaal, 1988; Coonen, 1987; Grossman, 1992; Hofer & Pintrich, 1997; Jaworski, 2001; Nettle, 1998; Richardson, 1989). Student teachers are frequently of the opinion that they are not offered the theory they need to prepare for their school practice (Knol & Tillema, 1995) and often appear not to be able to integrate the offered theory with their practice (Kagan, Freeman, Horton & Rountree, 1993; Cohen, 1998; Lampert & Loewenberg Ball, 1998).
A prominent function of theory is providing an orientation base for reflection on practice. Studies into research of professional knowledge for teachers, particularly into views on the knowledge-practice link, describe a range of ideas and tools for teachers that are seen as useful for fruitful recognizing and analyzing matters of practice. For example, Tom and Valli (1990) describe one of four ways to portray knowledge as related to practice: "knowledge as a source of schemata that can alter the perception of practitioners" (p. 384). Grimmett & MacKinnon (1992) analyze in their review study among other topics the research of Kohl (1986, 1988), who "(…) is committed to teachers taking control of their work through the refining of their craft" (p. 419). According to Grimmett and MacKinnon, the essential focus of Kohl's books is developing *teaching sensibility*, which finds its expression in the idea of loving students as learners.

Fenstermacher (1986) and Fenstermacher & Richardson (1993) introduce the idea of practical arguments. Practical argument is the formal elaboration of practical reasoning: laying out a series of reasons that can be viewed as premises, and connecting them to a concluding action. Practical reasoning describes according to Fenstermacher and Richardson (p. 103), the more general and inclusive activities of thinking, forming intentions and acting. The authors contend that the process of eliciting and reconstructing practical arguments allow teachers to take control of their justifications, and therefore take responsibility for their actions. Practical argument seems a usable concept. For student teachers it is a reason to use theory in practice, and so for teacher education it is a reason to 'feed' student teachers' learning environment with relevant theory. Pendlebury (1995) agrees on Fenstermacher's and Richardson's assertion that good teaching depends upon sound practical reasoning, but she doesn't agree with their statement that an improvement in teachers' practical arguments results in better practical reasoning. She thinks that sound practical reasoning requires situational appreciation, a way of seeing which is better nurtured by stories than by formal arguments (Pendlebury, 1995, p. 52). It is a relevant comment on Fenstermacher & Richardson's statements. The learning environment of (student) teachers does in any case need the feeding – both implied and explicit – with a variety of theories and theory laden stories and furthermore, the guidance of an expert in order to level up the student teachers' practical reasoning. Moreover, the expert has to be aware of the importance of learning by interaction (Elbers, 1993) and of 'constructive coaching' (Bakker, Sanders, Beijaard, Roelofs, Tigelaar & Verloop, 2008)[1].

An important question is to what extent the underlying intentions of theoretical reflecting, namely: understanding, formulating, describing, explaining, and improving practice can be realized for student teachers.

2.3.2 Practice in teacher education

The concept of practice can perhaps be best translated as 'professional situation.' It is a (learning) environment – with materials, tools and actors – in which a profession is practiced. The professional worker in that environment has been trained to act professionally, that is to say to act adequately on the basis of (practical) knowledge. A teacher can also be considered as someone who practices a profession (Verloop, 1995). Practice has many representations, which can be based on a number of views. For example, within the Dutch primary education system there are the views of Montessori, Dalton, Freinet, Jenaplan and the Free School. In the case of teacher education, school practice is an important representation of practice, being a learning practice for prospective teachers. In the report of the visitation for the Teacher training colleges (Pabo) in the Netherlands (Inspectie van het Onderwijs, 1989), seven functions of school practice have been described, for example, the function as a training area for

learning to teach or the function of the practice school as a laboratory to review and improve student teachers' educational designs. The functions illuminate the contribution of school practice to the learning environment of the Pabo. A specific elaboration of a learning practice for primary mathematics student teachers is the Multimedia Interactive Learning Environment MILE, that has been a part of the Pabo learning environment for a number of years (Dolk et al., 1996), in the shape of a digital representation of primary school practice for mathematics (chapter 3).

2.3.3 The knowledge base of the (prospective) teacher

In recent years there has been much attention to two characteristics of professionalism, namely monitoring the level of professional actions by experts or by the teachers' network and, secondly, working from a knowledge base which gives direction to professional actions (Verloop, 1999). We will look at the second characteristic. Since the 1970s, the study of the teacher's professional knowledge base has received new impulses as a result of increased attention to factors that guide the actions of the teacher, such as cognitions, aims and beliefs. Up to that time the emphasis was on process-product research and on studies into effective teaching (Rosenshine & Stevens, 1986; Shulman, 1986a; Creemers, 1991). While Rosenshine & Stevens in the Handbook of research on teaching placed a heavy claim on the role and the outcomes of process-product research, in the same book Shulman criticized those studies (p. 13). From the 1970s on, after the 'cognitive shift' (Clark & Peterson, 1986), researchers became more and more aware of the distance between research in academic settings and everyday practice (Schön, 1983, 1987; Richardson, Anders, Tidwell & Lloyd, 1991). That applied in particular to teacher training (Beijaard & Verloop, 1996; Harris & Eggen, 1993; Guyton & McIntyre, 1990). New theoretical conceptions were developed in educational research, such as situated cognition (Leinhardt, 1988; Brown, Collins & Duguid, 1989; Borko & Putnam, 1996; Herrington, A., Herrington, J., Sparrow & Oliver, 1998), constructivism (Piaget, 1937, 1974; Kilpatrick, 1987; Cobb, Yackel & Wood, 1992; Von Glasersfeld, 1995; Gravemeijer, 1995), narritivism (McEwan & Egan, 1995; Oonk, 2000), metacognition (Brown, 1980; Boekaerts & Simons, 1993) and learning styles (Vermunt, 1992).

The value of the new conceptions is not proven so much through (comparative) research in terms of effective education, but the new concepts serve especially as a rich source of inspiration for reform (Verloop, 1999). The source provides a cognitive tool with which teachers can improve the formulation and recognition of the teaching-learning process (Fenstermacher & Richardson, 1993; Tom & Valli, 1990). Also, in recent years an entirely new direction in the study of the professional base of knowledge for teachers has emerged. The study of the so-called practical knowledge or teacher knowledge wants to honor the insights into professional practice developed by the teachers themselves. Moreover, there is an intention to examine teacher cognitions more

in context, for example without taking a priori defined variables and analysis categories of researchers as a starting point.

2.3.4 Teacher practical knowledge

It was particularly Elbaz, with her case study called 'The Teacher's practical knowledge: Report of a Case Study' (1981; 1983), who marked the change from research of teachers' thinking to research of teachers' practical knowledge (Calderhead, 1996). Elbaz came to her study especially through dissatisfaction with what she saw as incoherence in the approach of research into the work of the teacher. She considers practical knowledge particularly as personally colored, situational knowledge. In the Netherlands, Verloop (1991) gave an initial interpretation of the concept of 'practical knowledge' in his inaugural lecture 'Practical knowledge of teachers as part of the educational knowledge base.' He referred to teachers' practical knowledge as a blind spot in educational research, as this type of knowledge had not yet been given a place in descriptions of knowledge that teachers should either have or have to acquire. This is generally implicit knowledge concerning all kinds of aspects of learning and teaching. Theoretical notions can be a part of it, but also images and ideas of experiences, for example from teachers' own educational history. International literature of educational research shows us different names for practical knowledge, such as craft knowledge, wisdom of practice and personal knowledge (Grimmett & MacKinnon, 1992). We follow Verloop, Van Driel & Meijer (2001, p. 446) by using the labels 'teacher knowledge' – or 'teacher practical knowledge' – to indicate the whole of the knowledge and insights that underlie teachers' actions in practice. The concept of 'knowledge' in 'teacher knowledge' is used as an overarching, inclusive concept, summarizing a large variety of cognitions, from conscious and well-balanced opinions to unconscious and unreflected intuitions. We will stress that teacher (practical) knowledge is not opposite to theoretical or scientific knowledge. In fact, knowledge gained from lectures, self-instruction and other sources of teacher education may be absorbed and integrated into (student) teachers' practical knowledge. Because practical knowledge is often not simply discernible in teachers' actions, it needs expertise to make practical knowledge explicit. Elbaz outlined characteristics of that 'tacit knowledge' and made a plea under the motto 'giving voice to the tacit' for research into the possibilities of making that knowledge explicit (Elbaz, 1991). Meanwhile research results of study of practical knowledge have been published; this concerns mainly study of the practical knowledge of (prospective) teachers in secondary education (Leinhardt & Smith, 1985; Peterson, Fennema, Carpenter & Loef, 1989; Wubbels, 1992; Meijer, 1999; Korthagen and Kessels, 1999; Verloop et al., 2001). In that research two important research lines can be distinguished. The first not only aims at conscious knowledge realized by reflection, but also at less conscious knowledge (Wubbels, 1992). The terms 'image' (Calderhead,

1989) and 'Gestalt' (Korthagen, 1993) are core concepts in that approach. The second research line concerning study of teachers' practical knowledge, is the study of domain-related cognitions. This direction has in fact been launched with Shulman's well-known article (1986b), in which is contended that a fundamental component of the expertise of teachers is a matter of translating content knowledge to knowledge that is suitable to educational situations. He studied the kinds of teacher knowledge that teachers possess and that underlie their actions, and developed an overview of domains and categories of teacher knowledge (Shulman, 1987).

- content knowledge;
- general pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter;
- curriculum knowledge, with a particular grasp of the materials and programs that serve as 'tools of the trades' for teachers;
- pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding;
- knowledge of learners and their characteristics;
- knowledge of educational contexts, ranging from the workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures;
- knowledge of educational ends, purposes and values, and their philosophical and historical grounds.
  (Shulman, 1987, p. 8).

Since then, much attention has been given in the international research literature to this 'pedagogical content knowledge' (e.g., Cochran, De Ruiter & King, 1993; Even, 1990; Even, Tirosh & Markovits, 1996; Lerman, 2001; Grossman, 1990; Gess-Newsone & Lederman, 1999). We follow Van Driel, Verloop & Vos (1998), who consider pedagogical content knowledge as a specific type of practical knowledge. In comparison with experienced teachers, student teachers' practical knowledge will be different, supposedly more extreme, which means either more theoretical (formal) or more of a 'practical wisdom' character (informal). Experienced teachers select (filter) useful knowledge on the basis of their teaching experience; student teachers mainly have to draw from experiences from their own educational history or from knowledge that they acquired in 'colleges' (Cohen & Ball, 1990; Stipek, Givvin, Salmon & MacGyvers, 2001).

In section 2.4 we discuss the meaning of the phenomenon 'theory and practice' in teacher education, firstly in the more general sense, subsequently aimed at mathematics teacher

education and, finally with respect to the specific situation in the Netherlands (2.5).
In section 2.6 we discuss the characteristics of the knowledge base for the subject area of learning and teaching mathematics at primary teacher training colleges (Pabo), at the center of this study.

2.4 The relationship between theory and practice in teacher education

Teacher training colleges have already struggled for decades with the problem of how to define the theoretical dimension of the training programs (Kennedy, 1987). The simplest approach was: you will learn theory during lectures and will then apply it in practice. Drever & Cope (1999) had to say the following about that: "Theory, in this context, was presented as a kind of pseudo-scientific justification for practitioner action, the implication being that, by using it to generate hypothetical solutions to problems, it could be 'applied in practice.' " Student teachers often indicated that knowledge acquired in teacher training did not enable them to handle the uncertainty, the complexity and the instability of actual practice situations (Coonen, 1987; Corporaal, 1988; Zeichner & Gore, 1990; Harris & Eggen, 1993; Oosterheert, 2001). By now one can state that the training philosophy slogan 'Learning theory at academy and applying theory in practice' is outdated. Over the last few years a number of researchers have brought up the problem of the relationship between theory and practice (e.g., Freudenthal, 1987; Bengtsson, 1993; Beattie, 1997; Beijaard & Verloop, 1996; Eraut, 1994a,b; Griffiths & Tann, 1992; Korthagen & Kessels, 1999; Leinhardt et al., 1995; Ruthven, 2001; Jaworski, 2001). Some authors express – often implicitly – the belief that there should be no gap between theory and practice in an appropriate teacher training program. Beattie describes a component of a teacher education program based on the principles of reflective practice and inquiry, where "the theory and practice of teaching and learning to teach are inseparable (…)" (Beattie, 1997, p. 10). Leinhardt et al. stress the important role for teacher education to facilitate the process of linking theory and practice.

> Future practitioners should be given the opportunity to construct their own theories from their own practice, and to thoughtfully generate authentic episodes of practice from their own theories. We have proposed that the university should take on the task of helping learners integrate and transform their knowledge by theorizing practice and particularizing theory. We believe that the university can facilitate this process because it can create opportunities for time and pace alteration, reflection on practice, and examination of consequences. Ideally, such episodes of integration and transformation should be systematic and comprehensive rather than arbitrary and piecemeal (Leinhardt et al., 1995, p. 404).

Freudenthal contends in an article (1987) concerning theoretical frameworks (e.g., learning lines, structures) and theoretical tools (e.g., mathematizing, didactisizing, context) that the gap between theory and practice can be avoided.

> From the wish to understand practice, theory from its side grows and purifies and improves practice. And if theory has been described efficiently enough to re-occur, it will likewise influence the practice of outsiders who did not directly experience the development process. After all, that is the sense and the aim of theory. The proverbial gap between theory and practice does not occur there – as I just said, perhaps somewhat too proudly and too prematurely. I should have been more cautious and say: the gap should not have to exist (translated from Freudenthal, 1987, p. 14).

Van Eerde notices as a result of an analysis of interviews, that Freudenthal for example interpreted observing learning processes as an intuitive process with a more or less implicit role for theory. His observations have been theory-guided, in the sense that theory is only made explicit afterwards, as a reflection on the mathematics teaching that actually occured (Van Eerde, 1996, p. 43). In his last work (1991) Freudenthal chiefly viewed the theory-practice relationship as derived from the level theory of Van Hiele (1973, 1986). He formulated his own, more extended interpretation of the level theory, both concerning subject matter and concepts (levels of learning, practice, theory)[2].

Concerning this thesis we already advanced our conceptions concerning the function of theory in teacher training (section 2.3.1). Our assumption is that reflection of student teachers concerning jointly observed and discussed practical experiences, or reflection as a result of investigations in a (digital) practice, will start a process in which they link theory and practice in a meaningful way. We define 'linking theory and practice' as the adequate use of theoretical knowledge when considering a (current) practice situation. The situation is the starting point of that activity. Therefore the learning environment has to be 'charged' theoretically. The expectation is that theoretical knowledge – as part of the professional knowledge base – will manifest itself in several qualities and gradations. This study takes place within the context of the formerly outlined problems. There are interesting developments in primary teacher education, which might generate answers to the questions that have mentioned in section 2.2. Digital applications such as multimedia learning environments seem to be able to fulfill a useful function within the area between theory and school practice (Lampert & Loewenberg Ball, 1998; Goffree & Oonk, 2001). An environment such as the Multimedia Interactive Learning Environment (MILE, cf. chapter 3) – developed for primary mathematics teacher education but also usable in the field of general education and language teaching – offers a possibility for student teachers to study intensively the authentic practice within the primary school. Student teachers' own school practice, where 'survival' takes first place, is less appropriate for such activities (Ball & Cohen, 1999; Daniel, 1996). Such a learning environment offers the advantages of both 'reflective practices orientation' and the 'development of professional knowledge orientation' (section 2.2). Unhindered by everyday concerns, student teachers can reflect on authentic situations, whereas in the

same learning environment all kinds of content-related and organizational components can be created that will 'feed' the learning environment with theory. In such a learning environment theory can fulfill the desired function of laying an orientation base for reflection on practice (section 2.3.1). We suspect that teacher education arranged in this way should enable student teachers to acquire 'theory-enriched practical knowledge' (EPK; section 2.6.5.5 and 3.9).

2.5 Theory and practice in primary mathematics teacher education in the Netherlands

2.5.1 Introduction

The history of primary mathematics teacher training in the Netherlands shows how the concept of theory has changed and evolved in the course of time from a limited subject matter concept to a more extensive concept that aims at the ongoing development of (prospective) teachers' professionalism (Goffree, 1979, 2000; Freudenthal, 1984a, 1991; Goffree & Dolk, 1995; Dolk et al., 1996; SLO/VSLPC, 1997; PML, 1998; Dolk & Oonk, 1998; Goffree & Oonk, 1999, 2001; Oonk, 2000, 2005; Dolk, Den Hertog & Gravemeijer, 2002; Van Zanten & Van Gool, 2007). Next, in a historical context, we will describe in brief how integrating theory and practice in teacher education developed, in particular concerning mathematics teacher education. First we describe (section 2.5.2) the characteristics of that development before 1971, the year that the Institute for the Development of Mathematics Education (IOWO, nowadays the Freudenthal Institute for Science and Mathematics Education (see also section 3.2) was established. Section 2.5.3 reports on some developments that are characteristic for the last decades.

2.5.2 History

The first Dutch primary teacher training college was established in 1813 by the government. Before 1800, Dutch primary teachers were not specifically trained for teaching as such. For centuries – until the fourteenth century – the profession of teacher was practiced by conventuals. Through the establishment of 'city schools' the convent schools gradually disappeared. However, the teacher's profession changed little: education was mainly seen as memory training. The teacher's work regarding mathematics was generally limited to explaining instrumentally; arithmetical procedures were described and then exercised through an impressive quantity of problems. Providing insight was seen as unnecessary and a waste of time. The quality of teachers differed widely (Kool, 1999). In those days the best pupil from the graduating class of a primary school would be chosen to assist the head teacher on a regular basis and, after additional lessons at home from the head teacher and demonstrating a sound understanding of the subjects, he or she was expected to teach. In a later period the

private lessons by the head teacher became more systematic or normalized and came to be seen as normal lessons. The so called 'Normal Schools' that evolved from this practice became later – around 1800 – the teacher training colleges.
From 1800 up to the present time, development can be seen in views concerning the relationship between theory and practice in the curriculum for primary teacher education. Van Essen suggests:

> Opposite the belief that the prospective teacher had to be an especially 'smart fellow' with a lot of general 'book knowledge' or a sound theoretical, subject matter stock-in-trade, the belief existed that benefit had to be expected in particular from a direct confrontation with school practice (...) (translated: Van Essen, 2006, p. 15).

Nevertheless, it would not be until the introduction of the New Training College Act in 1952, that real change was realized in teacher training. Up to then, the curricula of these training institutes were essentially the same as those for higher secondary schools, albeit with the addition of pedagogy and teaching methodologies and with half a day a week allocated for working in the practice schools. For example, the contents of primary mathematics teacher education in 1923, were arithmetic, algebra and geometry, with the following components:

> *The art of arithmetic:* Knowledge of the central issues of the art of arithmetic: basic operations with whole numbers and fractions; smallest general multiplicator and largest general divider of numbers; geometric proportionalities; determining square roots.
>
> Knowledge of the central issues of commercial arithmetic.
>
> *Maths:* Algebra. Knowledge of the central issues of algebra up to and including the equations of the second degree with one unknown variable.
>
> *Geometry:* Knowledge of the central issues of two and three-dimensional geometry. (Goffree, 1979, p. 19)

As a result of the New Training College Act of 1952, teacher training was changed. The school subjects of secondary education were replaced with the teaching methods for the subjects taught in primary schools. For example, mathematics was replaced with teaching methods for arithmetic (Van Gelder, 1964). However, because the teacher educators remained the same, little changed in practice. The teaching methods for arithmetic were frequently augmented with tough calculations for the student teachers, supplemented with some hints for working in the classroom. Most of these hints were of a general educational nature, they referred to for example teaching with visual aids, and elements of educational psychology such as different levels of thinking (Goffree & Oonk, 1999). Although theory and practice were closer in terms of curriculum development, subjects remained isolated and methods were far from the real practice. Goffree (1979) gives an example of a mathematics method book for teacher education

the title of which, 'Theory and Practice,' raises high expectations. In the explanation by the author J.H. Meijer (1963), it turns out that his concept of 'theory' encompassed the art of arithmetic and that 'practice' implied skills of arithmetic. This belief is characteristic for the lack of vision in teacher education during the 1950s and 1960s. The reorganization of 1968, when teacher-training colleges were to be named by law 'Pedagogical Academies,' did not in general lead to important changes. Because of the idea that the methods of 'all subjects of the primary school' should be taught, the matter of domain-specific instructional theory barely existed. As a result, teacher-training curricula remained fragmentary, with consistency and commonality towards goals lacking. In fact, 'theory' for the student teachers comprised mainly general educational theory. In 1984, the teacher training colleges were reorganized to a four year course, but it was only in the early 1990s that any significant changes occurred. Research by among others Coonen (1987) and Corporaal (1988) indicates that the desired consistency between theory and practice was still absent. While there was a shift towards practice, the need to do so did not necessarily arise from motives and considerations based on teacher training philosophy. Coonen wrote for instance the following about that:

> The respondents [teacher educators; w.o.] mentioned that the stronger orientation on practice also originates from the resistance of student teachers to everything that is associated with theory. Student teachers appear to show little interest for knowledge of a more abstract, deepening and explanatory nature. Because of this lack of interest, one fears that student teachers acquire a too naive, and too subjectively colored repertory of action, as a result of which their reflecting capacity is also limited. Many teacher educators experience the gap between theory and practice as a large problem (translated from Coonen, 1987, p. 236).

The cause of the changes in the 1990s lays in a large scale inspection of all Pabos in 1991 – the first inspection of its kind. The judgment of the inspection was scathing. The criticism was mainly directed towards the lack of a good academic background for primary school teachers and of a clear training concept involving teaching methods. In the following years, a variety of publications appeared with recommendations for improving the quality of primary teacher education (Inspectie van het Hoger Onderwijs en Basisonderwijs, 1996; SLO/VSLPC, 1997; PML, 1998). Problem-based learning, self-instruction and thematic education were espoused, and teacher educators from all disciplines were expected to develop their own materials according to these concepts. Again, the colleges were required to leave behind the paradigm of a program dominated by the school subjects and to look for themes, case studies, and problems that would have obvious validity to the study of teaching per se.

2.5.3 New developments

Although until the 1990s little change occurred generally in the training curricula concerning the relationship between theory and practice, it appeared to be different for the subject area of mathematics teacher education. With the establishment of the IOWO in 1971 (see section 3.2), in the Netherlands a bottom-up development in primary and secondary mathematics education and the related teacher education started. For primary mathematics teacher education, a model for learning to teach was developed (Goffree, 1979; Goffree & Oonk, 1999).

The main idea was that mathematics education, both for student teachers and pupils, should take concrete situations and familiar contexts as its starting point. While mathematization of those contexts plays an important part in the learning processes of children, for the student teachers it is a process of both mathematizing and didactisizing (Freudenthal, 1991). Student teachers carry out pupils' mathematical activities at their own level and then reflect on and discuss the results of those activities. These reflective discussions create a foundation for learning how to work with children. Freudenthal saw reflective thought as 'a forceful motor of mathematical invention,' i.e. *guided* reinvention for the pupils and the student teachers. The guide should provoke reflective thinking (1991, p. 100). In his view, the theory of the level structure of learning processes (Van Hiele, 1973) shows what matters in such processes, namely the discontinuities, 'the jumps' in learning (Freudenthal, 1991, p. 96).

The model for learning to teach was elaborated in books for primary mathematics teacher education (Goffree, 1982, 1983, 1984; Goffree, Faes & Oonk, 1989), which were used in more than 80% of the teacher training colleges. Following the Standards for primary mathematics education and the Standards for mathematics evaluation and teaching (NCTM, 1989, 1992), in 1990 a group comprising ten mathematics educators started developing national standards and presented the results to colleagues as a handbook for teacher educators (Goffree & Dolk, 1995). The philosophy of teacher education elaborated in the handbook is founded on three pillars: a teacher education adaptation of the socio-constructivist vision of knowledge acquisition, reflection as the main driving force of the professionalization of teachers and the interpretation of practical knowledge as a way of narrative knowing (see section 3.1 and 3.2). Discussions between the developers and fellow teacher educators of four teacher training colleges from the United States, who were interested in the Dutch MTE-standards, had major consequences for the training of Dutch primary school mathematics teachers. The Dutch teacher educators became acquainted with Magdelena Lampert and Deborah Ball's MATH project (Lampert & Loewenberg Ball, 1998). The student learning environment developed in their project coincided with ideas developed in the Netherlands about the training of student teachers and the way they learn. It was

decided to develop an environment for student teachers in the Netherlands similar to that developed in Lampert and Ball's project. And so the MILE-project in the Netherlands was born (see chapter 3), in which the developers saw a position between theory and practice in teacher education and, by which the statement "Real teaching practice has to be the starting point of teacher education" became emphasized.

2.5.4 Perspectives

The knowledge base of primary mathematics teacher education (Pabo), the area in which this study takes place, can be distinguished in two ways from other fields in teacher training. In the first place the nature of the mathematical knowledge requires a constructive commitment and much effort to become 'owner' of the specific insights and procedures. Furthermore, the developments over the last thirty years in this area at the Dutch Pabos led to an approach of integrating subject matter, pedagogical content matter and school practice (Goffree, 1979; Goffree & Dolk, 1995; Goffree & Oonk, 1999). However, such an approach does not lead naturally to student teachers' integration of theory and practice. That will perhaps happen if student teachers can use a 'rich' learning environment, for example the multimedia interactive learning environment MILE. Further research should show how student teachers link theory and practice if they have such a learning environment at their disposal; further study will also express the quality of their activities.

Next, as mentioned in section 2.3.4, we will elaborate further on the role of theory in mathematics teacher education.

2.6 Characteristics of a domain-specific instructional theory. Implications for the learning environment of mathematics teacher education[i]

2.6.1 Introduction

2.6.1.1 Focal points for a theory enriched learning environment

To get get an image of the focal points that are essential for the function of theory in teacher training, first an attempt is made to map core characteristics of theory. To achieve this, the domain-specific instructional theory for learning and teaching to multiply is analyzed, the theory that is part of the student teachers' learning environment in both the small scale study (chapter 4) and the large scale study (chapter 5). Examining existing

[i] Section 2.6 has been published in adapted form as: Oonk, W. (2007). Kenmerken van vakdidactische theorie. Implicaties voor de Pabo. [Characteristics of domain-specific theory. Implications for mathematics teacher education]. *Reken-wiskundeonderwijs: onderzoek, ontwikkeling, praktijk* [Primary mathematics education: research, development, practice], *26* (3), 19-32.

theories can make ideas about the ways prospective teachers use theory become manifest (Oonk, 2002). The underlying thought is that focal points for theory in teacher education can be derived from the characteristics that have been found. As a consequence, these points of interest can be important for developing a 'theory-enriched' learning environment for student teachers.

2.6.1.2 Working definition of a domain-specific instructional theory

Earlier (section 2.3.1), the large variation of definitions and beliefs concerning the meaning of the concept of theory was mentioned, as well as the concept's origin in Greek philosophy. According to Plato and Aristotle the purpose of being lay in the theooria or contemplation. In Plato's opinion the highest purpose is to rise above the low. Nevertheless he finds it necessary to return to the low (Bor, Petersma & Kingma, 1995, p. 49). Aristotle, Plato's brilliant student, is even more attached to the relationship between knowledge and reality than his teacher. According to him, knowledge also exists to enrich the everyday world. Scientists often refer to these different modes of thought. Korthagen (2001), for example, clarifies the term theory using two concepts of knowledge as developed by Plato and Aristotle, namely the epistème and the phronèsis. Epistème represents academic, conceptual knowledge, the phronèsis stands for perceptual knowledge, and this is practical wisdom that has been based on the perception of a situation and the reflection on that situation. Korthagen thinks that the development of the last type is the most important for teacher training.
The entirety of assumptions, arguments and conclusions – what the ancient Greeks saw as contemplation (theooria) – provides theory and new knowledge. A changed view on the original problems affords new perspectives and presuppositions and causes a cycle of self-renewing theories. The development of the theory of gravity – with successively the conceptions and findings of Aristotle, Newton and Einstein – is an example of such a course.

For the benefit of the analysis below it is necessary to provide a 'working definition' of a domain-specific instructional theory. It seems possible to formulate such a definition as an extension to what has been discussed in section 2.3.1 and, which is also in line with existing ideas about domain-specific instructional theories (Treffers, 1978, 1987; Freudenthal, 1991; Gravemeijer, 1994). As soon as a collection of descriptive concepts displays consistency, and the coherence has been underpinned, one can speak of theory. Such a system can contain statements and arguments as well as explanations, assumptions, conjectures, predictions and proofs. Very different concepts can be an object of establishing a theory. For example, educational researchers have been looking for years for theoretical constructs which can throw a different light on the development of children. The development of children's numeracy is one such example[3].

2.6.1.3 Selection of the theory to analyze

Considering theories on a continuum from 'pure,' formal (e.g., the Boolean Algebra) to empirical (e.g., Gestalt theory), the theory of learning and teaching multiplication seems to provide a 'rich' situation for analysis. This theory is characteristic of both a formal, mathematical background theory, evolved by deduction, and a more empirical theory developed by induction. The two types distinguish themselves to a certain degree also by objective and subjective characteristics. The characteristic 'objectivity' refers in its most extreme form to scientific theories which borrow their 'status' from acceptance within the paradigm of a scientific community. A scientific theory is considered as (tentatively) true on the basis of the methodology chosen within that paradigm (Koningsveld, 1992). In the development of the method for learning multiplication, there are also subjective characteristics, colored by individual beliefs.
The formal and empirical components of learning and teaching multiplication also differ in the relationship that they have with (associated) practice. That relationship is partly determined by the way in which theory is produced or used (Fenstermacher, 1986), for example 'in action' or 'on action' (Schön, 1983), or by the degree in which theory influences or even directs practice (Eraut, 1994b).

An attempt will be made to identify in reflective analysis features of the theory which are characteristic of the theory, for example the coherence between concepts or the grounding of assertions. This allows to make a distinction between intrinsic characteristics, which are aspects of the internal structure of the theory and the remaining, extrinsic characteristics, which appear in the context in which the theory is used or developed. While analyzing the theory, the aforementioned working definition (section 2.6.1.2) of theory is used as a tentative framework. We will start with a description of the history of the theory, i.e. its genesis and development. On the basis of an analysis of the theoretical characteristics we will come to points of interest for the place and function of theory in teacher education.

2.6.2 The theory of learning and teaching to multiply

2.6.2.1 The origin of the theory in the Netherlands

Learning and teaching to multiply occurs within the world of education, at the level of teaching students and, at the level of the school book authors who provide content and formats to teaching, and at the level of science, where theorists are studying the background of learning multiplication. The 'birthplace,' the field where work was being done at these different levels (student/teacher, developer/author, scientist/theorist) and from different points of view (e.g., mathematics and psychology) can therefore be seen as the context in which the theory was initiated. Learning (memorizing) the tables of multiplication, is traditionally the core of learning to multiply. In primary education

before 1970 – before Wiskobas[4]– memorizing the tables was therefore given most of the attention there was for multiplication. Many children already knew table products before they understood the meaning of the operation of multiplication. Teachers and schoolbook authors considered multiplying as no more than repeated addition. Teachers were taught that way in teacher training. They learned from a mathematical (arithmetical) point of view that 3 x 7 is the same as 7 + 7 + 7. That was calculated step by step as 7 + 7 = 14 and 14 + 7 = 21. This approach became the basis for learning the table products; so, through continuous recitation, students gradually learned more and more answers by heart. The smarter students rapidly realized that 6 times 7 could be calculated by adding up 2 x 7 and 4 x 7. Only those who had to do too much calculating did sometimes lose track and forgot where they were in the list; but without this clever calculating the tables of multiplication became a line of meaningless objects for the students. In terms of educational psychology, this way of learning the tables was based on ideas from the theory of association psychology[5].

Developer and researcher Hans ter Heege tells us about his own past experiences in this field both as a student and as a teacher. Among other things, working with children brought him face to face with his own conceptions. Those conceptions were colored by his own experiences in primary school, and had been confirmed in teacher training college and were reinforced further by working as a primary school teacher using the 'mechanistic' textbook 'To independent arithmetic.' Reflecting on working with children, his own conceptions and his own learning process, partly fed by the discussions in the Wiskobas team, stimulated him to develop a theory in which mechanistic characteristics were lacking. Ter Heege (1978, 1985, 1986) developed a new theory of learning and teaching to multiply.

2.6.2.2 The ingredients of the theory

The theory of learning and teaching to multiply contains the following components:

- Concepts, such as multiplication (with associated notation x), multiplication strategy, informal strategy, repeated addition, structured multiplication, formal calculation, memorization, automation, properties, anchor points, reproduction, reconstruction, models, line structure, group structure, rectangle structure, contexts, practice, apply.
- Indications for teaching which are related to phasing of the learning process of students in levels of acting and thinking, and indications for shortening that process with examples of student activities in the field of concept attainment, memorization, practice and application.
- Directives for entwining learning trajectories and for interweaving learning and teaching (Treffers, 1991; Van den Heuvel-Panhuizen, Buys & Treffers, 1998, p. 41).

In explaining this theory we will limit ourselves to the core concept of multiplication; other parts will brought up in section 2.6.2.4.

2.6.2.3 The concept of multiplication, considered phenomenologically and mathematically

The sum 3 x 4 = is bare (formal) multiplication, and every practitioner will know the answer. This multiplication can be the mathematical representation of a number of everyday situations. The practical value of multiplication is undisputed; we know that the operation of multiplying numbers already occurred in ancient times[6]. It is a means to understand, analyze and communicate situations that have a multiplicative structure.

Situations (manifestations) of the multiplication 3x4 or 2x3x4 are for example:

- 3 pieces of rope of 4 meters;
- 3 pants and 4 shirts, how many combinations?
- 3 golden stars per decimeter on 4 decimeter long Christmas garlands;
- there are 3 of us; everyone gets 4 sheets of paper;
- a square of 3 by 4 tiles;
- 4, three times enlarged;
- 3 routes from the town of Hilversum to the village of Laren and 4 routes from Laren to Huizen. How many routes from Hilversum to Huizen via Laren?
- on the calculator, 3 times 4 becomes…?
- the size of the pond is 2 by 3 by 4 meters. How many liters of water do we need to fill the pond?

The fact that one in three children at the end of primary school will not see a multiplication in some of these situations (Carpenter, 1981), particularly the situations in which combinations appear, illuminates the hidden character of the operation (multiplying) in those situations.

In 1998, developers at the Freudenthal Institute distinguished three types of structures in multiplicative situations: the line structure (3 strips of 4 meters), the group structure (3 boxes with 4 balls each) and the rectangle structure (3 rows of 4 tiles). It is a phenomenological description of mathematical structures, which means a description of concepts and structures (in this case multiplication) related to the phenomena for which ‘they are created and to which they were extended in the learning process of humanity’ (Freudenthal, 1984a, p. 9). The fundamental mathematical background of multiplying cannot be interpreted as readily as its phenomenological aspects. The operations of adding and multiplying and – with them other operations with numbers – are mathematically grounded in number theories. Euclid (approx. 300 before Chr.) was the first who developed a theory of natural numbers, with the main idea that a natural number exists of units.

Euclid says about that among other things:

> Unit is which to which each thing is called *one*. Number is a collection composed from entities. (Euklides VII, def. 2. in Freudenthal, 1984a, p. 86; Struik, 1990)

Until the beginning of the twentieth century this was the dominant concept. Around 1900 this view was formalized by the Italian mathematician Giuseppe Peano (1858-1932) in his design of an axiom scheme for natural numbers. The fifth and last axiom of that scheme, the so-called axiom of complete induction, stipulates in fact the complete, ordered and infinite collection of natural numbers (Loonstra, 1963, p. 18). Complete induction is seen as one of the tools to define addition and multiplication. The definition of multiplication by complete induction can be seen as repeated addition (table production).

A second definition originates from the cardinal approach to multiplication. The German mathematician Georg Cantor (1845-1918), inventor of the Set Theory, lay the foundations for that theory at the end of the nineteenth century by considering the number as a quantity of something. Considered mathematically according to that theory, number is a property of a finite collection, the so-called cardinal number. Adding up can then be defined as uniting disjunct sets, i.e. sets that have no common element. In non-mathematical language this means as much as merging quantities. Multiplication into the cardinal approach is defined as the cardinal number $v = m$ x n of the product set [A,B] of number pairs (a,b) with $a \in A$ and $b \in B$, where set A consists of m elements (cardinal number m) and, the set B has the cardinal number n. Concretization in a grid (or rectangle) model, demonstrates that this definition of multiplication – different from that according to complete induction – easily exposes its properties, because the structure of multiplication becomes visible.

As an example, we choose the set of products $[A,B]$ with $A = \{a1, a2, a3, a4\}$ and B = $\{b1, b2, b3\}$.

With $m = 4$ and $n = 3$ we construct then the matrix (AxB), that visualizes the number 4x3 in a grid structure.

$(a1, b3)$	$(a2, b3)$	$(a3, b3)$	$(a4, b3)$
•	•	•	•
$(a1, b2)$	$(a2, b2)$	$(a3, b2)$	$(a4, b2)$
•	•	•	•
$(a1, b1)$	$(a2, b1)$	$(a3, b1)$	$(a4, b1)$
•	•	•	•

For example by turning the grid over ninety degrees, it becomes visible that 3 x 4 = 4 x 3 (formulated generally: the commutative law applies, m x n = n x m).

What is notable is the effort necessary – even for a trained mathematician – to recognize the two formal approaches of multiplication in the phenomenology of mathematical multiplicative structures. Freudenthal acknowledged that indeed. In his 'Didactical Phenomenology of Mathematical Structures' (1983, 1984a), he writes about the relationship between the two mathematical approaches and about their didactical (methodological) relevance. One of the points he contends is that the set-product, as reflected in the grid model, is didactically (methodologically) inadequate. What he intends to say is that students' learning processes do not need to be a reflection of those mathematical (re)structuring processes[7]. The mathematical foundation does play a background role, but the phenomenology of mathematical concepts and structures is more appropriate for guiding the teacher towards the learning process of the student. Concerning the phenomenological character of multiplication, Freudenthal talks about the obviousness of the operation of multiplying: "No operation – not even adding or subtracting – presents itself so naturally." (Freudenthal, 1984a, p. 120). He then talks about the way in which young children construe the concept of multiplying. Before they are actually confronted with the arithmetical operation, there are years of experience in which they are already building a meaningful foundation with 'multiplicative words' such as 'double' and 'time.' Some examples: I have double; you already heard it three times now; you can take three times four beads or four beads and four beads and four beads. We will see that Freudenthal's views have had much influence on the development of the theory of learning to multiply in the Dutch educational situation. Elsewhere he discussed (Freudenthal, 1984c) acquiring basic knowledge through exercises on the computer. He experienced that determination of outcomes was possible with the computer, but it was impossible to make an analysis of errors or justify an arithmetic procedure; he considered the last to be aiming too high. He could not suspect that three years later Klep (together with Gilissen) would develop a software bundle, called 'a world around tables' which let the computer do much more than merely produce answers. In his dissertation Klep (1998) showed subsequently that it is possible to develop software with which the computer can follow the students' process of meaningful practice.

2.6.2.4 Genesis and development of the theory

The period until 1970: the mechanistic theory

It is only recently that changes have appeared in the vision on learning multiplication. Until the1970s 'drill and practice' were considered obvious for learning the tables of multiplication. In fact, for centuries, learning to multiply had consisted of little more than learning a number of arithmetic rules which had be acquired through 'demonstrating, imitating and practicing.' From the research of Kool (1999) into Dutch arithmetic textbooks from the fifteenth and sixteenth century, we know that at that time teaching multiplication mainly involved giving the students a definition of multiplying

and a table with table products. The purpose in laying the foundation for multiplying was memorizing knowledge that could be used for arithmetic algorithms. In fact understanding of the procedures was not required for using algorithms either, as can be seen in the work of Willem Bartjens, who is an example of a Dutch schoolmaster from the seventeenth century. He became famous with his textbook 'Cijfferinge' (published in 1604; see Bartjens, 2005). His books reflected the arithmetic habits of that time and would remain influential for centuries to come. The expression 'according to Bartjens' typifies the arithmetic of that period; the schoolmaster had to explain how the rule was 'according to Bartjens,' this is 'according to the rules of the art of arithmetic.' This meant instrumental explanation: the arithmetic procedures were first told and then practiced through doing an impressive amount of sums. Teaching understanding was considered unnecessary, and even a waste of time. Students had to learn to do arithmetic as quickly and as well as possible for daily life. Adults needed to train for professions such as clerk, teacher or surveyor; these involved skill and certainty, rather than the underlying explanations. There was no benefit in understanding arithmetic, that was for scientists. Associative psychology and behaviorism have had long and intense influence on that 'practice.' Barely thirty years ago, there were still textbooks in use that were based on a mechanistic foundation. One example of a much-used mechanistic method is 'Naar zelfstandig rekenen (Towards independent reckoning)' from the 1950s. Even today's education still contains elements of that approach. The following page from 'Naar zelfstandig rekenen' illustrates that textbook's approach (fig. 2.1).

2	1x2=2
2+2	2x2=4
2+2+2	3x2=6
2+2+2+2	4x2=8
2+2+2+2+2	5x2=10
2+2+2+2+2+2	6x2=12
2+2+2+2+2+2+2	7x2=14
2+2+2+2+2+2+2+2	8x2=16
2+2+2+2+2+2+2+2+2	9x2=18
2+2+2+2+2+2+2+2+2+2	10x2=20
Learn the two-times table by heart	

figure 2.1 A page from the textbook

'Learn the two-times table by heart' is the only instruction given to the students, while there is no further information on possible didactical approaches for the teacher in the manual either.

In summary, the mechanistic theory of learning to multiply can be characterized as a collection of mathematical symbols, definitions, procedures and views, including:

- the symbols +, x and = ;
- the definition that the repeated addition *b* + *b* + *b* +…(and that *a* times) can be written as *a* x *b* with the silent assumption that both have the same solution (for instance 2 + 2 + 2 = 3 x 2);
- the procedure of learning to multiply through the use of table charts;
- the view that learning to multiply occurs through memorizing table products and applying standard procedures.

That the theory survived for so long is probably linked to the authority of the just as tenacious underlying ideas about learning (associative psychology and behaviorism), possibly in combination with the consensus about the approach among teachers and pedagogues in those days, or the lack of communication about criticism of the approach[8].

The change from 1970 onwards: a new theory

In fact the theory of learning to multiply only significantly changed in the 1970s. Influenced especially by Piaget, who developed and described the clinical interview as a method, an interest in learning and developmental processes in children arose. Such methods were used in mathematics education by several researchers[9]. Ter Heege – a pioneer in the Netherlands on learning to multiply – used it when he discussed their knowledge of tables with children (Ter Heege, 1978, 1986)[10]. Using the title 'Johan, een afhaker haakt aan,' (Johan, a dropout drops in) Ter Heege analyzed seven interviews with Johan, a twelve-year old boy in grade 5. He discovered that Johan, who was known as being weak in arithmetic, applied a great amount of flexibility in calculating basic multiplications. Johan used his own arithmetic strategies which had not been taught before, becoming aware that it was allowed to do so. The yield of these interviews was both the reason and the means for the development of a new theory on learning multiplication.

Looking back on the development, it starts as a personal theory based on personal experience and analyses. What the 'theory' – phrased in conclusions and a recommendation (Ter Heege, 1986, p. 56) – comes down to, is that children find it much harder to learn the products from the tables of multiplication than is suggested by the traditional view on education. Also, the way that children do learn the products, deviates greatly from what teachers and authors of textbooks assume; children often use their own calculation strategies for multiplication. The recommendation is: "Children should learn the basic multiplications by flexible use of a number of characteristics and calculation strategies, such as the commutative property, the strategy of 'one time more' or 'one time less' and the strategy based on the factor 10." Ter Heege finds support for this and other conjectures that have been developed into hypotheses through analyzing of prevailing textbooks and an exploration through the literature on the psychology of memory[11]. This is the case especially for the fact that children will make use of mental strategies on their

own, and that they construct their own cognitive network for basic multiplication. Reflection on that 'practice' of research into memory evokes the next development question in him, namely: "(…) whether it is possible to develop an approach that maximizes the opportunity for children to construct an adequate network for the basic products they have to learn as arithmetical facts and have to be able to apply." (p. 133). In answer to that question, Ter Heege designs a teaching unit for multiplication. He then tests that unit in the practice setting of primary education. In another reflection on practice, which we can now describe as the practice of curriculum development, he describes the theoretical foundations that the development of the program is based upon. In fact, Ter Heege formulates a provisional final version of the theory of learning to multiply. The most important innovation compared to the previous version is its systematic underpinning. He creates the structure for doing so by placing three 'fundamental elements' (p. 171) in a central position: children's own constructions, children's own productions, and so-called horizontal and vertical mathematization (Treffers, 1978, 1987). This occurred against the background of the current domain-specific instructional theory for realistic mathematics education in the Netherlands (Treffers & Goffree, 1985).

Ter Heege clearly looks for connections with that theory and for consensus with its basis and principle. To do so, he further grounds the earlier-mentioned three 'theoretical elements' from his own research. For instance, his research shows that for the role of 'own constructions' in the learning process of students, that children, in their struggle to learn multiplication, look for their own solutions and find their own way. It turns out that the solutions the child itself 'invents' are better and more profoundly understood than solutions that are 'taken' from the teacher.

In 'de Proeve van een nationaal programma voor het reken-wiskundeonderwijs op de basisschool' [Standards for primary mathematics education], part II, chapter 3 (Treffers & De Moor, 1990), we can see how the theory on learning multiplication that Ter Heege developed is adopted and elaborated. To legitimize the 'Proeve,' it was submitted to a large number of experts. This testing has been further strengthened by previous publications in specialized magazines and by peer discussions during conferences. After another eight years the TAL brochure[12] is published (Van den Heuvel-Panhuizen, et al., 1998, 2000, 2001), describing intermediate goals for mathematics education in the lower groups in primary school. The intermediate attainment targets for multiplication and their justification (p. 59-63), echo the spirit of the theory of learning to multiply that is discussed above. The goals in fact present 'the theory in activities,' partly through the nuanced underpinning of the goals and their illustration with core examples.

The description of the domain-specific instructional theory in the TAL-publication leads towards a learning-teaching trajectory in three stages. In the first stage, most children operate at the level of multiplication by counting, helped by the use of jumps on the

number line. They are taught with situations requiring repeated addition. During the second stage they work at the level of structural multiplication. This should put the children into a position in which they can build up or reconstruct table products for themselves (informal strategies). Multiplication can take on the following appearances (models) in context:

- a *line* structure: chain, strip, number line;
- a *group* structure: groups of varying types (bags, boxes, coins);
- a *rectangular* pattern: grids, weave patterns.

Children are able to recognize, partly with the help of the rectangle model, the two core properties of multiplication, namely the distribution property (6 x 8 = 5 x 8 + 1 x 8) and the commutative property (5 x 8 = 8 x 5) (see also Buijs, 2008, p. 41 and note 14).

In the third stage, formal multiplication is used and the tables are gradually automated and finally memorized. Application of the network of table-knowledge will take place through calculating by heart and through algorithmic calculations, keeping the knowledge current.

We can characterize the most recent theory (in 'De Proeve' en 'TAL') as follows:

- As in earlier theories, the students' learning process is seen as a process of reconstruction[13] and defined in stages. Here, however, the stages are substantiated with more nuance and detail.
- Where Ter Heege's theory only gives little attention to models, their role has become much stronger in the Proeve, and even more so in the TAL brochure.
- A special focus on levels appears. Other than in earlier theories on learning to multiply, rises in the level of children's learning process are described that allow for recognizing and utilizing differences between children.
- In general the justification for approaches and choices is sharper than in the earlier described theory.

There are however still clear signs of previous theories[14].

2.6.3 Characteristics of the theory of learning and teaching to multiply

In this section we examine characteristics of the theory of learning and teaching multiplication previously discussed, with the intention to derive focal points for theory in teacher education (section 2.6.4).

We distinguish *intrinsic* characteristics (2.6.3.1), that are aspects of the internal structure of the theory, and *extrinsic* characteristics (2.6.3.2), that appear in the context in which the theory is used or developed.

2.6.3.1 Intrinsic characteristics

Grounding

The domain-specific instructional theory is grounded in reflections on practice (observations) that are colored by the theory-developers' own experiences and opinions.

Along the way the foundation becomes more 'objective' and a more systematic approach to development is taken, among other things through the use of the results of other development researchers and existing theory. The systematic approach can be seen in the cycle from reasoning processes on designing curriculum content, analysis of and reflection on the results from tryouts and the subsequent development of new theory. The quality of the foundation is determined to a large extent by the persuasiveness of the reasoning (e.g., 'multiplicative reasoning').

The range of application of theory

The cyclical process that was referred to before, indicates a strong, reciprocal relationship between theory and practice. We therefore speak of the domain-specific instructional theory of multiplication and the practice of curriculum development. Theory and practice in a way 'question' each other. Practice, through examples and counterexamples, can confirm, clarify or refute the theory, show dilemmas or evoke new connections for the theory. The theory provides understanding of practice by describing, explaining or predicting it, provides solutions for practical problems, is of assistance in justifying choices and more. The concept of multiplication takes its meaning from a number of practices. These are 'daily practices'[15], which initially – until the fifteenth century – involved a limited number of professionals such as merchants and the clergy, the practice of mathematicians for whom the concept of multiplication has a fundamental mathematical meaning and the practice of didacticians, those involved in learning and teaching multiplication. For the latter practice, there is a small distance between practice and theory.

The 'truth' of theory

A typical characteristic in the development of this theory is the designers' search for consensus. It is not just a case of keeping on trying, but also of negotiation. There can be two types of consensus, one relating to finding common ground with other theories, the other relating to the desire to find common ground in the discourse with fellow-scientists or with professionals in the relevant field. Here, development of theory is a subtle combination of individual and collective effort. Not only do such attempts at integration of the newly developed theory with existing theories strengthen that theory, they also facilitate the theory's ongoing development. The threshold of access to the existing paradigm[16] is lowered, along with the opportunity of extending the theory and widening consensus. That wider consensus means that the theory gains in strength and validity.

2.6.3.2 Extrinsic characteristics

Theory in action

The realistic theory in the Netherlands started as a response to mechanistic theory and to the 'New Math' movement in the 1960s. The various prototypes of the realistic theory are a result of adapting, modifying and expanding the previous versions. The designer uses and develops theory when he is working on designing and testing his teaching package. He develops theory as a reflection on his practice[17] and makes use of the latest developments in domain-specific instructional theory. In Schön's terminology (1983) we can speak of interaction between 'theory on action' and 'theory in action.' This is a cyclical process. The process of curriculum development, as we can see it in Ter Heege's work, but also with developers following him, can be named a 'theory-led bricolage.' The term has been used by Gravemeijer (1994, p. 110) to describe such a process, namely as the creative work of a kind of 'bricoleur' (handyman) who brainstorms, invents, improves, adjusts and adapts continuously. He uses appropriate tools and materials, whatever is available, to realize a specific function. While such a process is theory-led, theory is also developed in it. In all facets of the process the views of the designers – including their views on the concept of theory itself – help to determine the dynamics of (the development of) the theory.

The developmental process of the theory shows that it has been a process of trial and error, and that it involves continuous development, the theory is never done. Ter Heege (2005) makes that point yet again when he analyzes the developments in teaching multiplication. He describes how Freudenthal's view on memorization processes for learning arithmetic and mathematics developed. This refers to views that are still recognizable, at various levels and in different educational settings, views that have had a lasting influence on the development of theory in this field.

Adoption of the theory

The theory is influenced by a number of factors, which can make or break the adoption of theory. To begin with, there is the influence from other theories. This domain-specific instructional theory contains elements from mathematical theories and from theories of learning. What is unique is the way in which elements from domain-specific instructional theory, from didactical phenomenology and from learning theories are integrated.

Second, there is the influence of the designer-developer with his knowledge, understanding, values, standards and opinions. For example, earlier we discussed the confrontation with his own opinions that designer-developer Ter Heege experienced. His position as learner, teacher researcher and developer all color his theory. Finally, there is the user who influences (the development of) theory, teachers and students provide important impulses for change and are in the end the determining factor in what

becomes of a theory. These three factors reflect the open character and the dynamism of the theory as well as the complexity[18] of the educational setting in which the theory is to be adopted.

2.6.3.3 Summary of the characteristics

As described before, the history of the development of the theory of learning and teaching multiplication shows a drastic evolution. The content of the theory varies from a collection of simple concepts, rules and views, dominated by associative psychology and behaviorism, to a refined, content knowledge based system of concepts, suggestions and guidelines. In all cases there is a strong connection between the theory and 'the practice of education.' The relationship with that practice becomes a solid and specific one only after the 1970s. Freudenthal's mental legacy, specifically his didactic phenomenology of mathematical structures, stimulates the experts involved (developers, researchers, supervisors, teachers) to change their views. Thinking in terms of increasing complexity of the material offered to students changes to thinking about progress in students' learning processes. Development of theory on that largely occurs as reflection on practical experiences during developmental research. In that respect the theory can be seen as beginning with reflection on practice, which then evolves into a theoretical foundation and justification of that practice. The reciprocal relationship between theory and practice is determined to a large extent by a theory-led, cyclical process of brainstorming, designing, trying, analyzing and reflecting (from the theory); the designers themselves, as an exalted kind of 'bricoleurs' keep this process going, while positioning themselves as learners.

The description in the TAL brochure, even more than for Ter Heege and in 'De Proeve,' is characterized as a practical theory; that character is partly determined by the practice-oriented goals, as represented in activities, and the immediately included theoretical justification (called 'theory in activities'). It is a characteristic that is associated with the description Boersma and Looy (1997) give of practical theory[19].

The evolution of the theory for learning to multiply is influenced by its relationship with practice from another point of view as well, as debate and looking for consensus among those involved play an important part in the relationship between theory and practice. In the course of the process of the development of the theory there has been an increase in the consensus about the theory being developed among experts. From the 1970s onwards a close network of experts and interaction with practice develops, which leads to theoretical statements becoming gradually more profound and better understood and accepted in an ever-wider circle. That process – which as a 'democratic development of a curriculum' is of great importance for innovation – leads to a more and more balanced and widely-accepted theory.

The *cohesion* of the theory of learning and teaching to multiply is largely determined by views and goals, for example by learning-teaching principles (Treffers, 1991; see also section 3.2.1).
Looking back to the developmental process of the theory, we notice that its cohesion played an important part in the acceptance of the theory.

2.6.4 Focal points for theory in mathematics teacher education

We can deduce focal points for the use of theory in teacher education from the above-mentioned intrinsic and extrinsic characteristics of theory (see figure 2.2). The relevance of the focal points for this study is that they guided the thinking and designing of the student teachers' learning environment (chapters 3, 4 and 5). To illustrate this, some of these focal points will be discussed here.
The focal point for theory *'Underpinning (intended) actions, seeing patterns in student behavior'* (ad. 1 in figure 2.2), derived from the intrinsic characteristic 'Grounding: reasoning, arguing; patterns,' is a first example. In training colleges where interaction is considered of paramount importance, (learning) reasoning for students and their pupils in their practice schools is a core activity. Considered at a higher level the essence of this focal point for the student is especially learning to motivate (intended) actions and seeing patterns in their students' acting and thinking, and learning to capitalize on that. Lampert, in an article called 'Knowing, Doing and Teaching Multiplication,' already points this out in 1986:

> In the lessons [grade 4] my role was to bring students' ideas about how to solve or analyze problems into the public forum of the classroom, to referee arguments about whether those ideas were reasonable and to sanction students intuitive use of mathematical principles as legitimate (Lampert, 1986, p. 339).

Furthermore, she contends that the teacher needs a comprehensive repertoire of content and pedagogical content knowledge to be able to provide that style of teaching. Her colleagues recently brought notice to that necessity again (Ball, Hill & Bass, 2005) by pointing out the specific character of the mathematical knowledge required for teaching.
Another focal point of theory we found, derived from the intrinsic characteristic 'the truth of theory,' is *'Cogency and consensus'* (ad. 8; figure 2.2). Whether something is 'true' or not in pure, formal multiplication is determined by 'the answer,' and in the case of the teaching method of multiplication based on arguments from experts. In teacher education it is an important part of the discourse that students debate, for example on a didactical principle directed by their teacher educator, and try to *convince* each other and come to a *deliberated consensus*.

Characteristics of the theory	**Focal points for theory in mathematics teacher education**
Intrinsic characteristics	
1. Grounding: reasoning, arguing; patterns.	ad. 1. Underpinning (intended) actions, patterns in student behavior.
2. The nature of theory.	ad. 2. The nature of the theory that is brought into play (practical wisdom, the 'common sense caliber,' perceptual, conceptual, prescriptive, formal).
3. Value of reality.	ad. 3. Reality as a source for the development of theory (in the sense of didactical phenomenology).
4. A relationship to other theories.	ad. 4a.The context for the development of theory (construction of knowledge). ad. 4b.Personal theoretical network.
5. The beauty of theory.	ad. 5. Beautiful reasoning and constructs.
6. Level of formalization.	ad. 6. Levels of thought and action (concrete-abstract, material-mental, use of models).
7. Subjectivity.	ad. 7. Subjective concepts and theories. Personal preferences and beliefs.
8. The 'truth' of theory.	ad. 8. Cogency and consensus, e.g., in the discourse.
9. The presence of theory-charged examples.	ad. 9. Narratives as a means to acquire practical knowledge.
10. The range of application of theory.	ad. 10. (General) validity of theory, generalizing the knowledge and actions of students and teacher.
Extrinsic characteristics	
1. The genesis and dynamics of theory.	ad. 1a.'The Spark,' intuition and creativity. ad. 1b. Effort and success stories. ad 1c. A theory's history as an object of study.
2. Theory-in-action; theory-on-action.	ad. 2. Theory as a basis for pedagogical reflection on practice (describing, interpreting, clarifying, predicting, capitalizing on situations).
3. The discourse in the 'school' of scientists.	ad. 3. The discourse (deliberation) as the motor of constructing theoretical knowledge.
4. Adoption of theory.	ad. 4a. Appreciation of theory. ad. 4b. Becoming aware of one's own beliefs. ad. 4c. The tendency to apply theory (developing sensitivity for the use of theory). ad. 4d. 'Appropriating' theory. ad. 4e. Confidence through applying theory.
5. Judging the merit of theory.	ad. 5a. Applicability of theory. ad. 5b. Explanatory and predictive value of theory.
6. Justification of theory.	ad. 6. Justification: empirically-grounded norms of how the theory works.
7. Development of theory on the basis of research.	ad. 7. Acquiring theory through (re)searching.

figure 2.2 Characteristics of the theory and focal points for mathematics teacher education

An *extrinsic* characteristic that came to the fore in the theory of learning to teach multiplication is *the genesis of the theory* (ad. 1; figure 2.2). The history of the development of multiplication makes it abundantly clear that theory is a human invention, meant to make situations (phenomena) transparent and describe them (conveniently). Core ideas for an innovative, realistic didactic of multiplication arose partly because of *coincidental observations or experiences of success* (ad. 1b; figure 2.2) from a developer, albeit that he created a well thought-out 'design environment.'[20]
The stories about the genesis of multiplication and its didactics provide us with a number of focal points for the use of theory in teacher training. They make it clear that especially an inspiring (learning) environment may lead to *the 'spark,' a discovery or an aha-experience* (ad. 1a; figure 2.2). Experiences of success in their turn may lead to a chain reaction of targeted activities (Janssen, De Hullu & Tigelaar, 2008), increasing the chance that students will use and further develop theory as a matter of course. Finally there is *the history of (learning) multiplication itself* (ad. 1c; figure 2.2) that deserves attention as an object of study within teacher training courses. Ultimately it is knowledge of history that gives students an insight into the foundations of theory, and, by extension, of what they themselves and their students must be able to do and know to become 'competent' in this field (Fauvel & Van Maanen, 2000). History does not only show the knowledge, skills and insights that people gradually acquired, it also shows how important motivation and (work) attitudes are for the process of (learning) the acquisition of theory.
Finally, another (extrinsic) focal point has been derived from the characteristic '*theory-in-action, theory-on-action'* (ad. 2; figure 2.2), two expressions that are often cited in literature, and that have been coined by Schön (1983) to distinguish (theoretical) reflection during and outside practical activities. Theory serves a *clarifying and explanatory function* in nearly all stages of the learning and teaching process. We have seen how the designer uses and develops theory while he works on designing, trying and evaluating his teaching package.
For teacher education we can consider (theoretical) reflection – before, during and after practical activities – as an important derived focal point from the above-mentioned characteristic of theory. A good (future) teacher can be recognized partly from the ability to 'look ahead through looking back.' In training *theory mainly functions as a basis for orientation for reflection on practice*. Through theoretical knowledge *practice can be understood, explained, predicted or even improved*; in reverse, practice can also shed new light on theory.
For a complete overview of the remaining characteristics of theory and the focal points for teacher education that have been derived from that, we refer to the outline of characteristics and focal points that has been provided earlier (figure 2.2).

2.6.5 Conclusion

2.6.5.1 Development of theory

Theories often owe their origin to the creative discovery of one individual. Development of theory can begin where there is a need for explaining or predicting phenomena, for elaborating ideas or for disproving the findings of others. Often, these findings are 'spin-offs' of search processes with other goals. Statements that are made have the character of logical arguments, which may be more or less influenced by intuition or opinion. The theory of learning and teaching multiplication shows what might be called 'signs of an evolution' namely the growth of a collection of simple rules and beliefs into a sophisticated system of concepts, suggestions and guidelines.

2.6.5.2 The relationship between theory and practice

We characterized the theory of learning and teaching multiplication as a practical theory, on the basis of its practice oriented goals and the manner in which the developers founded their theory and justified their views. What this means is that a reciprocal relationship between theory and (teaching) practice is maintained. The history of the theory of learning to multiply shows continuous development, the theory is never finished. Field and thought experiments provide new concepts, principles and guidelines, from which a new version is developed and tested, in a process that shows similarity to the empirical cycle as described by Koningsveld (1992, p. 27). This is of course different from a pure, formal mathematical theory. This is a closed system of concepts, relations, axioms and theorems. The confirmation of hypotheses and the 'truth' of statements are derived from mathematical proofs. In learning to multiply the theory is lent persuasiveness through theoretical reflection on the outcome of experiments; the developers – at least those from later than 1970 – attempt to strengthen that persuasiveness even further by the continuous pursuit of consensus between one's own practical experience, the experience of professionals in the field and existing theories, with crucial justification coming from the theory's effects on students' learning processes. Each theory, to a certain point, arises from a practice situation and maintains a reciprocal (reflexive) relationship with that practice. Theory makes it simpler and more efficient to perform in practice, while practice in its turn, as an application in reality, clarifies thought about the theory.

'Production and use' of theory (Fenstermacher, 1986) are not as strongly interwoven with each other in every theory as they are in the (development of) theory of learning and teaching multiplication, where the professional practice of teaching provides the source for the development of theory and where – in reverse – theory provides direction to practice. We see here a parallel between the development of curricula in primary education and for training teachers in primary education (Goffree, 1979).

An important factor that determines differences in the relationship between theory and practice is the way in which theory is used. Theory can be used for instance to test

practice (and the other way around) or to anticipate on practice; in the latter case, it is a matter of 'theory in action' or 'theory on action' (Schön, 1983). If the practician concludes a 'transaction with the situation' (Schön, 1983), it is a matter of working and researching in practice as a 'reflective practitioner' with no division between knowing and doing. Schön distinguishes that method of working from a technical-rational approach, where activities take place on the basis of more external considerations that have been derived from scientific study.

Other factors that determine the relationship between theory and practice are the extent to which theory influences or even guides practice (Eraut, 1994b) and the extent to which professional characteristics of the practician, such as knowledge, insights, skills, attitude and beliefs (Hofer & Pintrich, 1997; Lampert, 2001; Thiessen, 2000; Verloop et al., 2001) encourage or inhibit the integration of practice and theory.

2.6.5.3 Legitimizing theory

As well as by its relationship to practice, the character of a theory is determined by the way in which statements are underpinned. That foundation in fact determines the 'strictness' of the theory. A (formal) mathematical theory is in that sense a strict theory that all statements can be verified (proven) based on the given concepts, relations and axioms. Statements in a purely empirical theory – for instance Gestalt theory – are often founded on and tested through field experiments. The validity however is of a different caliber than that of a formal theory, where statements can for instance be labeled with one of two values: true or untrue. Statements in an empirical theory have a degree of truth that can at best be expressed in a degree of probability (smaller than 1).

According to the empirical part of the theory of learning and teaching multiplication, statements are mainly empirically justified by experiments and achieving consensus within a paradigm, against the background of learning-teaching principles and goals. Within such a practical theory 'clarifying and predicting' as well as 'explaining and understanding' have therefore often the function of theoretical justification of practice-oriented goals and activities. The yield for 'users' of the theory is high; teachers who have such a theory, can 'see' more in similar teaching situations and can therefore think and talk about them in a more differentiated manner (Tom & Valli, 1990; Fenstermacher & Richardson, 1993).

The consensus and validity together with the relationship to practice – especially the relevant causality process (Maxwell, 2004) – , provide the scientific basis for the theory of learning and teaching to multiply.

2.6.5.4 Revision of the working definition

In the introduction to this chapter (section 2.6.1.2) we formulated the definition of theory as a collection of cohesive, descriptive concepts. Such a system of grounded coherence can contain statements and argumentation, as well as explanations,

assumptions, conjectures, predictions and proofs. While this working definition contains the main earlier-mentioned intrinsic characteristic ('grounding'), it lacks – certainly as a domain specific instruction theory – important elements such as the relationship to practice, and characteristics we described as extrinsic. Also, in this definition the concept of cohesion is linked to one-sided underpinning, which is a rather meager interpretation of the concept of grounding as described earlier. In the case of the theory of learning to multiply cohesion derives from the underlying concepts of the operation multiplication and from consensus concerning the view on learning (to multiply). In our view, which is colored by our affinity with teacher training, theory arises from reflection on reality. That can be translated as reflection on practice, with reality being seen as a 'collection of practices.' That reflection can represent itself in many ways and on many levels, from a personal practical theory full of individual views, to a scientific theory that can be expressed in theoretical concepts and laws.

In summary we can describe the revised definition of a domain-specific theory – which will never be 'definitive' – as a collection of descriptive concepts that show cohesion, with that cohesion being supported by reflection on 'practice.' The character of the theory is determined by the extent to which intrinsic and extrinsic characteristics manifest themselves.

2.6.5.5 Practice as a starting point, theory as the basis for orientation for reflection on that practice

Personal experience (as teacher, teacher educator and researcher) teaches that, particularly in the domain-specific instructional theory of multiplication, there is a parallel between the activities of developers and users. Presumably, this is partly due to the practice relevance and the phenomenological character of the theory, and the specific approach of the developers' activities. Possibly that parallel is a measure for the practical value of an educational theory. It is a characteristic of activities that is to some degree related to that of teacher educators in this specific professional field. In our view on teacher training, practice is the starting point for the professional development of students and the theory of realistic mathematics education is the basis for orientation for reflection on that practice. Students create their own (practical) knowledge, that will generally have a narrative character (Goffree & Dolk, 1995). The underlying thought is that, through reflection on 'theory-laden' practical situations, students will integrate theory into practical knowledge, and will so acquire 'theory-enriched practical knowledge' (EPK; Oonk, Goffree & Verloop, 2004).

2.6.6 Perspective

For the study of the way in which students deal with theory, it is essential to probe the characteristics of theory and their meaning for teacher training. For that reason, for the theory of learning and teaching to multiply we have considered the characteristics we

found from the perspective of theory in training. We have not (yet) considered the question of how teachers (in training) can deal with theory or theoretical knowledge. As a prelude to that exploration, there are some suitable statements from Freudenthal, who was not only a designer of groundwork for pure mathematics, but also a developer of didactics. He tried to bring attention to essential elements of theory through the use of stories from practice. He believed that you cannot comprehend theories from hypotheses and theorems, but from concrete examples[21]. He thought the selection of those examples was of eminent significance, as can be seen from his statement: "It is much less difficult to overwhelm the learner with a shower of countless examples, than to look for that one – paradigmatic – example that works." (Freudenthal, 1984b, p. 102). Those who want to familiarize themselves with theory can learn from this in so far that practice itself, or typical stories and other representations of practice, form a rich source and a useful starting point. Translating this to teacher training, that starting point can be elaborated into an approach in which students develop a repertoire of 'theory-enriched practical knowledge.'
The focal points for the use of theory in training serve as benchmarks for designing and improving the learning environment of the student teachers in the different stages of the research process.

2.7 The learning environment

2.7.1 Orientations for designing learning environments

The concept of a learning environment can be interpreted differently, but is frequently described as a context that has to yield, to guide, and to keep going learners' required learning processes in order to reach the desired learning results. A learning environment is part of the curriculum (Lowyck & Terwel, 2003). Already at the beginning of the previous century, scientists wrote about the context in which learning should take place. For example, Dewey expounds in 1916 his ideas about the characteristics of an educational (learning) environment. (Dewey, 1916; Hansen, 2002). Vergnaud (1983) describes the context in which students learn in terms of 'mastering situations' and calls a mastered collection of situations a 'conceptual field.' The learner (e.g., math student) 'masters' a conceptual field if he or she masters several concepts of a different nature. The learner has to invent how different formulations and symbols should be used for concepts and subjects that appear in different contexts. Lampert (2001) applies the idea of 'conceptual fields' not just to learning but also to teaching. Other than in curricula where concepts are offered linearly or spiral shaped, she considers her approach of teaching as facing students with conceptual fields. In that way the frequently capricious learners' learning – and also student teachers' learning, is given chances to develop by using a conceptual field that anticipates on the learning process. The conceptual field as such can be considered as a component of the learning environment.

In the Netherlands sometimes the word 'learning landscape' is used as a synonym for learning environment. It is a concept that became fashionable at Pabos and elsewhere in the 1990s; it means little more than a material interpretation of a part of the student teachers' learning environment. At some stage the word has been used as a substitute of the term 'werkplaats' (workshop), a name for a study space where student teachers can work freely with textbooks and teaching materials. Fosnot & Dolk (2001) give another interpretation to the term learning landscape, one that shows affinity with the idea of conceptual fields. They consider a learning landscape for primary mathematics education as formed by big ideas, strategies and models; those can help teachers with developing hypothetical learning trajectories. This idea was developed by Simon (1995), who considers it as a 'learning and teaching framework.' The framework is hypothetical, because you can never be sure about students' doing or thinking.
Gradually, as a teacher you have to adapt to the learning trajectory that students show. The big ideas, strategies and models give the teacher guidance in the landscape; sometimes they become means to choose the environment or the context. For students they provide another view and the progress in their mathematical development. According to that view, the learning landscape for student teachers can be considered as an ordered collection of important moments in the student teachers' development. Thus, at the level of teacher education, the concept of learning landscape can be seen – as in primary education – as a learning and teaching model and so as a component of the learning environment Pabo.
Verschaffel distinguishes five basic principles for designing powerful[22] learning environments:

- many types of content matter, cognitive and meta-cognitive knowledge and skills have to be acquired coherently, because they play a meaningful, complementary role;
- acquiring knowledge and skills actively and constructive by the learners have to be guided by adapted forms of aid or support by the educator;
- among other things, the idea that learning is embedded in a context implicates for the development of the learning environment the need for a process of decontextualizing;
- not only the role of the educator is of consequence for acquiring knowledge and skills actively and constructive, but also the interaction and cooperation with fellow-students;
- there should also be systematic attention on the dynamic-affective aspects of the teaching-learning process (Verschaffel 1995, p. 181).

Lowyck & Terwel subscribe to Verschaffel's view that designing learning environments concerns not only content matter knowledge, but also strategic knowledge and meta-

cognitive knowledge. That can be achieved through orientation, through supporting the construction of knowledge, through making the learner aware of the performed cognitive activities and through further support on self-regulation (Lowyck & Terwel, 2003, p. 296). The present study will take the aforementioned orientations into consideration, in particular the intention to create a balance between:

- the individual and social aspects of knowledge and knowing (Tough, 1971; Kieran, Forman & Sfard, 2001);
- subject content, pedagogical content, general content education and competences (Klep & Paus, 2006);
- self-regulation by student teachers and aid and support from their teacher educators (Vermunt & Verloop, 1999);
- the school practice of the prospective teachers and the desired professional practice of the teacher training institutes (Richardson, 1992).

Thinking in terms of learning environments respects the individual student teachers' own identity and own development. According to this framework, we interpret the student teachers' learning as a cognitive process of individual construction in a social process of participation in a group and of interaction with the learning environment. In section 2.3.1 we stated that the learning environment of (student) teachers will in any case need the feeding – both implied and explicit – with a variety of theories and theory laden stories, and furthermore also requires the guidance of an expert in order to level up the student teachers' 'practical reasoning' (Fenstermacher, 1986). Also, we mentioned the important role of the expert, who has to be aware of the importance of learning through interaction (Elbers, 1993) and of 'constructive coaching' (Bakker et al., 2008).

To move students to practical reasoning, it is important to look for content and ways of working that naturally inspire them. These may be for example questions that are confrontational or that evoke discussion, leading to 'didactical conflicts' that require cognitive flexibility (Spiro, Coulson, Feltovich & Anderson, 1988). A challenging or even confusing situation may arise, leading to a natural need to find a solution, a phenomenon that Piaget describes (1974, p. 264) as a process of reaching equilibration from perturbation. According to Ruthven (2001, p. 167) an 'ideal' level of reasoning is reached when students have achieved the ability of 'practical theorising.' In his description of that ability he refers to Alexander (1984, p. 146) who believes that "students should be encouraged to approach their own practice with the intention of testing hypothetical principles drawn from the consideration of different types of knowledge."

2.7.2 Design research

The described orientations in section 2.7.1 – combined with the other implications mentioned in this chapter – served to inspire the design of the learning environment in both the way it was shaped and its use by pre-service teachers. The learning environment

– necessary to develop in favor of elaborating and answering the research questions – was gradually refined in accordance with the approach to developmental research or educational design research (Gravemeijer, 1994; Cobb, 2000; Gravemeijer & Cobb, 2006). Educational design research is the systematic study of designing, developing and evaluating educational programs, processes and products. The process can yield considerable insight into how to optimize educational interventions and better understand teaching and learning. In the whole development process of designing the learning environment(s) – cyclic in nature – four stages are distinguished.

In the first stage of designing the learning environments for the series of the four research projects in this study ('The first exploratory research'; see section 3.5), the learning environment of the student teachers was composed of ten lessons on CD-roms, information about the lessons (Oonk, 1999) and the first version of a computer search engine. The study was mainly focused on knowledge construction. The actors were two student teachers and their participating teacher educator-researcher. The learning environment in the second stage ('The second exploratory research'; see section 3.8) comprised the new MILE course 'The Foundation' for second year student teachers (Dolk, Goffree, Den Hertog & Oonk, 2000). The teacher educator gave his students a list of 150 key theoretical concepts from previous courses, to serve as a theoretical framework to help student teachers to value their theoretical knowledge. Ten two-hour meetings were held. Following the method of triangulation (Maso & Smaling, 1998), four pairs of student teachers within two classes of 25 student teachers were observed and interviewed, and a participating study of the group work with two student teachers was conducted. The third stage was the small scale research (see chapter 4). The learning environment for the 14 participating student teachers – a group of 6 respectively 8 – was composed of a variant of MILE, including one CD-rom 'The Guide' (Goffree, Markusse, Munk & Olofsen, 2003; see section 4.2.2.2). The improvement to the learning environment in comparison to the previous one was first of all a change with respect to the student teachers' possibilities to relate theory to practice, a change to a 'theory-enriched' environment. The development of this third stage of designing the learning environment was preceded by a try-out of components, in fact an extra 'cycle' in terms of design research.

For the fourth and last stage of designing – conducting the large scale study (see chapter 5) – the learning environment was furthermore optimized to enable the research concerning the refined search questions.

In each of the four stages one can recognize the three broad phases in the process of conducting a design project, as described by Cobb & Gravemeijer (2008), namely: preparing, experimenting in class or group, and conducting a retrospective analysis. For example, components of preparing were: clarifying goals for student teachers and their teacher educators, designing concepts lists and 'initial assessments' as an orientation

basis for both student teachers and teacher educators and not the least: deliberating on and choosing the theoretical foundation of the related research. An example in the stage of conducting a retrospective analysis is the development of a reflection-analysis tool to analyze student teachers' reflective notes.

This component of the cycles of design and analysis started with deliberating and gathering notions of possibilities to gauge student teachers' reflections in the exploratory researches and, ended in the design of a reliable reflection-analysis tool.

The next chapters will provide more details of the process of designing the learning environment.

3 The exploratory studies[i]

3.1 Introduction

The two exploratory studies, the first two links in the chain of four studies in this thesis, were part of the national Multimedia Interactive Learning Environment (MILE) project for primary mathematics teacher education in the Netherlands (Dolk, Faes, Goffree, Hermsen & Oonk, 1996). Both studies are described below in the context of the development of the MILE-project, with a focus on the two exploratory studies in sections 3.5, 3.8 and 3.9.

When designing learning environments in primary teacher education, there is an attempt to represent real teaching practice in an authentic and natural way to prospective teachers (see sections 2.4 and 2.7). When constructing these environments, teacher educators have to consider how to best motivate the student teacher, identifying the most relevant practice-based principles and the ways in which the theory and practice can be bridged (see the sections 2.4 up to 2.7). There are other considerations as well. For example, in the Netherlands, as in some other countries, teacher education is changing drastically. Controversial teacher education curricula, consisting of primary school subjects originated after more than one hundred years of reflection on the subject matter of primary education and the ways teachers have taught, have been replaced. The new curricula intend to improve the general professionalization of the prospective teacher, for the most part neglecting the school subjects (section 2.5). More specifically, the new objective is to adequately prepare students to become competent beginning teachers.
In this chapter, we will describe how a learning environment focused on representing various teaching practices to prospective teachers, known as the MILE project, was inspired, designed, implemented, tested, and refined. Exploratory studies were important in the support of these processes.
Before describing the making of MILE and presenting details about its pedagogy and technology (section 3.3), we first provide some theoretical background on the development of MILE in the context of the theory-practice discussion of primary mathematics teacher education, following what has been argued in section 2.4 and 2.5.

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[i] This chapter has been published in adapted form as: Oonk, W., Goffree, F., & Verloop, N. (2004). For the enrichment of practical knowledge. Good practice and useful theory for future primary teachers. In J. Brophy (Ed.), *Using video in teacher education. Advances in Research on teaching. Vol. 10* (pp. 131-168). New York: Elsevier Science.

MILE is rooted in the Standards for Primary Mathematics Teacher Education (Goffree & Dolk, 1995). The process of developing these standards and subsequent discussions provided the contributing teacher educators with opportunities to articulate pedagogical ideas and expand their repertoire of theoretical orientations. Furthermore, these developments appeared to provide new inspiration (Barnett, 1998; Goffree & Oonk, 1999, 2001; Herrington et al., 1998; Lampert & Loewenberg Ball, 1998; Mousley & Sullivan, 1996), so the process was generative.

One of the areas that dominated team discussions was the concept of practical knowledge (see section 2.3.4), used to indicate the network of knowledge and insights that underlie teachers' actions in practice (Elbaz, 1983; Fenstermacher, 1994; Verloop, 1992). Within this concept of practical knowledge, the concept of educative power, used by Cooney (2001b) and Jaworski (2001), is particularly applicable. How to help prospective teachers acquire educative power is an important question for educators. The factors that motivate teachers often remain hidden as tacit knowledge (Elbaz, 1991), even if researchers ask about them, although sometimes they are revealed in teachers' talk about practice. Those who listen well to reflective practitioners describe their teaching (see for example Lampert, 2001) will get a sense of (situated) practical knowledge. Such practical knowledge can be considered as a "narrative way of knowing" (Gudmundsdottir, 1995, 1996; Carter, 1993). The designers of MILE (Dolk et al., 1996) adopted this idea, which would have consequences for the content and format of MILE and also for the use of the learning environment by the student teachers.

Given the assumptions that practice plays a central role in teacher education curriculum and that acquiring practical knowledge is the main learning goal, it follows that teacher training colleges should incorporate useful representations of real teaching practice. What real teaching practice means has been described in many ways (Lampert & Loewenberg Ball, 1998; Masingila & Doerr, 2002; Herrington et al, 1998; Barnett, 1998), but in general, it means that all aspects of mathematics education in school are present, including such things as the teacher's preparation and reflections, students' notes, and when possible, interviews with teachers and students regarding their experiences of the lessons.

Many publications are available about representations of practice, recently in relation to the use of cases in teacher education (Walen & Williams, 2000). Some authors emphasize showing 'good' practice in these cases (Goffree, Oliveira, Serrazina & Szendrei, 1999). This appears to force one to choose between 'authentic' and 'good,' a dilemma that can be reconciled if authentic is considered as 'full professional practice' and good as 'good for student teachers.' Selecting practical situations evokes the concept of situated cognition about knowledge within the situation and the idea of situated learning about eliciting knowledge from practice (Brown et al., 1989; Borko &

Putnam, 1996; Herrington et al, 1998; Leinhardt, 1988; Tzur, 2001).

New technology creates possibilities for adding an extra dimension to narrative representations of practice. Beyond written cases are multimedia narratives, in which educational stories are told in sound and picture, sometimes connected with text. Student teachers have opportunities to reflect on this recorded practice and to write their own interpretations and analyses, eventually related to their own teaching practice (Pi-Jen Lin, 2002; Mason, 2002). How to make the learning environment of student teachers educative (Lampert & Loewenberg Ball, 1998) has been discussed recently (Cohen 1998; Sullivan, 2002; Masingila & Doerr, 2002; Pi-Jen Lin, 2002). One strategy for making the environment educative is to engage learners in investigations with the support of an on-line tutor or expert. Following this initial process, student teachers have the opportunity to constitute a community of prospective teachers in order to discuss their investigations of their own classroom practice, which will provide motivation to learn from it (Brophy, 1988). Teacher educators and researchers assume that this process leads to reflective practice (see section 2.2) as output (Beattie, 1997; Griffiths & Tann, 1992; Jaworski, 1998, 2001; Krainer, 2001; Schön, 1983). However, the process does not always produce the intended results. For example, stories of other teachers' practice often do not stimulate meaningful discourse. Student teachers often believe that they can manage their professional work as a matter of common sense (Lampert & Loewenberg Ball, 1998), so they fail to appreciate the need to articulate a theory of practice.

Researchers (and educators) look for ways to put student teachers in touch with relevant theory outside of practice (see chapter 2). For example, Donald Schön (1983; 1987) suggested linking theory to practice during 'reflective conversations' with practical situations. Others have developed the concepts of 'practical theorising' (Alexander, 1984; Ruthven, 2001; see section 2.7.1) or 'theoretically grounded reasoning' (McAninch in Masingila & Doerr, 2002, p. 241). Lampert (1998b) speaks of 'thinking practice' (p. 53) that is 'integrating reasoning and knowing with action' (Loewenberg Ball, 2000, p. 246). Professionals cannot constrain themselves to telling stories about practice using only the language of practice (Verloop, 2001). Theory cannot be omitted (Cooney, 2001a). Teachers need flexible cognitive structures (theory) to understand the information they derive from their complex and uncertain teaching practice (Spiro et al., 1988).

The next question for the designers of MILE was which theoretical framework student teachers needed to help them adopt a professional approach to practice. By starting modestly and leaving higher levels of theory in action for later (Leinhardt et al., 1995; Oonk, 1999), teacher educators can focus attention on a framework of theoretical concepts to use for deriving meanings from practice (see also section 2.6). Student teachers must be provided the opportunity to assimilate the theoretical framework into

their practical knowledge, so that practice and theory will be integrated naturally (Dewey, 1933; Daniel, 1996; Leinhardt et al., 1995; Selter, 2001; Thiessen, 2000).
Having discussed the roots of MILE in research on cognitive structure and the assimilation of a theoretical framework, we describe in the next sections our efforts to make MILE'99 educative on the basis of continuing development, including exploratory investigations. In the last section (3.9) we explain how theory completes thinking on practical knowledge in the context of a new learning environment and the perspective of new research.

3.2 Prior development and research

3.2.1 Developing good practice

We begin by providing some background on our view of good practice in mathematics education.
The contents of good practice were developed as a response to reaction to the problems with the world-wide New Math movement in the 1960s and inspired by Freudenthal's ideas (1978) about a new approach to mathematics education, embodied in what is now called as Realistic Mathematics Education (RME) (Treffers & Goffree, 1985; Treffers, 1991; Streefland, 1993; Gravemeijer, 1994).
Three late twentieth-century developments provided the foundation for MILE:

- developmental research in the Wiskobas project;
- the formulation of national core objectives in the Netherlands;
- the discussions on a new publication about 'a National Programme for Mathematics Education on Elementary Schools.'

Developmental research
In the Wiskobas project (1970-1980; note 4), a new mathematics curriculum for primary schools was developed with the support of Freudenthal. It resulted in a concrete realistic program that describes a clear image of good practice in five learning-teaching (L-T) principles (Treffers, 1991).

L.1 Construction. Learning mathematics is a constructive activity.
T.1 Concrete basis for orientation. Make mathematics concrete. Create recognizable contexts to which children can assign their own meanings.
L.2 Raising the level. Learning mathematics takes place somewhere between the informal mathematics of the children themselves (intuitive notions and self-invented procedures) and the formal mathematics of adults.
T.2 Models. To be able to achieve the required raising in level during the teaching-learning process, the pupils must have at their disposal the tools for bridging the gap between informal and formal mathematics (Gravemeijer, 1994).

L.3 Reflection. Learning mathematics is stimulated by reflection. Reflection is, as it were, the engine for raising the level (Freudenthal, 1991).

T.3 Reflective moments. The teacher finds the right times to bring reflective moments into mathematics teaching. Good occasions for reflection include any cognitive conflicts that might occur and anything the pupil may have thought of independently ('own productions') (Streefland, 1991; Selter, 1993).

L.4 The social context. Children learn more often than not in the company of adults or other children. This means that other actors in the learning environment can provide the impulse for learning. As the different actors communicate with each other about mathematical concepts and procedures, they argue about them and come to insights collectively.

T.4 Interactive mathematics lessons. The teacher organizes mathematics education such that interaction becomes a natural part of it. This, in turn, creates a pedagogical climate in which all the pupils can take part in the interaction. The concept of a classroom as a sort of 'mathematical community' gives it an extra dimension, as does the Mathematical Conference in the class described by Selter (1993).

L.5 Structuring. If children construct their own meaningful mathematics, then new knowledge and insights become incorporated in what they have already learned. This means that the available mathematical knowledge (think, for example, of cognitive structures) is subject to constant upgrading. The new knowledge is fitted into the existing cognitive structure (assimilation) or the total structure is adjusted to accommodate the new insights (accommodation). Also, one aspect of learning is the task of bringing structure to what is being learned.

T.5 Interweaving the strands of learning. The teacher bases mathematics teaching on real-world situations, both as sources of ideas and as places to apply them. The first case would be an example of 'horizontal mathematizing.' Further, the mathematical ideas being used can themselves form the subject matter (vertical mathematizing). This brings connections with other mathematical ideas into the picture, partly as a result of the concrete background.

At the very end of the century (Goffree & Frowijn, 2000) 'good practice' had to be defined again, but this time with the intention to create an instrument for self-evaluation in schools. For this goal the principles were elaborated into more refined statements, called 'indicators' of realistic mathematics education, used to observe and analyze mathematics teaching in classrooms.

- The teacher is teaching mathematics by problem solving.
- Problems are introduced in familiar contexts.
- A substantial part of the effective learning time is used to explore the context.

- While exploring the context of a problem the non-mathematical aspects mentioned by students are also considered.
- The context gives meaning to the mathematical activities.
- Introduction, problem setting, problem solving, and subsequent discussion are realized in interaction with the whole class.
- In order to stimulate mathematical activities in cooperative groups, the teacher creates reasons for the students to discuss, to explain, to cooperate, to convince each other, and to distribute tasks properly.
- Sufficient learning time is spent on the introduction and exploration of 'models'.
- The use of concrete models (e.g., schemas such as number line, reckon rack, or fraction strips) results in the use of mental models.
- The teacher continuously anticipates students' reactions during interactive class discussion.
- The pedagogical climate allows children to make mistakes and the teacher to overtly discuss these errors and their possible causes.
- The teacher takes time for reflective moments during the mathematics class.
- Students are stimulated to create mathematical problems themselves (e.g., for peers) and also to solve these problems reflectively.
- Teacher and students have an open mind for other people's solutions.
- Frequently asked questions are "Why?" and "Are you sure?".

The provoking character of 'cognitive conflicts' is used to challenge children's thinking.

The formulation of national core objectives

Increasing attention to quality management in primary education is the second development to consider. The National Institute for Curriculum Development in the Netherlands (SLO) published, after a national debate in the different domains, a list of core objectives for the school subjects (SLO, 1993; Treffers, De Moor & Feijs, 1989).

National Programme for Mathematics Education on Elementary Schools

A subsequent publication showed how to teach 'in the spirit of Wiskobas' in order to realize the core objectives: Standards for primary mathematics education (Treffers et al., 1989; Treffers & De Moor, 1990). These standards fueled a broad debate about 'realistic mathematics education,' that resulted in widely accepted and theoretically founded views of 'good practice in realistic mathematics education.'

3.2.2 Good practice for teacher education

During the Wiskobas project, teacher educators participated in the research and development. Freudenthal supported these activities; he participated in field tests and increasingly viewed student teachers' learning processes as an emerging outcome of

mathematizing and didactisizing (Freudenthal, 1991). Thus in the years of Wiskobas a new approach to primary mathematics teacher education was designed, in close connection to the creation of realistic mathematics education (Goffree, 1979).

Following the Standards for primary mathematics education and the Standards for mathematics evaluation and teaching (NCTM 1989, 1992), the Dutch Association of Primary Mathematics Educators (NVORWO) submitted a request to the National Institute of Curriculum Development (SLO) to draft a similar publication specifically for Dutch teacher education. In 1990, a group comprising ten mathematics educators started developing national standards and presented the results to colleagues as a handbook for teacher educators (Goffree & Dolk, 1995).

The philosophy of teacher education elaborated in the handbook is founded on three pillars: a teacher education adaptation of the socio-constructivist vision of knowledge acquisition, reflection as the main driving force of the professionalization of teachers (Schön, 1983, 1987) and the interpretation of practical knowledge as a way of narrative knowing (Gudmundsdottir, 1995). The statement "Real teaching practice has to be the starting point of teacher education" is emphasized. In the attempt to elaborate this principle into concrete curriculum materials for student teachers, an essential question still remained: How can curriculum designers give a learning environment a 'natural' aura? And next: what do we mean by 'natural'? Student teachers' fieldwork practice is natural by definition, but when they discuss this practice, they often stick to a superficial interchange of ideas and opinions (Verloop, 2001). Rarely do these discussions reach a level of theoretical reflections.

Learning in practice is mostly a solo task because student teachers not often have the opportunity to discuss common experiences and observations. Moreover, they usually focus on fulfilling responsibilities and on survival issues, so talk about actions dominates their reflections on the profession. As a result, they do not acquire practical knowledge that can be generalized across situations or organize their narratives of teaching into a broader framework.

The group of ten Dutch teacher educators got a new perspective on this problem when they visited the School of Education of the University of Michigan. They were introduced to the Student Learning Environment (SLE), created by Lampert & Loewenberg Ball (1998), which became a source of inspiration for the making of MILE. Using the records of real teaching practice the Michigan student teachers could access a whole year of mathematics teaching, with options to observe teaching and learning from different points of view (teacher, students, subject matter, curriculum, classroom climate, et cetera.).

Although MILE would become a quite different learning environment than SLE was in 1995 (Goffree & Oonk, 2001), MILE is based on a similar philosophy about the

presentation of real teaching practice in teacher education: "good practice for student teachers learning about teaching primary mathematics in the spirit of realistic mathematics education." Knowing this, it was important to make video recordings of practice without losing quality.

3.3 The making of MILE

3.3.1 Introduction

We began with the intention to create a similar multimedia learning environment for Dutch teacher education. The funds for a quick start were provided by the Dutch Government, and a project team (four math teacher educators and two technicians) (Dolk et al., 1996) worked on the first stage of the MILE project over the next 18 months.

The 40 Dutch Colleges of (Primary) Teacher Education were informed and invited to participate[23]. The majority expressed a desire to do so. In November, 1996 the first mathematics lessons were recorded and the MILE team again visited Michigan, this time to investigate the Student Learning Environment in action and to discuss the theoretical background, the making of, and specifically the use of the Student Learning Environment in the framework of ongoing methods courses at the university. Student teachers there perform 'open' investigations based on personally formulated problems to investigate and questions to answer. The tutor (a teacher educator or graduate student) supervises these open investigations, and regularly annotates (via comments in 'Word') the reflective reports that the student teachers submit. Student teachers' learning is optimal during these electronic discussions about observations and interpretations (Lampert & Loewenberg Ball, 1998).

3.3.2 Preparing the recording of good practice

Before the first video could be recorded in the classroom, decisions had to be made about subject matter (what is relevant?), the teacher (who is representative?), the school (as typical as possible and within easy reach). We wanted the school, the teacher, and the textbooks and manuals in use to reflect the situations student teachers ordinarily meet in primary schools. Toward this end, we asked all mathematics teacher educators in the Netherlands to answer three questions: (1) Describe the most appropriate practice school for your student teachers learning to teach mathematics, (2) Sketch a profile of the ideal primary school teacher to be a mentor (tutor), and (3) Consider the best teaching-learning situations you like your student teachers to see, to experience, to investigate, and to practice.

Responses to these questions suggested that good practice for student teachers should present interactions between actors in the classroom, the teacher's numeracy and mathematical attitude, his/her pedagogical and didactical expertise, and the teacher's

and students' attitudes towards mathematics. There also were many ideas about the supervision and coaching of student teachers during fieldwork. With the results of this questionnaire in mind, the team members visited elementary schools, talked with headmasters and teachers, and made additional observations in classrooms. In discussions after each math class, specific attention was paid to the degree to which the teachers were able to reflect on their actions and to put their thoughts into words.

An elementary school in Amsterdam, appeared to be a good location for the first five weeks of shooting video in grade 2. Some time after the choice had been made, a lucky incidental circumstance was recognised: two teachers in grade 2, sharing one job, created a 'natural' setting for observing reflective practice (Jaworski, 1998, 2001; Krainer, 2001). In order to keep continuity in students' learning processes, they discussed subject matter and the performances of children regularly.

Also, thanks to the headmaster's efforts, the MILE project was adopted by all (41) teachers of the school and members of the MILE team were accepted by the parents (grade 2) as well. The wish to do something in return for the school and also the intention to create a positive working climate for filmers, educators, teachers, and students was realised by offering four workshops as part of an in-service course. 'Enlarging your practical knowledge of realistic mathematics teaching' was the workshops' focus. The teachers mentioned four themes: explaining, contexts, models, and interaction in a math class. This focus on practical knowledge typified the MILE team's discussion and study. Practical knowledge is tacit knowledge; it becomes visible only in the actions of the teacher and during rare moments when teachers are telling stories of what happens in and around their classrooms. In the latter cases, the practical knowledge is hidden (wrapped) in their personal narratives (stories). Our two grade 2 teachers, sharing one job, would be able to reveal much of the practical knowledge that had been inspired by their transfer meetings.

The parents (grade 2) got serious attention from MILE. One parents' evening was recorded in its entirety and became an integral part of MILE. On that evening the parents watched an earlier recorded grade 2 lesson. Then both teachers presented themselves as reflective practitioners and explained what happened in the classroom, told about underlying objectives, and invited the audience to ask questions about their children, the teachers' actions, and the subject matter.

The lessons to be recorded were prepared in general outline by teacher and project director, not to model the ideal teacher, but to increase teachers' self esteem. It is comparable to the approach of Lampert and Loewenberg Ball (1998): they didn't want to be the model of a teacher to imitate, but indeed they want to be good teachers with their own (personal) practical knowledge of which student teachers could learn from. So the idea was that good practice for teacher education is not good practice to imitate.

The teachers agreed to follow the textbook and manual, but also add a 'problem of the week.' Every Wednesday during the five weeks of recording, the teacher would present an 'open' problem in order to stimulate interaction, problem setting, and problem solving in whole-class discussions and in small cooperative groups.

3.3.3 The scenario

To prepare the real time recordings, a shooting script (scenario) was compiled describing sequences of students' and teacher's actions during a lesson. The parents gave permission to take videos of their children and a professional film institute with experience in school classes was engaged.

The script appeared to be a useful advance organizer for all concerned, although once the lessons began, it unfolded naturally and was taped without interruption or script consultation. Scripting the scenario offered another advantage: becoming aware of the different positions of the cameras, the use of zooming in and out, the need for a clear lesson structure, the visibility of learning aids in the classroom, and the planning of fixed time periods (linked to the video tape length).

Three professional filmers operated two mobile cameras, one fixed camera, and the audio apparatus. One mobile camera just followed the teacher; the other focused on individual students or small groups. The fixed camera continually recorded wide-angle shots of the whole classroom. Because children in grade 2 do not speak 'loud and clear,' small microphones were set on the tables, hidden in small pots with plants.

The MILE educators planned with the filmers about how to capture the essentials of math classes and accomplish the general goal of making records of good practice every day during the next five weeks. The filmers had to be prepared to capture interesting events such as: children handling manipulatives, the teacher using models on the blackboard, subtle moments of help, interactions between people in the classroom, rising levels, spontaneous expressions of pupils, et cetera. Special attention has been asked for the narrative aspect of registrating, that means 'thinking in stories' while taping, i.e. anticipating the narrative that might be developed later to weave together the segments they were taping. Furthermore the 'integrity of practice' was a point of attention to the preparation of the lessons; the filmers' stance towards teacher and children was discussed: they should find a balance between commitment (to taping children and their learning processes) and distance (in order not to influence the events in the classroom). Keeping a distance proved difficult, because the camera operators became popular guests. In no time they learned the children's names and personal characteristics.

The project team chose to compromise between 'shooting a Hollywood movie' and simply recording everyday classroom interaction, searching to achieve a balance between recording authentic practice and recording representations of practice that would be optimal for use in educating student teachers.

3.3.4 The screen-test

One day was spent finding out the most appropriate camera positions, the audio management, the director's tasks, and rehearsing the recording of a math class. Teacher and students in the try-out lesson rapidly got used to the presence of three cameras, an audio installation and three filmers, but it is difficult to estimate how such circumstances influence the daily routine. Children sometimes showed awareness of being taped, even after weeks of daily recordings. Also, the teachers confessed to being more careful during their interactions with children, particularly when disciplinary measures had to be taken or when values and standards had to be discussed.

One day the teacher had to comment on the negative attitude of one of the students in a small peer group. Afterwards she explained: "I do not like approaching Sandra in front of the camera the way I do in ordinary circumstances. I do not want to hurt either Sandra or myself." Her colleague agreed: "You and I act less naturally in situations like that. Usually I say, raising my voice, 'Stop it now!' In front of the camera I first count to ten before speaking." And, referring to a recent experience celebrating Santa Claus in the classroom, she said: "I usually act the fool with the children, but this time I found myself rather reserved." In his report, the director reflected on these confessions: "It appears that the recording violates the intimacy of the classroom atmosphere. Keeping the situation natural requires specific preparation and coaching of the teachers involved." (Oonk, 1997).

The rehearsal was very helpful in organizing the communication between the director outside and the filmers inside the classroom. Three purviews on classroom teaching had to be considered: the teacher's point of view, looking over a student's shoulder, or watching as an outside researcher does. Because our focus was on researching good practice for future teachers, the teacher's viewpoint was considered to be the most relevant. In zooming in and out of the scenes, we used overall shots, half-total shots, and close ups. The half-totals usually were the most informative, such as a half-total shot of a small group at work or the teacher questioning one or two students. Close ups clearly show non-verbal expressions or the details of students' seatwork. Overall shots were used afterwards, when editing the tapes to show transitions between activities or when inappropriate half-totals or close ups had to be replaced.

3.3.5 Recording and editing

Creating a representation of real teaching practice, such as the Student Learning Environment in Michigan required recording more than the math classes themselves: the teacher before beginning the lesson telling what has been prepared; the 'transfer' discussions between the two job-sharing teachers; individual students practising basic facts in the workstation with a computer; interviews with students immediately after finishing a class, reflecting upon the lesson; a parent talking with the teacher about a

child; a teacher discussing a low achiever with the remedial teacher; a team discussion about interaction as a means to discuss good practice of realistic mathematics education; celebrating Santa Claus at school, et cetera.

Sitting in a corner in the corridor, specially equipped with headphones, a microphone, and three monitors, the project director coached the filmers in the classroom. As a didactic expert, former teacher, and teacher educator, this director/researcher was focused on capturing the children's learning processes, with an eye toward using the video with future student teachers. In contrast, the filmers were concerned mostly about the quality of the pictures.

After directing the recording of the math class and the attached activities to complete real time teaching, the director wrote notations into a reflective report, describing the events and adding ideas about how the tape might be used with student teachers. In other words, he already was thinking about making MILE educative, drawing inspiration from reflecting on recently directed recording. His report also was helpful for the editorial work. Three videotapes, recording the same events but from different points of view, had to be merged into one record of (good) practice; to maintain three different 'streams' as an alternative for the student teachers' learning environment was too complicated as a representation of real practice. The report provided guidance for making the right choices and not missing any essentials. The director's report kept focus on the narrative character of the practical knowledge perspective. A classroom plan and photographs of the pupils were necessary to keep orientation while editing. The reflections on recording and editing recorded in the director's report were published (Oonk, 1997)[24].

3.3.6 Making the records of real teaching practice accessible

After editing, the videotapes had to be digitized so that the benefits of IT-technology could be used. Then an essential problem had to be dealt with: how to make this voluminous and still growing video material transparent and accessible for students in teacher education (Hermsen, Goffree & Stolting, 1997). IT-technology solved the problem by offering a search function capable of full text retrieval. Also, much time had to be spent writing texts close to the existing visual material, and IT-technology once again was of great help. The technicians in the team designed software to divide the videos into short sections (video fragments). The events in these sections could be described (in what formerly were called 'titles') as mini-stories, making use of keywords that could guide student teachers to these sections later. The desired length of the video fragments and linked mini-stories was considered in the context of facilitating the development of practical knowledge. The minimal size of a section was called 'a narrative unit' and the rule of thumb became 'complete meaningful mini-story (narrative) with size between one and three minutes,' as small as possible to create the possibility for a subtle search, but not so small that the

fragments would lose their meaning. Experience using the search engine indicated that students typically began with one mini-story and then wanted to view video fragments that came immediately before or after the mini-story just viewed. Consequently, the system was improved to make it easier for students to shift from the segment just played to adjacent segments of the same lesson.

An example: In one mini-story, grade 2 student Chantal is called on to draw the position of the number 45 between 40 and 50 on the number line. The mini-story says: "*Chantal is measuring to find the position of 45 on the number line. She first estimates the middle between 40 and 50 to be sure. Minke gives praise and asks the other students to look carefully to the jump of ten.*"

Student teachers, making their investigations in MILE, can watch the previous and next within a sequence of mini-stories that together represent a substantive part of the math class. It is also possible to combine stories from different places to create whole cases. Thus the search function can be used in two different ways. The first brings the MILE investigator to the archive, in which dated lessons divided into sections (and other data) are organized chronologically. One can browse through the mini-stories and, if one seems interesting, click to the linked video fragment. It is also possible to find a fragment by typing a keyword (all mini-stories containing this word are shown). Each mini-story in this list is linked with the archive again, so it is possible to see the whole lesson in which it was embedded.

Making the records of real teaching searchable in this way converts 'good practice' into 'good practice for use in teacher education.' It also connects 'good practice' to 'practical knowledge' as a way of narrative knowing (Gudmundsdottir, 1995). In the next section, MILE is discussed in terms of good practice, as a starting point for exploring and thinking about making MILE educative.

3.4 MILE, a digitalized teaching practice

The heart of MILE can be considered as a digitalized 'representation of full practice' that provides examples of 'good practice' for use in teacher education. Looking back on the process of development we notice some characteristics of good practice.

Showing authenticity with real practice in schools. MILE deals with daily life in classrooms with 'ordinary' teachers, teaching in a realistic mathematics education vein, confronted with problems and dilemmas (Lampert, 1985), and with pupils both doing good things and making mistakes. It shows authentic practice, comparable with the practice that student teachers will experience.

Representing the complexity of real teaching practice. The 'full practice' represented in MILE reflects the complex reality of teaching (Lampert, 2001; Uhlenbeck, 2002).

Nearly all of the components that Lampert mentioned in her extended model of the didactic triangle (2001, chapter 14) are included, and there was an attempt to represent all areas of subject matter as well. The same attention to completeness applies to general pedagogic issues and the mathematical learning processes of pupils. Teachers reflect on their practice and pupils react during classroom discussions and in small group work. Textbooks and teachers' guides are available and in relevant situations one can meet other teachers of the school team, the headmaster, the counselor, or parents.

In the vein of realistic mathematics education. The five previously mentioned learning-teaching principles and the indicators of realistic mathematics education (section 3.2.1) continually played a part in preparing, recording and making MILE accessible.

Exemplary for the program of primary education. The digitized primary mathematics education that has been stored in MILE has been characterized as a representation of 'real teaching practice'. However, because it is not possible to capture the full range of practice in tapes taken during limited periods in different grades, the representation of the subject matter in MILE has to be considered as exemplary for the primary school. What is meant by 'exemplary' is illustrated in the next examples. MILE contains:

- Recordings of a learning strand for percentage in grades 5 and 6, with special attention to going through a learning strand, the procedure of problem-oriented education, the evaluation of certain subject matter, and differences between children.
- A theme (the restaurant) in grades K-1, with special attention on teaching young children, the development of number sense, learning in context, and interaction in classroom discussion.
- Mathematic activities in daily life, 'the problem of the week' in grades 2, 4 and 6.
- Practicing the multiplication tables and a clinical interview with two pupils of grade 2 who differ in insight and attitude about the tables.
- A pupil explaining a mathematical production.
- A pupil taking over the leadership of a group.

Through outlines. MILE offers two kinds of outlines. The first are concerned with learning strands such as multiplication tables or percentages. The second follow individuals' learning of mathematics, especially in the recordings of grades 2, 4 and 6, where the same children are shown at intervals two years apart. One can observe the children during classroom interaction, individual activities, group work, and interviews.

Reflective practice. MILE includes recording of the professional talks between the two job-sharing teachers about the progress of mathematics education in their grade 2 class. Much of the (tacit) practical knowledge of these teachers became explicit as they reflected on their practice before and after the lessons.

Theoretical charge. Without violating the authenticity of the digitized practice, the director emphasized certain situations during recording; hence sometimes there was exclusive attention to the teacher or to a pupil. For instance, the director could decide to tape one child for a longer time, to capture interesting elements such as the specific effect of a context or a clever or invalid use of a model by a pupil trying out a strategy.
Next, we describe what investigations of good practice, characterized in the previous section, brought to student teachers during their first expeditions in MILE.

3.5 The first exploratory research[ii]

3.5.1 Research question

To get insight into the (optimal) possibilities of MILE as good practice for teacher education, exploratory research was carried out using ten lessons on CD-roms and the first version of the search engine (Oonk, 1999). The study was focused on knowledge construction, but also with an attempt to provide insight into the investigation process experienced by two student teachers and the benefits of their related discourse and collaboration.
The research question was: *What is the character of the investigation process of student teachers in MILE and what is the output of their learning process in terms of knowledge construction?*
A total of 15 meetings, eight of which were two-hour sessions with the researcher, were audio recorded.

3.5.2 Learning by investigating the recorded teaching practice

Our approach resembled that used in the Student Learning Environment approach developed at Michigan rather than the more academic approach of Mousley and Sullivan (1996). But in contrast to the Student Learning Environment approach, student teachers could investigate the digitized practice of MILE by means of keywords derived from their own research questions. There was little steering: the student teachers could formulate, reformulate, and revise their own questions. The rationale for this approach was that investigating practice by focusing on one's own pedagogical problems, especially in tandem with the coupled reflective activities, will contribute to self-regulated professional development (Zeichner & Noffke, 2001).

[ii] This section is based on: Oonk, W. (1999). *Pioniers in MILE. Een exploratief onderzoek* [Pioneers in MILE. An exploratory study]. MILE-reeks nr. 9. [MILE series nr. 9] Utrecht: Freudenthal Instituut.

3.5.3 The process

In this section we describe an overview of the investigation process of two excellent student teachers of the primary teacher training college at Amsterdam (Hogeschool van Amsterdam) and their highly motivated coach/researcher.

The fifteen meetings developed through three stages, which naturally flowed from the process of the student teachers' investigation. The first was orientating to MILE by getting acquainted with the teachers Minke and Willie of grade 2, the pupils, the subject matter, and the techniques for searching in the learning environment. The student teachers Dieneke and Hayet had at their disposal a computer, ten lessons of grade 2 on CD-rom, the 'Telling stories of grade 2' (Oonk, 1997), the pupils' textbooks, and the teachers' guide for the mathematics textbook series 'De wereld in Getallen' [The World in numbers][25]. The first meeting started with a short discussion about the goal and the schedule of the learning project and continued with an orientation to the technique and content of MILE. The following quotes from the student teachers give a representative idea of their reactions to the first meeting:

> Beautiful, such a transfer you never can see in your school practice, that happens mostly by phone.
>
> Children can suddenly have an 'aha' experience, as in an active moment after a passive period of language acquisition.

In this first period, student teachers got used to the styles and personalities of the MILE teachers. Their comments gradually shifted from impulsive perceptual reactions to reflective discussions. Often a remark led to (sometimes heavy) discussions, which frequently led to re-viewing of the video.

The second stage was learning to investigate. The publication 'Telling stories of grade 2' (Oonk, 1997) offered the student teachers a point of departure for searching and studying video fragments. However, this was not sufficient, and the students soon reverted to trial and error. This led Hayet to create a step-by-step plan for searching and observing the ideas and for formulating (new) questions.

The third stage was directed research. It began when the student teachers decided to make a video to orient imaginary peers to MILE. They thought that this would allow them to show their own learning, make available an 'orienting adventure in MILE' for young future teachers, and inject an element of originality to their report and presentation. By this time, Dieneke and Hayet had acquired experience in using MILE and investigating their own learning questions.

The culmination of the investigation consisted of an oral exam, a written report, and a presentation. Audio recordings during the discourse, e-mail communications, and written reflections document the collaboration and the individuals' learning and thinking processes.

3.5.4 Incentives in the learning environment

We next discuss what the first, simple version of MILE offered the two student teachers and their coach/researcher, and focus on the question: What makes MILE educative? We probe to answer that question by elaborating main incentives in the learning environment.

Search words, search questions, and learning and investigation questions.
In their investigations in MILE, the two student teachers were limited by the simple search engine and their own search questions. In the first stage they often used 'Telling stories of grade 2' (Oonk, 1997) as a source for finding search words (e.g., mental action, 'playing Dumb August'[26], 'egg box'[27]). To some extent the 'The World in numbers' textbook and the teacher's guide served the same function. To prepare for meetings, they used the textbook to plan the same lesson as the lesson taught by the MILE teacher, and then observed that lesson. This made the subsequent discussion more reflective than it had been previously. They raised questions such as: Did I devote sufficient time in my lesson to dividing numbers? Did I leave too much to the children? Can they actually estimate prices?
After seeing how the video teacher approached the lesson, they would draw comparisons and perhaps revise their own lesson plans.

Revealing practical knowledge. The student teachers were impressed by what they saw in MILE. The discourse often ended in personal analyses from different points of view. They called upon mathematics aspects but also took pedagogical and content pedagogical positions. From the beginning they were convinced that they could learn a lot from MILE teachers. Dieneke explained: "What are the 'good questions' that Minke asks? I noted how well she formulated an exercise: 'If you have thought and drawn one way, try to think of another one.' I would not mind hearing this remark a thousand times so I can imprint it in my mind. Instead of a compulsory exercise such as 'Give at least 3 solutions,' Minke shows she appreciates one solution and she encourages the pupils to think further. In my opinion, her choice of words is a clear example of good teaching."
However, Dieneke did not accept Minke's practical knowledge unquestioningly. She analyzed, interpreted, and put forward arguments to substantiate what she believed she had observed. In doing so, she touched on the practical knowledge that appeared to guide Minke's actions (in effect, unpacking this practical knowledge). In addition, she recognized 'usable material' even in the routines of the daily classroom activities. For example, Dieneke learned that Willie – Minke's peer teacher – could quickly quiet the children and get their attention by remarking: "Everyone turn the calculator in your head on."
From the beginning Hayet philosophised about the additional value of MILE as compared with teaching practice or lectures, and she also recognized the practical

knowledge of the expert teachers in MILE. She was especially impressed by the interviews, in which the teachers told about their plans for the next lesson: "Of the three types of video (transfer, interview, lesson), the interview made the most impression on me. It was like looking into the head of the teacher and finding out secret information. Going through a lesson step-by-step in this way is very practical and concrete. None of my tutors ever did this for me."

Theory from practice. Searching for a direction to their investigation, Dieneke and Hayet got the idea to design a video for their peer first year student teachers, inspired by a lesson in which teacher Willie introduced the five times table. They became interested in the ways that Willie translated concrete material to the children, the children worked with that material, and the material precipitated mental action. By connecting to relevant theory, Dieneke and Hayet would make a statement about encouraging the rise from material to mental level in children's learning. They formulated questions, made notes, then started theorizing: "We have made the following statement based on this video: 'If the transition from concrete to mental action does not take place in sufficiently small and logical steps, the (material and mental) actions will remain separated from each other. The main objective is to couple these actions together (…).' "

Theory of practice. After the above mentioned discussion, the teacher educator wrote an extensive (electronic) annotation to make the student teachers aware of the distinction between a mechanistic 'step-by-step' approach and realistic didactics in which properly conceived 'learning' jumps in teaching are encouraged. He also focused on theoretical views on different levels of subject matter and learning processes applied to the structure of mathematics courses. The student teachers became very interested as they addressed these problems. Dieneke recognized the danger of misunderstanding rules from her own past education. In the next meeting, she revised a previous statement about a video fragment in which a pupil shows that thinking of egg boxes helps her interpret 43 as 40 (four egg boxes) + 3 (separate eggs): "This statement applies to small logical steps, raising pupils' levels, and direct support. Support and raising pupils' levels are similar concepts. The material is first used as a support. When pupils no longer need it and begin to construct mentally, their level of competence rises. Materials always help and support, provided you introduce them properly. If you do not do this, they are only extra ballast and lead to confusion. The fragment 'Think of egg boxes' is a good example of this since Minke had not referred to them before and 'suddenly' introduced them without moving through a sequence of small logical step towards them (…)."

The practical meaning of theory. The discussion about how to raise levels of thinking required student teachers to find relevant theoretical knowledge. They questioned the teacher educator and studied relevant articles and textbooks. They realized that MILE

contains information (practical knowledge) that cannot be found in textbooks or teachers' guides and compared the theory taught in lessons at college with the observed theory linked to the real-life situations in MILE. For example, by relating theory to practice, Hayet became aware that the terms concrete, abstract, mental, and formal initially put her on the wrong track: "In my opinion, 'the abstract level' means formal mathematics. 'Mental actions with material' contains the word mental, but does not belong on that abstract level. It should be placed on the concrete/material level. Actions with material are performed 'in your head' (it is not a physical action), but they are concrete, or in other words, conceivable and meaningful (…)."

The discourse as driving the learning process. The discourse about the practice in MILE was elaborated in corridor chats and e-mail communications. The student teachers were aware of the influence of the discourse on their cooperation and their learning processes. As Hayet put it: "I find it rather comical. If I had watched the video on my own, I would probably have missed the whole scene. The discussion that was brought about by that 'small interesting incident' is for me just like the didactics involved the most instructive part of the whole meeting. Another good thing about the discussions is that we start with critically thinking about what Minke does and why she does it. Is it part of her conscious strategy? And then the emphasis shifts to our own experiences and didactic considerations."

One's own practice as reflective. The practice in MILE engendered student teachers' linkages to their own teaching practice. This happened in a natural way as they analyzed and compared the actions of the MILE teacher to their own experiences or anticipated future practice. Hayet compared her failed experience with 'Playing Dumb August' of her pupil Keltoum to the successful experience of MILE teacher Minke. Placing the 'Dumb August' approach in a wider perspective, she started formulating questions and hypotheses. She wondered about the relationship between one's approach to 'Dumb August' and learners' attitudes about making mistakes: "When you want to teach children to investigate mathematics (i.e., try out more than one strategy), it is important that you see making mistakes as part of the process. If you do not, you go straight back to mechanistic viewpoints: there is one way to solve a problem – the right way, mistakes are bad, and children who make mistakes are stupid. This MILE episode made me realize the strength, but also the possible dangers of the 'Dumb August' method."

Final presentation. Both student teachers tell a lot of stories about their pioneering in MILE (Blikslager & De Bont, 1997; Oonk, 1999). In the closing presentation of their assignment, Hayet explained how her investigation in MILE had made her aware of what is behind theory in the lectures she hears in teacher education courses. She believes that MILE stories will help her keep in mind the connection between theory and practice.

3.5.5 The main findings of pioneering

The pioneers' experiences during their investigations of the first version of MILE can be interpreted from four points of view.

The investigating process. In the course of the exploratory research, the investigating process was portrayed as the vehicle of the student teachers' learning process. In other words, carrying out investigations kept the learning process in motion. The investigating process in MILE is a cyclical process of planning, searching, observing, reflecting, and evaluating. Its relationship to action research (Carr & Kemmis, 1986; Jaworski, 1998) is obvious, although there are differences in reasons, goals, and content.

The discourse. The discourse that was put together during the meetings can be seen as the driving force behind the learning process. The discourse can provide opportunities for the emergence of new insights, incentives for searching further, and impetus for choosing a theoretical line of approach or developing one's own theorizing. The discussions observed during the investigations of Dieneke and Hayet have the characteristics of 'reflective discourse' (Cobb, McClain, & Whitenack, 1997), which we treat as a socio-constructive adaptation of Schön's reflective conversation (Schön, 1983).

Levels of knowledge construction. The analysis of Dieneke and Hayet's investigation process has provided insight into knowledge construction at four levels. These levels are observable particularly in the discussions and the reflective notes.
Firstly, knowledge can be 'taken over' from the teachers in MILE; student teachers expand their own didactic repertoires through assimilation of the practice knowledge contained in MILE. Assimilation occurs if the student teacher indicates that he or she would like to implement the knowledge of the MILE teacher (as observed, without adaptation to his or her own purposes).
Secondly, adaptation and accommodation of practice knowledge is a second level of knowledge construction. Users of MILE can modify the repertoires of the MILE teacher to suit their own purposes; they expand their own repertoires by modifying the MILE teacher's repertoire. Knowledge construction at this second level can have a greater impact on the student, especially if the student has to adjust his or her personal beliefs. In this case, something changes in the cognitive or affective structure that we call practice knowledge. For this reason, this is called accommodation. It also has some of the characteristics of what Perry calls relativism. Knowledge on the first level that is constructed on the authority of the 'model teacher' has more of the characteristics of Perry's 'dualism' (Hofer & Pintrich, 1997).
Thirdly, the student teachers display an even higher level of knowledge construction when they establish (new) links between the events in MILE and events from their own trainee practice and related theory. This is the level of 'integrating theory,' in which

they might (re)consider didactic insights and points of view. They ask themselves questions about the situation observed in MILE and make links with what they find in the literature. The teacher educator has a task in this respect, namely to respond effectively to the questions posed by the student teachers. Using theoretical reflections – as annotations – about given real practice situations that intertwine the investigation of practice with the examination and development of theory, the teacher educator puts students on the track of explanatory theory. The theory is understood and remembered in the context of a scene in MILE, usually in the form of a story of the events that occurred in MILE.

The fourth and highest level of knowledge construction – the level of theorizing – manifests itself when the investigators in MILE design their own local theories. They build up ideas about causes and consequences through the observation and interpretation of fragments they find themselves. The ensuing discourse can have a specific theoretical orientation and provide motivation for follow-up investigations.

3.6 Larger scale field tests

Since the seventies, Dutch primary teacher training colleges have created a strong infrastructure for mathematics education, in which teacher educators and curriculum designers collaborate in a Mathematics and Didactics program for future teachers. The teacher educators meet annually at conferences. This infrastructure was used to introduce MILE and invite math teacher educators to use and assess it. From the beginning more then twenty educators were participating in the workshops, discussing the state of the art of MILE, and anticipating its future use in their own colleges.

Shortly after the first exploratory research at the primary teacher training college in Amsterdam (section 3.5), five pilot studies were started at fourteen other Colleges (Goffree, 1998). In these pilots, three aspects of MILE were field tested: using MILE within the local curriculum, the optimal use of MILE by student teachers, and the relationship between teaching practice in schools (fieldwork) and MILE. All pilots addressed the issue of MILE's position between theory and practice in teacher education. In the meantime the basic MILE program had been increased to 30 gigabytes consisting of 23 lessons of grade 2 and five lessons of grade 5 (percentages) at a different elementary school. Student teachers could use the search engine and the archive now, without having to change disks.

Although approaches to MILE varied at the different locations (size of groups, the scale of the research, the amount of face-to-face instruction, collaboration, coaching, assignments, and reports), the common output provided a lot of reflection on the creation of a learning environment for future teachers. Studying the output, an obvious question arose: What should be added to MILE in order to help student teachers to

enlarge and deepen their practical knowledge? Assignments and reports provided vehicles for addressing this question.

The outcome of the analysis yielded answers to main questions according to the desired improvement of MILE.

What practical knowledge do student teachers have available at the start of their investigation? Like the pioneers (Dieneke and Hayet), the participants of the pilot projects were (mainly) last year students. Their practical knowledge became apparent during observations of lessons when they were asked to question MILE. For instance:
Would the teacher indeed be able to teach this subject matter interactively?
How does the teacher deal with low achievers?
Does the teacher focus students' attention on different strategies?
These student teachers apparently know a lot about math teaching; now they want to see concrete examples in practice. Sometimes their practical knowledge seemed to hold theoretical elements.

How was MILE experienced as a learning environment?
Their subsequent statements indicated that the student teachers in the pilots experienced MILE as:

- A reservoir of instructive events.
- Storage of all sorts of aids to use in math classes.
- A place to work for teachers and pupils.
- Practice in which 'the theory' becomes apparent.
- A set of opportunities to show one's professionalism.
- 'Something else' rather than teaching practice.

Which self formulated learning questions initiated student teachers' investigations?
In most pilots, the teacher educators presented good reasons to start an investigation. A successful reason was created by having student teachers prepare a lesson that would be observed subsequently in MILE. Other reasons were found in fieldwork, in lectures, in the storybook (Oonk, 1997), or in the theory. An example of such a learning question, which subsequently led to investigation questions and searching words, was "What is the nature of collaboration between a gifted student and the other students in a small cooperative group?" Other learning questions addressed using the blackboard, the teacher's behavior, the learning process of one selected pupil, classroom interaction, group work, mental arithmetic, and estimation.

What did the video-pictures of MILE evoke in student teachers?
MILE sought to represent practice, so it is interesting to know which aspects of practice student teachers attend to in MILE. The reports mention a number of favorite fragments,

both from student teachers during their first exploration and from teacher educators who wanted to illustrate practice using MILE.

What did the student teachers say they had learned from MILE?
The student teachers' statements about what they had learned from their investigations in MILE were fairly vague. Second-year student teachers believed that learning from the MILE teacher is the same as imitating the MILE teacher. Older year student teachers pointed to the transfer talks between the job-sharing MILE teachers and to the number line used as a model during instruction. More profound was the reaction of a fourth-year student teacher who wrote: "I learned that during a math lesson more happens than you could imagine in advance—much more than the teacher can see. Because of MILE you become aware how children think, more so than you do from just reading theory books."
The learning attributed to MILE varied with the context in which the question was asked, the nature of the studied fragments, and the student teachers' reflective abilities. Just as in the exploratory research on the pioneers, the pilot projects suggest that methods for learning about practice need to be learned along with practice itself. Both projects underscore the value of discourse for the learning process. Furthermore, having some basic practical knowledge at one's disposal, taking initiative, posing one's own learning questions, considering one's own past in math education, and moving beyond an initial critical attitude towards the MILE teacher benefits learning by investigating MILE.
In order to transfer MILE into 'good practice for future teachers,' it appeared to be necessary to supply MILE with 'reasons' to start investigations, to help student teachers to engage in the desired activities and reflections. Actually from this moment the need for structuring the learning environment became more and more manifest.

3.7 Making MILE educative

In section 3.5.4 we discussed the question: 'what makes MILE educative?' by elaborating the main incentives in the learning environment. The analyses of the research results as described in section 3.5.5 and 3.6, led to refining of the answers to that question. We defined therefore 'educative' as enabling future teachers to acquire practical knowledge, and investigated the conditions, circumstances, and means needed to accomplish that goal. The following inventory shows the main part of the observerved conditions and means.

Initial achievement level of the student teachers:

- It appeared that student teachers learned something more than imitating in MILE if they already had a basic practical knowledge; obviously, student teachers cannot start investigations without at least some reflected experience in classrooms.

- Searching in MILE has to take place in an investigation context, preferably on the basis of searching words that arise out of a learning question.
- Various reasons to start an investigation can be constructed by tutor or tutee in order to formulate learning questions.
- Formulating personal learning questions needs directed support by a coach who is familiar with the contents of MILE.
- Student teachers who want to start an investigation in MILE should first have practiced learning by investigating and observing and interpreting MILE fragments.
- New student teachers begin with a personal belief about good practice.

Acquiring practical knowledge as narrative knowing:

- The meaning of practical knowledge can be clarified by discussing and analyzing the transfer talks between the two MILE teachers at grade 2.
- The preliminary talks (interviews) with the teachers, when they put into words what they intend to do during the next lesson, provide attention points for observing.
- The book 'Telling stories of grade 2' (Oonk, 1997) can be utilized as a source book for investigating MILE.
- The investigating process is the vehicle for the student teachers' learning.
- The discourse about the practice drives student teachers' investigating process.
- Meaningful assignments are needed to support student teachers' progress with MILE.
- Teaching stories about the events in MILE can be best written by the student teachers as case studies.
- Studying pedagogical and didactical actions of the teacher in MILE motivates student teachers to compare MILE with their own fieldwork.
- Surprising moments of pupils' learning might help student teachers to connect with their own primary school experiences.
- To stimulate narrative knowing, student teachers may collect favorite narratives linked to MILE videos.
- Input from the MILE tutor usually deepens student teachers' learning.
- Student teachers have to learn to participate in reflective discourse.
- Collaborating generally stimulates the continuity and profundity of an investigation.
- Developing 'didactical productions' creates an appropriate context for investigating in a narrative, constructive, and reflective way.
- Preparing a MILE lesson before observing it helps students to make sense of the discourse.

- Student teachers report valuable experiences when theoretical concepts or reasoning are actualized through observation of practice, as when the MILE teacher organizes a classroom discussion about pupils' mistakes or when the student teachers recognize differences between children.

Limitations and bottlenecks.

In this period the MILE team tried to adapt the idea of a Knowledge Forum (Bereiter & Scadamalia, 1997) in order to improve the discourse of the student teachers, using its functions such as special interest groups and e-mail. The Knowledge Forum provides a structure for knowledge construction by remote cooperating students. However, this attempt had to be dropped because of technical problems and the problem of the narrative organization of practical knowledge.

The research revealed some important bottlenecks in the way to enable future teachers to acquire practical knowledge:

- Although the MILE team supported the open approach of the student teachers' environment, the field tests pointed towards the need for more structuring and scaffolding of the student teachers' learning by investigating.
- Student teachers usually require support in learning to relate theoretical knowledge to the practice in MILE.
- Student teachers do not realize immediately that theoretical knowledge is a valuable addition to practical knowledge.

Research and development of the educative component of MILE had to be continued, because the program did not yet enable student teachers to carry on reflective conversations with practice (as mentioned by Schön, 1983) or relate known theory to observed practice. The dangers of superficial reflection on practice are pointed out by Verloop: "It is worthwhile to reflect on one's teaching, but at some stage questions should be asked about the quality of that reflection. For example, to what extent does the reflection take relevant external knowledge into account, what exactly does reflection improve in the actual teaching process, and to what extent is the reflection open to external scrutiny and critique?" (Verloop, 2001, p. 436).

The next section will describe the research that goes into the question of whether student teachers indeed use theory when studying practice.

3.8 The second exploratory research[iii]

3.8.1 Research question and method

The second exploratory study was designed to explore the nature of relating theory to practice. The research question in this context was: Which signals of utilizing theory do student teachers show in their reflections on studied practices of MILE?
MILE has been expanded with video records of mathematics activities in Kindergarten, group discussions in grade 1, and interviews and mathematics lessons in grades 2, 4 and 6, to an amount of 70 gigabytes.
Research at the teacher training college at Helmond was designed to find out how prospective teachers make connections between theory and practice in MILE. This research involved two classes, each with 25 student teachers. The teacher educator gave his students a list of 150 key theoretical concepts from previous courses, to serve as a theoretical framework to help student teachers to value their previous theoretical knowledge when they start the new MILE course ('The Foundation') for second year student teachers (Dolk et al., 2000). Ten two-hour meetings were held. Following the method of triangulation (Maso & Smaling, 1998), four pairs of student teachers were observed and interviewed, and a participating study of the group work with two student teachers was conducted.

3.8.2 Identifying theory in action

Schön (1983) has demonstrated that theory in action is primarily implicit, so the researchers generated a list of possible signals of theory in action to support the observations of student teachers at work (see appendix 1). Following are some examples, in which each signal is coupled with a representative case of theory in action and references to its sources:

- While observing practical situations, student teachers can *refer to the theory that comes to mind.* Example: student teacher points to a teacher who interprets the product of 2 x 5 and, in doing so, employs the rectangle model (Treffers & De Moor, 1990, p. 75).
- Theory is used to *explain* (as a means to understand) *what occurred in the practical situation* observed. Example: student teacher explains the method employed by the pupil who is using MAB (base ten) material as a working model (Gravemeijer, 1994, p. 57).

[iii] This section has been published in adapted form as: Oonk, W. (2001). Putting theory into practice: Growth of appreciating theory by student teachers. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th conference of the International Group for the Psychology of Mathematics Education 4* (pp. 17-24).

- The student *interprets the intention of the teacher or pupil(s*) with the help of theory. Example: student teacher points out the 'mirroring technique' applied by the teacher as a means to help the pupil reflect his own actions (Van Eerde, 1996, p. 143).
- The theory *generates new practical questions*. Example: student teacher wonders at which level (stage) of learning multiplication the pupils are (Goffree, 1994, p. 280).

3.8.3 Theory in action. An example

Student teachers indeed showed certain predicted signals of utilizing theory (Oonk, 2001). However, these signals were generally rather weak and ambiguous to localize, and it was hard to determine how and when utilized theoretical knowledge had been acquired.

We will portray the essentials of the research in three steps: the characterization of a fragment in MILE, the theory the researcher/teacher educator linked to that practice, and what student teachers had to say about the fragment.

- The MILE fragment shows a pupil, Fadoua, and her teacher, Minke, sitting at the instruction table during an independent working session in grade 2. Using a diagnostic interview, Minke seeks to identify the thinking behind Fadoua's mistake (18 - 6 = 11) in her seatwork. It appears that Fadoua counts backwards starting from 18 and in the process also skips two numbers (12 and 14).
- The theory that makes this practical situation more comprehensible is a result of research into subtraction strategies employed by young children, in particular the method of counting backwards. Initial errors, counting mistakes, and counting too far are well-known problem areas. To avoid problems in the transition from manipulative to mental calculations when learning to shorten procedures, structural models based on visualising 'fives structures' can be employed to learn to subtract numbers to twenty (Gravemeijer, 1994; Van den Heuvel-Panhuizen, et al., 1998).
- After watching and analysing the video, the student teachers Denise and Marieke discuss the most appropriate way to assist Fadoua. Denise initially suggests solving the problem using 18 blocks (units). Marieke rejects this idea however, because she believes that it doesn't solve Fadoua's counting problem. She also rejects a second suggestion – using the number line – for the same reasons. Marieke ultimately agrees with Denise and suggests using the reckon rack. Denise's foremost argument is that the fives structure of the reckon rack can help Fadoua to address the problem by directly subtracting 6. "And that doesn't involve counting anymore," she says.

In this discussion between the student teachers Denise and Marieke we see theory in action. They compare, face, and consider, on the basis of theoretical perspectives, which material or model is (or is not) appropriate and why. A similar process occurs when they design an explanatory approach for pupil Fadoua, partially on the basis of theoretical considerations (the reckon rack teaching method).

3.8.4 Some results

The results of this study revealed that student teachers used theory as a means to understand and explain practical situations. The frame of reference of second-year student teachers appeared somewhat diffuse and fragmented. It remained difficult to separate practical wisdom from pedagogical theorizing (cf. Shulman, 1987; Sockett, 1989; Fenstermacher & Richardson, 1993; Pendlebury, 1995). The ability to articulate observations of and reflections on practical situations in theoretical terms remained largely undeveloped. The culture of teacher training colleges also seemed to hamper this development. As a result, there is a real danger that student teachers will hang on to their personal (subjective) theories instead of learning to integrate theory and practice to attain Theory-Enriched Practical Knowledge (EPK). To improve the educative character of MILE it was concluded that accommodations in two directions were necessary. On the one hand, theory should be stated more explicitly in the learning environment. On the other hand, in mathematics teacher training more time should be spent on discourse and tutoring, less at the level of 'coaching at a distance.'

In the next section we will go into the intended accomodation of the learning environment for the following research.

3.9 Practice based professionalization and enriched practical knowledge

3.9.1 The necessity of enriching practical knowledge theoretically

In the previous sections involving activities with student teachers, we investigated real teaching practice in MILE, to explore how and what student teachers learned using MILE as well as how they acquired different levels of practical knowledge. The results of our first exploratory research and the pilot projects provided us with a clearer understanding of how student teachers can construct practical knowledge, the value of discourse, and the nature of the investigation process. The second exploratory research provided some evidence of student teachers making use of theory when studying real teaching practice using MILE.

The research evoked questions about the quality and depth of students' learning from practice. Theory seemed to be missing in some observations. It was clear that when a tutor/expert was included in the discussion and when common observations ended in a reflective conversation, the use of relevant theory (in action) was guaranteed. In the

absence of an expert, the use of theory in action appears to be more difficult. Reflecting on real teaching practice does not ensure theoretical thinking or practical reasoning. Student teachers' learning processes easily remain on a superficial level and rarely move beyond basic common sense. Although the use of common sense is necessary, it is insufficient for a teaching professional.

Similar considerations apply to the practice events offered to student teachers: good practice judged as such by a school inspector is not necessarily good practice for use in teacher education courses. The latter should give student teachers the opportunity to make connections between their theoretical frame of reference acquired in earlier courses and the (new) teaching practice observed in the study. Student teachers can organize practical knowledge narratively in a natural way, but this knowledge only contributes to professionalization if it is activated in appropriate situations along with connected theory. Research on teacher education has begun to pay a lot more attention to the need to combine theory with practical reasoning (Loewenberg Ball, 2000; Lampert, 1998a; McAninch in Masingila & Doerr, 2002, p. 241; Verloop, 2001).

Being a reflective practitioner, the teacher educator plays an important role when student teachers' practical knowledge has to be enriched theoretically. In doing so, he deploys the instrument of reflective conversations, applying it during discourse, supervision, and coaching. But if necessary (when experts are not available), reflective conversations also may be an important element of the learning environment, in written form or otherwise.

The exploratory investigations showed student teachers gradually applying theoretical considerations to commonly studied teaching practice. But it also became clear that connecting theory to practice seldom appeared spontaneously, without a tutor giving support. Reflective conversations help to avoid the learning paradox as stated by Bereiter (1985), which in the context of learning from practice says: Who does not know much, does not see much.

3.9.2 The learning environment for the next research

A tentative conclusion may be drawn: it is evident that theoretical enrichment is necessary for the intended learning of student teachers in our digitalized learning environment. We have been studying the possibilities of theoretical enrichment and also the conditions of an appropriate learning environment for acquiring practice-based professionalism. One study that has yielded focal points for theory in teacher education originated from an analysis of the characteristics of theory (section 2.6). The characteristics were helpful to develop a new learning environment for the next research. Furthermore, part of that environment is the so called 'Guide' for learning to teach multiplication in grade 2 (Goffree et al., 2003). The Guide is a CD-rom and can be considered as an adapted version of MILE, following our ideas about making the

program educative and emphasizing the theory-to-practice connection (see section 4.2.2.2).

It is assumed that the new learning environment will prompt student teachers to engage in practical reasoning (Fenstermacher, 1986; Pendlebury, 1995) to a greater degree than the earlier versions of MILE. Oral and written, videotaped expressions in student teachers' interviews, reports, assignments, and presentations can reveal theory-enriched practical knowledge (EPK).

4 The small scale study

4.1 Introduction

On the one hand the small scale study of fourteen third-year students of the fulltime primary teacher education described in this chapter was a logical continuation of the two preceding exploratory studies, and on the other hand the study prepared for the large scale study that would follow it.
The main goal of the small scale study was to map the possible variation and depth of the student teachers theory use in a theory-enriched learning environment.
The intention was to further optimize and chart the theory use of students, and generate an optimal collection of data, both in this study and the large scale study. A connected goal was to develop a suitable reflection-analysis instrument, if possible on the basis of the fifteen 'signals of theory use' that had been formulated in the second exploratory study.
What follows explains the way the study has been set up and conducted. Central to that description is the case study of the student teacher Anne, who represents the variations among the student teachers in reasoning and depth of theory use.

The preceding exploratory studies provided a first insight into the use of theory by students. The first exploratory study (section 3.5) identified student teachers' levels of knowledge construction and their investigating process in the learning environment MILE. Especially at the so-called third and fourth level of knowledge construction, the 'integration of theory and practice' occurred when students asked themselves questions about situations they observed, when they made a connection with the literature or when they formulated their own 'local theory.' In the second exploratory study, the MILE-learning environment had been extended with lessons from various grades to an amount of 70 gigabytes and an advanced search engine to enable the students to search the lessons and additional materials. Furthermore, the students had at their disposal a list of theoretical concepts to serve as a theoretical framework to help them to value their theoretical knowledge, and a textbook with learning and investigations assignments. It turned out that the students were only able to rise above the level of reacting in terms of 'practical wisdom' in situations where the teacher educator participated. Analysis of the results based on fifteen 'signals of theory use' (section 3.8; appendix 1), showed the need for a more structured and 'theory-enriched' learning environment (section 3.9). Based on that conclusion the learning environment was adapted in service of the small scale study. The assumption was that a more structured and enriched learning environment would lead to stronger use of theory in both a qualitative and quantitative sense.

The research question for the small scale study was:
In what way and to what extent do student teachers use theoretical knowledge when they describe practical situations after spending a period in a learning environment that invites the use of theory?
A sub-question to this question in the small scale study was:
To what extent is there a relationship between student teachers' use of theory and their level of numeracy?

4.2 Method

4.2.1 Context and participants

Fourteen third-year students were involved in the small scale study, six from IPabo in Amsterdam and eight from IPabo in Alkmaar. All fourteen students were female and aged between eighteen and twenty. Previous education varied between mbo-level (senior secondary vocational education) to vwo-level (pre-university education). They were part of a larger group of twenty-two students in Amsterdam (twenty-one women and one man) and twenty-five students in Alkmaar (twenty-four women and one man) who had chosen the special subject 'The young child,' one of the special subjects they could select in the third year of the four-year, full-time teacher training college they were following. This special subject targets the education of children between the ages of four and eight, and involved kindergarten and grade 1 and 2 in primary education. One of the subjects the students took for this special subject was mathematics education; the formal study load for that area was 80 hours of study with 18 hours contact time for meetings led by the teacher educator (6 x 3 hours).
After four meetings in four consecutive weeks, all students entered a period in which the emphasis was on independent study, orientation on teaching practice, and planning and design for the three weeks of teaching practice to follow. During that period the students received individual guidance and were stimulated to work in groups. After the period of teaching practice there was to be one more meeting for (theoretical) reflection, aimed at linking the student teachers' experiences from teaching practice and their achievements from the course at the training college. The course was closed with a presentation and a final reflective note from each student.
The course in both programs has – from March to June 2003 – been given by two experienced teacher educators, both authors for 'The Guide' (see section 4.2.2.2), part of the learning environment for the student teachers.
The fourteen students volunteered to take part in the study, after being informed by their teacher educator and the researcher. That choice was partly determined by their preference for teaching children between the ages of six and eight, the age group the course was targeting. The five meetings that were part of the study each consisted of a

one hour lecture for the whole group of third-year students, followed by separate ninety-minute meetings for the students in the study-group and the other students.

4.2.2 The learning environment

4.2.2.1 The design of the learning environment (see also section 2.7)

The meetings for the students in the study group were prepared by the researcher in close cooperation with both teacher educators. There was advance consultation on the global set-up of the course for the students. In a try-out with four groups of students (63 student teachers), four components of the learning environment for the course were tested; those were the 'list of concepts' (appendix 2A and 2B), 'The Theorem' (appendix 3A and 3B), 'The Guide' (section 4.2.2.2) and the numeracy test (appendix 18). Afterwards, the researcher developed a first version of the course. There was a debriefing after each meeting and the researcher did suggestions for the continuation of the course. Elements were added to the learning environment with the intention of challenging students to 'practical reasoning' (section 2.3.1 and 2.7.1) to make the theory-loaded practical knowledge present in the multimedia learning environment explicit and to analyze it, in order to allow construction of 'theory-enriched practical knowledge' (EPK; section 2.6.5.5). Examples of such elements were: practical narratives with theoretical reflections and literature, a multifunctional list of defined concepts, discussions based on propositions that had been formulated in group sessions, a 'game of concepts'[28], research in one's own field placement and writing 'annotated stories' and reflective notes. A general characteristic of the curriculum development was the multiple embedding of theory (intrinsic, extrinsic; section 2.6.4.) and the attempt to achieve a balance between content components (Klep & Paus, 2006), between self-guidance and guidance by the teacher educator (Vermunt & Verloop, 1999) and between 'school practice' and professional practice (Richardson, 1992) (see also section 2.7.1). These characteristics became visible in the discourse under the direction of the teacher educator, when cooperating in small groups[29] and in working under guidance on the basis of a personal learning question and independent study. The students' learning environment – this is the course including 'The Guide' – had the character of a 'learning landscape' (Vergnaud, 1983; Lampert, 2001; Fosnot & Dolk, 2001). In that learning environment practical knowledge of experienced teachers was made visible. Directed by the teacher educator practical knowledge was made explicit and enriched with theoretical notions in cycles of observation, analysis and reflection. Next, a global overview of the students' activities during the course is given.

Programme of the Course

Meeting 1. Course introduction

- Introduction to the programme.
- Filling in list of concepts (individual, 30 min.).
- Numeracy test (one hour; individual, supervised).
- Independent study: self-assessment ('work concept').

Meeting 2. Introduction of 'The Guide' and the personal learning question

- Introduction of 'The Guide' (CD-rom); discussion under the direction of the teacher educator.
- Thinking up and formulating a personal learning question: introduction by teacher educator; plenary discussion.
- Independent study: study with the aid of 'The Guide' and writing a commentary on a personally selected teaching narrative (initial assessment); elaborating the personal learning question.

Meeting 3. Presentation and discussion

- Students present and defend their comments on the selected practical situation from 'The Guide.'
- Reflection on the situation based on a comment selected by the teacher educator.
- Assignment for the practice environment: introduction and discussion.
- Independent study: with the aid of 'The Guide'; continuing to work individually and in small groups on the personal learning question and prepare for the teaching practice.

Meeting 4. Game of concepts and concept map

- Game of concepts: introduction and discussion directed by the teacher educator.
- Defend or refute a 'theorem.'
- Learning strand of teaching multiplication: theoretical reflection by teacher educator.
- Concept mapping.
- Independent study: work on the personal learning question; preparing a study of a student's table network.

Meeting 5. Final assessment (supervised)

- Filling in list of concepts (concepts that have received meaning, including teaching narratives for two of these concepts).
- Writing a reflective note for an (unknown) MILE-situation.
- Questionnaire (anonymous): filling in questionnaire.
- Hand in final assessment and report of teaching practice.

4.2.2.2 'The Guide'

The Guide for mathematics in grade 2 (Goffree et al., 2003), a CD-rom and main part of the learning environment during the course for the student teachers, can be considered as an adapted version of MILE, in the sense that the educative component has been extended twice. 'The practice' is available in a website structure in three layers. In the first (narrative) layer, twenty-five narratives (stories of real teaching practice, in texts and videos) are linked to twenty-five matched reflective conversations in the reflective layer. It is possible, starting from either layer, to find explanations and working definitions of the keywords used, using links to a dictionary consisting of 59 key concepts from the theory of learning and teaching multiplication. It is intended that these key concepts, which have been given meaning through the reflected narratives, become part of the student teachers' theoretical frame of reference. The CD-rom also contains pages from students' textbooks and teachers' manuals, relevant articles from professional journals, and other texts, pictures, and videos of interest. Student teachers can navigate (surf) through this 'workplace' as they would do on the internet.

The starting page of the CD-rom shows six main entries:

- an instruction, containing suggestions for working with the CD-rom;
- an 'introduction,' containing subject-specific content information about the work of the teacher in grade 2 of primary school;
- an 'archive,' containing the 59 elaborated key concepts mentioned before;
- 'teaching narratives,' containing twenty-five narratives from practice in text and video;
- 'reflective notes,' containing theoretical reflections on each of the twenty-five narratives and the integral use of the theoretical concepts in these notes;
- a 'thematic entrance.' This gives students the chance to approach the work of the teacher in grade 2 from both a pedagogical content perspective and from a more general methodical one; there are twelve themes to choose from.

4.2.2.3 The substrate for 'the use' of theory in the learning environment

The inviting character of the learning environment regarding the use of theory has been realized by operationalizing the theory in several ways:

- as a written list of concepts with general methodical and pedagogical content concepts, offered to the students as a frame of reference at the start of the course (appendix 2A,B). The list served as an advance organizer for the students and was available to them during the course in digital form as well. During the course the list provided support and insight into progress;
- as an index of elaborated defined theoretical concepts on the CD-rom, but also incorporated in the twenty-five reflective notes in The Guide;
- by 'feeding' the process of reflection with theoretical information (Verloop, 2003) by the teacher educator and by (fellow) students: during discussions, through

introductions, through annotations in students' work, through email or informal contacts. This feeding – especially on the part of the teacher educator – was accompanied by focusing and filtering; i.e. focusing on desirable learning processes and learning questions, both for individuals and for groups of students, and filtering, by redirecting inadequate ideas towards contents that did offer a perspective.

The addition of reflections from a theory-based educational, psychological, or pedagogical point of view, enriched the work of the student teachers. In this way, the student teachers became involved in the creation of theory-enriched practical knowledge.

4.2.3 The instruments

4.2.3.1 Initial and final assessment

The first part of the initial assessment consisted of completing the 'list of concepts.' The list had been tested earlier in an extensive try-out in four groups with a total of 63 students (appendix 2B). The students were asked to indicate for each concept whether they knew what the concept meant, whether they could tell a teaching narrative that contained that concept and from which categories (own teaching practice, video/film, literature, lectures/workshops) the narrative was taken. During the course the list provided guidance and insight into progress. At the end the students were asked which concepts had gained meaning for them and for which they could provide a fitting 'narrative.' For the teacher educator the yield of the list of concepts at the start of the course was an indication which concepts would need more attention. The yield for the researcher was extra information about the theoretical knowledge the students assumed (un)familiar for themselves at the start and at the end of the course.

The second part of the initial assessment was meant to see at the start of the course meetings how students described practical situations and to what degree they used theory. For this purpose, after a first introduction of The Guide, the students were given the assignment to write a reflective note of one typed page for one of the twenty-five narratives in The Guide, with the narrative to be chosen freely. Further data were taken from video observation of the group of students during the discussion based on these reflective notes. This part of the initial assessment yielded two types of data. In the first place the *number* of theoretical concepts and theoretical notions each student used in doing the assignments, and secondly *statements* from students using theoretical concepts or notions of theoretical concepts.

The final assessment consisted of three parts, namely filling in the 'list of concepts,' writing two teaching narratives for two (newly) familiar concepts (e.g., appendix 5), and writing a reflective note on a teaching situation in MILE that had not been used in that

course (e.g., appendix 6). This final reflective note had an essential function in the study for *comparing* the use of theory by the students.

The list of concepts in the final assessment differed from that in the initial one in the sense that students were now asked to indicate which concepts had become familiar (appendix 2A,B).

The two teaching narratives gave information to the researcher on how the students gave meaning to the two theoretical concepts. The narratives also indicated whether the student made a connection with other theoretical concepts.

The reflective note provided information on *the nature of using theory* by the student teachers (factual description, interpretation, explanation or responding) and *the level of use of theory* (number of concepts; number of units with meaningful relationships between concepts) (see section 4.3.9).

4. 2.3.2 Observation

All meetings were recorded on video tape. The video material was used to analyze the discussion for the use of theory by students, to make an inventory of interventions by the teacher educators and to obtain other important data on the use of theory and for setting up the learning environment for the large scale study. Transcripts of the video recordings were made.

4.2.3.3 Video stimulated recall

This study used a variant of the stimulated recall procedure. During a stimulated recall interview (Krause, 1986; Verloop, 1989) the students made explicit their thinking in reaction to watching video sequences of the discussion in which they participated. In the penultimate lesson, during the so-called 'game of concepts' (section 4.2.2.1), students, directed by the teacher educator, discussed whether there was a demonstrable connection between given theoretical concepts and four practical situations. The video recordings of these discussions were used for stimulated recall sessions with individual students after that meeting. The students were given some general instructions before the interview[30].

The approach of the stimulated recall procedure in this study differs slightly from the standard procedure; this concerns the time interval (max. 4 days) between video recording and interview, as it was not always possible to have the interview immediately following the recording.

4.2.3.4 Concept mapping

This study used the technique of concept mapping (Novak, 1990; Morine-Dershimer, 1993; Zanting, Verloop & Vermunt, 2003) to verify to what degree students were able by the end of the course to make connections between ten theoretical concepts that had come up in the course[31]. To this purpose the teacher educator asked them to each individually order the ten cards with each one theoretical concept, according to their

own insight. The assignment was: "Order the cards according to your own insights and glue them on the large sheet of paper; draw lines between cards when you think the concepts are related and add short explanations if you think it is necessary. This is not about doing something right or wrong, but to gain insight into the connections you see between the concepts that have been discussed in this course."

4.2.3.5 Questionnaire

The questionnaire that had been developed for this study was also used in this small scale study to allow adding, removing or adapting questions for the large scale study.
For the design of the (anonymous) questionnaire, the list of 'constructs and their contrasting poles' from the study by Verloop (1989, p. 188) was used as a source. Furthermore, the categories found by Holligan (1997), in student responses in a similar study of appreciation of theory, have also been taken into consideration. The fourteen questions relate to the evaluation of the course, and especially to student appreciation of theory as it is expressed in the course (appendix 13). The written response to the questionnaire was set before the interview, to achieve as clear an impression of the students' opinions as possible. Descriptive statistics of the data (mean and std. deviation; appendix 13) have been determined using the computer software SPSS, version 15.0.

4.2.3.6 Final interview

After the course ended, the students were interviewed individually. This was a semi-structured interview (Kagan, 1990; Fontana & Frey, 2000). The researcher targeted five topics (the lists of concepts, concept mapping, the reflective note, the numeracy test and the questionnaire) and accompanying key questions (see appendix 8). The intention was to gain extra information about the students' theory use, particularly the character and size of the network of theoretical concepts the students had available. Posing additional questions was determined by the student's responses; criterion for such use of new questions was the expectation that there was a chance to gain a deeper insight into the meaning the student gave to the concepts and into the quality of the relationships the student made between concepts.

4.2.3.7 Numeracy test

The students did – outside the framework of the course on offer – a numeracy test (appendix 18); the students own numeracy serves as an independent control variable in the study marked by the research questions. A positive correlation is suspected between the ability of the students to solve mathematical problems and their level of use of pedagogical (content) theory. Students who have a high level of skill in solving mathematical problems are functioning at a high cognitive level. That quality is likely also important for using pedagogical content theory at a high level, not least because being able to solve problems in mathematics teaching is a basis condition for one's

functioning in relation to the pedagogics of content. The parallel between the level of one's numeracy and that of the use of pedagogical theory is likely to be even more prominent in older students than in younger ones, as a result of the growth in experience for older students, particularly where the pedagogical use of theory is involved. The written test contained ten problems and was derived from tests for the subject of mathematics, which were in common use at many Pabos (Teacher training colleges) in the Netherlands at the time (2003) of the numeracy test (Faes, Olofsen & Van den Bergh, 1992; Goffree & Oonk, 2004; Oonk, Van Zanten & Keijzer, 2007). The standard for the test (0-100) was established for the whole group in consultation between the teacher educators and the researcher.

4.2.4 Procedure

In the first meeting of the course the initial assessment and the written numeracy test were taken. The numeracy test was made by all forty-seven students in the third-year group to be able to compare the results of the study group to those of the group as a whole.

All meetings were videotaped by the researcher. If invited to do so by the teacher educator or the students, the researcher would participate in discussions. He would sometimes also intervene with a question if he wanted to provoke an (additional) opportunity for the use of theory (section 4.2.2.3).

A forty-five minute stimulated recall interview was held with each student after the third meeting (section 4.2.3.3).

In the penultimate meeting the teacher educator instructed the students about the concept map and let them do the accompanying assignment (section 4.2.3.4).

The final meeting involved the final assessment, ending on the (anonymous) questionnaire (section 4.2.3.5).

Soon after the course the final interview was held with each of the students (section 4.2.3.6).

4.2.5 Data collection and triangulation

In this study a choice has been made for a multi-methodical design for collecting data, not only because that allows triangulation of data (Denzin & Lincoln, 2000; Maso & Smaling, 1998; Black & Halliwell, 2000), but also because that allows for better expressing the complex and varied aspects of learning to teach, and because it allows for continued refining of data (Baxter & Lederman, 1999).

The data, student expressions in which they used theory or notions of theory and obtained from observations, concept mapping, stimulated recall interviews, reflective notes, the final interviews and the numeracy test have been collected per student. The data were structured into meaningful units, i.e. 'thought units' in the form of a

paragraph on a subject or theme (Bales, 1951, Krippendorf, 1980; Rourke, Anderson, Garrison & Archer, 2001). In this study thought units were ordered on the basis of the theoretical concepts.
The filled in forms of the anonymous questionnaire were collected per group.
The data for the whole group of fourteen students form the source material for the coherent description of the case of student teacher Anne[32], as an illustration of the way the study has been set up and conducted, and as an inventory of the variations in student teachers' reasoning and differences in the depth of theory use. Moreover, these data were necessary for the large scale study to get an optimal picture of all possibilities for enriching the learning environment and for acquiring the varieties of theory use.
The choice for student teacher Anne was based on three criteria, which have been inspired by the wish to have an optimal data yield for the researcher. The preference was for a student:

- who made a relatively large contribution during the course meetings and who performed the assigned tasks with dedication;
- whose thought and reasoning processes gave an optimal insight into the use of theory in practice situation;
- who was sensitive to interventions – from the teacher educator or from fellow students – that intended to enrich either discourse or practice.

The assumption was that analysis of the data so obtained would produce the maximum amount of information across the whole bandwidth of theory use and all facets of the learning environment that played a part in that. The collection 'signals of theory use' (appendix 1) was used as an analysis instrument and could possibly be used as well for the comparison of theory use between all fourteen students. During the study, however, the instrument turned out to be unsuitable for that purpose and a new reflection analysis instrument was developed (section 4.3.9). Data analyses of the reflective notes of the final assessment were used to compare the theory use of all fourteen students (table 4.2 and 4.3). The output of these data analyses and that of the case together provided an insight into what was still missing from the learning environment and the reflection analysis instrument to be able to achieve a maximum yield in the large scale study.

4.3 Anne's use of theory: a case study

4.3.1 Anne's work plan

Introduction
As part of the preparation for her special topic, Anne, like all third-year Pabo students, has made a work plan, based on self assessment. This is done on the basis of a number of questions and assignments and should give insight into the knowledge, skills, insights and attitude that have been acquired over the preceding years. For this, a distinction into

three areas is made, these areas being domain knowledge, practical skills and educational vision. The intended goal is acquiring an overview of the effort still required within the chosen subject to gain starting level skills as a primary school teacher. The results must lead to a targeted choice for spending the time that is available for study, teaching practice and guidance for – in this case – the specialisation in mathematics.

Domain knowledge

Anne herself says she did not experience primary school arithmetic as particularly difficult. After her secondary school (vwo met wiskunde A; pre-university education with mathematics A), she did find at the start of her Pabo education that much of her primary school arithmetic knowledge was no longer readily at hand. She is now aware that her domain knowledge at that point was mainly formal and that a teacher's professional domain knowledge also contains students' informal strategies.

> After my VWO (with mathematics A) I did feel far removed from primary school arithmetic. Particularly fractions had faded very badly. I used a lot of formal calculation methods. At Pabo I gradually returned to informal methods. You need them to explain certain calculations to the children. I have regained a lot of my primary school arithmetic.

She points at gaps in her knowledge and puts this self-knowledge into words using appropriate wording:

> I am not good at real mental arithmetic. I always need to use paper, to formulate the various steps. Many answers I do not have readily at hand. You could say that I have not yet achieved memorisation.

She uses examples of mental arithmetic strategies to clarify what she thinks is important domain knowledge, and she connects that with the importance of domain-specific pedagogical knowledge in the area of learning trajectories for mental calculation.

Where the pedagogics of fractions is concerned, Anne lacks key concepts such as measuring strip and mediating quantity. For example, she cannot immediately give an adequate response to the question regarding a suitable context and model for the problem $\frac{2}{3} + 1\frac{1}{4} =$. At the same time she has apparently enough pedagogical feeling and know-how that she can didacticize a reasonable solution on the spot.

> When you place $\frac{2}{3}$ pizza on top of the pizza that has been divided into twelve slices, you can see that $\frac{2}{3}$ is the same as $\frac{8}{12}$. You can do the same for $\frac{1}{4}$. Once the students understand this, you can determine how many twelfth parts $\frac{2}{3} + 1\frac{1}{4}$ are together. Now that I think about it longer, chocolate bars may be even clearer, since they are already divided into 12 parts.

Practical experience

Anne has gained a variety of practical experience in the two previous years of study. She speaks of diagnostic interviews with students who have problems with arithmetic, of research into calculation strategies, of series of lessons, designing themes and more.

Anne is positive about the role played by her mentors. Among other things, she gives examples of ideas and educational strategies she has copied from them. That does not mean that she is not critical about her mentors' action. If she disagrees with her mentor about a problem, she will not refrain from offering her view as an alternative, as for instance in the case of a student in grade 1, who persistently clings to a counting approach for adding and subtracting.

> My mentor suggested speed assessment. The child will discover that its method is too slow, and will have to use a faster strategy. I am not too certain that this is the right solution. I think it is too negative an approach. I would like to help him get rid of his counting approach by doing flash games with him. The flashed images of egg boxes, fingers or reckon rack contain the five structure that makes it easy to quickly recognize numbers. By playing these games with him, he can practice counting in groups (...).

View on education

She also shows herself to be a student who can justify her own opinions where her views on education are involved in terms of educational activities and underlying theory.

> (...) There was very little room for other strategies. The result was that all children used the strategy that was offered and they were not motivated to find their own solution. The children were also unfamiliar with the various names of the strategies (friends of ten, doubling, etc.). This is where I reach the point where I would act differently from my mentors. I want to give much more room to different strategies. I also want to use the names of the various strategies within teaching.

Anne believes that the realistic approach to mathematics teaching (see section 3.2) fits in her view on education. She finds the attention to meaningful context and the opportunities students get for their own solutions of essential importance. Her experience is that it is not always easy to fit these ideas into existing mathematics education. She is aware that she has a long way to go, but is motivated to take that road.

> I still have a lot to learn about planning my time. I often plan too much for one lesson. I do find that it works better when I am teaching a series of lessons. I have only worked with older textbook series myself (Wereld in getallen and Pluspunt). I think the structure of Pluspunt is good. I would like some experience with newer methods. Perhaps realistic mathematics can be included better there. In my next work practice I will come into contact with 'Wis en Reken.' I am curious if this method suits my preferences more.

All in all the image appears of a motivated student, who is aware of the development she has undergone in the two preceding years of study as a teacher in training. She is

capable of naming knowledge, insights and skills regardless of whether she has gained them and shows the attitude and opinions that are needed to further work on her professional development. She looks upon the continuation of the course as a challenge.

Motivation for the research project

Anne is one of the group of eight students from the IPabo in Alkmaar who are voluntarily taking part in the research project, after they were informed by their teacher educator and the researcher. As well as by her preference for teaching children between the ages of 6 and 8, the group targeted by the course, Anne's decision is determined by three opportunities the course can offer her. First is the opportunity to learn more about differences between children, a topic that will at a later stage be a key part of her learning question. Second, she sees it as an opportunity to study the developments that can be seen in children of grade 1 as a prelude to the concept of multiplication, which she refers to as 'awakening multiplication.' Third, the approach of the course appeals to her: the mixture of cooperative learning and individual study on a specific theme, in this case learning to teach the tables of multiplication. Further, she 'just wants to learn a lot.'

> I also look really forward to working in the classroom, but I feel that I still have a lot to learn. I will just go to work, it is fun to collect all the knowledge that is offered to you.

4.3.2 The initial assessment

Inventarisation of (un)familiar concepts

On the list of theoretical concepts (appendix 2A,B) Anne indicates at the start of the course that she knows 38 of the 59 concepts. For 27 of these 38 concepts she indicates that she knows a 'story' from her own practice and from lectures and workshops at Pabo, and for 5 of those 27 a story from MILE. For six concepts she (also) knows a story from literature. It can be deduced which concepts have become (more) familiar for her during the course from the list she completed at the end of the course (section 4.3.6).

First assignment: reflective note

All students in Anne's group are given – after some general information about the goals and the approach of the course – the assignment to write a personal commentary (reflective note) on one of the twenty-five available teaching narratives from 'The Guide.' Anne selects – based on her interests – a story from the series in the theme 'What children may differ in.' The story, entitled 'Swinging Marella playbacks' ('Swinging Marella playt back'), is about a student in grade 2, who swings along with the rhythm of practicing the two-times table, but of whom it is suspected that the yield for her is minimal.

Knowledge and views

In her commentary on Marella's story Anne shows that she grasps the practical situation. She sees what the teacher intends and how she tries to achieve her goals.

> As a result of the questions she [the teacher; w.o.] asks, the children are continually approaching the multiplication problems from this table in a different way. The attention to different strategies also contributes to more understanding of the problems. The table does not turn into a rhyme to be recited, and where you have to start at the beginning. Work occured on the ability to solve the problems independently of each other.

Her view on mathematics teaching is expressed in the final sentence of the previous quotation. With 'independently of each other' she means flexible strategy-based solutions, rather than 'reciting a rhyme.'

Anne also shows that she already has a repertoire of pedagogical (content) knowledge at her disposal at this stage, as is seen among other things in adequate use of concepts such as teaching methods of multiplication tables, active learning, handy counting with two at a time, ways, strategies, understanding, doubling and memorizing. Of the 292 words in her reflection 23 are theoretical concepts that have been used in a meaningful way, 4 of which are pedagogical content concepts. This is the highest score for her class. Anne assesses the teacher's actions against her own views on good mathematics teaching, particularly where interaction and reflection are involved.

> Once the series up to ten has been completed twice, she [the teacher; w.o.] looks together with the children again at everything that can be seen on the edge of the blackboard. She asks questions so that the children think along actively.

Constructive criticism

Anne critically follows the teacher, but never without motivating her opinion. Sometimes she adds an observation as 'proof' of her 'hypothesis.'

> It is not fully clear to me why she finishes by saying the table one more time. When I look at her goals for this lesson, it was unnecessary. It is mentioned in the narrative ['The Guide] that not all the children are actively taking part yet, but they also do not do so in the recital. Just look at Marella. She cannot give the answers herself and moves her mouth almost for 'show.'

She also takes advantage of situations by providing alternative solutions.

> But I do not want to imply that reciting a rhyme is not useful. I do in fact think that as a part of the complete learning process it can have some point. Hearing it again and repeating it oneself may help with internalizing. Reciting in smaller groups, without support from the teacher, may work better. There will be less opportunity for a child to submerge itself in the group and playback. As well as reciting a rhyme, other ways to automate and memorize the tables must be used. This teacher does do that. One approach suits better than the other. By using a large diversity of

> approaches there is more opportunity to allow for a individual child's style of learning.

Another remarkable thing is the originality of Anne's reflective note. It is a personal, critical reaction on the teaching situation. Of course there are some similarities between her reaction and the expert reflection, such as for instance 'giving meaning to multiplication by making it concrete,' but Anne is not tempted into 'copying.' Of the 23 theoretical concepts she uses, only three also appear in the expert reflection on this narrative.

The causal arguments in the final four quotations are examples of Anne's focus on explanatory descriptions which she often ends with a suggestion for a possible alternative or a continuation of the situation. She occasionally shows a tendency to think and reason hypothetically. When she does so, she makes adequate use of theoretical knowledge.

4.3.3 Anne's use of theory in class

Varied theory input

It can already be seen in the first meeting with the group of eight students that Anne gives much input in the discourse within the group. In the three discourses that are led by the teacher educator and that are most important for the study, she gives an oral response to the teacher educator or her fellow students 99 times, twenty-one of which in a narrative of more than five sentences. Her input has several forms. The most frequent ones are content-related or are responses that have to do with the planning and organisation of her own work.

We can describe her use of theory during these meetings with a number of 'signals of theory use' as formulated in the second exploratory study (see section 3.8 and appendix 1). We will give some examples. In the following text the signals from appendix 1 that manifest most clearly will be italicised and marked with the corresponding signal number. The italic text is sometimes a paraphrasing of the text in the appendix.

In the first two meetings for the course the students are mainly orienting themselves on the learning environment and they try to find out what they want to emphasize in their studies and what learning question they intend to formulate. Anne is doing teaching practice in grade 1; learning the tables of multiplication will not yet come up there. Even so, from the start she takes initiatives *to make a connection between theory and practice* (nr. 10) in stories and literature from 'The Guide' and her own practice. She expects that the *theory will help her to create clarity* (nr. 5) on the preparatory skills that students in grade 1 already develop for multiplication.

> Can the CD-rom perhaps also explain what skills the children in grade 1 must already have before they go to grade 2? We [doing teaching practice in grade 1; w.o.] have to deal with the question of what students in grade 1 have to do once they are ready to start learning the tables of multiplication, so to say preparatory multiplication or awakening multiplication.

Anne formulates several learning questions and in the end chooses: "What connection is there between the strategies being offered and the strategies used by the children?" She works on that learning question by studying the stories and literature in 'The Guide' and by discussions with her fellow students and the teacher educator. For the study of student multiplication strategies and their teacher's opinion about these approaches, she interviews eight students and the teacher of grade 2 at her practice school.

Anne's views on theory

During the third meeting the students present and defend their commentary on the practical situations they selected from 'The Guide.' One of the items being discussed is the reflection provided by their student peer Susanne. The reactions to Susanne's note are positive. The students are of the opinion that it is a clear, well-considered response, in which, moreover, Susanne clearly gives her own opinion. When the teacher educator asks whether the piece can be seen as a theoretical reflection, author Suzanne is the first to respond: "I don't think so, especially because I included my own opinion and because I didn't really involve the theory, theoretical facts, certain views. I'm not certain if this is correct." The teacher educator intervenes with a question about accounting for one's own approach. At that point Anne comes up with a reaction to Susanne.

> This is I think also what you [Susanne; w.o.] mean with theory; that you don't give arguments for why you think it's a good approach. You are missing some theory, and with theory you mean arguments from views, from all kinds of things to underpin them.

And a bit later:

> Attractive [material; w.o.] it's fantastic and it must be, but just attractive is of course not good, it also has to be suitable for the nine-times table. When you say 'well-chosen,' why is it? It is attractive, but it is also very suitable because it contains the structure of nine (...). This is really about what conditions the material and the context for times problems have to meet.

Here, Anne implicitly indicates that *theory can provide the tools* to *underpin and so justify* ideas and choices for content and design of teaching (nr. 9).

In the penultimate (fourth) meeting, theory has a double function when the students hold a discussion led by the teacher educator about the question of whether there is a demonstrable relationship between six given theoretical concepts (context, informal procedure, mental model, anchor point, structure and strategy) and four practical situations (the 'concept game'; section 4.2.2.1). These functions are '*connecting theory and practice*' (all signals) and '*making meaningful mutual connections between theoretical concepts*' (all signals, particularly 15). One of the situations is about Werner who has difficulties with the five-time table and who is given extra help by his teacher at the instruction table while the rest of the group (grade 2) is working independently.

One thing Werner has to do is the multiplication 4x5 and put it into words with the aid of materials (4 jars with each 5 pencils; later on also 'tiles') and then has to do the problem 5 x 5 using materials.
The first student teacher who responds doubts between the concepts *context* and *structure*, but cannot offer convincing arguments to defend either of the two concepts. The second student considers the concepts *mental model* and *anchor point* and chooses the latter, based on the argument that she thinks student Werner knows the problem 4 x 5 = 20 by heart and can determine 5 x 5 from that.
Anne responds to the last-mentioned student and doubts her conclusions by *setting her own conclusions against them based on theoretical considerations* (nrs. 1-3; 11). Here too the hypothetical character of her reasoning is remarkable.

> *Student teacher 1:*
> It's about the structure, they're in fact groups of five each time.
> But there's also context. Umm, I don't know.
>
> *Student teacher 2:*
> I'm doubting between mental model and anchor point. I really want to know how he is doing his calculating, what does he see in the four and what does he see in the five. How does he calculate that there is one time [five; w.o.] more. That brings you to a different point: he knows the four times five, he calculates on from there. Then you could say about that very well that his anchor point is four times five. He knows that and goes on from there.
>
> *Anne:*
> Does he really know it? He sees it in front of him, but he doesn't immediately say: 'Oh, that is twenty.' He first settles down for it, then he counts. I do get what you refer to. Four times five does seem to be his anchor point, but is it really an anchor point for him? I don't think so (...). I think she [the teacher; w.o.] is working on having him internalize the model: they are groups of five, you have four groups of five, so if you now have one more group of five, you make a jump of five. So she is trying to have him internalize the step he has to make in his head, say the model. So I would choose mental model.

Defending or refuting a 'theorem': connecting one's own practical experience with theory
During another activity in the same meeting – defending or refuting a 'theorem' (appendix 3A,B) – Anne offers arguments that are based on her own practical experiences and the theory related to them. The teacher educator puts forward the following 'theorem': "There is no point in lumbering a student who already knows the tables with all kinds of multiplication strategies that occur in the textbook." The group of eight students is divided into two groups of four, one group being given the role of opponent and one group in favour of the 'theorem.' Anne is one of the members of the group that has to try to refute the 'theorem.'

> Anne: When a student knows the tables, it usually means they can recite them. But ummm..., there is still the question of whether the child understands them. I have a good example from my practice. A child had no problem at all reciting the tables of one, five and ten. She had learned them, and she knew them. But when I asked a question from a different table, she didn't know it. Why not, because she did not know the strategies of halving, doubling, and all those other strategies. That's why I think, okay, they know them [the tables; w.o.], but it's still limited. For example, if you have to calculate 50 x 42, you have never learned the 42 times table. But if you have learned those strategies, you can also learn these other problems. So just knowing the tables isn't enough to really be able to multiply.

Here, Anne uses *convincing arguments based on an example from her own training practice* (nrs. 7, 9) at an appropriate moment. After some discussion, the 'theorem' is rephrased with some more nuance: "There is no point in lumbering a student who already knows the tables from 1 to 10 and can apply them in various mathematical situations, with all kinds of multiplication strategies that occur in the textboek." However, Anne still hangs on to her position and implicitly brings up the meaning of the concept 'apply.'

> (...) Okay, he knows the tables from 1 to 10 and can apply them in various situations. But my question was, can he use them to also calculate 50x42? Because they will have to learn to do that at some point as well, so to say, to get beyond that ten.

Reaction at meta level

Anne also gives input at meta level. That can be seen for instance in the rounding off of the discussion about the 'theorem.' Two of the students make it clear that they do not see much sense in taking a position they cannot support. The teacher educator points out that it is good to realize that outsiders, such as parents, will often have a negative view of the didactical approach with strategies, they believe 'drilling' the tables is a better approach. Anne gives a general reaction to her two student peers:

> You do need it [defending points of view you don't support yourself; w.o.], you have to know what the counter arguments are, because you have to have another argument for them in return.

It is a line of thought that Nelissen (1987, p. 160) would characterize as the description of an internalised dialogue. You try to position yourself in the ideas and arguments of your dialogue partner or 'opponent' and on the basis of that you construct a (counter) reaction.

4.3.4 Video stimulated interview

Anne's first reaction in the video stimulated interview is about students' knowledge and understanding. In the video fragment she states that as a teacher you should not just pay attention to whether students know the answer, but that you should also verify whether

they understand it. Next, she watches as she herself reacts negatively to a fellow student who feels that in the lesson about the boxes with 3 x 3 brownies – the topic under discussion – the teacher should, for the sake of variety, also be able to ask questions about different material shapes (boxes), for instance about rectangular boxes of two by five. Anne feels that different boxes would distract from the goal – achieving insight in the structure – and discusses the structure of the material in relation to the rectangular model and the learning of multiplication strategies. In the recall interview Anne goes on about this and is looking for an e*xample of a situation to interpret students 'understanding'* (nrs. 2, 3). Furthermore, she *underpins an assumption* (nr. 4) about student Bastiaan's notion (3 rows of 6 instead of 2 groups of 9) that she had as a result of an earlier observation.

> I don't know where my thoughts were exactly then, but umm, the way I see it now, if you work on that, there 3 and there 3, then you are in fact working with umm…, the tables, but not just the nine-times table (...). Then you get for instance 2 x 5, a row and another row, together 10 (...). So you can use that structure to go on with those tables, only you have a different goal, not just the nine-times table. 1 x 9 is a box, 2 x 9 is two boxes, but you don't do that, you are heading more in the direction of tile squares [as a notion of the rectangular model; w.o.]
>
> Now that I'm listening on, I think this is what I meant, that they [the students; w.o.] will see the logic of 3 x 3 = 9 and that they will see that as a tile square [notion of rectangular model; w.o.], other than just a box of brownies. That has at the moment [for them; w.o.] nothing to do with the nine-times table. So I think it is correct what I just said. This is what I meant before, that he [student Bastiaan; w.o.] sees three rows of 6, instead of these [two; w.o.] boxes of 9. He sees the boxes next to each other and 3 rows of 6 (...). This is correct, but then you no longer are working with boxes, but with tile squares. I think...

Constructive criticism of the teacher in terms of an alternative approach

The following is Anne's response to a video fragment of the discourse with the statement from student peer Susanne who thought that student Bastiaan did not see the box of 3 x 3 brownies as a whole (1 x 9) because the teacher emphasised the 3 by 3 structure (A = Anne; O = researcher).

> O: You seem to agree with that [argument of student peer Susanne; w.o.].
>
> A: Yes, I'm also thinking now, that start went a bit wrong because she [the teacher; w.o.] involved the three-times table when she wanted to discuss the nine-times tables. Because she in fact... what Susanne said just now... 3 x 3 is apparently what Bastiaan is stuck on. While this isn't important, the 3 x 3, because she wants to know 1 x 9. I think. But what is an advantage with those rows, is that you can realize that 2 x 9 is the same as 6 x 3 or 3 x 6, so this is very good, but I think that it's a bit too complicated still for Bastiaan. Or the teacher should have picked up on it better. I think. Because he can explain and point out 6 x 3 very clearly. I think she

> should just have asked how do you get that problem. And indicate it, what does the 6 mean, what does the 3 mean. I think he would have worked it out and that Bastiaan isn't really as dumb as he appears here.

In her reaction to the fragment, Anne speaks in defence of student Bastiaan and *provides an alternative for the teacher's approach* (nrs. 8, 12). She mathematises (you can see 6 x 3 = 2 x 9 through the structure of the rows) and didacticizes (thinks of different questions using the material). She continues this in a subsequent reaction to the same video fragment, where it can be seen that the teacher educator wants to close the discussion about student Bastiaan, and Anne rushes to make one final remark about Bastiaan's teacher: "You can also say she [the teacher; w.o.] gives a support problem [anchor point, already known table product; w.o.], because three times six is the same as two times nine. Or am I saying it wrong?" In her recall response to that last statement, Anne uses the concepts 'support problem' (anchor point) and 'doubling' in *underpinning her (changed) opinion* (nr. 11).

> A: You see, ummm, that you can use a different problem to calculate it, he does know 6 x 3, he already has the answer, and because you have the boxes, he can also calculate 2 x 9. Because he already knows how many brownies there are. It isn't as clear a support problem if you have for example 2 x 9 = 18 and then double that 4 x 9. This here isn't that clear, because you really must have the boxes of brownies in front of you to see it. This isn't a very clear support problem. I think this is what I meant, that because he does see 6 x 3, that he will find it easier later to get that 2 x 9, I think.

Reasoning about relationships in the personal concept network

In the following video fragment from the discourse during the 'concept game' Bastiaan's story is referred to again at the moment where Anne considers whether the concept context or structure best fits the story. She says: "I was also... yes, context, this is a concept that is in nearly every story, here too. Only I thought like..., yes, why is she using that context. I thought that she wanted to show the structure clearly of the nine-times table. It [the structure; w.o.] is more the goal, the context is more the means. That's why I chose structure."

In her recall reaction to the video fragment Anne now *looks closer at her earlier statements* (nrs. 6, 11) about context as means or goal (A = Anne; O = researcher).

> A: I think it's umm..., she [the teacher; w.o.] uses the umm..., brownies, the boxes as a means to a goal. The goal is understanding of ummm..., of what 1 x 9 is, so to really get an image for that problem. You can also provide a context, umm..., yes, how do I say it..., complicated, I thought the brownies, the brownies had umm...., a subordinate role. It was like using a reckon rack[33]. Or an...., umm blocks, to quickly calculate a problem, I think yes, a goal, context as goal.
>
> O: Subordinate role you say, for the brownies...

> A: Yes, I think that, how can I put it, maybe if a context plays a big part, you will have more that children... yes, how do I say it, I'm not really sure. I think more of a larger story, that you can take more problems from and where children can make their own stories... that you're working more with the context. Here, I have more of an idea that, yes, there is a context there as such, but I would have seen it more as material, the box of brownies, more than a real context that makes it clear what kind of problem it is. I think that's it. You cannot really take a problem, ummm.... a strategy from the box. If you have a question like ummm..., a bag of marbles costs € 2,50 and I buy four bags how much do I have. I think you are referring more to strategy. You are using the money to make it easy to calculate. If you set a problem: 'how much is 4 x 2,5...' many of the children will take that as 'how do I calculate that.' But if you say: "Use euros," they can combine it much easier, you first do 4 x 2 euro, and then you have 50 eurocent and that times two, is a euro together. I think you have the context as a goal more than with the boxes of brownies. I think this is what I meant (...). I look at it more like an aid, an ummm...., small part of the bigger whole.

Anne differentiates between the use of a context and of a material, particularly the structure of the material. She looks on the box of brownies mostly as a material and less as a context. She gives the example of a money context to clarify the difference. The 'money context' evokes a strategy, and according to her the box of brownies does that less clearly. To Anne, the concepts of context, story, material, structure and strategy form a relational network. The objects take their meaning from the situation in which they are used, but also from the network that Anne has apparently created. She *in fact reflects at a higher level than that of the network by reasoning about the relations within the network* (nr. 15).

In the next reaction, Anne goes into the question of a possible relation between structure and context.

> O: What other concepts did you think of for this situation, there were more: context, informal procedure, mental model, anchor point, structure and strategy.
>
> A: Ummm..., I don't quite remember, but I do know that I thought this was one of the most difficult ones to decide what it was. I think I would have picked structure, because it, yes..., you can really see the shapes, also because Bastiaan says 3 x 6. You do have the structure of the brownies in the box, which is how you get those problems. If it hadn't had a structure, they [the students; w.o.] couldn't have made times problems with it, I think.
>
> O: Do you think there is a relationship between context and structure? You considered both.
>
> A: Yes, of course. From the context...at least... you need it. You need the context of the brownies to get the structure. So you cannot really see it separately. Only if you just looked at it as blocks you put in place. Then it doesn't really have a context.

Further underpinning earlier conclusions (nrs. 9, 11)

The following recall shows what Anne adds to her own statements in the discussion about student Werner who is given extra instruction about the five-times table. The student teachers are asked which theoretical concept best suits that narrative. Anne examines her choice for the concept 'mental model' and rejects the concept 'anchor point,' mentioned by her student peer.

> A: I hesitated about that because ummm..., he doesn't mention it even once, he only knows once the jars are already there. Then I think, in my opinion it isn't an anchor point, because I think he doesn't know it yet. I hesitated about what Marlon said [about anchor points; w.o.]. See, they do use it later to calculate 5 x 5. But it isn't an anchor point yet for him. For the teacher, yes, but not for him.
>
> O: Do you feel that or see it? Is it your intuition or do you say: I can see it in this and in this.
>
> A: I ummm..., I believe she [the teacher; w.o.] asks to..., here she asks... you are good at times problems, write down 4 x 5. If that was an anchor point for him, he would already have said 4 x 5 = 20. Because you know it and if you know it, you will say it. Werner does know the teacher want to know the answer to 4 x 5, or he wouldn't have to write it down, at least like that. Or he would have to be so shy and quiet that he wouldn't dare to say it. But I don't think so. So this is why I don't think it's an anchor point. For him.

Revision of an earlier conclusion (nrs. 9, 11)

In the same discussion a student peer defends the concept 'strategy' for Werner's story. She says: "It is a strategy. He was asked: 'How do you do it.' Then she [the teacher; w.o.] made him take another step ahead." Anne tries to discover the student peer's thoughts in the video fragment.

> A: I think that she [the student peer; w.o.] means that it shows that they are working on strategies, that the word that she ought to select for this story is 'strategy.' I think this is what she means. The teacher is trying, based on the jars, to first do 4 x 5, and then 5 x 5, this is 'one time more.' I think the teacher's goal is to explain that strategy to Werner. I think this is what she meant with what she said.

After that statement by the student peer in the discussion about Werner, the teacher educator/chair of the discussion reacts to Anne with the comment: "So in the end you chose mental model." Anne says: "Of course, they are also using that strategy, but I'm more like: she is working on having him internalize that model, they are groups of five so if you have one group of five more then you make a jump of five. That's why I have mental model. So she is trying that, the steps he has to make in his head, that model so to say, to have that internalised." In a reaction to the video fragment Anne changes her position:

A: Sounds logical, but now I would say strategy. The longer I think about it, the more I don't think that was her intention... I think her intention was indeed to go one further from 4 x 5 [Anne means the strategy 'one time more'; w.o.]. Of course she's also using groups and forming the understanding that you have it as a model in your head. But yes, I think that in that mental model... it contains strategy. I think. So yes, it is connected with each other... But now I would have said 'strategy.'

O: How did you see that mental model, what gave you the idea?

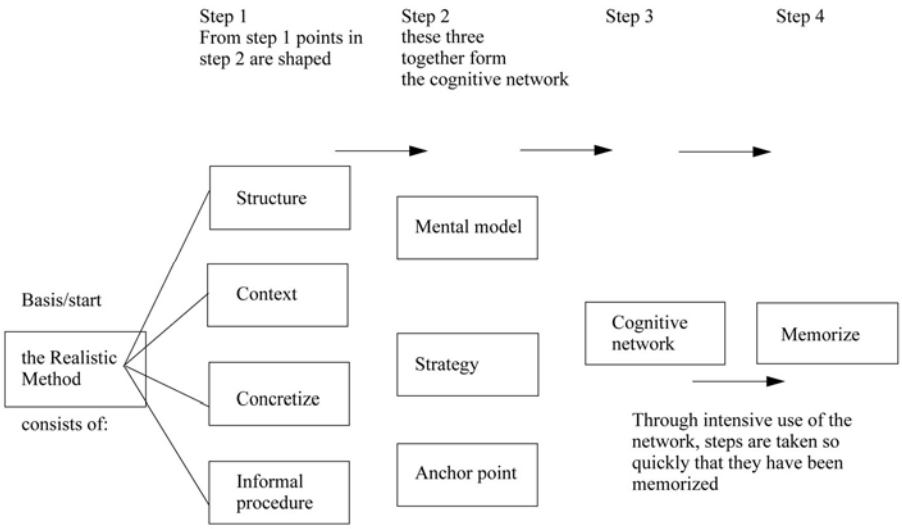
A: Yes, I had chosen the word mental model, because I see that as a picture in your head, which helps to solve the problem so to say, thinking in groups. The groups are a mental model to umm... calculate it so to say. I think this is why I chose mental model. I think mental model is a very broad term, I think there's a lot that can be included in it.

In this final fragment, Anne conveys the core of the concept mental model, namely that it gives a (mental) representation of a mathematical action. Furthermore, she shows she *has awareness of the overarching function of mental models (nr. 15).*

4.3.5 Anne's concept map

Anne uses the ten concept cards (fig. 4.1; see also section 4.2.3.4) to make the following concept map. The italic text is the explanation she herself wrote.

figure 4.1 Anne's concept map



At the request of her teacher educator she provides an *oral* explanation for her scheme.

I really like step-by-step plans, so that's why I made it as one. I first examined like, hey, what can I combine. We just talked about levels. I didn't call it that for myself, but I did in fact lay out in a kind of levels. You start with the realistic method, that's

> the starting point so to say, that's what the four parts are: structure, context, concretize and informal procedure [step 1; w.o.]. Because I looked at informal procedure as a strategy of the children themselves. And this is very important in a realistic method, that you let the children find out things for themselves. From these activities, the stories in context, so to say from the structures, you get a mental model, a strategy and an anchor point. This is step two. And these together form step three, the cognitive network. Together with the children, you construct that in their heads. And if you use the cognitive network often, it is used often enough that it is memorised. This is the final step. This is how I did it.

She is able to put her choice, the (logical) ordering of her map, into words clearly. Later, in the final interview with the researcher, she briefly returns to the ordering in the concept map.

> I think yes, with this [step 1; w.o.] structure, context and concretize I'd say, this is what the teacher contributes and informal procedure is I think more what the child itself contributes, at least... contribution is perhaps not the right word... the things the children themselves know already. Step 2 is more analyzing what is on offer.

What stands out here is the difference she makes between activities by the teacher and the student.

She considers the activities according to structuring, use of context, concretizing and informal procedures as a start of the realistic method, and 'what is on offer.' These activities may lead to a cognitive network in students in which mental models, strategies and anchor points play a crucial role. Intense use of that network leads to memorizing knowledge of the tables of multiplication.

In the same interview the researcher asks her to apply the ten concepts for a teaching situation from the course. In the following quotation she holds forth on the concepts structure and strategy.

> I was just watching structure and strategy (...). You can find a strategy through the structure in the concrete material. But in the strategy, if you start looking from that position, you can also see a certain structure, perhaps they are very close together. In the ummm.... strategy of repetitive adding there is a structure of these groups of 5.

Here, Anne makes meaningful use of the concepts structure and strategy and connects the concepts (nrs. 2, 4, 15; cf. figure 4.1, table 4.1 and appendix 7).

4.3.6 The final assessment

The list of concepts at the end

In the final version of the list of 59 concepts Anne indicates for six of the concepts that they were unknown to her at the start of the course (anchor point, cognitive network, own construction, informal procedures, narrative to a problem, time table method), but that she can now – at the end of the course – tell a teaching narrative in which that concept has

meaning. These narratives are for the most part derived from all four categories (own practice, 'The Guide,' literature or the introductions and discussions during meetings at the Pabo). In addition, she indicates for three other concepts that they have been further broadened during the course (structured material, strategy, multiplication). This is the case particularly for 'multiplication' in general. 'Proofs' for these concepts having made a lasting impression on Anne come from the triangulation of data as described in the previous sections, but also from reporting on activities in her teaching practice and during individual study. For instance, she reports comprehensively on a study of eight students in grade 2, where she maps the cognitive (times-table) network of the children (appendix 4), describes the learning trajectory for multiplication in the mathematics textbook series 'Wis en Reken' and interviews the teacher on learning and teaching the tables. All activities are related to the learning question she chose: "What is the connection between strategies that are offered and strategies used by the children?" In the analyses and reflections she makes meaningful use of theoretical concepts and makes connections between concepts.

Narratives for selected concepts, the practical study and the personal learning question
The two stories she has to write within the framework of the final assessment for two concepts (appendix 5) are yet another proof of her competence to *meaningfully clarify concepts from her own practical experiences (nrs. 1-3, 10).* Anne chooses the concepts 'anchor point' and 'strategy.' In both cases she provides a clear argument, successfully integrating eight concepts into each of them.

In addition, Anne performs an extensive study in her practice school about the knowledge children have of tables.
As is the case with the two narratives of practice, these activities are placed in the context of her own learning question. She supports her findings with the examples, schematic representations of the children's table networks (appendix 4) and theory-laden notes. Even in the summary of her findings (see below) she uses many theoretical concepts.

> My findings. There are several things I noticed during the interviews.
> - The strategy the children add to the table is counting with jumps.
> - The supporting problem [anchor point; w.o.] and counting with jumps are used often by these children. These strategies may be useful as well for solving other problems, but can also be hard to use.
> - Some of the children knew they had to make jumps, but were uncertain about how to determine the size of the jumps.
> - An incorrect supporting problem was used a number of times.
> - I also found that several strategies were used in one problem.
> - Children do have a preference, which differs per child.
> - Doubling and halving are not used much.
> - A number of children have automated the problems that were looked at, others find it difficult to apply the strategies.

In the reflection on an interview with the teacher of the class where she conducted the above-mentioned study, she *considers the teacher's actions while using theory meaningfully and in mutual connection (nrs. 3, 15).*
As a result of her study in the framework of her teaching question she examines in detail the character of the connection between the strategies on offer and their use by the children.

> (...) Particularly the supporting problem [anchor point; w.o.] and the reversing [commutative; w.o.] strategy are important, in addition this class uses counting with jumps (repeated addition) to ultimately reach the answer. Though the teacher follows the teachers' manual, she does place her own accents. The children follow the teacher, but also use their own ways and names. The various strategies do turn out to be somewhat complicated for some children. They find it difficult to choose the right supporting problem or the right size of the problem (…). Strategies are needed to automate and memorize the problems. You can also reverse this. That memorised problems are needed for the strategies. Think of the supporting problems. They can calculate new problems through problems they already know. Strategies, automating and memorizing are inextricably linked. The use of strategies is not limited to multiplication, but occurs in all other areas of mathematics education. Another reason to offer strategies is the opportunity for checks. In practice you encounter children who have memorised problems wrong. By calculating problems using the strategies you can check the answers. Provided the strategies are used correctly, which is sometimes difficult for the weaker students.

In her final conclusion Anne emphasises the importance of developing strategies in students, but she also asks to what extent weaker students will profit from being offered a variety of strategies if that offer is not accompanied by sufficient individual attention.
Here, Anne shows that she possesses a network of meaningful relations between theoretical concepts (nr. 15).

Reflective note for the multimedia teaching situation (the 'suitcase full of balls')
Anne writes – as do her student peers – a reflective note (A4) to finish the course. Her note (appendix 6) consists of 628 words and seven meaningful units (section 4.2.5; table 4.2). It turns out that Anne use more than one theoretical concept in six units and also meaningfully connects these concepts. She also touches upon explanatory reasonings of what happens in the situation in all units. Nowhere does she limit herself to *factual describing* of occurrences or *interpreting* the situation without providing some founding for her opinion. In three units she goes beyond *explaining or clarifying* what happens and she in fact *responds to* the situation by reflecting on the teacher's actions and by considering alternatives for the approach she observes. This happens for example in unit 6 (see the quote below), where she differentiates between shortened counting as an activity and as a strategy. This is an increase in level in her thinking and reasoning about the situation.

> (...) Afterwards they count together in the same way that the class started.
>
> You might ask whether this is the correct approach. If you want to work from the children's view, it's good to first follow the children, the problem can be stated afterwards. The emphasis will be only on shortened counting as an activity, not as a strategy.
>
> If you want to offer shortened counting as a strategy for multiplication, you must start with the times problem. Then you can use shortened counting to solve it, with jumps of 5. I think this was her [the teacher's; w.o.] goal, and she achieved it. It's good she offers this for large numbers. You avoid that children know the answer immediately. Then the strategy wouldn't come across so well.

She starts from the actual, observed situation, and *questions* (nr. 6) what the consequences of different approaches might be. Yet she still tries to think along with the goal that the teacher *presumably* (nr. 4) has in mind. She reaches a *concluding consideration* through if-then and cause-effect reasoning.

4.3.7 The final interview

In the interview that was held after the course meetings, five topics and the corresponding questions were highlighted (appendix 8). The data relating to the lists of concepts, the concept map, the final assessment and the numeracy test have been included in the descriptions of the preceding sections; this is not the case for the data relating to the written questionnaire. Anne – like her student peers – filled in the questionnaire (appendix 13) anonymously. The researcher asked her several questions about the difficulty of the course as she had experienced it and her views on (the use of) theory. Anne said the level of difficulty of the course was neutral for her (score 3). She supported that judgement with the argument that while she could understand the content, she found the working method rather difficult. That last point gave rise to some doubts about the yield for herself (A = Anne; O = researcher).

> O: Did you find the course easy or hard or somewhere in the middle? Do you still recall where you set the mark [on the five point scale; w.o.]?
>
> A: I think it was in the middle [neutral; w.o.]. It was a bit hard and it was a bit easy… What you had to do exactly, more the way of working, I found hard. The content I found easy to understand. Okay, new things are mentioned, but not so that you had to use a dictionary. Too easy, no, I don't think so. There were concepts you had to think about longer, like the cognitive network and that type of words. The working method I did find difficult; how it worked, how I had to approach it, what the intention was.
>
> O: Was it useful to do?
>
> A: Yes, it was for me. I did hear other reactions, and I started to doubt myself and I asked myself: did I get enough from it. I now find in this kind of interview, that I can explain things and put things into words that I have an image of, something that I never used to think about. I think that's very important, like the structure of

> multiplication, the approach, the materials, I know more about them, I think that's useful, yes. I also find it enjoyable to talk about it like this, to explain it, to order things for myself. I already found that out in secondary school. It's also, yes… one of my ways of learning, it's really part of me. It's important to know that in teaching, so that you can also use other styles of teaching.

Her answer to the question what she thought of for the word 'theory' was:

> The first thing I think of is a piece of text, books, the literature so to say, that's what I think of. Then, I also think of umm…, yes, like what kinds of strategies there are. For example, you have a series [of strategies; ed.] that you list, and working them out is then the practice. How the children use them, is practice. The series of strategies, these and these strategies exist, this is the theory.

She felt that theory was mainly 'covert' in the course. Theory, with the concept of strategy as its representation, she considers as a means of supporting her own learning and her own practice.

> A: (...) it was more covert also, for example in the teaching narratives. Through the reflective notes and the umm… wordlists [concept list; w.o.], you could look up the theory, but initially it was covered up in the teaching narratives about practical situations. And umm… theory was discussed during the lectures. But… we often started with practice and then took things from that.
>
> O: What did you find a remarkable example of theory, something of which you can say, this to me was a real example of the theory of learning to multiply that was discussed in the course.
>
> A: Umm…, the example of the strategies, which strategies there are, and ummm… all the strategies that were mentioned, and the explanations. I thought that was theory, because, well, you don't use those concepts in the classroom, it's more background. For myself as well, my own learning question, the structure, yes…, the structure of a learning trajectory, this is theory too.

Anne appears *aware of the function of theory* (nr. 14).

Working on the basis of a personal learning question was experienced as rather difficult by Anne (and most of her student peers). Though there was a reasonable amount of guidance, perhaps lack of experience in setting a learning path fuelled this uncertainty. Perhaps the information about formulating and elaborating the personal learning question was not fully adequate yet. Apart from that, Anne did understand soon that she should not be too hasty in devising the learning question:

> I think that when you are writing down a learning question, you should think: "How am I going to approach this and do I have enough sources and things like that (…)." Your question shouldn't be too general or too vague. You must be specific and write down exactly what you are looking for, or else your research will be very difficult.

She showed herself aware of the importance of literature as a source for (reflection on) her investigations (nr. 14).

4.3.8 The numeracy test

Anne scored 88 out of a possible 100 on the numeracy test, the highest score in her group. The average score for her group of eight students was 65. She missed a problem where she had to multiply algorithmically and divide by decimal numbers. In the final interview she commented that multiplication and division with numbers which contain zeroes and decimals is a gap in her knowledge and skills.

For Anne there clearly is a positive correlation between the ability to solve mathematical problems and her level of (content-related) pedagogical use of theory (see table 4.2).

4.3.9 Data analysis and results of Anne's use of theory

First analysis with the signals

In this study qualitative analysis of the data was initially done using the signals of theory in actions as developed in the second exploratory study (section 3.8 and appendix 1) and as described in the italic text in Anne's case study. The quality of the arguments was specifically analyzed for the type of reasoning and for whether there was meaningful use of theoretical concepts. Furthermore, the number of theoretical concepts used in the reflective note of the final assessment was scored. The concept map provided information about the cognitive network of theoretical concepts. Data triangulation was based on data sources that had been collected from reports and transcripts using the instruments described before (list of concepts; reflective note; video observation; video stimulated recall; concept map).

In Anne's oral and written expression 13 of the 15 signals of theory use (appendix 1) were demonstrable. The most frequent signal displayed by Anne was '(re)considering opinions and action on the basis of theory' (signal nr. 11; e.g., section 4.3.4).

Although the 15 signals allowed a nuanced consideration and discussion of the use of theory, it became gradually clear that the instrument was hardly efficient as a means of comparing expressions of theory use (section 4.2.5) to raise understanding of that use. There was overlap and not enough coherence in the collection of signals. Differences in the level of theory use could not be unambiguously defined. Partly with the large scale study in mind, the decision was made to develop a new instrument for a comparative analysis of student statements.

Towards a new reflection-analysis instrument: nature and level of use of theory

The reflective note in the final assessment – produced by all students at the end of the course and under the same conditions – functioned as a data source. Data analysis of the case study and earlier studies (Oonk, Keijzer, Olofsen, De Vet, Tjon Soei Sjoe, Blom, Ale & Markusse, 1994) on 'The Framework for Reflective Pedagogical Thinking' from the study by Sparks-Langer, Colton, Pasch, Simmons & Starko (1990, p. 27), inspired

the formulation of initially five, but finally four categories to describe the *nature* of theory use, namely factual description, interpretation, explanation and 'responding to' situations[34]. The categories show similarities with levels 2 to 6 of the seven descriptions given by Sparks-Langer et al. The four categories A, B, C and D form an inclusive relationship (B contains A, C contains B, et cetera.) and have the following characteristics:

A: Factual description: the student teacher describes factual occurrences.
B: Interpretation: the student teacher relates what he or she thinks happens, without any foundation (I think…, in my opinion…).
C: Explanation: the student teacher relates/describes why the teacher/student acts in a particular manner.
D: Responding to situations: the student teacher relates or describes – for example in a design/preparation/evaluation – what could be done or thought (differently), what actions he or she as stand-in for a virtual teacher would (want to) take (I expect, predict, I would do, I make, I intend to, with the intention of…).

Using this scheme, the descriptions and reasonings, except for one or two exceptions, could be named unequivocally (see examples in section 5.3.6.4, table 5.3). The descriptions mainly had the character of 'explaining' situations (category C) and 'responding to' situations (category D). A number of meaningful units of Anne's reflective note in the final assessment (appendix 6) showed an A-C-D structure. This is a description starting with a factual description, followed by a reflection in which she for example asks a question about the teacher's approach. Subsequently an alternative is provided and the consequences of that other approach are weighed. Such a unit is mainly characterised by reasoning relationships (arguing and explaining; Pander Maat, 2002). The 'response' often takes place by describing alternative approaches or by responding to the observed learning and teaching trajectory with suggestions for the continuation of the actions of teacher or students.

Of the seven units in the reflective note in her final assessment Anne scored four times in category C and three in category D (see table 4.2, student 1). Nowhere did Anne limit herself to merely factual description or interpretation (without founding) of an observed situation. In some cases (e.g., section 4.3.2 and 4.3.3) Anne showed a tendency to thinking and reasoning hypothetically (section 2.7.1).

The level of theory use

As far as the number of theoretical concepts used in the reflective note of the final assessment, Anne scored the highest in her group. Furthermore she used two or more theoretical concepts in a mutual, meaningful *relationship* in six of the seven units of the note (table 4.2). So she used the theoretical concepts at a *different level.*

Anne's concept map (section 4.3.5) gives an overview of the relationships Anne made between ten theoretical concepts. The difference with her student peers lies mainly in her underpinning her (logical) ordering in the map (appendix 7). She has used all key concepts meaningfully at least once, or more. At a higher level she used 13 *meaningful relations between concepts*. The table 4.1 below gives an overview of these 'proven' meaningful relations between concepts.

1. Realistic method
2. Context
3. Concretizing
4. Structure
5. Informal procedure
6. Mental model
7. Anchor point
8. Strategy
9. Memorizing
10. Cognitive times-table network

Table 4.1 Meaningful relations between ten key concepts on the basis of concept mapping.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1										
2	R									
3	R									
4	R	R								
5	R									
6										
7										
8		R	R	R	R	R	R			
9										
10							R	R		

R = (meaningful) relationship

Taking into account Anne's intentions in formulating her learning questions, the relatively high frequency of the concept 'strategy' (nr. 8) is understandable. Anne made meaningful connections between that concept and seven of the other nine. While she did use 'memorizing' meaningfully, Anne did not relate that concept to any of the others in the concept maps. Triangulation using the data collection from the observation, the interviews and the reflective notes does however show that Anne did at least connect the concept of memorizing to the concepts anchor point, strategy and automating. Apart from that, the network would be more extended if the analysis had also taken *notions* of theoretical concepts into consideration. Looking at 'memorizing' that would have been the case if 'remembering' and 'know quickly' could have been scored.

In some cases level increases were observed in Anne, for example in the video stimulated interview (section 4.3.4) where she reacted to the discussion in the meeting surrounding the concept game. The concepts context, narrative, material, structure and strategy turned out to form a *relation network* for her. The relations between the objects (concepts) took their meaning from the situation in which they occurred, but also from the network that Anne had acquired. *She was in fact reflecting on a higher level than that of the network, by reasoning about the relationships in the network.* The relations between concepts in the relation network became subject of reflection at a higher level.

4.4 Results and conclusion of the small scale study

4.4.1 Results

The research question for the small scale study was: In what way and to what extent do student teachers use theoretical knowledge when they describe practical situations after spending a period in a learning environment that invites the use of theory? A sub-question to this question in the small scale study was: To what extent is there a relationship between student teachers' use of theory and their level of numeracy?

The hypothesis was formulated that the adapted learning environment would allow the student teachers to apply varied reasoning to practical situations and that there would be demonstrable differences in the depth of theory use. The goal of the study was to map that variation and depth.

The considerations at the base of the hypothesis had been inspired by reflection on the results of the exploratory studies (section 3.9) and the turn of thought on the character of the new learning environment resulting from that (section 4.2.2). That character was mainly determined by three characteristics that were intended to motivate the students to use the learning environment and that could evoke the use of theory by them.

First, one of the most important characteristics in that context was the practical orientation of the learning environment in combination with the theoretical import of the available practical situations. Second, the structured yet open character of the learning environment allowed a variety of working methods. The third characteristic, the theory-focused interventions by the teacher educator, was intended to optimize both the 'practical reasoning' (Fenstermacher, 1986; Pendlebury, 1995) and the 'feeding' with theory (section 4.2.2.3).

The hypothesis in the sub-question was that students who use a relatively large amount of theory would score higher on the numeracy test than students who use little theory. The assumption was that students who use a lot of theory will more often reason at a higher level (function at a higher cognitive level) and should therefore do better on the numeracy test (section 4.2.3.7 and 4.3.8).

Table 4.2 below gives an overview of the use of theory by Anne (student 1) and the student teacher peers from the group at the end of the course. The table contains comparable data from the reflective note from the final assessment, a text with (on average) seven meaningful units. There are relatively large differences between students.

Table 4.2 Theory use by Anne and her seven student teacher peers in the reflective note from the final assessment (IPabo-Alkmaar)

	Anne	St 2	St 3	St 4	St 5	St 6	St 7	St 8	Mean
1. N (meaningful units)	7	8	7	8	8	9	6	6	7,4
2. N (theoretical concepts)	26	21	21	20	15	6	5	16	16,3
3. N (units of factual description); (Cat. A)	0	0	0	0	0	3	1	0	0,5
4. N (units of interpreting); (Cat. B)	0	0	0	0	0	4	1	0	0,6
5. N (units of explaining); (Cat. C)	4	5	3	2	3	2	2	6	3,4
6. N (units of responding/gearing to situations); (Cat. D)	3	3	4	6	5	0	2	0	2,9
7. N (units with meaningful relations between concepts)	6	5	3	5	1	0	0	3	2,9
8. Percentage of units with meaningful relations between concepts	86	63	43	63	13	0	0	50	39,8
9. Score numeracy test	88	52	49	84	59	55	51	78	64,5

Table 4.3 Theory use by the group of six student teachers in the reflective note from the final assessment (IPabo-Amsterdam)

	St 9	St 10*	St 11	St 12*	St 13	St 14	Mean
1. N (meaningful units)	10	-	7	8	8	7	8,0
2. N (theoretical concepts)	16	-	6	10	18	21	14,2
3. N (units of factual description); (Cat. A)	0	-	2	2	0	2	1,2
4. N (units of interpreting); (Cat. B)	0	-	3	1	0	0	0,8
5. N (units of explaining); (Cat. C)	1	-	1	1	2	2	1,4
6. N (units of responding/gearing to situations); (Cat. D)	9	-	1	4	6	3	4,6
7. N (units with meaningful relations between concepts)	3	-	0	1	4	3	2,2
8. Percentage of units with meaningful relations between concepts	30	-	0	13	50	43	27,2
9. Score numeracy test	66	45	38	51	74	58	55,3

(*) Students 10 and 12 did not make a concept map and student 10 missed also the final assessment.

The first row gives, per student, the number of meaningful units the researcher has split the note into. To allow for a good, unequivocal comparison of data, four ways in which student teachers used theory in describing situations were formulated during the study. Rows three to six of the table contain the number of units scored for those four categories for the *nature* of the theory use: factual description (A), interpreting (B), explaining (C) and responding to situations (D). Analysis of the reflective note from the final assessment showed that most meaningful units were scored as category C (explaining) or D ('responding'), usually preceded within the unit by text with an A or B signature.

The second row contains the number of theoretical concepts used in the note.

Row seven gives the absolute number of units that has been scored as 'unit with meaningful relations between concepts,' row eight registers the percentages of units with meaningful relationships.

Anne did not score *factual descriptions* (A) or *interpretations* (B). It is remarkable that the three weakest students (st. 6, st. 7 and st. 11) only or mostly score units in 'factual description' and 'interpreting.'

Furthermore, table 4.2 shows that Anne also distinguishes herself from her student teacher peers where the *level* of theory use is involved. She uses *more concepts* than the other students and scores the *most units with meaningful relations between* two or more theoretical concepts, both in absolute numbers (6) and relatively (86%). On average 16 theoretical concepts were used in her group, a minimum of 5 and maximum 26 (row 2: median 18; std. deviation 7.4). There were also large differences (range 86%) between the number of 'units with meaningful relations between concepts' in the students' reflective notes (row 7 and 8).

A number of units show a rise in level. Statements are made in the first or second sentence containing a theoretical concept, while the following sentences contain different concepts which have a meaningful relationship with the previous concept. Sometimes a transition to a higher level occurred in the thus *created network of relationships*, when there was *reasoning about the relationships* of that network (e.g., section 4.3.4 and 4.3.9). In the meeting level increases mainly occurred in the discourse led by the teacher educator.

The research data according to the sub-question in this study, about the student teachers' use of theory and their level of numeracy (section 4.2.3.7 and 4.3.8), support the hypothesis of a connection between these two variables. Regression analysis points to a significant positive correlation (beta 0,730; sig. 0,005).

Regarding the data from the questionnaire for the fourteen students from Amsterdam and Alkmaar (appendix 13), it can be concluded from questions 10 to 14 that the students appreciated the theory on offer, integrated into practical situations, mainly as a support for

their practice. The score for question 14 ('theory and practice are far apart/are integrated') is remarkable, with the highest average score of the 14 questions (3,92). The difference between the result for question 7 ('the course is boring/challenging'; average 3,08) and question 8 ('the course is vague/concrete'; average 2,15) can perhaps be explained from the fact that some students had problems with the individual study based on a personal learning question. The relatively large standard deviation in the output from question 8 supports that explanation.

4.4.2 Conclusion and discussion

The first conclusion from the study was that all students used theory in both their oral and written reactions to practical situations. The differences in both the *nature* of the theory use and the *level* of using theory were relatively high.

The nature of using theory can be distinguished in *four categories* (A to D; section 4.3.9).

The difference in level of the student teachers' theory use in their reflection on practical situations, can be characterised as follows, based on the data (table 4.2 and 4.3) from the final assessment. Primarily there were differences in *the total amount of theoretical concepts* the students used in their reflective notes. Secondly, the on average eight meaningfull units per student teacher, consisted partly of units which did *not* contain theoretical concepts. Thirdly, there were units with *at most two theoretical concepts and with no mutual relationship*. Finally, there were units in which *theoretical concepts were meaningfully connected to each other* (table 4.2 and 4.3, row 7 and 8). Three of the thirteen students who did the reflective note of the final assessment did not reach the last mentioned level in any of their described units.

Anne had the highest score in her group on all counts. She used theory in nearly all her arguments. In comparison to her student peers, Anne uses the most theoretical concepts in the reflective note of the final assessment (appendix 6; table 4.2). She showed teaching sensibility (Grimmett & MacKinnon, 1992), an obviousness of practical reasoning (Fenstermacher, 1986) and reflecting (Freudenthal, 1991) and an appreciation of the situations (Pendlebury, 1995). She familiarised herself with the situation by explaining it, followed by responding to it with her own considerations. Theory helped her to formulate pro and counter arguments. Sometimes she showed a tendency to hypothetical thinking and reasoning, similar to what Ruthven (2001) labelled as 'practical theorizing' (section 2.7.1).

In a number of cases Anne took a step back from the situation to give a more general conclusion. The level increase she experienced in that case – *from reasoning about the concepts in a network to reasoning about the connections between these concepts* – seems related to the type of level increase Van Hiele (1973) describes in his theory about levels in mathematical thinking. This level theory has had a strong influence on

other scientists in the Netherlands in their development of theory about mathematical learning processes in students (Gravemeijer, 2007) and teachers in training (Lagerwerf & Korthagen, 1993; Korthagen & Lagerwerf, 1995). Gravemeijer for instance describes level increase in the framework of the design heuristic of emergent modelling as the development of a network of mathematical relations. In that sense he sees abstracting as a form of constructing (Gravemeijer, 2007). And this is what Anne does: she in fact constructs abstraction by reflecting on the relations she has distinguished. Analogous to *vertical mathematizing* (Treffers, 1978, 1987), the increase in level that Anne makes (e.g., sections 4.3.4 and 4.3.6) can be interpreted as *vertical didacticizing* (Freudenthal, 1991).

Other indications of an increase in level concerned integrating theory and practice were – for Anne's student teacher peers as well – visible in the reflections on the study in their teaching practice. A number of students experienced a kind of Aha-Erlebnis (Bühler, 1916) in their discovery of the multiplication strategies used by the children. This led to a reflection on that success experience (Janssen et al., 2008) in which their argument gradually shifted from a discourse about the strategies to underpinning it with other theoretical concepts which were related to multiplication strategies. The previously described experiences with 'level increases' were reason to consider that phenomenon as a focal point for the large scale study (see the final point of attention in the examples of implications in the next section).

The hypothesis posed in this study according to the sub-question is plausible. The large scale study will show if there is a significant positive correlation there too between the student teachers' use of theory and their level of numeracy.

Appendix 13, finally, contains the output of the questionnaire. It can be interpreted as an appreciation of theory by students and of the integrated approach in the offer of theory and practice in the small scale study.

4.5 Implications of the small scale study for the large scale study

The previous section 4.4 gave a description of the findings regarding the answers to the research questions of the small scale study. Below the most important implications of the small scale study for the large scale study will be described. As mentioned in section 4.3.9 and 4.4, the small scale study provided a new insight into the character of the use of theory by students. Two dimensions were distinguished, namely *nature* and *level* of theory use (section 4.3.9). On the one hand the use of theory appears to manifest in the way students describe situations with the help of theory; this we refer to as the *nature* of theory use. This can take place by *factual description*, *interpreting* or *explaining* a situation, and by *responding to* a situation. On the other hand the use of theory can be expressed in the presence or absence of a *meaningful connection between the theoretical*

concepts used by students, the *level* of theory use. These new insights were reason to focus the research questions for the large scale study on nature and level of theory use and on the connection between these two dimensions. A related implication was that the reflection-analysis tool had to be developed further and tested on validity and reliability. We assumed that the learning environment of the small scale research would prompt student teachers to engage in practical reasoning to a greater degree than the earlier versions of the learning environment. That hypothesis appeared to be confirmed by the small scale study. The results showed a large variety in theory use, though the conjecture arose that theory use was insufficiently evoked in some of the students. The study gave indications for a further optimisation of the learning environment. The ideas arose from the analysis of the experiences, based on focal points described in the theoretical framework, particularly in sections 2.6, 2.7 en 3.9. Below we will briefly describe five of the desired changes. Each will mention the most important motivation for the proposed change, followed by the desired design activity and (between brackets) the relevant focal point for theory use (see section 2.6.4).

- Some of the students felt uncertain about the tasks they were set, and the requirements attached to them, particularly the ones relating to independent study. Design activity: designing a detailed manual for teacher educators, with hints and tools for both students and teacher educators, such as criteria for good learning questions with examples and guidelines for support from teacher educators (focal point for theory use: finding confidence).
- After two meetings the researcher and the teacher educators felt the need to make student teachers even more explicitly aware of their own learning, respectively learning yield. Design activity: designing a 'logbook activity' called 'What (else) was learned in this meeting?' (focal point for theory use: gaining awareness of one's own views and develop sensitivity towards theory use).
- Within the time available for the course, a choice had to be made from several student activities that had taken place within this study, but also in some cases in the preceding try-out. Regarding the use of theory, more was expected of the concept game than from the activities relating to 'the theorem.' Design activity: decision to further work out the 'concept game' to the detriment of 'the theorem' (focal point for theory use: underpinning opinions).
- The output of the questionnaire and statements by students during meetings and interviews showed that the students appreciated the lists of concepts. Some of the students did however report that they could not indicate in the list that a concept had gained *more* meaning as a result of the course. Others thought at first that they did know a concept and could give meaning to it in a narrative of practice, but became aware during the course that their earlier interpretation had been incorrect

(appendix 2A,B). These experiences implied that it would be necessary in the large scale study to change the text in the list of concepts for the final assessment. The experience with the second part of the initial assessment, the reflective note for a narrative from 'The Guide,' led to another idea for changing the assessment. Students were in the opportunity to choose the narrative themselves, which was intended to give them a motivating first orientation on the learning environment. This approach would be inefficient for the large scale study, though, and endanger unambiguousness within the data. Design activity: adapting the lists of concepts and designing a new assessment (focal point for theory use: from subjective concepts to general applicability).

- In this study, the interventions by the teacher educator turned out to be of crucial importance for stimulating the use of theory by students, particularly in relation to raising the level of that theory use. Design activity: organize a one-day training for teacher educators and write a teacher educators' manual containing detailed guidelines for the treatment, containing examples of key insights, key questions and pitfalls (focal points for theory use: e.g., particularly 'developing sensitivity for the use of theory' and 'the function of persuasiveness, justifiability and usefulness of theory').

The assumption was that the proposed changes would optimize the use of theory by students, and that their 'enriched practical knowledge' could be mapped and analyzed systematically.

5 The large scale study

5.1 Introduction

The large scale study of the question to what level primary school teachers in training are able to make connections between theory and practice that is described here, is a continuation of the two exploratory studies (chapter 3) and the small scale study (chapter 4). The latter study mainly served as a preparation for this large scale study.
The small scale study showed a large variety in use of theory by students. There was, however, a suspicion that not all students were encouraged to optimally use theory. It therefore seemed advisable to adapt the learning environment and a part of the research instruments for the large scale study. Furthermore the small scale study provided a new view of theory use by students, which led to the development of an instrument to analyze the research data on the basis of these new insights.
This study focuses on accurately charting the way that students use theory, after a period in which they are confronted with theory-laden practical situations in a multimedia interactive learning environment. The conjecture is that at the end of that period the students will show signs of 'Theory-Enriched Practical Knowledge' (section 2.6.5.5 and 3.9). This large scale study – in contrast to the small scale study – is quantitative in character and aims at making visible patterns in the use of theory by student teachers.

5.2 Research questions

Chapter 2 explained the considerations that led to the central problem definition and the research questions. The main goal of this study can be broadly described as gaining an insight into the phenomenon of 'theory use' by students in primary teacher education. The previous research led, among other things, to an interpretation of the use of theory in two dimension, namely *nature* and *level* of theory use (e.g., see section 4.3.9). The nature of the use of theory manifests in the way in which students use theory in describing situations. This can for example occur through factual description or by explaining a situation; in this case, theoretical concepts will strengthen the factual description of a situation or the explanation of what is happening. The three levels of theory use are expressed by the degree to which students use the theoretical concepts meaningfully.
The study targets gaining an insight into the nature and the level of theory use (see the explanation below, as well as the examples in table 5.3). Therefore the learning environment is set up to optimize conditions for theory use, partly through 'feeding' the process of reflection with theoretical information (see also section 5.3.2).

One part of this large scale study is the refining and testing for reliability of the instrument used to analyze nature and level of theory use. The first version of that

instrument was designed during the small scale study (section 4.3.9). The large scale study aims at three main questions, with the third question split into two sub-questions.

The first research question focuses on the *nature* of use of theory:
In what way do student teachers use theoretical knowledge when they describe practical situations after spending a period in a learning environment that invites the use of theory?

Rationale and explanation for question 1.
During the five meetings in the course (see section 5.3.2) the students are given the opportunity in various forms and in several locations (Pabo, practice school, individual study, et cetera) to gain – notions of – theory. To that purpose, they are 'fed' through an amalgam of interventions by the teacher educator, reactions from student peers and the material on offer in the learning environment. One of the things asked of the students in the final meeting is to write a reflective note on a practical situation that was not a part of the learning environment in the previous meetings. These notes are the most important research data for the first research question. The assumption is that all student utterances (descriptions) can be interpreted using four different types of theory use, namely *factual description, interpretation* or *explaining* practical situations or *responding to* practical situations (see also section 5.3.6.3). These four types represent the *nature* of the use of theory; the relation between the four types is seen as an inclusive relationship, meaning that the next type includes the preceding type. The '*using theoretical knowledge in describing practical situations'* meant in this research question, can also refer to the gradual development of new theory or theoretical notions or the further development of already existing ones.
The division into the four types of theory use mentioned here, is partly inspired by the work of Sparks-Langer et al. (1990) and is created as a result of the experiences in both the exploratory studies and the small scale study (section 4.3.9). The expectation is that every statement (reflection) by students can be categorised as one of the four types of theory use and that analysis of data can lead into insight into the *nature* of theory use and into the differences that distinguish students from each other when using theory.
Three hypotheses have been formulated for this research question. Section 5.4.2 provides an extended description of the considerations according to hypothesis 1.1, 1.2 and 1.3. Data are obtained from the final written assessment in the last meeting (see section 5.3.4.1). In aid of the analysis of the data the first version of the reflection analysis instrument from the small scale study (section 4.3.9) is refined, validated and tested for reliability (section 5.3.6).

The second research question focuses on the *level* of use of theory:
What is the theoretical quality of statements made by the student teachers when they describe practical situations?

Rationale and explanation for question 2.
In the introduction the two dimensions that interpret the use of theory were referred to the *nature* and the *level* of theory use. This research question is aimed at the second dimension, the level of theory use. One of the studies that preceded this study, the second exploratory study (section 3.8), led to the formulation of 'signals for theory use.' Although the signals allowed a nuanced consideration and discussion of the use of theory, differences in the level of theory use by student teachers could not be unambiguously defined (section 4.3.9 and 4.4).
In this study the *levels* of theory use are expressed by the degree to which the students use the theoretical concepts meaningfully (see also section 5.3.6.2 and 5.3.6.4). That thought was partly inspired by the ideas of Van Hiele (1973) and Freudenthal (1978, 1991) on levels of mathematics learning processes[35]. Furthermore, the *number* of theoretical concepts occurring in a statement is also considered, with a distinction being made between general pedagogical and pedagogical content concepts.
The small scale study revealed that students can construct a network of theoretical concepts and that rises in level do occur, for example if student teachers make a transition to a higher level in a created network of relations, when they reason about the relationships of that network (section 4.4.1). This second research question of the large scale study mainly investigates the degree to which students differ in their levels of theory use and to what degree variables such as prior education and study year correlate with those levels. Two hypotheses (2.1 and 2.2) have been formulated. Section 5.4.3 provides an extended description of the considerations according to hypothesis 2.1 and 2.2.
Data are obtained from the reflective notes of the initial and final assessments, which were held in the first, respectively last, meeting within the framework of the course (see section 5.3.4.1). To aid the analysis of data the previously mentioned reflection analysis instrument is used (section 5.3.6).

The third research question focuses on the *coherence* between the nature and the level of use of theory, and zooms in on the relationship between the use of theory and the level of numeracy:
3a. Is there a meaningful relationship between the nature and the level of theory use? If so, how is that relationship expressed in the various components of theory use and in various groups of students?
3b. To what extent is there a relationship between the nature or the level of the student teachers' use of theory and their level of numeracy?

Rationale and explanation for question 3a.
This third research question is aimed at the dual dimensions of theory use, and particularly at the question of to what extent and in what way there is a connection

between nature and level of theory use. Both nature and level of theory use are structured hierarchically (see section 5.3.6).
The expectation is that there is a relationship between the dimensions of nature and level of theory use and that variables such as prior education and the students' study year will give an insight into that relationship. Other variables that may do so, are the *total* number of concepts and the number of *different* concepts (see hypothesis 3.1, section 5.4.4). The small scale study has already shown that some students use a relatively small number of concepts more often, while other students used more concepts once. The question is whether these differences – and the relations between them – correlate with differences in nature or level. Another focal point is the difference between the use of general pedagogical and pedagogical content concepts. While the general pedagogical concepts are used within a pedagogical content context, they have different or multiple meanings. Students have often developed them from multiple domains (compare for instance general concepts such as learning-teaching trajectories, practice or pedagogical climate with pedagogical content concepts such as shortened counting, number line, or the pedagogy of learning to multiply). The question is whether the students who use comparatively more pedagogical content concepts than general ones show a different nature or level of theory use compared to students for whom this is not the case (see also hypothesis 3.1). Section 5.4.4 provides an extended description of the considerations according to hypothesis 3.1.
To aid the analysis of data, the reflection analysis instrument that was referred to in the first and second research question, is used (section 5.3.6).

Rationale and explanation for question 3b.
The development of numeracy is placed in a pedagogical perspective in the training of their prospective primary school teachers (Goffree & Dolk, 1995). The growth of the development of numeracy can be seen as an amalgam of four components (Oonk, Van Zanten & Keijzer, 2007), where along the way mathematizing is intertwined with didacticizing. It is therefore likely that the content and pedagogical content components of numeracy will be more developed in later year students, which is a reason to expect a positive correlation between the degree of numeracy, the nature and level of theory use and the variable 'study year.' More generally, someone who has a high level of numeracy is likely to function at a relatively high level of reasoning and arguing. In view of the nature and level of theory use, for that reason a positive relation can be expected between *explaining* and numeracy and between *the level* of theory use and numeracy (hypothesis 3.2). Section 5.4.4 provides an extended description of the considerations according to hypothesis 3.2.
To aid the analysis of data a suitable instrument is being developed and tested for this research question (section 5.3.6 and appendix 20 and 21).

5.3 Method

5.3.1 The context and the participants

Eleven Pabos, with a total of 269 students in groups of at least 10, participated in the large scale study. This involved first, second and third year groups from the full-time and part-time courses, the dual course and the so-called shortened course (table 5.1).

The students were offered a course, in which 'The Guide' (see section 4.2.2.2) was an important component of the learning environment. The Guide is a CD-rom for mathematics in grade 2 (Goffree et al., 2003) on which 'the practice' of mathematics teaching is available in a website structure.

The five one and a half hour course meetings were directed by the teacher educator. The next section shows an overview of the course. Part of the first meeting was used for an initial assessment; the fifth and final meeting contained a final assessment, which required an hour and a half extra. In total, the course consisted of forty study hours, nine of which were contact hours with the teacher educator. Less than two weeks after the final meeting the students took the numeracy test, which took one hour.

The group of 269 students that was part of the study consisted of 249 women and 20 men between 18 and 20 years of age (table 5.1). The spread for gender, prior education and type of course (full-time, part-time, dual, shortened) is comparable for that of the national population of Pabo students[36].

The students' prior education varied from mbo level (senior secondary vocational education) to vwo level (pre-university education) and higher education. The groups were taking the course that was offered in the framework of the study as a part of the regular programme. The students were informed in advance of their participation in the national research project 'Theorie in Praktijk' (TIP – Theory in Practice). One Pabo group offered the course as an optional course, giving the students the choice of whether or not to follow the course, in other groups the teacher educator determined, in consultation with the researcher, in what group and at which time the course would best fit into the curriculum.

The teachers teaching the course were experienced teacher educators in at least the subject area Mathematics & Pedagogics; they had taken part in the training course (section 5.3.3) that was developed within the framework of this study and taught by the researcher. The study occurred in the school year 2003-2004, with an extension in the autumn of 2004.

Table 5.1 Complete overview of the population of the eleven Pabos

PABO		1	2	3	4	5	6	7	8	9	10	11
	total	Number of students per study year										
Year 1	42						24			18		
Year 2	79				12				19		9	39
Year 3	148	27	49	36		28		8				
Total	269											

PABO		1	2	3	4	5	6	7	8	9	10	11
	total	Number of students per type of course										
Full-time	146		49	36		28					9	24
Part-time	54	27						8	19			
Three yr, dual	15											15
Three yr. shortened	12				12							
Three yr. vwo	42						24			18		
Total	269											

PABO		1	2	3	4	5	6	7	8	9	10	11
	total	Number of students per prior education										
Mbo without maths	62	8	20	10	1	7	1	1	5			9
Mbo with maths	17	3	3	2		2		3	2			2
Havo without maths	31	4	4	6	1	5		1	4	1		5
Havo with maths	96	9	20	15	6	12	9		3	1	6	15
Vwo without maths	5							2	2			1
Vwo with maths	51	1	1	3	4	2	14		3	15	3	5
Higher ed.	2									1		1
Other	5	2	1					1				1
Total	269											

5.3.2 Design of the learning environment

The learning environment for the students who participated in this large scale study was largely similar to that in the small scale study, and was based on the same theoretical principles (section 2.6, 2.7 and 3.9). The most important components of the content were – as in the small scale study – the Guide (section 4.2.2.2) with stories from practice, along with theoretical reflections and literature, a multifunctional list of concepts, discussions framed by a so-called 'game of concepts,' research within one's own practice school and writing 'annotated stories' and reflective notes. The character of the learning environment was mainly determined by characteristics that were intended to motivate the students to work within the learning environment and which could evoke the use of theory. The most important characteristics were the learning environment's focus on practice in combination with the theoretical load of the practical situations that were available, the environment's structured, yet open character, and the

teacher educator's interventions aimed at the use of theory. These interventions were intended to optimize 'practical reasoning' (Fenstermacher, 1986; Pendlebury, 1995) and 'feeding' with theory.

A few components of the learning environment were adapted or added based on the experiences from the small scale study (section 4.5). This involved for example the list of concepts (appendix 2A,B), the initial assessment (section 5.3.4.1) and the numeracy test (appendix 19). In addition, a 'logbook activity' – called 'What (else) did you learn in this meeting?' – was designed with the goal of making students even more emphatically aware of their own learning or increase in learning (appendix 9). The teacher educators were provided with a detailed manual and were given a day of training (appendix 22). The assumption was, that with these adaptations and additions to the learning environment as used in the small scale study, the students' use of theory could be optimized and their 'Theory-Enriched Practical Knowledge' (EPK) systematically mapped and analyzed.

Next a short overview is given of the programme of the course for the students taking part. The teacher educators' manual (appendix 22) contains a detailed version of the programme.

Overview of the five meetings in the course

Meeting 1. Initial assessment (supervised)

- Filling in list of concepts (individual, 30 min.).
- Initial assessment 'reacting to practice situations' (assignments for four MILE situations on CD-rom, individual, one hour).
- Individual study: becoming familiar with the Guide (CD-rom).

Meeting 2. Introduction of the Guide and the personal learning question

- Introduction CD-rom ('the Guide'); discussion directed by the teacher educator.
- Individual notes: 'What did you learn?'
- From learning question to study assignment: thinking about and formulating a personal learning question.
- Individual study: study of CD-rom and formulating personal learning question; making a note for a teaching story selected from the CD-rom.

Meeting 3. Network for tables of multiplication and learning trajectory (cooperative lecture).

- Interview of two primary school students, Paul and Necmiye (CD-rom): analysis and discussion about the students knowledge of the tables of multiplication.
- Preparatory instruction for individual research by student teachers into primary school students' network of tables of multiplication.
- The teacher educator gives an overview of the four stages of the learning trajectory for multiplication as a theoretical reflection on the practical situations on the CD-rom.
- Individual notes: 'What did you learn?'
- Individual study: working on the individual learning question. Preparing and elaborating the individual research into students' table network.

Meeting 4. Game of concepts

- The game of concepts: looking together for identifiable connections between given theoretical concepts and four practice situations (group discussion led by the teacher educator).
- Individual notes: 'What did you learn?'
- Individual study: working on the individual learning question.

Meeting 5. Final assessment (supervised)

- Filling in the list of concepts (the concepts that have gained meaning, including teaching narratives for two of those concepts).
- Writing a reflective note for (an unfamiliar) MILE situation.
- Questionnaire (anonymous): filling in the questionnaire.

As soon as possible after the final meeting:

- Numeracy test (individual, supervised).

5.3.3 Training the teacher educators

5.3.3.1 Introduction

The researcher presented his research plan in front of an audience of over a hundred mathematics teacher educators, representatives of the 39 training colleges for primary school teachers, at the annual conference of the network of experts involved in primary mathematics teaching in the Netherlands. In addition to information about the content of the research, information was given about the conditions for participation in the study, being: the mandatory and conscientious use of the course materials offered, including the course in the regular curriculum, mandatory training for teacher educators, certification, and at least two years of experience as a mathematics teacher educator, the size of student groups, the number of meetings, the number of contact hours and the participation of the teacher educator in a closing interview. These conditions were mentioned again in the flyer that was handed out as well as distributed electronically over the national network. At first, seventeen Pabos responded to the invitation for participation. In the end, twelve met the conditions for participation. The Pabos that participated showed a geographic spread across the Netherlands: they were located in Amsterdam, Breda, 's-Hertogenbosch, Eindhoven, Gouda, Hengelo, Leeuwarden, Meppel, Rotterdam and Zwolle. One Pabo dropped out during the study due to organisational problems.

5.3.3.2 Training

A (mandatory) day of training in advance of the study for the teacher educator was organised under supervision of the researcher. The goal of the training was optimizing the analogy in working with students by the various teacher educators. Results from the foregoing development and research were used to inform teacher educators about how to introduce the Guide as a tool for the discourse and for the student teachers' self-regulated learning, as well as to create an appropriate investigation context for student teachers, help them find reasons to get a successful start, formulate inspiring learning questions, et cetera. The information was described in a teacher educator's manual (Oonk, 2003; appendix 22). The quotations, given below, from the manual (p. 12) are examples of a general guideline.

> *General*
> In the interest of the study it is necessary that the general and specific guidelines are followed. The material offered must be as 'natural' as possible for all student teachers, while at the same time it must vary as little as possible between the various locations (...).
>
> *The students' logbook*
> The term 'logbook' is interpreted in a diversity of ways at the Pabos. Therefore this course doesn't use the term for student teachers. For the purpose of the discussions in meeting 2, 3 and 4, the students' reflections are aimed at 'What did you learn?' (see appendix 5 of the manual).

> *The students' individual learning question*
> It is the intention that all coursework is guided by the students' individual learning question, which runs through the course like a guideline. The reflective notes from the research into the network for the tables of multiplication and the final assessment are both assumed to be guided by the individual learning question (...).

The manual contains detailed guidelines for each meeting. These guidelines concern the goals of the meeting, the organisation, the subject-specific and course-pedagogical content, suggestions for the students and aspects that are vital for obtaining valid research data, such as the exact instruction for filling in the lists of concepts and handing out the assignments for the assessments. The following quotation from the manual is an example of an instruction for the teacher educators (manual, p. 28; appendix 22).

> *Instruction final assessment, unit 3*
> Writing a reflective note for a situation from MILE ('The suitcase full of balls'). See appendix 9 from the manual.
>
> It was shown in the small scale study that the students, even if they did not yet know MILE, could easily find the fragment in question. Only help the students, if needed, with the procedure, that is to say: if they don't understand or can't find the fragment concerned. Do not offer the fragment in a whole-class setting, as students have the understandable tendency to react to the content of the images when viewed in a plenary session. Furthermore, the assignment contains a subtle impulse to stimulate searching at the end – in the italic text; the intention is to find out to what extent students show searching behaviour, partly on the basis of that impulse, and will for instance look to see if there is relevant information to be found before or after the selected video fragments. To remind the students of the focus on their own learning question and to inform the teacher educator, the individual learning questions are noted at the top of the reflective note (see appendix 11, p. 61 in the manual).

The prescribed teaching material has been planned chronologically and in detail in terms of activities for teacher educator and students. During the training day the guidelines were discussed, and crucial interventions by the teacher educator – for instance specific questions – have been practiced in the meetings based on video material from the small scale study. The teacher educators also had tasks, assignments and texts for students that had to follow the letter of the manual pointed out to them. Some guidelines concerned the right moment to show students certain papers or forms, with one of the intentions being to avoid influencing open-minded reflections.
Finally, the training day gave attention to collecting the research data and there were opportunities to ask questions.
Based on the experiences during the training days the researcher revised the manual and provided it to the teacher educators who participated.

Shortly after the end of the course, every participating teacher educator was interviewed in a session that lasted about an hour. Goal of this interview was to obtain as much information as possible about experiences with the course, particularly information relating to interventions by the teacher educator and striking reactions from students that were related to the use of theory. The teacher educators received a list of nine questions from the researcher not long before the interview, intended for use as a guideline.

5.3.4 The instruments

5.3.4.1 Initial and final assessment

Experiences from the small scale study led to changes in the initial assessment for the large scale study. The change involved a changed set-up for the way in which the reflective note made by the students was done, resulting in a better match between the initial and final assessments (appendix 11 and 12).

The reflective note at the start of the course was intended to test at what *level* the students used theory within a *specific* category of the nature of theory use (factual description, interpretation, explain and respond to) (section 5.3.6.3 and 5.3.6.4). The four assignments for four different situations from MILE had been phrased so that they would consecutively evoke these four types of theory use. For instance, in the first assignment the student teachers were asked to observe student Chantal, and then in their own words give a factual description of what occurred in that situation (appendix 11). This part of the initial assessment yielded two types of data: primarily the number of theoretical concepts that each student used in doing the assignments, and also statements by students in which theoretical concepts were used. The four practical situations presented in the video material and the situation for the final assessment were selected from the lessons about learning the tables of multiplication in MILE-grade 2, with the same students and teachers for all situations (appendix 23). The situation that was selected for the final assessment was new to the students. They were given a short explanation about the context of the situation, and where the video clip of the situation could be found in MILE; there was also some advice on writing the reflection (appendix 12).

The large scale of the study necessitated limiting the use of tools and data to those of the written reflections in the initial and final assessments, the numeracy test (section 5.3.4.2) and the questionnaire (section 5.3.4.3).

5.3.4.2 Numeracy test

After the course had ended, the students did a numeracy test (appendix 19). The written test consisted of ten problems, and had been derived from tests for the subject area Mathematics & Pedagogics as used widely at Pabos in the Netherlands at the time (2003) the test was set (Faes et al., 1992; Goffree & Oonk, 2004). Individual numeracy was used as an independent control variable in this study that is marked by the research questions. A positive correlation is supposed between the ability to solve mathematical problems in

students and their level of pedagogical (content) theory use (section 4.3.8 and 4.4.2). Triangulation of the data resulting from the numeracy test with the data produced by the other instruments, had as its goal to generate answers to research question 3b (section 5.2) to the extent of a correlation between the student teachers' level of numeracy and the nature or the level of their use of theory. To have the problems in the numeracy test target aspects of numeracy even more emphatically, the test as used in the small scale study (appendix 18) has been slightly changed for the benefit of the large scale study (appendix 19). This change involves an extended explanation with the problems in order to evoke reflection. Furthermore, the students could also rate each problem on a five point scale to evaluate how hard they thought a problem was.

A personal evaluation index (PEI) was also determined, to define the relation between the level of difficulty and numeracy score (difficulty total score times two, minus the total score of the numeracy test). The underlying idea is that the index can be a measure for the confidence in one's own numeracy.

The standards for determining the level of numeracy have been developed in three sessions, with the researcher's first proposal being discussed with other expert educators[37], tried out and revised (appendix 20). The second version of the standards was subjected to a random sample (n = 15; appendix 21). The fifteen tests were scored independently by two judges in three sets of five with analysis in between. Independent assessment of the whole random sample yielded an interrater reliability (Cohen's kappa) of $\kappa = 0{,}91$.

5.3.4.3 Questionnaire

Section 4.2.3.5 describes the backgrounds, the purpose and the set-up of the questionnaire. The experiences with the questionnaire in the small scale study did not lead to adapting the questionnaire for the large scale study. The fourteen questions relate to the evaluation of the course, and particularly to how the students appreciated the theory as expressed in the course.

Descriptive statistics of the data (mean and std. deviation; appendix 14) have been determined through the use of the computer software SPSS, version 15.0.

5.3.5 Procedure and data collection

During the first meeting of the course offered to the students, the initial assessment was done, with the purpose of determining the number of theoretical concepts used, as well as the level per category for the nature of the theory use (factual description, interpretation, explanation, response to). The data yield consisted of the number of theoretical concepts and of statements by students in which theoretical concepts or notions of theoretical concepts were used.

For the number of concepts, a distinction was made into the total amount of concepts, the number of different concepts, the number of pedagogical content concepts and the number of general pedagogical concepts.

In the final meeting of the course the final assessment was performed. This established the nature and level of theory use at the end of the course, as well as the number of theoretical concepts used. The data yield consisted of the number of theoretical concepts and of statements by students in which theoretical concepts or notions of theoretical concepts were used. Just as for the initial assessment, subcategories were made for the number of concepts, the number of different concepts, the number of pedagogical content concepts and the number of general pedagogical concepts.
Also in the final meeting, the students had to fill in the anonymous questionnaire for the evaluation of the theory on offer in the course. The output from the questionnaire consisted of quantitative data on the appreciation of the way that – and the degree to which theory was treated in the course (appendix 14) and statements by students who provided an explanation for their answers to the questions in the questionnaire.
The numeracy test was set shortly after the course, with the intention of determining the students' level of numeracy. The data were the levels of numeracy (on a scale of 0-100) and the level of difficulty of the problems as experienced by the students, indicated on a five point scale; these data were used for research question 3b.
The following variables served as background variables for the large scale study: the institute (the Pabo) at which the student studied, the student's prior education, the kind of course the student was taking (fulltime; part-time; shortened), the study year, the group (class) the student was in, small or large group, gender and, the primary school group in which the students did their teaching practice.

5.3.6 Data analysis

5.3.6.1 Analysis instrument

Based on the knowledge and experience gained from the pre-study (focal points for theory, section 2.6.4), the exploratory studies (e.g., signals for theory use, appendix 1), the small scale study and discussions with colleagues (appendix 10), an instrument has been developed (see 3x4 matrix, table 5.2) to systematically order and analyze the data. The use of theory is expressed in four types of the nature (factual description, interpreting, explaining and responding to) and in three levels. The development of the instrument has the character of design research (Gravemeijer, 1994; Cobb, 2000; cf. section 2.7.2). The concept of the instrument, which arose as a result of the analysis in the small scale study, was further refined in the large scale study, partly influenced by ideas found in literature (Van Hiele, 1973, 1986; Freudenthal, 1978, 1991; Zeichner & Liston, 1985; Sparks-Langer et al., 1990; Simon, 1995). Finally that process of refining and revising (Bales, 1951; Miles & Huberman, 1994; Krippendorf, 1980; Rourke et al., 2001) led to a reliable instrument.

5.3.6.2 Meaningful units

The students' reflective notes[38] were split into meaningful units (see examples in table 5.3), and each unit was evaluated for the use of theory. A meaningful unit is a complete segment within a text, a 'thought unit' in the form of a paragraph on a topic or a theme (Bales, 1951, Krippendorf, 1980; Rourke et al., 2001). This study defined units as determined by 'completed' stories, trains of reasoning, or thoughts about an occurrence, or by transitions in the type of theory use, for instance from factual description to interpretation of the situation being observed. Where possible, the structure imposed on the text by the student when (re)constructing (sub)situations was taken into account. Sometimes the units to be distinguished were already visible through white lines or paragraphs. The syntax also offered support for separating the text[39]. Considerations related to the use of concepts and the structure used by the students themselves were taken into account before those based on a type (nature) of theory use.

Where there was doubt about a separation, a choice was made for the larger unit without that separation.

The definition of a meaningful unit as presented here is the revision of an earlier version; the discussions about the revision occurred during two sessions between the researcher and a second expert on validating meaningful units. The conversations have been transcribed.

Using a random sample of 15 students out of 269, the interrater reliability was determined at 81% (appendix 15). The discussion on the remaining differences led to full agreement between the judges.

5.3.6.3 The nature of the use of theory: defining the four concepts horizontally (cumulative/including)

The relationship between the four categories for the nature of the use of theory is seen as an inclusive relationship. It is assumed that each subsequent category will contain one or more of the preceding ones, whether or not explicitly visible in the student's description. Descriptions that do clearly not satisfy the criterion of the inclusive relationship are not scored.

A: Factual description

The student describes actual events only; no opinion is given, nor are any operations or expressions by the teacher or a student explained. The student's statements show in no way that the situation has been thought about or that it has been responded to/geared to.

B: Interpreting

The student tells what he or she thinks is happening and gives his or her own opinion without adding any explanation. For instance, a 'bare' assumption is made, or a judgment is made without a foundation, or the situation is simply labeled.

Indicator words[40] for this type of description can be for example: *I think that* (...) *or according to me is* (...). Also *adjectives* can give an indication of interpretation, but one

should be alert: adjectives – and also adverbs – can confuse the scoring; only adjectives which give a subject-specific interpretation of the associated noun, can be qualified as indicators. Compare 'the student has a nice solution' and 'the student has a time-consuming solution'; the first case (nice), might express a meaningless 'filler,' the second (time-consuming) can be an indication of an interpretation, more so if there is no foundation for the statement.

C: Explaining

The student explains why the teacher/student acts or thinks in a certain way. This concerns an unambiguous, 'neutral' explanation on the basis of (previously mentioned) facts or on the basis of interpretations or factually observed events. For example, it does not concern what could have happened before, during or afterwards, but *why* it was (probably) done or what might have thought to cause to the visible action; in the latter case it involves a conjecture of an idea together with an explanation ('proof').

Indicator words for this type of description are for example*: why, for this reason, because, as, as... if, probably, it could be possible that…..* In terms of interpreting text (Pander Maat, 2002) this often involves causal relationships and reasoning (argumentation and explanatory) relationships.

Both last mentioned indicators (probably, it could be possible that) also might be indicators words for B (interpreting), but the difference is in the further elaboration: here, "after the 'why' is the 'because' " (Freudenthal, 1978).

The connecting word 'so' in a reasoning relationship can point to a conclusion or a possible explanation (Pander Maat, 2002). If 'so' can be left out without the sentence changing meaning, there is usually no C-description.

D: Responding/Gearing to

The student teacher can respond to the situation in several ways. It can be commonly stated that responding to a situation by the student teacher appears as what one could call a 'design activity' by the student teacher. Below, the different forms of 'responding' will be named.

The student tells what, in his or her opinion, the teacher could have thought or done (differently) in preparation for the given situation, or what reaction by the teacher or the student one might expect after the end of the given teaching situation. In that last case there may be a kind of hypothetic learning trajectory involved (Simon, 1995).

Indicator words in this case can be for example: *I expect that..., I predict that..., I should…, I suspect that*. Although it could start as an explanation (C), the description is characterized as an idea about the possible consequence of operations or as a possible continuation to the given situation.

The student teacher can also take a position as a virtual substitute of the teacher in the observed situation: the student teacher tells or describes – for example in a preparation,

a design or a review – which action he himself, or he as a substitute of the teacher, wants or would want to take, for example to try out an observed activity or an alternative action in his own field placement group.
The reflective note can also take the form of a critical, deliberated response to the actions of the teacher.
Indicator words in those cases can be for example: *I do..., I make..., I intend for..., I should..., my intention would be....*
The student asks himself a question.
Indicator words in this case are among others: *I wonder..., the question is...*
The student reflects on his own thinking; the student's own learning process is taken into consideration.
Indicator words are for example: *I have learned from this, that..., when I think of my learning question…*

Observation

It is imaginable that a D-observation turns out negative, in the sense that a student for instance starts his or her reflection by giving a non-supported alternative for the given teaching situation. In that case, the inclusive relationship between the horizontal categories, with D more or less following A, B and C, is absent, and – depending on the content – the negative D-score can be seen as a D1-score (lowest level) or a B-score (incorrect interpretation).

5.3.6.4 Levels of use of Theory

Theoretical concept

A 'theoretical concept' is defined as a concept from a list of 59 general pedagogical concepts and pedagogical content knowledge concepts which is part of the learning environment of the student teachers (appendix 2A,B).

Notions of theoretical concepts and lay concepts

A *notion of a concept* is seen as a synonym or a description that within the given context lends the same meaning as the 'mother concept' – which is always the concept mentioned first – in the list of 59 concepts. These are words or expressions that occur in the descriptions of these same 'mother concepts' from the register of The Guide.
A theoretical concept or the notion of it manifests as factual information in a text, not as an interpretation of what a student might have been thinking.
It does happen that theoretical concepts are referred to by a name that is the same as concepts that occur in daily use, for instance the concept 'multiplication.' If no difference exists between use by students and use by lay people, it will *not* be considered as a theoretical concept. That difference will be expressed when there is a clear pedagogical surplus value, for instance when the concept is used in relation to another concept within the context of the given teaching situation or if the concept is

used in a more contemplative sense[41]. In the sentence: "Fariet is doing multiplications," 'multiplication' is taken as a lay concept if no further connection is made or explanation given, while in the sentence: "Fariet uses smart multiplication with tens," 'multiplication' is seen as a theoretical (pedagogical content) concept.

On the other hand there are also words that are not identical to one of the 59 'mother concepts,' but that can have the same meaning. Sometimes they have the character of a 'lay concept.' An example is the phrase 'make visible' with the mother concept 'visualizing.' These concepts are scored if they occur in the description of the mother concept in the register of The Guide; they are also included in the score list together with the mother concepts. For these concepts it is also the case that they are only scored if their use within their context lends a meaning that is equivalent to the mother concept.

Characteristics level 1: no visible use for theory

No visible and relevant use of theoretical concepts is observed; at most there is relevant use of notions of theoretical concepts.

The use of irrelevant theoretical knowledge occurs in case of incorrect or improbable statements[42] or intuitive judgments in which theoretical knowledge has no meaning and has only been 'mentioned.'

Characteristics level 2: reproductive or mechanical use for theory

Visible and relevant use of a theoretical concept can be seen in a sentence or in a cluster of sentences.

Where two or more theoretical concepts or theoretical notions are being used, there is no visible insight from the student teacher into the coherence between those concepts or notions of concepts. No use of relative language is observed, either on its own or in combination with demonstrative language.

Mainly reproduction of theory takes place.

Judging with the benefit of a theoretical concept occurs on the basis of simple reasoning.

Characteristics level 3: integrating and synthesizing theory

A visible and relevant use of two or more theoretical concepts is observed, with visible insight by the student teacher into the coherence between those concepts or notions of concepts.

Judgments and conclusions are made with the benefit of theoretical concepts on the basis of logical reasoning (if... then implications, use of arguments, (re)considering, making relationships, generalizing), among other things with reference to literature. Sometimes a student's 'own theory' is formulated and founded; reconstruction of theory takes place.

In section 3.9 the concept of 'theory-enriched practical knowledge' (EPK) was introduced as a derivation of the concept of 'practical knowledge.' Within the framework of this study, level 3 of the use of theory is seen as an important indicator for theoretical enrichment of practical knowledge.

Table 5.2 gives an overview of the twelve categories for the nature (horizontal) and the level (vertical) of theory use by students. Table 5.3 shows examples of each of these categories and table 5.4 describes some examples of doubtful cases encountered by experts when scoring the meaningful units.

Table 5.2 Reflection Analysis Tool. Brief description of the twelve score combinations

	A **Factual description** *facts: who, what, where, how*	**B** **Interpreting** *For instance opinion or conclusion without foundation*	**C** **Explaining** *For instance 'explaining why'*	**D** **Responding, gearing to** *For instance, anticipation, continuation or alternative design, meta-cognitive reactions*
Level 1	**A1** Factual description of events without use of theoretical concepts.	**B1** Interpretation of events without use of theoretical concepts.	**C1** Explanation of events without use of theoretical concepts.	**D1** Description, alternative event, continuation or meta-cognition without use of theoretical concepts.
Level 2	**A2** Factual description of events using one or more theoretical concepts without mutual connection.	**B2** Interpretation of events using one or more theoretical concepts without mutual connection.	**C2** Explanation of events using one or more theoretical concepts without mutual connection.	**D2** Description, alternative event, continuation or meta-cognition using one or more theoretical concepts without mutual connection.
Level 3	**A3** Factual description of events using one or more theoretical concepts with a meaningful connection.	**B3** Interpretation of events using one or more theoretical concepts with a meaningful connection.	**C3** Explanation of events using one or more theoretical concepts with a meaningful connection.	**D3** Description, alternative event, continuation or meta-cognition using one or more theoretical concepts with a meaningful connection.

The units (table 5.3) have been borrowed from reflective notes in the final student teachers' assessment (appendix 12), which was the closure of the course within the framework of the research.

Table 5.3. Examples of the combinations A1 up to D3

Score combination	Example of the meaningful unit
A1	At the front of the class there is a suitcase with tennis balls. *Explanation: This is an actual reproduction of the situation. No theoretical concept is used.*
B1	Before the teacher counted the balls in the suitcase together with the children, she probably told them the exciting story of how the suitcase got into the classroom. You can see that the children are very involved in this lesson. *Explanation: The first sentence is an interpretation of what might have happened before the observed situation; the word 'probably' is an indication, just like the expression 'very involved' in the second sentence. The word 'exciting' in the first sentence shows a 'weaker' signal.* *No theoretical concept is used.*
C1	It is precisely the choice for a large quantity of balls that evokes students' thinking. The large number of balls makes it less likely that they will just count them. *Explanation: It is indicated why teacher Minke sets the students thinking.* *No theoretical concept is used.*
D1	Placing the cylinders with balls might have been done at a slower pace, which could give space for doing arithmetic in between; this is how I would do it in any case. *Explanation: In the reflection the student teacher gears towards concepts of a possible alternative for the teacher's approach in the observed situation. No theory is used.*
A2	The suitcase with balls that was put down by 'Black Piet' is used by Minke as a reason to count (in a structured way) with the children. The fragment starts at the moment that the balls are snatched away and are put in transparent cylinders. *Explanation: It is a factual reproduction of a situation, in which one theoretical concept (structured counting) is used.*
B2	The children count once more up to 100 in the same way (strategy) Fariet did. Minke indicates that Fariets' way of thinking makes sense; that response will reinforce his self-confidence. *Explanation: The second sentence points towards an interpretation of the situation; one theoretical concept (strategy) is used. The final clause can be seen as a notion of the concept 'pedagogical climate.'*
C2	Minke is working with the whole group. Counting together with jumps carries the danger that not everybody participates in the activity. I can see that with two children who are doing different things while the class is counting. *Explanation: The student teacher postulates a 'thesis' and an associated 'proof' for it.* *Theoretical concepts are used (group teaching, to count with jumps); however, those concepts do not have a coherent meaning that is relevant for the third level of using theory.*

D2	Hereafter, it could be possible to let make the students a table network, starting with the sum 20 x 5 = 100; hang it up and discuss it. *Explanation: The student teacher gears to the given situation in terms of a possible continuation on the observed activities.* *One theoretical concept (table network) is used.*
A3	By moving the cylinders the teacher makes another grid model. Now there is a rectangle of 10 x 10. Next, she let the students give meaning to the new model, working with doubling and halving in a very concrete way. She emphasizes that the multiplication is/sounds different, but that the answer remains the same. She writes the new multiplication on the blackboard as well and again connects the concrete and the abstract sum. *Explanation: This is a factual reproduction of three successive events. Three pedagogical concepts (grid model, rectangle model and doubling and halving) are used coherently.*
B3	Fariet gives a handy solution for 13 x 5. He immediately thinks of the multiplication that is really represented by the 13 cylinders. So far his class has only done the tables up to 10 x 5 (I assume), but he already understands how to calculate the five times table above 10 x 5. *Explanation: The words and expressions 'handy,' 'he immediately thinks of,' 'I assume' and 'he already understands ... above 10x5,' indicate an interpretation of the situation.* *The concepts 'multiplication 13x5,' the '13 cylinders' (notion of material) and the 'tables up to 10 x' are used coherently.*
C3	The class already comes up with 2 x 5 followed by 3 x 5. Because she visualises the five times table for the children, they can also tell a story to accompany a problem. 1 x 5 will be possible to see as 1 tube times 5 balls. She also makes a connection between concrete material and a grid model. At one point Clayton is counting 10 x 5, the teacher confirms this for the class. In fact a transition is being made here from multiplication by counting to structured multiplication. *Explanation: the whole text has the character of an explanatory description, with the words 'because,' 'also' and 'in fact' functioning among other things as signal words. Seven concepts are used in connection (five times table, visualises, story to accompany a problem, concrete material, grid model, multiplication by counting and structured multiplication).*
D3	Do the children really see the tens in the rectangle model? The teacher could have asked on with Fariet: "Fariet, how do you see the 10, 20...? Can you tell me or point it out, Fariet?" *Explanation: The student teacher anticipates the situation in terms of a possible alternative for the teacher's approach. The concepts 'tens,' 'really see' (notion of structure), 'rectangle model' and 'asking on' are used coherently.*

In some cases there was some doubt about the score combination for a unit. This doubt was expressed in differences between expert scores or by both experts finding at first that a unit qualified for two or three scores.

Below (table 5.4) some examples of such doubtful cases are described, together with the considerations that led to an unequivocal score combination.

Table 5.4 Examples of doubtful cases of scoring units

Example 1	
Unit	In this fragment we see teacher Minke, teaching grade 2. Using a splendid context, 'The trunk full of balls,' she teaches learning to multiply.
Doubt between	A2 and B2
Considerations by the experts	The first sentence has an A-character. However, the expressions 'splendid context' and 'learning to multiply' in the second sentence indicate interpretation (B). It is not clear how 'learning' is taken here: learning to multiply in general? Has the first introduction been meant, the conceptual attainment of learning to multiply? Because of the link to the 'trunk full of balls,' the application of the concept 'context' can be considered as meaningful use of a theoretical concept. Level 3 is not reached: no meaningful relationship has been made between theoretical concepts.
Conclusion by the experts	B2
Example 2	
Unit	At first, you would think that this fragment belongs to the introductory stage of learning to multiply, because it starts out with a context (with the balls), in which the five times table of multiplying is hidden. One can watch the children counting smoothly by fives and even making a link between the table of 5 and 10, doubling, halving and, counting with jumps on the number line. Soon after this, strategies are used and links are made. Therefore, you could also say: these activities belong to the memorizing stage of learning to multiply. This is strange: contexts, materials, but nevertheless the memorizing stage (or not?).
Doubt between	C3 and D3
Considerations of the experts	The first part of the text reflects a C-character: the assumption that the fragment belongs to the introductory stage of learning to multiply, is founded with an argument. On the other hand the expression 'at first you would think' has a fairly subjective, non-neutral character, which is typical for D (and for B). The same could be said about the sentence 'Therefore, you could also say: these activities belong to the memorizing stage of learning to multiply.' In the last sentence the author of this unit asks himself a question, as a result of the situation; that underpins the choice for score D. Concerning the level of theory, it is obvious that this text characterises level 3, showing meaningful relations between several theoretical concepts, namely some stages of learning to multiply, materials, context, the five times table, counting with jumps and doubling and halving.
Conclusion by the experts	D3

5.3.6.5 Scoring procedure

The procedure, done by two experts[43], was aimed at improving and validating the instrument in a cycle of random samples coding, discussion and revision. Finally the reliability was established in a random sample of 15 students (see next section and appendix 17).

- The students' assignments for the initial and final assessment (appendix 11 and 12) were studied, along with the accompanying video fragments and transcripts.
- The theoretical backgrounds (theoretical concepts, practical knowledge) of the situations were mapped.
- A score was made for each individual student based on the definitions from the analysis instrument.
- The student's reflective memo was split into meaningful units; each unit was checked to see if it contained theory use.
- Each unit was assigned a letter and a number to indicate nature and level of theory use (initial assessment: number; final assessment: combination of letter and number), as well as the number of theoretical concepts (6 categories for the initial and the final assessment each; see section 5.4.1). The data were collected and ordered on a score form (Guidelines for rating nature and level of use of theory; appendix 16)
- If one unit could be assigned several (intermediate) scores, the highest one was counted. The highest score in such cases was determined by the following order of combinations: A1, B1, C1, D1, A2, B2, C2, D2, A3, B3, C3, D3. Note that the final score in that case was not necessarily determined by the final sentence in the unit, nor by the final score within the unit.
- The judge was focused on possible level 3 scores going across more than one unit.
- When there was hesitation between two possible scores, the video fragment or the transcript of that fragment from the initial or final assessment was studied and scored again; the last score was considered final (see examples table 5.3).
- Inaccurate, irrelevant or judged to be unlikely statements were scored as level 1.

5.3.6.6 Reliability of the instrument

The reflection analysis instrument has been revised a number of times on the basis of comments made by experts in the Netherlands and abroad (appendix 10). Finally, in a random sample of 15 students out of 269 the interrater reliability was determined (appendix 17). That led to the four following results. The Cohens Kappa for the level of the initial assessment was 0,85. The Cohens Kappa for the nature and the level of the final assessment was 0,80 respectively 0,86 and for the combination of nature and level of the final assessment the outcome was $\kappa = 0{,}77$.

The next step in the procedure was for the researcher to score the reflective notes and assessments of the remaining 254 student teachers.

5.4 Analysis and results

5.4.1 Introduction

The data collection of the large scale study involves the data of 269 students over 11 Pabos (table 5.1). According to the procedure described earlier, the initial assessment has been scored for the level of theory use and the final assessment for nature and level of theory use. The initial assessment consisted of four situations, each aimed at one of the categories for the nature of theory use. For the purpose of scoring, the final assessments had been divided into a total of 1740 meaningful units, on average seven units per student (table 5.5).

Table 5.5 Statistics units in final assessment

N	Valid	246
	Missing	23
Mean		7,07
Median		7,00
Mode		6
Std. deviation		1,794
Minimum		4
Maximum		13
Sum		1740

For nature as well as level of theory use, each student has been scored on the number of theoretical concepts used. Here, a division into six categories was made for both the initial and the final assessment.[44]

The scores of the numeracy tests have been included in the data collection both for the individual problems and in total. This is also the case for the students' own evaluation on a five-point scale of the difficulty of each problem. A variable 'personal evaluation index' (PEI) has also been created (see also section 5.3.4.2), which is the difference between 2M-S, where M is the total of the difficulty scores and S the total of the problem scores. PEI gives a positive result if students overrate themselves and a negative result if they underrate themselves. It might be possible to see PEI as a measure for self-confidence.

In the following overview all variables are mentioned that have data stored in SPSS.

The variables:

Pabo (Primary Teacher Training College), class, group size, study year, type of study, prior education, gender, practical experience, number of concepts (pedagogical content knowledge, general pedagogical knowledge, different concepts, for begin and end, comparing number of similar concepts (pck, gpk) start and end, level initial assessment, number of units, nature of theory use (also in percentages), level of theory

use (also in percentages), number of combinations (also in percentages), assessment numeracy, difficulty, Personal Evaluation Index (PEI).

Using various SPSS (version 15.0) tools, equations have been created to make the variables and data accessible for analysis.
Next, an account is given of the analysis and the results for each research question.

5.4.2 Analysis and results of the first research question

The first research question is:

> In what way do student teachers use theoretical knowledge when they describe practical situations after spending a period in a learning environment that invites the use of theory?

Considerations for research question 1
The expectation is that, where the nature of theory use is involved, students will relatively often '*factually represent*' (category A). A number of arguments can be provided for that assumption. To start with, factual description – as the first in the inclusion relationship – has an obvious function as the start of a description. Moreover, it seems likely that a number of students will not get beyond factual description in their reflection. Those will mainly be the students that score highest in category A. It is plausible that this will largely involve students in lower years or students with a lower level of prior education, primarily because the pedagogical (content) jargon of lower year students will generally be less developed than that of higher year students. Without a sufficiently large pedagogical repertoire it is harder to let reflection rise above the level of factual description. Moreover we expect that factual description will occur more in the group of students with a lower level of prior education than in students with a higher level of prior education, because factual description requires less cognitive abilities than explaining and responding to situations.

The two final statements – regarding a limited jargon and a lower level of prior education – are also valid for *interpreting* (category B). In addition, experience teaches that students at the start of their study have a tendency to be faster to reach a judgement about the teacher or the students they are observing than students in later years. This is an added reason to expect interpreting to occur more with first year than with later year students.

To *explain* teaching situations (category C) students must possess a sufficient pedagogical repertoire and cognitive ability. It is therefore likely that we will find students who tend to apply this type of theory use among the later year students or among students with a higher level of prior education.

Responding to situations (category D) is likely to be scored relatively little, since that activity requires a high cognitive level and some creative input.
Regarding the use of concepts the study distinguishes between general pedagogical and pedagogical content concepts. Students tend to react spontaneously, and primarily in general terms to teaching situations. This is understandable, since the general pedagogical jargon is aimed at the whole of the action taken by teacher and students and is more often used in both course and teaching practice. Often, intervention by the teacher educator or by student peers is required in the discourse to focus on *content-specific* aspects of the situation that has been observed.
The expectation is therefore that students will more often use general rather than content-specific concepts in their reflections on teaching situations. It is also plausible that students who explain or respond to situations more, will also use more theoretical concepts, and vice versa; if you have more theoretical concepts at your disposal, there is more of a chance for explaining or 'responding.'
The above considerations lead to three hypotheses regarding the nature of theory use.

Hypothesis 1.1
The characteristics of the nature of theory use will manifest to various degrees, with 'factual description' as a category with a relatively high frequency.

Hypothesis 1.2
The characteristics of factual description and interpreting for the nature of theory use will occur most often with lower year students or with students with a lower level of prior education, while explaining and 'responding to' will mostly occur with later year students or students with a higher level of prior education.

Hypothesis 1.3
Students will mainly use theoretical concepts to explain teaching situations and to respond to situations. This will involve general pedagogical concepts more often than pedagogical content concepts.

Data analysis and results hypothesis 1.1
To be able to research the hypothesis, first the variables 'percentage X' (X = A, B, C or D) for the four categories of the nature of theory use are entered. The different numbers of units per students necessitate the creation of a comparable measure.
Quantitatively speaking the first hypothesis can be answered simply by giving the four percentages that arise from the descriptive analysis of the percentages X. The output of that indicates a division into respectively 25, 12, 42 and 21 as the average percentages scored by students in the categories A to D (factual description, interpretation, explanation and 'responding to'; see table 5.6).

Table 5.6 Statistics mean percentages for the nature of theory use

		percentage A factual description	percentage B interpretation	percentage C explanation	percentage D 'responding to'
N	Valid	246	246	246	246
	Missing	23	23	23	23
Mean percentage		25,30	11,62	41,96	21,11
Std. deviation		25,320	18,077	28,423	20,898

Furthermore, 38% of the students starts the reflective note with a factual description of the teaching situation, 19% even scores category A on both unit 1 and 2.
The results partly confirm the first hypothesis. While factual description (A) does score high (25%), explanation (C) has a score of 42%, which is by far the highest percentage, and category D also scores higher than expected.
Because the percentages mentioned for category A to D relate to the average percentages scored by students (with standard deviations of 18 to 28%), and not to percentages of the population or numbers of students per category, we also look at groups of students where the nature of the use of theory is relatively often aimed at one specific category. This gives us an extra opportunity to look for specific student characteristics that belong with certain characteristics for the nature of theory use. For this purpose we define the concept 'characteristic dominance' as that characteristic of theory use where the students scores at least 50% of the total number of units in the category that occurs most often[45]. The seven students who score 50% in two categories are left out of consideration (table 5.7).

Table 5.7 Students with two 50% scores for the nature of theory use

	number	Student nr.
Perc A = perc B = 50%	1	164
Perc A = perc C = 50%	3	75, 163 and 238
Perc A = perc D = 50%	1	107
Perc C = perc D = 50%	2	260 and 262
Total	7	

Data selection and frequency analysis provide the following view of the percentages and the numbers of students in the four categories:

Table 5.8 Statistics characteristic dominance

		perc A ≥50 (FILTER)	perc B ≥ 50 (FILTER)	perc C ≥ 50 (FILTER)	perc D ≥ 50 (FILTER)
N	Valid	239	239	239	239
	Missing	23	23	23	23
Percentages of population		17,6	6,7	43,5	11,7
Number of student teachers		42	16	104	28

It turns out that as much as 79,5% of the students dominates in one of the four categories (190 out of 239 students; table 5.8). It may be possible to explain that result from the differences in learning or writing style between students (Kolb, 1984; Vermunt, 1992). The ranking of the category percentages X ≥ 50% matches that of the category percentages X. Here too the relatively high frequency of category C stands out. It turns out that 43,5% of students belongs to the category percentage C ≥ 50.

The higher-than-expected percentage C and 'C-dominance' may be related to the fact that the student population consists of a relatively large number of higher-year students (84% second and third year) and students with a relatively high prior education level (havo – senior general secondary education – with mathematics 36%; vwo with mathematics 19%). We will look further into this conjecture for hypothesis 1.2. The influence of the learning environment may be another factor that has reinforced the explanatory character of students' reflections.

Data analysis and results hypothesis 1.2

Where *factual description* is involved, linear regression analysis points to a significant negative correlation between both category A and category percentage A > 50% and students' prior education (respectively sig. 0,041; beta –0,131 and sig. 0,003; beta –0,195).

Further analysis on prior education indicates a significant positive correlation between both category A and category percentage A ≥ 50% and the students with an mbo education without mathematics (respectively sig. 0,041; beta 0,130 and sig. 0,013; beta 0,160; table 5.9).

Table 5.9

Correlation nature and prior education	**Beta**	**Sig.**
Percentage A (factual description) and prior education	-0,131	**0,041**
Percentage A ≥ 50 and prior education	-0,195	**0,003**
Percentage A and mbo without mathematics	0,130	**0,041**
Percentage A ≥ 50 and mbo without mathematics	0,160	**0,013**
Percentage A and vwo with mathematics	-0,099	0,127
Percentage A ≥ 50 and vwo with mathematics	-0,106	0,103
Percentage B (interpretation) and prior education	-0,129	**0,043**
Percentage B ≥ 50 and prior education	-0,042	0,514
Percentage B and mbo without mathematics	0,092	0,151
Percentage B and vwo with mathematics	-0,043	0,498

Table 5.9

Correlation nature and prior education	**Beta**	**Sig.**
Percentage C (explanation) and prior education	0,243	**0,000**
Percentage C ≥ 50 and prior education	0,275	**0,000**
Percentage C and mbo without mathematics	-0,202	**0,001**
Percentage C ≥ 50 and mbo without mathematics	-0,246	**0,000**
Percentage C and vwo with mathematics	0,138	**0,031**
Percentage C ≥ 50 and vwo with mathematics	0,149	**0,021**
Percentage D ('responding to') and prior education	-0,051	0,426
Percentage D ≥ 50 and prior education	-0,037	0,573

There is a *negative trend* for the correlation between the percentage *interpretation* (category B) and the students' prior education (sig. 0,043; beta –0,129).
For *explaining* (percentage C and percentage C ≥ 50), linear regression analysis points towards a significant *positive* correlation with students' prior education, likewise for the specific case of vwo with mathematics. Conversely, mbo without mathematics has, as expected, a significant *negative* correlation with percentage C and percentage C ≥ 50.
For '*responding to*' (percentage D and percentage D ≥ 50) there is no significant correlation with students' prior education.

As far as the correlation between the nature of theory use and the year in which students are, linear regression analysis only shows a significant relation for explaining (category C). Other than what was expected, that correlation is negative (table 5.10). A more detailed analysis shows that the correlation is positive for the first year – and negative for the third year. It seems likely that a combination of the following factors can explain these correlations. First, a negative correlation has been found for *explaining* and *mbo without mathematics* as prior education, and a positive one for *explaining* and *vwo with mathematics* (table 5.9). In addition, the first-year students are mainly *vwo with mathematics* students and many of the third-year students have *mbo without mathematics* (table 5.1).

Table 5.10

Correlation nature and year of study	**Beta**	**Sig.**
Percentage A (factual description) and year of study	0,096	0,134
Percentage A ≥ 50 and year of study	0,079	0,222
Percentage B (interpretation) and year of study	0,010	0,879
Percentage B ≥ 50 and year of study	-0,054	0,408
Percentage C (explanation) and year of study	-0,169	**0,008**
Percentage C ≥ 50 and year of study	-0,130	**0,044**

Table 5.10

Correlation nature and year of study	Beta	Sig.
Percentage C and study year 1	0,128	**0,046**
Percentage C ≥ 50 and study year 1	0,079	0,226
Percentage C and study year 2	0,070	0,273
Percentage C ≥ 50 and study year 2	0,086	0,186
Percentage C and study year 3	-0,157	**0,013**
Percentage C ≥ 50 and study year 3	-0,136	**0,036**
Percentage D ('responding to') and year of study	0,105	0,101
Percentage D ≥ 50 and year of study	0,068	0,292

In summary we can say that the characteristics for the nature of theory use that were assumed in hypothesis 1.2 mainly occur for factual description (category A) and explaining (category C). This happens particularly for students with a prior education of *mbo without mathematics* (more factual description, less explaining) and students with *vwo with mathematics* (more explaining).
As far as the variable *year of study* – and particularly for years 1 and 3 – the results (table 5.10) confirm the characteristics that were mentioned for the category *explaining*, keeping in mind the specific composition of the student population (first year mainly vwo with mathematics, many mbo students without mathematics in the third year).

Data analysis and results hypothesis 1.3
Linear regression analysis shows a clear confirmation of hypothesis 1.3 for *explaining* teaching situations (table 5.11). A significant positive correlation appears between the percentage C – and percentage C ≥ 50 as well – and the number of theoretical concepts. The significant *negative* correlation between *factual description* – and to a lesser degree *interpreting* – and the number of theoretical concepts, can be seen as additional support for that confirmation.
No significant relationship has been shown for *responding to* situations. Perhaps the learning environment has been an influence to the extent of responding to situations shown by students, although it seems unlikely that this influence is dominant, since the meetings and the individual study materials did not just give attention to explaining, but also to responding to situations. It is also possible that responding to situations is a habitual action, something that appears to require no theory.
Also remarkable are the strong correlation between *explaining* (both percentage C and percentage C ≥ 50) and the amount of *general pedagogic* concepts used, and the absence of any correlation between the nature of theory use and the amount of *pedagogic content* concepts. There was an expectation of a difference in use between general pedagogic and pedagogic content concepts, but not this large. This point

requires closer analysis. Perhaps the level of theory use plays a part; we will look into this in research question 2.
We see another 'opposite' in the significant negative correlation between *factual description* and the number of *general pedagogic* concepts used, as well as a negative trend in relation to the correlation between the percentage *interpreting* and the amount of theoretical concepts.

In connection with the outcomes of hypothesis 1.2 (table 5.9), it is to be expected that mbo students without mathematics will use fewer concepts and vwo students with mathematics will use more. Linear regression analysis does in fact show a significant negative correlation between students who have mbo without mathematics as their prior education and the number of *general pedagogic* concepts used (table 5.12). No correlation exists between *vwo with mathematics* and the number of *general pedagogic* concepts.

Table 5.11

Correlation nature of theory use and number of used concepts	**Beta**	**Sig.**
Percentage A (factual description) and number of theoretical concepts	-0,197	**0,002**
Percentage A ≥ 50 and number of theoretical concepts	-0,197	**0,002**
Percentage A and number of *general pedagogical* concepts	-0,256	**0,000**
Percentage A ≥ 50 and number of *general pedagogical* concepts	-0,217	**0,001**
Percentage A and number of *pedagogical content* concepts	-0,050	0,435
Percentage A ≥ 50 and number of *pedagogical content* concepts	-0,101	0,118
Percentage B (interpretation) and number of theoretical concepts	-0,130	**0,041**
Percentage B ≥ 50 and number of theoretical concepts	-0,101	0,120
Percentage B and number of *general pedagogical* concepts	-0,107	0,094
Percentage B ≥ 50 and number of *general pedagogical* concepts	-0,169	0,287
Percentage B and number of *pedagogical content* concepts	-0,116	0,070
Percentage B ≥ 50 and number of *pedagogical content* concepts	-0,106	0,101
Percentage C (explanation) and number of theoretical concepts	0,218	**0,001**
Percentage C ≥ 50 and number of theoretical concepts	0,159	**0,014**
Percentage C and number of *general pedagogical* concepts	0,262	**0,000**
Percentage C ≥ 50 and number of *general pedagogical* concepts	0,209	**0,001**
Percentage C and number of *pedagogical content* concepts	0,086	0,181
Percentage C ≥ 50 and number of *pedagogical content* concepts	0,039	0,552
Percentage D ('responding to') and number of theoretical concepts	0,054	0,400
Percentage D ≥ 50 and number of theoretical concepts	0,003	0,961

The final result can be explained as follows. The group of students with vwo-with mathematics as their prior education mainly consists of first-year students who have an

as yet undeveloped pedagogical (content) jargon. Furthermore, they have not yet gained much experience in reasoning about teaching situations, which is expressed in the fact that these students *explain* less (see table 5.9) than might be expected on the basis of their prior education.

Another remarkable point is the significant *negative* correlation between students with prior education mbo *without* mathematics and the used number of general pedagogical concept in opposition to the significant *positive* correlation between students with mbo *with* mathematics as their prior education and the number of general pedagogical concepts used (table 5.12). It puts the group of students with mbo *without* mathematics in a special light.

Table 5.12

Correlation pre-education and number of concepts used	**Beta**	**Sig.**
MBO without mathematics and number of *general pedagogical* concepts	-0,142	**0,026**
MBO with mathematics and number of *general pedagogical* concepts	0,154	**0,016**
VWO with mathematics and number of theoretical concepts	0,051	0,424

5.4.3 Analysis and results of the second research question

The second research question is:

> What is the theoretical quality of statements made by the student teachers when they describe practical situations?

Considerations for research question 2

As far as the level of the average percentages scored by the students for the levels of use of theory, it is impossible to give a considered opinion. At most it seems reasonable to predict that the average percentage for level 3 will be the lowest, simply because level 3 is the hardest to reach, highest level.

Concerning the use of theoretical concepts, it is obvious to assume that there is a relationship, both in the initial and in the final assessment, between the level of theory use and the number of concepts that is applied. For both the initial and the final assessment, for the teaching situations, respectively the units, the levels of theory use have been determined on the basis of a definition where theoretical concepts are the determining factor for the level (section 5.3.6.4). The more concepts are used, the higher the chance to score level 3 and vice versa. It might also be the case that students who possess more theoretical knowledge, are challenged to higher cognitive activities or perhaps the difference in levels comes from a potentially present difference in cognitive capacity between students. Being able to (re)construct a meaningful relationship between theoretical concepts, this is use of theory at level 3, demands, along with possession of a pedagogical (content) repertoire, the ability and the experience to adequately use or refine the cognitive network.

Finally, the random sample has also shown that there usually is a meaningful relationship between concepts in units where two or more theoretical concepts occur. Our conclusion is that use of theory at level 3 will be achieved more often by students with a higher cognitive level (read: higher prior education) or students from a later study year.
Furthermore, we expect the relationship between the number of concepts and the level to manifest stronger in the final assessment than in the initial one, as the students have had the opportunity in between, that is to say within the learning environment of the course, to expand their repertoire.
The above considerations lead to two hypotheses regarding the level of theory use.

Hypothesis 2.1
Students who use more theoretical concepts reflect at a higher level and vice versa. This will be more strongly expressed in the final assessment than in the initial one.

Hypothesis 2.2
The first level of theory use will mainly be found in first year students or in students with a lower level of prior education, while level 3 will mainly manifest in third or second year students or in students with a higher level of prior education.

Data analysis and results hypothesis 2.1
Descriptive analysis of the levels reveals that the average percentages of the three levels are not far apart, with an average of 35%, 29% and 36% for respectively levels 1, 2 and 3 (see table 5.13).

Table 5.13 Statistics mean percentages for the level of theory use

		percentage level 1	**percentage level 2**	**percentage level 3**
N	Valid	246	246	246
	Missing	23	23	23
Mean percentage		34,64	29,45	35,92
Std. deviation		24,951	17,950	25,575

The average percentage for level 3 is higher than expected. As was assumed earlier, that higher percentage may have been caused by the relatively large number of second and third year students or the percentage of the student population with a relatively ‘high’ prior education.
For the same reasons as with the categories for the nature of theory use, here too we will look at the groups of students for whom the level of theory use is relatively often aimed at one specific level. For that purpose we define the concept of ‘level dominance’ as the level of theory use where the student scores at least 50% of the total number of units on the level that occurs most. The five students who scored just 50% on two levels are left out of consideration (table 5.14).

Table 5.14 Students with two 50% scores for the level of theory use

	number	student nr.
Level 1 = level 2 = 50%	1	121
Level 1 = level 3 = 50%	2	46, 159
Level 2 = level 3 = 50%	2	127, 186
Total	5	

Data selection and frequency analysis give the following view of the percentages and numbers of students (table 5.15).

Table 5.15 Mean percentages levels 1, 2, 3 ≥ 50%

		Perclevel 1 ≥ 50 (FILTER)	Perclevel 2 ≥ 50 (FILTER)	Perclevel 3 ≥ 50 (FILTER)
N	Valid	241	241	241
	Missing	23	23	23
Percentage of population		30,3	15,8	30,3
Number of student teachers		73	38	73

Remarkable are the – coincidentally identical – relatively high percentages for levels 1 and 3. Linear regression analysis confirms hypothesis 2.1 on several points (see tables 5.16a and 5.16b). First, the analysis reveals that there is in fact a significant positive correlation between level 3 of theory use and the number of theoretical concepts used, both for percentage level 3 as for percentage level 3 ≥ 50%. Moreover, a significant positive correlation is valid for the number of *different* theoretical concepts used in the initial and final assessments.

Table 5.16a

Correlation level 3 and number of concepts in the initial assessment	Beta	Sig.
Percentage level 3 and number of theoretical concepts	0,204	**0,002**
Percentage level 3 ≥ 50 and number of theoretical concepts	0,171	**0,011**
Percentage level 3 and number of *pedagogical content* concepts	0,126	0,061
Percentage level 3 ≥ 50 and number of *pedagogical content* concepts	0,062	0,352
Percentage level 3 and number of *general pedagogical* concepts	0,220	**0,001**
Percentage level 3 ≥ 50 and number of *general pedagogical* concepts	0,205	**0,002**
Percentage level 3 and number of *different* theoretical concepts	0,222	**0,001**
Percentage level 3 ≥ 50 and number of *different* theoretical concepts	0,147	**0,029**
Percentage level 3 and number of *different pedagogical content* concepts	0,085	0,205
Percentage level 3 ≥ 50 and number of *different pedagogical content* concepts	-0,002	0,980
Percentage level 3 and number of *different general pedagogical* concepts	0,264	**0,001**
Percentage level 3 ≥ 50 and number of *different general pedagogical* concepts	0,226	**0,001**

Table 5.16b

Correlation level 3 and number of concepts in the final assessment	**Beta**	**Sig.**
Percentage level 3 and number of theoretical concepts	0,820	**0,000**
Percentage level 3 ≥ 50 and number of theoretical concepts	0,681	**0,000**
Percentage level 3 and number of *pedagogical content* concepts	0,661	**0,000**
Percentage level 3 ≥ 50 and number of *pedagogical content* concepts	0,524	**0,000**
Percentage level 3 and number of *general pedagogical* concepts	0,720	**0,000**
Percentage level 3 ≥ 50 and number of *general pedagogical* concepts	0,618	**0,000**
Percentage level 3 and number of *different* theoretical concepts	0,755	**0,000**
Percentage level 3 ≥ 50 and number of *different* theoretical concepts	0,611	**0,000**
Percentage level 3 and number of *different pedagogical content* concepts	0,587	**0,000**
Percentage level 3 ≥ 50 and number of *different pedagogical content* concepts	0,436	**0,000**
Percentage level 3 and number of *different general pedagogical* concepts	0,685	**0,000**
Percentage level 3 ≥ 50 and number of *different general pedagogical* concepts	0,575	**0,000**

As expected the correlation is stronger in the final than in the initial assessment.
A remarkable difference can be found between the *general pedagogical* concepts and the *pedagogical content* concepts. There is a significant positive correlation between the number of general pedagogical concepts and level 3 of the initial assessment, and that correlation is stronger in the final assessment for this category too. However, for the number of pedagogical content concepts and level 3 there is only a significant (strong) correlation in the *final assessment*, both for the percentage level 3 and the percentage level 3 ≥ 50%. It can be taken as a given that the learning environment of the course is the cause of this correlation. It is true in all cases that the significant correlation with the number of general pedagogical concepts is stronger than that with the number of pedagogical content concepts.
Within the framework of hypothesis 1.3 we discussed the strong correlation between *explaining* (both percentage C as percentage C ≥ 50) and the number of *general pedagogical* concepts used and the absence of any correlation between the nature of theory use and the number of pedagogical content concepts used (table 5.11). That there is a positive correlation between level 3 and the number of pedagogical content concepts, albeit not as strong as with the number of general pedagogical concepts, may be linked to the more dependant relationship between the number of concepts and the level definition than is the case for the number of concepts and explaining. Also, the content-related differences and the difference in the range of general pedagogical and pedagogical content concepts have been highlighted before (hypothesis 1.3).
We can find one more confirmation of hypothesis 2.1 in the outcome of the linear regression analysis in relation to the correlations between level 1 and the number of concepts used.

These correlations with respect to the final assessment are almost a mirror image of those for level 3, that is to say, virtually the same strength, but in a significant negative direction.
For the initial assessment the (negative) correlation is weaker in the absolute sense, moreover – in contrast to the initial assessment of level 3 – there is a significant correlation between percentage level 1 ≥ 50% and the number of *different pedagogical content* concepts. That correlation is negative, like the other numbers of concepts related to level 1.

Data-analysis and results hypothesis 2.2
Linear regression analysis shows that study year one has a significant positive correlation with percentage level 1 and significantly negative with percentage level 3 (table 5.17a and 5.17b). So this is a confirmation of the hypothesis, though with the following comment. Taking into account that the first year students are all in the three-year programme for vwo students, it apparently means that here the study year (experience, available knowledge repertoire) and not prior education (for instance cognitive ability), is the determining factor. Apart from that, there is *no high* correlation between study year one and level 1, which may be linked to the relatively high prior education. A weak correlation has been found between *prior education* and percentage level 1 and percentage level 1 ≥ 50% (table 5.18a).
Study year two has a significant negative correlation with level 1 and a significant positive one with level 3, another confirmation of the hypothesis (table 5.17a and 5.17b).
There is no significant correlation between study year 3 and percentage level 1 and a negative correlation between study year 3 and percentage level 3 ≥ 50% (table 5.17a and 5.17b). So study year 3 is out of line with hypothesis 2.2. An explanation is the relatively high percentage of students with mbo without mathematics as their prior education and that group scores significantly negative on percentage level 3 and percentage level 3 ≥ 50% (table 5.18b).
Noteworthy is the significant *positive* correlation between percentage level 3 and prior education mbo *with* mathematics (table 5.18b).
The positive correlation between level 3 and prior education vwo (table 5.18b) also confirms hypothesis 2.2, but the correlation is lower than expected for the same reason mentioned for level 1, namely that the vwo students are for the larger part first year students.

Table 5.17a

Correlation level 1 and study year	**Beta**	**Sig.**
Percentage level 1 and study year 1	0,153	**0,017**
Percentage level 1 ≥ 50 and study year 1	0,170	**0,008**
Percentage level 1 and study year 2	-0,187	**0,003**
Percentage level 1 ≥ 50 and study year 2	-0,159	**0,013**
Percentage level 1 and study year 3	0,062	0,329
Percentage level 1 ≥ 50 and study year 3	0,034	0,601

Table 5.17b

Correlation level 3 and study year	Beta	Sig.
Percentage level 3 and study year 1	-0,124	**0,053**
Percentage level 3 ≥ 50 and study year 1	-0,078	0,220
Percentage level 3 and study year 2	0,221	**0,000**
Percentage level 3 ≥ 50 and study year 2	0,253	**0,000**
Percentage level 3 and study year 3	-0,115	0,072
Percentage level 3 ≥ 50 and study year 3	-0,141	**0,027**

Table 5.18a

Correlation level 1 and prior education	Beta	Sig.
Percentage level 1 and mbo without mathematics	0,094	0,143
Percentage level 1 ≥ 50 and mbo without mathematics	0,047	0,468
Percentage level 1 and mbo with mathematics	-0,137	**0,032**
Percentage level 1 ≥ 50 and mbo with mathematics	-0,132	**0,040**
Percentage level 1 and havo without mathematics	0,068	0,287
Percentage level 1 ≥ 50 and havo without mathematics	0,035	0,588
Percentage level 1 and havo with mathematics	0,038	0,552
Percentage level 1 ≥ 50 and havo with mathematics	0,029	0,652
Percentage level 1 and vwo without mathematics	-0,136	**0,033**
Percentag level 1 ≥ 50 and vwo without mathematics	-0,078	0,222
Percentage level 1 and vwo with mathematics	-0,032	0,615
Percentage level 1 ≥ 50 and vwo with mathematics	0,013	0,838

Table 5.18b

Correlation level 3 and prior education	Beta	Sig.
Percentage level 3 and mbo without mathematics	-0,145	**0,023**
Percentage level 3 ≥ 50 and mbo without mathematics	-0,133	**0,039**
Percentage level 3 and mbo with mathematics	0,159	**0,013**
Percentage level 3 ≥ 50 and mbo with mathematics	0,167	**0,010**
Percentage level 3 and havo without mathematics	-0063	0,322
Percentage level 3 ≥ 50 and havo without mathematics	-0,119	0,062
Percentage level 3 and havo with mathematics	-0,021	0,745
Percentage level 3 ≥ 50 and havo with mathematics	-0,006	0,931
Percentage level 3 and vwo without mathematics	0,143	**0,024**
Percentag level 3 ≥ 50 and vwo without mathematics	0,056	0,384
Percentage level 3 and vwo with mathematics	0,074	0,245
Percentage level 3 ≥ 50 and vwo with mathematics	0,127	**0,047**

5.4.4 Analysis and results of the third research question

The third research question is:

> 3a. Is there a meaningful relationship between the nature and the level of theory use? If so, how is that relationship expressed in the various components of theory use and in various groups of students?
>
> 3b. To what extent is there a relationship between the nature or the level of the student teachers' use of theory and their level of numeracy?

Considerations for research question 3a.

There are reasons to assume that there is a relationship between the nature and the level of theory use.

First, there are signals from the small scale study that students who function at a relatively high level do more often tend to explain and respond, and on the other hand factual description and interpreting mostly go with a lower level of theory use.

Also, the analyses resulting from the first and second research questions have shown that the differences in the size of students' theoretical repertoire are related to differences in nature and level of theory use. Factual description, interpreting, the first level, and to some degree the second level, have a negative correlation with the number of theoretical concepts used, while explaining and level 3 both correlate positively with the number of theoretical concepts used. Additionally factual description and interpretation are related to a lower level of prior education, particularly mbo without mathematics, while explaining correlates with a higher level of education, particularly vwo with mathematics. Finally it is known from literature that teachers who have less content knowledge are more oriented on facts and procedures, while teachers who possess a larger repertoire look for conceptual and problem solving aspects more (Putnam & Borko, 1997, p. 1232 and 1233).

Combining the above considerations leads to the formulation of hypothesis 3.1.

Hypothesis 3.1

The characteristics of factual description and interpreting for the nature of theory use mainly occur on the first and second level of theory use, while explaining and – to a lesser degree – responding to situations are related mainly to the third level of theory use.

Data analysis and results hypothesis 3.1

Table 5.19 gives an overview of the regression coefficients and the accompanying significances of the correlation between nature and level of theory use by students.

Looking at *factual description (category A)*, we see that the regression coefficient beta = 0,129 (sig. 0,043) for level 1 and beta = -0,230 (sig. 0,000) for level 3. *Interpreting (category B)* shows a similar picture for beta and the related significance. The reverse is

the case for *explaining (category C)*. Beta is negative for level 1 (sig. 0,001) and positive for level 3 (sig. 0,000). Barely any system can be found for *category D ('responding to')*.

Table 5.19: Correlation between nature and level of theory use

	A Factual description	B Interpreting	C Explaining	D Respond to
Level 1	**A1** Sig. 0,043 Beta 0,129	**B1** Sig. 0,096 Beta 0,106	**C1** Sig. 0,001 Beta -0,214	**D1** Sig. 0,506 Beta 0,043
Level 2	**A2** Sig. 0,020 Beta 0,149	**B2** Sig. 0,015 Beta 0,155	**C2** Sig. 0,105 Beta -0,104	**D2** Sig. 0,007 Beta -0,173
Level 3	**A3** Sig. 0,000 Beta -0,230	**B3** Sig. 0,001 Beta -0,212	**C3** Sig. 0,000 Beta 0,282	**D3** Sig. 0,212 Beta 0,080

So the linear regression analysis gives a clear confirmation of hypothesis 3.1, with the exception of *category D (responding to situations)* which deviates more than expected. In view of the inclusion relationship it would be logical for D to have a correlation that is similar to C.

Some explanations can be provided for the deviation from the expected result for category D.

First, the learning environment may have played a part. However, amply attention has been given in the common activities and in the expert notes (e.g., The Guide) to the aspects of responding to situations. Therefore, that does not seem to be the most logical explanation, although it is unclear to what degree for example individual learning styles have played a part (Vermunt, 1992). The tendency to take creative initiatives or to spontaneously develop metacognitive activities mostly suits an open, meaning-oriented learning style (Oosterheert, 2001) and not many students have developed that learning style. It is also not clear what earlier experiences by students in relation to 'responding to situations' have played a part.

Second, it can be questioned whether the definition of category D in the analysis instrument had been phrased sufficiently unequivocally. Although the random sample did not show problems with that definition, the number of explainers (Cat. C), including at high levels, is remarkable, particularly as there are few 'responders' at a high level. It might be the case that in category D the inclusion relationship is insufficiently expressed or is not made explicit enough in the definition. For example, the definition refers to a metacognitive component. Under that header, D is scored among other things when a student asks himself a question. In such a case there is however often no

obvious evidence of an inclusion relationship between C and D. Furthermore, the use of theoretical concepts in that kind of reflection is less obvious, and therefore the chance of scoring the highest level is small.
In summary we can come to a second explanation that the analysis instrument may not be optimal for category D, and the definition for 'responding to' situations may need to be adjusted.
A third explanation for the deviation from the expected result for category D is the character of the student population that was studied. The students with 'vwo with mathematics' as their prior education are concentrated in the first study year, while students who have done 'mbo without mathematics' are mainly third year students. Taking into account the analyses of the first and second research questions, it is well possible that this unbalanced spread across the study years results in a different description of category D than might have been expected based on the influences of prior education and study year.

Table 5.20 represents the average percentages that the students scored per category. The twelve average percentages confirm hypothesis 3.1 in still another way. We see for instance that A3 + B3 = 8%, while C3 + D3 = 27%.

Table 5.20 Mean percentages categories A1 to D3

Category	**A1**	**A2**	**A3**	**B1**	**B2**	**B3**	**C1**	**C2**	**C3**	**D1**	**D2**	**D3**
Valid	246	246	246	246	246	246	246	246	246	246	246	246
Missing	23	23	23	23	23	23	23	23	23	23	23	23
Mean perc.	12	8	5	5	4	3	12	12	18	7	5	9
Std. deviation	16	13	10	10	9	7	16	14	20	12	9	13

This means that the third level of theory use mainly occurs in explaining teaching situations and responding to situations, and that factual description and interpretation hardly occur at that level.

Considerations for research question 3b.
Question 3b is formulated as follows: *To what extent is there a relationship between the nature or the level of the student teachers' use of theory and their level of numeracy*?
It has been proposed earlier (section 5.2) that the development of numeracy in the education of primary school teachers in training is intertwined with the development of pedagogical insights and skills (Goffree & Dolk, 1995). The growth in the development of numeracy is seen by Oonk, Van Zanten & Keijzer (2007) as an amalgam of four components, namely the acquisition of elementary arithmetical skills, recognizing mathematics in one's own environment, being focused on solution processes in solving mathematical problems, and responding to pupils' solution processes. Along the way mathematizing is entwined with didacticizing. The pedagogical content aspect of

numeracy will develop fully in the later years of teacher training. This matches the fact that students in later years have access to a larger pedagogical (content) repertoire than students in earlier years.

Possessing solid content knowledge is seen in educational circles as an undisputed quality of teachers. Popular wisdom also subscribes to that necessary quality of a teacher: "If you cannot do math well yourself, how can you teach someone else to do it?" Research among secondary school teachers shows that lack of good understanding of the core concepts of one's own subject can lead to misconceptions about those core concepts in students (Putnam & Borko, 1997; Van Driel & Verloop, 1999). We have already mentioned before (section 5.4.4) the conclusions of Putnam & Borko (1997) in relation to the connection between content knowledge and the orientation – 'the level' – of a teacher's actions. Mandeville (1997) also found, in a large scale study of 9000 students, a positive correlation between the content knowledge of mathematics teachers and the performance of students, with here too the differences occurring mainly in relation to students' higher order skills.

As far as we know, no studies have been done into this kind of phenomenon in teachers (in training) in primary education. Research has been done into the level of the content skills of Pabo students. In the 1980s the low mathematical skill level of Pabo students was being linked to the mechanistic and insufficiently insight-based mathematics teaching these students themselves had received in primary school (Jacobs, 1986). Recent research into mathematics as a subject shows that it is mainly students with mbo as their prior education who score lower. Their content knowledge is deficient, while students with havo – senior general secondary education – as prior education who do less well, have a 'maintenance problem' (Straetmans & Eggen, 2005; Meijer, Vermeulen-Kerstens, Schellings & Van der Meijden, 2006).

Someone with a large amount of proficiency – or even numeracy – for mathematics, is likely to function at a relatively high level of reasoning. In view of the nature of theory use, for that reason a positive correlation between *explaining* and numeracy is to be expected. In terms of the inclusion relationship that correlation should be present also for '*responding to*,' although that conclusion is not obvious after the results of the previous analyses of category D.

Taking into account the positive connection that was found earlier between explaining, level 3 of theory use and the number of theoretical concepts used, it is plausible that there will be a positive correlation between level 3 or the number of theoretical concepts and the variable numeracy as well.

Based on these considerations for research question 3b, we formulated hypothesis 3.2.

Hypothesis 3.2

There is a positive correlation between the level of numeracy and these variables:

- nature of theory use 'explaining,'
- the highest, third level of theory use,
- the number of theoretical concepts used, and
- students' prior education.

Data analysis and results hypothesis 3.2

The very first thing we note is that linear regression analysis confirms the results of recent studies into the relationship between Pabo students' own proficiency and their prior education (table 5.21). Particularly strong relationships are the significant negative correlation between numeracy and mbo without mathematics and the significant positive correlation between numeracy and vwo with mathematics. Although not unexpected, these results are remarkable when it is taken into account that the mbo students in this population are mainly third year students and the vwo students are mainly first year students. A negative correlation has also been found between the personal evaluation index (PEI) and students' prior education (Beta –0,155; Sig. 0,034). This may indicate increasing reticence about assessing one's own level of numeracy as the level of prior education increases.

Table 5.21

Correlation numeracy and prior education	**Beta**	**Sig.**
Numeracy and mbo without mathematics	-0,342	**0,000**
Numeracy and mbo with mathematics	-0,014	0,826
Numeracy and havo without mathematics	-0,117	0,069
Numeracy and havo with mathematics	0,180	**0,005**
Numeracy and vwo without mathematics	-0,035	0,587
Numeracy and vwo with mathematics	0,323	**0,000**

The hypothesis is also confirmed for the nature of theory use '*explaining*' (table 5.22). The other categories relating to the nature of theory use do not show a significant correlation, including, as expected, for 'responding to situations.'

There is a positive trend between level 3 and numeracy. That the correlation with level 3 is less strong than that with explaining can be understood from the relationship between explaining and 'problem solving,' while the relationship between numeracy and level 3 is less obvious.

The negative trend between level 1 and numeracy (table 5.22) is virtually mirrored with level 3, and also supports the hypothesis.

The number of theoretical concepts correlates significantly positive with numeracy.

This is the case particularly for the *number of general pedagogical* concepts, and *not* for the number of *pedagogical content* concepts.

Table 5.22

Correlation numeracy, nature and level	Beta	Sig.
Numeracy and percentage C (explanation)	0,202	**0,003**
Numeracy and percentage C ≥ 50	0,201	**0,003**
Numeracy and level 3	0,135	**0,046**
Numeracy and level 3 ≥ 50	0,133	**0,050**
Numeracy and percentage C3	0,235	**0,000**
Numeracy and level 1	-0,136	**0,044**
Numeracy and level 1 ≥ 50	-0,129	0,056
Numeracy and number of theoretical concepts final assessment	0,166	**0,014**
Numeracy and general pedagogical concepts final assessment	0,175	**0,010**

Within the framework of hypothesis 1.3 and 2.1, the differences between the number of *general pedagogical* and *pedagogical content* concepts used have already been pointed out.

5.4.5 The role of the teacher educator

Section 3.5.5 and 3.9.1 indicate the important role of the teacher educator in making the connection between theory and practice by student teachers. In this study, the educator's activities stretch from 'feeding' the discourse with theory to giving lectures and coaching students in preparing and performing assignments in their teaching practice (see section 5.3.3).

In all their interviews after the end of the study, the educators showed themselves positive about content and work forms on offer in the course. Some referred to the increase in workload[46] that teaching the course and collecting the data brought with them. Two educators experienced organisational problems from fitting in the course, partly in connection with the need for using a computer room. One Pabo had to pull out halfway through the study as a result of the faculty moving.

Recently, the majority of the educators who participated in the study indicated, when asked, that the course as a whole, parts of it or ideas from it, had been included in their Pabo's curriculum, sometimes even for other subjects. One educator published an article in a specialist journal about her experiences with the study and the inspiration gained from it (Terlouw, 2005).

> (…) For me as a fellow researcher it was good to see that when discussions became heated, people would search feverishly for a common language to express themselves in. They were looking for the jargon to use. I noticed that they wanted to

> use that language to convince others of their point of view. In that way many strategies and models were referred to by name, and they suddenly recalled the basis of the reconstruction pedagogics. From the dialogue arose a need to learn. That need to learn also arose once the students started doing research in their own teaching practice. They found out they did not yet know enough about certain aspects of the didactics of multiplication and had grown curious. It was really only then that the learning questions gained in eloquence (Terlouw, 2005, p. 23).

She seemed to have undergone a 'conceptual change' in relation to her opinions about making connections between theory and practice by students. She considered the discourse led by the educator and the research in students' teaching practice as core elements in the learning environment; these, according to her, were the activities that evoked a need to learn in the students. The natural desire to learn did, for instance, cause spontaneous questions about background literature from students.

5.5 Conclusion of the large scale study

This study involved the following questions:

1. In what way do student teachers use theoretical knowledge, when they describe practical situations after spending a period in a learning environment that invites the use of theory?
2. What is the theoretical quality of statements made by the student teachers when they describe practical situations?
3. a. Is there a meaningful relationship between the nature and the level of theory use? If so, how is that relationship expressed in the various components of theory use and in various groups of students?
   b. To what extent is there a relationship between the nature or the level of the student teachers' use of theory and their level of numeracy?

The study was done among 269 students at eleven primary teacher training colleges in the Netherlands. According to the procedure, described in paragraph 5.3.5, the results were scored for nature and level of theory use, for the number of theoretical concepts used and for the students' level of numeracy. To determine the so-called meaningful units in which the reflective notes of the final assessment of the students were divided, for the nature and level of theory use in the notes and for assessing the numeracy tests the interrater reliability was determined.

First, the study shows that each meaningful unit described by a student, on average seven per student, can be interpreted using one of the four characteristics for *nature*, and one of three characteristics for *level* of theory use.

Nearly all students (98,5% of the population) use theory in their final reflection, on average 12 theoretical concepts per student. From the average percentages scored by the

students for each of the four categories for the nature of theory use (respectively 25, 12, 42 and 21 procent), it turns out that 'explaining' occurs most. Remarkable is that nearly 80% of the students dominates in one of the four categories (190 of the 239 students), that is to say that at least 50% of the total number of units is scored in one of these categories. It may be possible to explain that result from the differences in learning or writing style between students. The average percentages of the three levels (respectively 35, 29 and 36 procent) are close together. Just like the nature of theory use, for the level of theory use is a high percentage (76%) of the student population dominating on one of the levels.
38% of the students start their reflective note with a *factual description* of a teaching situation, with half of that group also scoring A for the second meaningful unit.
It turns out that the higher their prior education, the less student teachers use *factual description*. It is the case particularly that students who have *mbo (senior secondary vocational education) without mathematics* as their prior education use *factual description* more and *explain* less, while students who have done *vwo (pre-university education) with mathematics explain* significantly more. This may well be related to the fact that *factual description* calls less upon cognitive abilities than *explaining*. Moreover, 'explainers' use more theoretical concepts, while students use less theoretical concepts the more they use factual description or interpreting. What stands out here is the strong positive correlation between *explaining* and the number of *general pedagogical* concepts, and the absence of any correlation between – all four of the categories of – the nature of theory use and the number of *pedagogical content* concepts used (table 5.11). However, a positive correlation appears between level 3 of theory use and the number of pedagogical content concepts used, though not as strong as with the number of general pedagogical concepts. That positive correlation may be connected to the to a certain extent dependent relationship between the number of concepts and the *definition* of the levels. Also, content-related differences and the difference in reach between general pedagogical and pedagogical content concepts may play a part. Students tend to initially respond in general terms to teaching situations. This is understandable, since the general pedagogical jargon is aimed more at the whole of the pedagogical-didactical actions of teacher and students, and is also used more frequently in teacher training and practice.
As a matter of fact students who have mbo without mathematics as their prior education significantly use relatively few *general pedagogical* concepts, and the students with mbo *with* mathematics use these concepts significantly more. No correlation has been found between vwo with mathematics as prior education and the number of *general pedagogical* concepts. The group with that specific prior education consists mainly of first year students, and these have not yet sufficiently developed their pedagogical (content) jargon. Furthermore, they have not yet gained enough experience in reasoning about teaching situations.

The lack of correlation with level 3 for that group fits with the image of their as yet underdeveloped theoretical knowledge network.
As far as the correlation between the level of theory use and the use of theoretical jargon is concerned, it is the case for all (six) categories, that the more theoretical concepts students use, the hight the level they reflect at, and vice versa. This is expressed more strongly in the final assessment than in the initial assessment, and is the case particularly for the number of *pedagogical content* concepts used, which, as mentioned before, has a significant positive correlation with level 3 only in the final assessment. Possibly that difference between the initial and the final assessment can be ascribed to the influence of the learning environment.
According to the correlation between study year and the level of theory use, it appears that first year students mainly function at the first level. The second year students function more at the third level. The third year does not match up with the third level; there is even a negative correlation for level 3 ≥ 50 (table 5.17b). An explanation is the relatively high percentage of third year students who have mbo without mathematics as their prior education, which is a group that correlates significantly negative with the third level of theory use (table 5.18b). This is clearly not true for the group of students with mbo *with* mathematics (also later year students), who in fact *correlate positively* with percentage level 3 (table 5.18b).
The correlation between the two characteristics *factual description* and *interpreting* for the *nature* of theory use, and the third *level* of theory use is significantly negative. Factual description and interpreting mainly occur at the *first and second levels* of theory use and hardly at the third. The third level of theory use is mainly related to *explaining* teaching situations and *responding to* situations. The connection between the category '*responding to*' and the levels 1 to 3 resembles the relationship between the category *explaining* and the levels 1 to 3 less in its structure – from significantly negative to significantly positive – than one might expect based on the inclusion relationship. There are three explanations for that anomaly. In the first place the character of the learning environment, individual learning styles or previous experiences may play a part. Second, there is some doubt as to whether the definition in the analysis tool of category D for '*responding to*' situations has been optimally formulated. A third explanation for the deviation from the expected result for category D is the character of the student population that was studied.

Section 2.6.5.5 introduced the concept 'theory-enriched practical knowledge' (EPK) as a derivation of the concept of 'practical knowledge,' where level 3 of theory use is seen in this study as the most important indicator for theoretical enrichment of practical knowledge (section 5.3.6.4). We can therefore draw the conclusion that on average the students (re)constructed over a third of their reflections using 'theory-enriched practical knowledge.'

In relation to the level of numeracy we can draw the following conclusions. There is a positive relationship between the level of numeracy and the nature of theory use '*explaining,'* as well as between the level of numeracy and the third level of theory use. The correlation with *level 3* is not as strong as that with *explaining*. This may be connected to the relationship between explaining and 'problem solving.'
A significant positive correlation has also been found with the number of theoretical concepts used. The higher the level of numeracy, the more theoretical concepts students use. This is the case particularly for the number of *general pedagogical* concepts
The relationships that have been found between the level of numeracy and students' prior education fall within expectations, with students who had mathematics in their prior education standing out positively.
What is remarkable here, as for the results of theory use by students, is the weak position of the – generally third year – student teachers who have done mbo *without* mathematics as prior education. A further point that stands out is that this group with the lowest average score, scores the second highest PEI-value (read: selfconfidence).

The (anonymous) questionnaire was administered to 257 students.
Even more clearly than the data from the questionnaire in the small scale study (appendix 13), the data from the large scale study show that the students appreciate particularly the theory on offer, integrated into practical situations, as support for their practice (appendix 14). This is especially true for question 14 ('theory and practice are far removed from each other/are integrated'), with the highest average score on the five point scale of the 14 questions (4,22).
Another noticeable difference is that the average score of question 7 ('the course is dull/challenging'; mean 3,31) and question 8 ('the course is vague/concrete'; mean 3,47) is higher than the score of the same questions in the small scale study. A possible explanation for this is that the students appreciated the more structured learning environment of the large scale study.
The students had the option to clarify their score for each question in the questionnaire. Two typical examples of student statements are given below.

> *Question 14. Theory and practice are far removed from each other/are integrated.*
>
> Student 93. I recognize everything we learned in theory in my practice school now.
>
> Student 170. When you know a bit more of the theory you can understand better that children use different calculations than you think.

Generally speaking, the output of the questionnaire can be interpreted as an appreciation by the students of both theory and the integrated approach in the theory and practice on offer in the course.

6 General conclusion and discussion

6.1 Introduction

The purpose of the present study was to gain insight in the student teachers' process of integrating theory and practice, and particularly to find out *how* they are relating theory and practice and *to what extent* they are competent to use theoretical knowledge in multimedia education situations.
Motivation for the study was the still opaque and unresolved theory-practice problem in teacher education. There is still little known in the research area of teacher education about how student teachers link theoretical knowledge and practical situations. The question of how the integration of several elements of the knowledge base of (prospective) teachers can be realized, and in particular can be fostered in student teachers (chapter 2) is essential for this. A second reason for this study, related to this problem, was the development in the field of multimedia learning environments that started at the end of the nineties, particularly for the subject of mathematics education (chapter 3). This development appeared to offer an opportunity to focus student teachers on their own professional development in a natural way, particularly where learning to integrate theory and practice was involved.
This study was performed in such a – gradually more and more adapted – multimedia learning environment for student teachers.
In brief, the complete study can be considered as a chain of four links, two exploratory studies, a small scale study and a large scale study, with each of them having its own function. Every time the output of each link provided the material for the next study, with more refined questions and a better adapted design of the learning environment for the participating student teachers.
The main conclusion of this study is that 98,5% of the student teachers is able to relate theory and practice in the context of the learning environment offered. However, students differ strongly in *the way* in which they link theory and practice and in *the depth* to which they use theoretical concepts in their reflections on practice.
The instrument that has been developed in this study offered the opportunity to perform a systematic and nuanced analysis of the student teachers' reflections.

In the next section, some general conclusions of this study will be drawn.
Then, as an elaboration of the findings, in section 6.3 a proposal for a local theory of integrating theory and practice by student teachers will be presented.
Some limitations of the study have not been referred to, or only implicitly, in the analyses and conclusions; section 6.4 takes a closer look at some of these limitations.
In section 6.5 some suggestions for future research will be made. These are partly the

product of the supposed shortcomings of the study, but are in the main prompted by ongoing developments as a result of the outcome of the study.
Finally, in section 6.6 some implications for teacher education, the area that this study focuses on, will be discussed.

6.2 Conclusions

6.2.1 The exploratory studies

The first research question that was formulated in the (first) exploratory study, was about the existence and the type of theory use by students in a multimedia learning environment: do they use theory, and if so, what is the output of their learning process in terms of knowledge construction (section 3.5). Four levels of student teacher knowledge construction were observed. Their learning and research process turned out to be a cyclical process of planning, searching, observing, reflecting and evaluating. Especially at the third and fourth level of knowledge construction (section 3.5.5), integration of theory and practice occurred, in those moments where students asked themselves questions about situations they had observed, when they made a connection with literature or when they formulated their own conjectures.
The second exploration was designed to find out more explicitly how prospective teachers made connections between theory and practice, and particularly which signals of utilizing theory they showed in their reflections on studied practices of MILE. A list of fifteen signals of theory use was drawn up (section 3.8).
In short it can be said that the two exploratory studies provided a first insight into the use of theory by students and in addition the yield of these studies was a reason to set up a learning environment that was more structured and more focused on engaging student teachers in practical reasoning, in combination with the use of theory.

6.2.2 The small scale study

The CD-rom 'The Guide' that was used for the design of the new learning environment in the small scale study (chapter 4), can be considered as an adapted version of MILE. The research question for this small scale study was: "In what way and to what extent do student teachers use theoretical knowledge when they describe practical situations, after spending a period in a learning environment that invites the use of theory?"
The study showed that all students used theory in their oral and written responses to practical situations, and that the selected learning environment enabled them to reason diversely about those situations. The differences in both the way of describing and in using theoretical concepts were relatively large. The extent to which students used theoretical concepts in their reflections on practical situations could be differentiated into levels based on the students' ability to make meaningful connections between

theoretical concepts. Especially during the interaction under the supervision of the teacher educator and during interviews, reasoning leading to a rise in the level of theory use was observed. In a few instances that rise in level could be interpreted as vertical didacticizing (section 4.4.2).
On the other hand, the suspicion arose that an optimal use of theory was not being instigated in all students, which was the reason to adapt the learning environment for the large scale study. Furthermore, new insights into the use of theory by the students were reason to refine and further focus the research questions, as well as to design a first version of a reflection-analysis instrument.

6.2.3 The large scale research

Through the 'natural structure' (increasing refinement of focus and methodology) of the series of four studies, the developments and data from the previous studies lend a content-related, characteristic meaning to the conclusive descriptions in the final, large scale study. For instance, the insight that the use of theory by students can be distinguished in two dimension, the *nature* and the *level* of theory use (section 4.3.9), only arose during the small scale study, partly in reaction to the output of the preceding exploratory study.
In the large scale study these insights have been further elaborated into a reflection analysis instrument.
The nature is shown in four types of theory use: factual description, interpretation, explanation and 'response to.' For the level, three types have been defined, level 1 to 3, based on the degree to which theoretical concepts are used meaningfully (section 5.3.6). This approach based on both dimensions allowed to unambiguously visualize the use of theory. The matrix of twelve categories resulting from combining the two dimensions is shown in table 5.2 (section 5.3.6.4).
The students' reflective notes were divided into – an average of seven – so-called meaningful units, 'complete units' within a text (section 5.3.6.2). Every meaningful unit described by a student, could be interpreted using one of the four characteristics for the *nature* and one of the three characteristics for the *level* of theory use.
One general conclusion that can be drawn is that a large majority of the students, that is to say 98,5% of the population in the large scale study, used theory in the final assessment of the course they followed.

The first research question

The first research question mainly related to the nature of use of theory: "In what way do student teachers use theoretical knowledge when they describe practical situations after spending a period in a learning environment that invites the use of theory?" (section 5.4.2).

It turned out that 'explaining' was the most common; students scored an average of 42% for that category, against 25, 12 and 21 percent for respectively factual description, interpretation and 'responding to situations.' So factual description is placed second, rather than first, as was assumed in hypothesis 1.1: *'The characteristics of the nature of theory use will manifest to various degrees, with 'factual description' as a category with a relatively high frequency.'* A possible explanation for the higher frequency of the category 'explaining' is the relatively high number of older year students (84% second and third year) and students with a relatively high level of prior education (havo – senior general secondary education – with mathematics 36%; vwo – pre-university education – with mathematics 19%). The learning environment may be another factor that has strengthened the explanatory nature of student reflections. On the other hand it is the case that a large number of students (38%) started their reflective memo with 'factual description,' with a fifth of the students even scoring category A on both the first and the second unit. Roughly another fifth part *dominated* on category A, meaning they had a score of at least 50% of all units in that category. There was dominance for the other categories for the nature of theory use as well; 80% of the students did in fact dominate on one of the four categories (190 out of 239 students; table 5.8). Differences in learning or writing style between students (Kolb, 1984; Vermunt, 1992) may provide an explanation for that dominance.

The second hypothesis (1.2) for the first research question concerned the relationship between the nature of the use of theory and the variables prior education and study year: *'The characteristics of factual description and interpretation for the nature of theory use will occur most often with lower year students or with students with a lower level of prior education, while explaining and 'responding to' will mostly occur with later year students or students with a higher level of prior education.'* Analysis revealed that the hypothesis could be confirmed for factual description (category A), interpretation (category B) and explaining (category C), with as its clear exponents the students with as their prior education mbo (senior secondary vocational education) without mathematics (more factual description, less explanation) and students with vwo with mathematics as their prior education (more explanation).

The third hypothesis (1.3) in the framework of the first research question concerned the relationship between the nature of theory use and the degree to which concepts were used. The assumption was that '*students will mainly use theoretical concepts to explain teaching situations and to respond to situations. This will involve general pedagogical concepts more often than pedagogical content concepts.'*

The hypothesis received strong confirmation for 'explaining' and the number of *general pedagogical* concepts used and the absence of any relationship between the nature of theory use and the number of *pedagogical content* concepts. In relation with the result

of hypothesis 1.2 that *mbo students without mathematics* explained less, linear regression analysis showed a significant negative correlation between this group of students and the number of *general pedagogical* concepts used. No relationship exists between *vwo with mathematics* as prior education and the total number of pedagogical or pedagogical content concepts. That last finding can be explained as follows. The group of students with vwo with mathematics as their prior education mostly consisted of first year students, and the pedagogical (content) jargon of first year students is not yet very developed. In addition, they have as yet gained little experience in arguing about teaching situations, which was confirmed in the study by the fact that these students *explained* significantly less than might have been expected on the basis of their prior education.

In brief it can be put as the result of research question one that the theory use of students mainly manifested itself in '*explaining*' situations.
It also turned out that students with a higher level of prior education used less factual description and explained more. For students with mbo without mathematics as their prior education it was the case that they used significantly more *factual description* and significantly less *explanation*, and for the students with vwo with mathematics as their prior education that they *explained* significantly more.
Finally it became clear that students used significantly more *general* pedagogical concepts for explaining and significantly less for factual description of situations.
No relationship has been established between the nature of theory use and the number of *pedagogical content* concepts.

The second research question.
The second research question, concerning the level of theory use, ran as follows: "What is the theoretical quality of statements made by student teachers when they describe practical situations?" (section 5.4.3)
Below, first some general conclusions in relation to this research question are discussed.
The average percentages scored for the levels were 35, 29 and 36 percent for respectively levels 1, 2 and 3. Especially the percentage for the third level was higher than had been expected for that 'highest' level.
Also, the conclusion was drawn in the preceding studies that some students do in fact reach an even higher level than that of level 3. This happened for instance when a 'personal theory' was formulated in response to a practical observation ('theory from practice'; section 3.5.4) or when a student reflected at a higher level than the level *of* the network of theoretical concepts, by reasoning *about* the relationships within that network (section 4.3.4). The latter phenomenon has been named in section 4.4.2 as a level transition from horizontal to vertical didactization. Similar level rises were

observed in student teachers' reflections on the research in their teaching practice into children's multiplication strategies.

The highest level (3) of theory use by students can be seen as an important indicator for theoretical enrichment of practical knowledge (section 5.3.6.4). Upon consideration the conclusion can be drawn that, in on average well over a third (table 5.13) of the number of meaningful units in their reflections, students were (re)constructing 'theory enriched practical knowledge.'

The first hypothesis (2.1) of the second research question assumed a relationship between the level of theory use and the number of concepts: "*Students who use more theoretical concepts reflect at a higher level and vice versa. This will be more strongly expressed in the final assessment than in the initial one.*"

This hypothesis could be confirmed in several respects. An obvious explanation is the fact that the more concepts are used the higher the chance of scoring level 3 is and vice versa. Furthermore it is likely that in students who possess more theoretical knowledge, higher cognitive activities are evoked, or that a potential difference in cognitive capacity between students will lead to the differences in level. That the relationship between the number of concepts and the level manifested stronger in the final assessment, can be ascribed to the fact that between the initial and the final assessment – that is to say in the learning environment – the students had the opportunity to expand their repertoire.

One thing that stands out is the positive correlation between the number of pedagogical content concepts and level 3, especially since there was no correlation at all between the nature of theory use and the number of pedagogical content concepts, not even for explaining (hypothesis 1.3). An explanation is that the relationship between the number of concepts and the level definition has a more dependent character than is the case between the number of concepts and 'explaining.' There are also differences with respect to content between general pedagogical and pedagogical content concepts, and there is a difference in reach for both types of concepts. The general pedagogical jargon is aimed at all actions by teacher and students, and is also used more frequently in training and teaching practice. This study shows that in all cases that occur, the significant correlation with the number of general pedagogical concepts is stronger than with the number of pedagogical content concepts. It is the case for all groups of students that proportionally more general pedagogical than pedagogical content concepts are used.

The second hypothesis (2.2) for this research question into the level of theory use, focused on the relationship between the level of theory use and the variables study year and prior education: "*The first level of theory use will mainly be found in first year students or in students with a lower level of prior education, while level 3 will mainly manifest in third or second year students or in students with a higher level of prior education*."

This hypothesis has been inspired by the idea that the conditions for theory use at level 3 are mainly determined by having a pedagogical (content) repertoire at one's disposal, and the ability and experience to adequately use the cognitive network. It is argued that the third level of theory use will therefore be achieved more often by students with a higher cognitive level (higher prior education) or students from a higher study year.
The hypothesis has been confirmed by the variable study year, including the second study year, which has a significant negative correlation with level 1 and a significantly positive one with level 3.

The research within the framework of the second research question led, in summary, to the conclusion that students who used more theoretical concepts reflected at a higher level and vice versa. Remarkable is the strong, significantly positive correlation between the number of *pedagogical content* concepts and level 3 in the final assessment, against the absence of that correlation in the initial assessment. It was also the case that the first level of theory use mainly occurred in first year students, while level 3 mainly manifested itself in second and third year students or students with a higher level of prior education.

The third research question
The first sub-question 3a of research question 3 focused on a possible connection between the nature and the level of theory use: "Is there a meaningful relationship between the nature and the level of theory use? If so, how is that relationship expressed in the various components of theory use and in various groups of students?" (section 5.4.4).
Indeed, a meaningful relationship exists between the nature and the level of use of theory. It can be seen in the conclusions of the first and second research questions that the differences in the size of the theoretical repertoire available to students correlate with differences in nature and level of theory use. Factual description, interpreting and level 1 have a negative correlation with the number of theoretical concepts, while explaining and level 3 both correlate positively with the number of theoretical concepts. It is also the case that factual description and interpreting are related to a lower level of prior education, particularly mbo without mathematics, while explaining correlates with a higher level of prior education, particularly vwo with mathematics.
These results largely confirm hypothesis 3.1, which has been formulated as follows: "*The characteristics of factual description and interpreting for the nature of theory use mainly occur on the first and second level of theory use, while explaining and – to a lesser degree – responding to situations are related mainly to the third level of theory use.*"
Only for category D (responding to situation) there is no clear confirmation of the hypothesis.

Linear regression analysis shows a remarkable agreement with these results. For *factual description (category A),* beta = 0,129 (sig. 0,043) for level 1 and beta = - 0,230 (sig. 0,000) for level 3 (table 5.19). For *interpreting (category B)* there is a similar result for beta and the related significance. For *explaining (category C)* the reverse is the case. There, beta is negative for level 1 (–0,214; sig. 0,001) and positive for level 3 (0,282; sig. 000). For category D (responding to situations) there is a significant correlation between nature and level of theory use only for D2.
Another confirmation of hypothesis 3.1 can be found in the average percentages of the twelve categories A1 up to D3 (table 5.20). For instance, the averages for A3, B3, C3 and D3 are respectively 5, 3, 18 and 9 percent. This also confirms that the third level of theory use mainly occurs in explaining teaching situations and responding to situations, and that factual description and interpreting only occur at this level to a slight degree.
The deviation from the expected outcome for category D may have been caused by differences in students' learning styles, by a definition of category D that did not target the inclusion relationship enough, or by the special composition of the student population that was studied (see section 5.4.4).

The second sub-question 3b involved the relationship between the use of theory and the students' level of numeracy: "To what extent is there a relationship between the nature or the level of the student teachers' use of theory and their level of numeracy?"
The hypothesis (3.2) that was formulated for this research question, was motivated by the idea that students who possess a great deal of ability for numeracy, were likely to reason at a relatively high level. For that reason a positive relationship was expected between *explaining* and numeracy. In terms of the inclusion relationship, that relationship should also occur for '*responding to situations*,' although that conclusion was no longer self-evident after the results of the previous analyses relating to that category (D).
In addition the conclusion in relation to the positive correlation that was found between explaining, level 3 of theory use and the number of theoretical concepts used, led to the assumption that there would exist a positive relationship between the latter two variables and numeracy as well. Based on these considerations, hypothesis 3.2 was formulated as follows (see also section 5.2 and 5.4.4):
"*There is a positive correlation between the level of numeracy and the variables:*

- *nature of theory use 'explaining,'*
- *the highest, third level of theory use,*
- *the number of theoretical concepts used, and*
- *students' prior education*."

Linear regression analysis confirmed the positive correlation between numeracy and 'explaining,' while a positive trend was found between level 3 and numeracy. That the

correlation with explaining turns out to be stronger than the one with level 3 is likely when one takes into account the relationship between explaining and 'problem solving,' while the relationship between numeracy and level 3 of the use of theory is less self-evident.
The positive relationship between numeracy and the number of theoretical concepts used also turns out to be significant, though this concerns the number of *general pedagogical* concepts used, rather than the number of *pedagogical content* concepts.
This study also confirms a significant correlation between student teachers' numeracy and their prior education (table 5.21). This result is not unexpected, and corresponds with the results of recent studies into the relationship between individual skills and Pabo students' prior education. It is remarkable that the mbo students in this study's population were mainly in the third year, while the vwo students could mainly be found in the first year, and that for these groups of mbo and vwo students there were still significant negative, respectively positive, correlations being found. According to this result, the negative correlation between the so-called personal evaluation index (PEI; section 5.4.1) and students' prior education is remarkable (Beta –0,155; Sig. 0,034). It may indicate that more reticence regarding estimating one's own level of numeracy corresponds to a higher level of prior education.

The results from research question 3 can be summarised as follows.
A meaningful correlation appears between the nature and the level of theory use.
The characteristics of factual description and interpreting for the nature of theory use occur mostly at the first and second level of theory use, while explaining and – to a lesser degree – responding to situations, are on the whole related to the third level of theory use. Also, a strong relationship exists between the category 'explaining' for the nature of theory use and numeracy, to a lesser degree also between level 3 of theory use and numeracy.
There also is a strong correlation between the level of numeracy and that of prior education.
In a more general sense, it could be established, from the questionnaire the students filled in (appendix 14), that the students who participated in the study appreciated the learning environment aimed at integrating theory and practice.

6.3 Towards a local theory of integrating theory and practice

The results of the study and the analysis of the student teachers' activities in the course of the four parts of the study, provides the basis for reflection to a local theory for learning to integrate theory and practice by student teachers. On the one side this theory involves the student teachers' process of learning to integrate, on the other it involves the learning environment that is intended to support that process with teaching materials

and targeted interventions by the teacher educator. Both components are presented mainly in an integrated manner in the following description.
Below, first a description is given of the context in which the intended learning by the student teacher and the support of that learning process by the teacher educator took place, providing an overview of the ingredients of the local theory. After that, the theory will be further elaborated and finally presented in summary.

The first confrontation student teachers had with theory within this study, was the moment that the theoretical framework was presented as a multifunctional list of theoretical key concepts that would come up in the learning environment. At first this list functioned as an advance organizer. The students could indicate which concepts were (un)known to them in the context of a practice story, and the source of that story (own practice, literature, MILE, lectures and workshops). At this stage it was likely that for most of the students the theory of the domain in question was a disjointed collection of concepts, parts of which were, as separate elements, related to narratives of practice. The stories were not always meaningful to the students, sometimes they even turned out to be linked to concepts that were thought to be meaningful on the basis of misconceptions. A number of students indicated in the evaluation of the study that certain concepts had gained a different, or more, meaning for them during the course than their original ideas. The intention of the course was to evoke, in several ways, meaningful use of the concepts by the students, to expand and deepen their repertoire, with the highest goal attaining a cognitive network of 'theory-enriched practical knowledge.' The most important sources were theory-laden 'practice stories' from MILE and The Guide, and the 'research stories' from the students' own practice. The theoretical reflections by the teacher educator that were related to those narratives and the reflective notes in The Guide functioned as mirror and sounding board in the discourse and during individual study.
Multimedia learning environments as used in this study, give student teachers the opportunity to observe 'practice' alone or together, to discuss and study it, without being distracted by having to keep order or all kinds of organisational problems. The experience and identity of student teachers do place specific demands on that learning environment. Opinions about teaching and learning that students have acquired, also by earlier experiences, can easily lead to critical judgements and a focus on cut-and-dried answers in analysing practical situations. It requires extensive coaching to put the students on the investigative trail, and any approach must lead students towards an attitude that is marked by being prepared to ask questions of oneself and pronouncing cautious suspicions and preliminary conclusions. In such a learning environment, including sophisticated coaching, students can learn to integrate theory and practice.
The variety of data collected from both the small and large scale studies has shown how

students made connections at different levels between theory and practical situations. In the second part of the course on offer, the focus of student activities shifted more and more towards constructing a cognitive network of theoretical concepts. Examples are the reflections on the investigations about childrens' knowledge of tables of multiplication in the student teachers' teaching practice, the activities related to the game of concepts, the concept-map activities and the concluding 'collaborative lecture' in which the knowledge and experience that had been gained were positioned in the stages of the multiplication course under the teacher educator's supervision.

The search for answers to the student teachers' individual learning questions could lead to a more profound 'ownership' of the enriched practical knowledge. At the end of each meeting, students were invited to think, respectively become aware of, the theory-enriched practical knowledge they had gained, using the motto: "What (else) did I learn?" The practical knowledge that was gained could be further deepened and widened by writing reflective notes at some points during the course.

At that stage, the list of concepts gained two new functions, that of giving support and providing an overview, and providing an insight into progress with acquiring theory. In the final assessment, students could show the theory-enriched practical knowledge they had gained by writing a reflective note based on observation of a teaching situation from MILE that had not been brought up in the course.

The theoretical character of the course showed itself in the number of theoretical concepts that students used and their ability to meaningfully relate theoretical concepts to each other. In Dutch mathematics teacher education student teachers are faced with *subject specific* theory, with the realistic mathematics *domain-specific instructional theory* in that area (RME; e.g., section 2.6 and 3.2) and with *general pedagogical* theories. That complexity of teaching mathematics (Lampert, 2001) was reflected on a small scale in the study, through the learning environment, the *theory* in the list of fifty-nine theoretical concepts that were central to the course, together with the theory laden *practice narratives*. The study showed large differences in the way in which the students involved these theoretical concepts in their arguments. Two dimensions were distinguished, the *nature* and the *level* of theory use. The *nature* of theory use relates to four ways of using theory: factual description, interpretation, explanation and 'responding to.'

The *level* relates to the degree to which the concepts are expressed meaningfully and in relation to each other in the statements and notes of the students. The highest level (3) is reached when students express a meaningful relationship between two or more theoretical concepts in a written (meaningful) unit. In such level 3 units, the transition from the second to the third level can often be seen. A first or second sentence will contain statements using a theoretical concept, while the following sentences will

contain different concepts that correlate meaningfully with the foregoing concepts. There are also rises in level within the third level. One such rise in level has for instance been observed in student Anne, when she showed a tendency towards hypothetical thinking and reasoning (section 4.3.2 and 4.3.3). This could be defined as a fourth level of 'responding to situations' (D4), something that Ruthven (2001) might call 'practical theorizing' (section 2.7.1) and Simon (1995) as the start of developing a 'hypothetical learning trajectory' (HLT) (section 2.7.1). A rise in level from D3 to 'D4' also occurs when Anne reflects at a higher level than the level *of* the network of theoretical concepts, by reasoning *about* the relationships within that network (section 4.3.4).
Section 4.4.2 argues that these rises in level seem related to the kind of level-rise that Van Hiele (1973) describes in his theory on levels in mathematical thinking. That level theory has influenced many scientists both within and outside the Netherlands, among other things in the development of theory about mathematical learning processes in students. For instance, Gravemeijer (2007) describes rises in level within the framework of the design heuristics of emergent modelling as the development of a network of mathematical relations. And this is in fact what student Anne did, to construct abstraction by reflection on the relationships she distinguished. In section 4.4.2 this has been interpreted as the transition from horizontal to vertical didacticizing (Freudenthal, 1991).
The study has shown that the role of the teacher educator regarding the stimulation of rises in level is crucial. The teacher educator has the expertise to theorise, to evoke theory use and to stimulate it, among other things by selecting adequate video fragments, asking challenging questions, making use of differences in argumentations, presenting confronting situations (Piaget, 1974; section 2.7.1) and inspiring 'pedagogical conflicts,' sharpening the discourse with theory-laden summaries or by stimulating hypothetical thinking. It is exactly the combination of these ingredients that can lead student teachers to adopt theory (section 2.6.4) and construct EPK. The narratively oriented learning environment (Pendlebury, 1995) provides the EPK with a lasting meaning. The 'theory in narratives' leaves a lasting impression and can be recalled.

Taking the above considerations and their relation to the results of the study as its starting point, a local theory of integrating theory and practice in mathematics teacher education has been formulated, based on the concepts *theory, practice* and *the relationship between theory and practice* as they have been described in the sections 2.3 up to 2.7. There, theory is defined as a collection of descriptive concepts that show cohesion, with that cohesion being supported by reflection on 'practice.' For the acquisition of theory-practice relationships by students, the first step is to look for a connection with theoretical notions that students already have. This is done by making connections between theoretical

concepts with multiple definitions (definitions, notes, contexts; list of concepts) and practical situations students themselves have experienced. Afterwards practical knowledge is made explicit and theory-enriched through cycles of observation, analysis of theory-enriched practical situations and 'responding' to them. That enrichment occurs in the discourse, led by the teacher educator, in collective work, during individual study and by writing reflective notes. Impulses for enrichment are: the 'narrativised' theoretical framework of concepts, adequate literature, the learning and investigation assignments, confrontational situations, reflective conversations, challenging questions, reflection on successes, (collaborative) lectures, and reflective notes.
To some degree, the cycles of observing, analysing and 'responding,' are the detailed elaboration of the cyclical process that for example was observed in The Pioneers in the first exploratory research project (section 3.5.5). The 'theory-enriched practical knowledge' that student teachers acquire, contains the key insights in relation to learning and teaching mathematics.
The connections between theory and practice that students themselves make, become visible in the nature and level of theory use. A rise in level is caused by practical reasoning and reflection; it leads to an extension and refinement of the 'theory-enriched practical knowledge' network.
The reflection-analysis instrument can be used as a guidance or (self)assessment tool to establish the degree to which students are competent to integrate theory and practice.

In summary, and in line with what has been described about the definition of theory, it can be established that the local theory is determined by three main components, the *formulated concepts* of theory, practice and the relationship between theory and practice, *the theoretical knowledge base of the learning environment* for student teachers and the *guidelines for teacher educators,* to support the learning and developmental processes of students.
These lead to the theory gaining a function as an orientation basis for reflection on practice. The coherence of the descriptive concepts that was mentioned in the definition of theory, is determined by the learning and teaching theory of realistic mathematics education and the concepts for nature and level of the use of theory.
The research into theory use by student teachers has provided the reason in this study to design a learning environment that is optimized with respect to the possibilities for students to use theory. The research questions could be answered in this learning environment. The fact that the development of the learning environment was guided by theory, and that there are guarantees that the development can be traced, makes it possible to do a similar study in other domains and other subjects in teacher education. The design of the learning environment can be considered as a paradigmatic case of a broader class of phenomena (Cobb & Gravemeijer, 2008). The trackability also involves

the reflection analysis instrument that is part of this study and which can be used as a guidance and assessment tool.

6.4 Limitations

To a certain extent this study was limited by the context in which it occurred. The students' learning environment consisted mainly of practical situations that were represented in a multimedia form. While these situations were real teaching situations, they were not situations from the students' own practice. The student teachers' own practice experiences were to some degree involved in their activities, for instance through investigations on their field placement. One might ask whether having situations from the students' own practice as objects of discussion and reflecting would not have resulted in a better and more realistic insight into the process of relating theory and practice. It is after all 'real' practice where (student) teachers have to become aware of theory as a necessary instrument for reflecting on their own teaching, aimed at 'explaining' situations, and 'responding to' situations. This allows them to use their theoretical knowledge and develop it further, among other things by testing conjectures that are aimed at their own 'professional setting' (section 6.3) in various situations. The next section (section 6.5) contains suggestions for further research into this point.

Another limitation of this study was the selected portion of the available data collection from the large scale study. The nature of this collection – the students' reflective notes – may have limited insight into some aspects of theory use. Expressing thoughts in writing is something that requires specific skills in students, which may mean that input of potentially present notions of theory may be less than when thoughts are expressed orally. The yield of oral reflections is often higher than that of written ones (Jaworski, 2006, p. 188). In addition, theory use is particularly evoked by activities where oral input is natural, such as the interaction in the discourse and in interviews. As a result, the large scale study does not yield hard evidence in relation to for instance student reasoning leading to level rises, as were seen in the small scale study.

Other limitations of the study have already been described more or less explicitly in the analyses and conclusions of the various sub-studies. This concerns for instance the deviation from the expected outcome in category D ('responding to situations') and the nature of the research population in the large scale study.

During the course of the study, ideas also arose about desired, possibly more effective or more efficient research strategies. One example is the only partially fulfilled desire to have the teacher educators participate in the study as teacher educator-researchers. Another example is the need that arose for an interdisciplinary research team consisting of content specialists for mathematics, language, and general pedagogues. The use of such a team would be particularly profitable for analysing the data from different

angles, most likely leading to deeper insight into student reasoning than was the case now. In the next section, the suggestion to form such an interdisciplinary research team will be made.

6.5 Suggestions for future research

Before (in section 6.4) the limitations of practical situations in the multimedia learning environment in comparison to students' teaching practice were discussed. On the other hand, this study also shows that a multimedia learning environment gives students the opportunity to quietly observe, discuss and study 'practice' – also together. It turns out that they appreciate working in such a multimedia learning environment with its focus on integrating theory and practice and take advantage of it. The learning environment of their own teaching practice evokes 'survival' rather than study and reflection.

In any case, further study in the students' teaching practice will be necessary. Moreover, there is a need for long-term study to determine how both beginning and experienced teachers – consciously or subconsciously – use theory in daily practice and how the development of 'theory-enriched practical knowledge' takes place in the longer term. Particularly long-running research can provide more insight in for instance the ability of teachers to anticipate with consideration on students' learning and to respond to learning processes (category D for the nature of theory use in this study). Also, in-service training would seem a suitable venue for such a study. It is important to involve a varied group of experienced teachers for such long term research, with variables such as prior education, prior experience, the learning and teaching style or the extent of using professional literature. Taking into account the various angles from which the data have to be analyzed, it is desirable to form an interdisciplinary research team consisting of general pedagogues as well as content specialists in the fields of mathematics and language.

The use of the reflection-analysis instrument by teacher educators and (student) teachers requires further training. Recent use of the instrument, also for other subjects, has shown that there are minimum conditions that have to be met to allow its effective use. One example of such a condition is prior definition of a theoretical network of concepts, not only at meso-level but also at micro-level. If the available theoretical network of concepts is too limited, it will not be possible to establish whether students are capable of creating meaningful connections between concepts.

The reflection-analysis instrument offers the opportunity to perform a systematic and nuanced analysis of (one's own) video recordings of teaching practice. Do teachers have (spiral) levels of development, as is sometimes assumed? To what degree do teachers appreciate the use of theory in their reflection on their own practice, and can the usefulness of the effect be measured?

Based on the experiences from this study, a combination of small scale and large scale research is recommended. Triangulation of the results from both kinds of research can lead to deeper, coherent analyses, which will, as a result of the possibility to have more nuances within the data system, be more consistent and cogent than the analysis of data from individual studies. Examples of such results in these studies were the level rises in student teachers (cf. sections 4.3.2, 4.3.3, and 4.3.4).

6.6 Implications for teacher education

Implications for the curriculum design

The set-up of the designs for the learning environment in the four sub-studies, shows an increased structuring in the approach of the student teachers' learning processes. Seen in that light the learning study of the student pioneers in the first exploratory study was an 'open learning study' in MILE, and the studies that followed were more structured studies. It has been mentioned before (section 2.4 and 3.5) that open investigations as a part of the learning environment for students in mathematics teacher education have been seen already for years as a likely opportunity to have students focus on their own professional development in a natural way. The theoretical backgrounds that have been mentioned are recognisable in the design of the investigations activity, in the sense that the ideas have to be placed in the context of the development of theory for the pre-service mathematics teacher education (in the Netherlands). A large effort is asked from teacher educators to coach students in the pre-service training, for example from superficial observing and judging of teaching situations to the level of predicting occurrences or anticipating on and responding to situations. This is not an easy task; one thing teacher educators are confronted with is the dilemma of the learning paradox (Bereiter, 1985; section 3.7 and 3.9.1).

This is different for in-service training. The students often start the courses with their own, practice-oriented questions, and can put in their direct experience and mirror or test that against the experiences of others and against practice-relevant theory being discussed.

Particularly, and this is not the least important difference with the pre-service training, there is the direct, functional goal to improve one's own teaching. For example 'lesson studies' as proposed by Hiebert, Morris & Glass (2003), can fit the knowledge generation and improvement processes for teacher preparation.

For that reason, gaining and extending the repertoire of theory-enriched practical knowledge, seems a more easily attainable target for in-service training than for pre-service training. The question may be asked whether it is possible for the pre-service training to closer approach the concept of the in-service training. In both cases, we are in fact dealing with cycles of observation of practical situations and reflecting on them

(collectively), followed by using the newly-gained practical knowledge. For the curriculum of the pre-service training this could for instance mean a build-up of investigation activities in *four successive learning practices*:

- the 'multimedia practice' of expert teachers,
- the mentor's practice,
- the student's own practice under the mentor's supervision and
- independently in one's own practice.

In that phased continuum, multimedia applications for students have different functions, which gradually focus more and more on learning one's own, complex practice. Looking at the extremes, the scale runs from being able to quietly study the good practice of expert teachers – particularly aimed at learning to observe and analyze students' and teachers' activities – to studying and reflecting (also by others) on one's own practice in a 'community of learners' (cf. school team). For each of the four stages that have been mentioned, the basis for reflection on practice is provided by theory, with the 'theory-enriched practical knowledge' having the potential to develop into a theory of practice for the teacher in practice. In the final stages of the outlined course, video recordings of one's *own* practice are an important tool for reflection. The images alert (student) teachers to their own actions. This can be a confrontational experience, but for that reason it will also lead to greater involvement and reflection, visible in the level character of the local theory.

This study shows that multimedia may perform useful functions in the learning environment of primary mathematics student teachers, particularly in relation to the theory-practice problem. Primarily, there is the previously-mentioned possibility for students to concentrate on others' 'safe' good practice, away from the hectic of their own practice group. Secondly, it is possible to discuss the 'communally experienced' practice in small or large groups. Thirdly, the 'theory-enriched practice' that is offered, can be selected by teacher educators and be included in a sophisticated way in the curriculum. Important is that the discourse about that practice is led by the teacher educator, who, like no other, is able to make (hidden) practical knowledge explicit and enrich it with theory. While the mentor at the practice school cannot do that, he or she can play an important role in eliciting the mentor teachers' practical knowledge in prospective teachers. That ought to occur primarily in the third and fourth stage of the structure for using video practice mentioned above. It turns out that the obvious advice for student teachers to ask their mentor questions about a lesson they observed in practice, is often overlooked by them (Zanting, 2001), while that activity contains excellent opportunities to have the mentor's practical knowledge be made explicit, particularly if that 'mentor's practice' has been recorded on video. The teacher educator now moves to the foreground again, especially where enrichment of theory is involved,

or for instance in response to the student's reflective note. An important competence of the teacher educator is the ability to theorize practice. Being able to "intertwine the investigation of practice with the examination and development of theory" (Lampert and Loewenberg Ball, 1998) is a key element of the educator's expertise. There should be special attention to stimulating a rise in level. Section 2.4 and 4.4.2 point out, among other things, the relationship between the rise in the students' level of reasoning and the ideas of Van Hiele (1973) and Freudenthal (1991). Van Hiele sees rises in level as discontinuous, because the levels differ in the degree of abstraction and the way in which the learner thinks and acts in relation to objects and relationships. The discontinuous character of level rises may be one of the causes for the lack of understanding between student(s) and teacher, because they think and reason on different levels. The danger is present also in teacher education that teacher educator and student(s) misunderstand each other for that reason, especially since the phenomenon can occur at all 'levels' of education. Students encounter it in their relations with the children in their teaching practice, and in their contacts with teacher educators and student peers, the teacher educators encounter it in their contacts with student teachers and in contacts at the level of the methods of teacher education.
This study shows that a rise in the level of reflection by students mainly occurred as a result of direct or indirect interventions by the teacher educator. It will require great effort by student teachers and their educators to reach the level of practical theorizing (sections 3.1, 4.4.2 and 6.3), not in the least because that requires a specific knowledge base and attitude from both parties.

The reflection-analysis instrument from this study can support teacher educator and student in formative or summative assessment of the quality of the theory-enriched practical knowledge. The ability to reflect is one of the most important characteristics of a teacher's professionalism, and it is largely the component of learning to reflect in a systematic and functional way that gives the teacher education curriculum the appropriate level. The reflection-analysis instrument is one item that can help create that functionality and system. If necessary, it can be reduced to the level dimension of theory use, making it easy to apply in teacher education for both students and teachers, including those in other subjects than mathematics education.

Knowledge for mathematics teaching
The tendency by students that was found in the study to use general pedagogical, rather than pedagogical content concepts, has consequences for the curriculum design at teacher training colleges, in the sense that it is important to optimally use the meaning of general pedagogical concepts in subject-specific contexts. Also, when choosing teaching situations, subjects for discussions and interventions by teacher educators, it is

possible to focus (more) strongly on evoking and using domain-specific theory. In section 2.6.4 we already pointed at the importance of giving more attention to what Ball, Hill & Bass (2005) call 'mathematical knowledge for teaching,' a generic term for the subject matter knowledge and the pedagogical content knowledge that teachers need in the practice of mathematics teaching. The fact that *pedagogical content* concepts are used mainly at 'level 3,' albeit that the use of general pedagogical concepts also occurs more at that level, is another reason to stimulate students to as high a level of theory use as possible, i.e. meaningful use of theoretical concepts in mutual relationships.
The fact that students have a lot of affinity with the general pedagogical aspects of the development of children has other consequences for the design of the Pabo curriculum. Teaching situations within the subject of mathematics clearly are motivating learning situations for student teachers where the use of general pedagogical theory is concerned. This means that it may be possible to make even better use of the school subject of mathematics to develop (notions of) big ideas in the area of general educational theory. On the other hand, deep exploration and use of general pedagogical concepts (e.g., interaction; context) is very important for learning to distinguish the pedagogical content meaning, and particularly helps to avoid verbalism and separation of systems in thinking.

The use of theory and student teachers' prior education
It was to be expected that students with a higher level of prior education and students who were in later years in their study would reflect at a higher level. That students with mbo (senior secondary vocational education) without mathematics as prior education still differ negatively in *later years*, both in their own ability and for the competence to use subject-specific pedagogical knowledge, indicates that specific attention needs to be given to this category of students. The hypothesis that is occasionally heard that students with that kind of prior education and often weaker numeracy skills would not be able to function well in the upper grades of primary school, though they would do well at the lower levels, are not confirmed by the results of this study. After all, this group scored below average not just on numeracy, but their use of theory (both general pedagogical and pedagogical content) was also limited in nature and level, while the theory in the study was aimed at the lower grades, and the mbo students who participated in this study were mainly third year students. According to the results of the study, for students with mbo without mathematics as prior education, it seems essential to strengthen their theoretical knowledge network by reasoning about practice in terms of 'explaining' and 'responding to' situations. Such activities stimulate the use of theory, with respect to the number of theoretical concepts as well as to the level of using those concepts.
For the 'opposite poles' of the mbo students where prior education is concerned, the

vwo (pre-university education) students, extra attention is also needed. Out of the 'top six' among the student population, i.e. the students who scored 100% at level 3 of theory use, five had vwo as their prior education. Perhaps not surprising, taking their cognitive head start at the beginning of the course into account, but on the whole the vwo students perform not as well as expected. The outcome confirms the suspicion that these students also need special attention. Research has shown that vwo students, more than others, indicate that they miss a theoretical depth (Geerdink & Derks, 2007). One explanation for the fact that vwo students do not perform significantly better than their student peers for the (pedagogical) use of theory, may be the attitude that a part of them assumes at the start of the course. It does happen that these students underestimate the relevance and the level of domain specific pedagogy at the start of the course, possibly because they themselves can usually solve the mathematics problems to be taught quickly (albeit in a formal manner).

In summary we can say that this study shows that multimedia in a primary mathematics student teachers' learning environment can perform useful functions, particularly in relation to the theory-practice problem. If this learning environment is optimized for the use of theory in practical situations, students can learn to integrate theory and practice, and they may acquire 'theory-enriched practical knowledge.' An important criterion for that optimisation of the learning environment can be found in the input of the teacher educator, whose guidance for instance leads to a level rise in student reasoning about practice.
The reflection-analysis instrument from this study can support the student teachers' self-assessment, and can be a tool for formative and summative assessment.

7 Summary

This study concentrates on the theory-practice problem in primary teacher education, focusing specifically on the subject of mathematics education.
The main question is how student teachers can integrate theory and practice and how the organisation of their learning environment can contribute to that integration. Little is known yet about how student teachers at teacher training colleges gain knowledge or about how they connect theoretical knowledge and practical situations, both crucial components of learning to teach.

Chapter 1 of the thesis briefly describes background, context, problem, research questions, relevance and character of the study.
The aim of the study is to gain an insight into the way in which student teachers connect theory and practice and to what degree and at what level they are able to use theory in teaching situations.
The research consisted of four sub-studies, namely two exploratory studies, a small scale study and a large scale study, each with their own function in the whole. Each time, the result of the previous study provided the means for the next study, with more refined research questions and a more adequate design of the learning environments for the student teachers who participated in the study. The multimedia character of the learning environment allowed the optimisation of the use of theory.
The first three studies gradually provided new insight into the use of theory. In the small scale study that insight was defined as the *nature* and *level* of theory use by student teachers. Categorizing nature and level provided the tools for developing a reflection-analysis instrument for the large scale study.

Chapter 2 describes the theoretical framework of the study.
The necessity of prospective teachers learning to integrate theory and practice is generally acknowledged. Yet very little is known about the character of that process of integration. The realisation that the prescriptive 'transfer' of theory does not lead to the desired integration has existed for years in the field of teacher training, as well as the awareness that theory is often insufficiently attuned to reality and the complexity of acting in practice.
Teacher education colleges differ – also at a global scale – strongly in the way in which they try to shape integrating theory and practice. Of the various trends that can be distinguished, in the last several years the emphasis has been on the so-called 'reflective practice' orientation and on the 'development of professional knowledge' orientation (§ 2.2).

This last direction appears promising, especially because of the attention to the simultaneous and integrated use of theoretical and practical knowledge by teachers in training. However, this still does not answer the question of how integrating the various elements of the knowledge base by teachers-to-be takes place and how that integration can be developed and supported. A barrier in the search of an approach to a pedagogy for teacher education is the ambiguity over the concepts of theory and practice. There is a variety of views in the research literature. In § 2.3 and § 2.4 of this thesis an attempt is made to chart this diversity and to justify the choices made for the purposes of the studies. In that view the concept of *teacher practical knowledge* plays an essential part (§ 2.3.4). For the derived concept 'theory-enriched practical knowledge' developed during this study, this is the case especially. The quotation below shows the 'theory-enriched practical knowledge' of student teacher Anne as she reflects on the results of the study of multiplication strategies she performed in her practice school (§ 4.3). She describes practice using theoretical concepts (multiplication strategies, memorizing, automating, supporting problems) and uses these concepts in a meaningful way and with mutual connections.

> The various strategies do turn out to be somewhat complicated for some children. They find it difficult to choose the right supporting problem [anchor point; w.o.] or the right size of the problem (…). Strategies are needed to automate and memorize the problems. You can also reverse this. That memorised problems are needed for the strategies. Think of the supporting problems. They can calculate new problems through problems they already know. Strategies, automating and memorizing are inextricably linked. The use of strategies is not limited to multiplication, but occurs in all other areas of mathematics education. Another reason to offer strategies is the opportunity for checks. In practice you encounter children who have memorised problems wrong. By calculating problems using the strategies you can check the answers. Provided the strategies are used correctly, which is sometimes difficult for the weaker students.

The history of Dutch primary mathematics teacher education, particularly after 1970, shows how the relationship between theory and practice developed there (§ 2.5). The founding of the Institute for the development of mathematics education ('Instituut voor ontwikkeling van het wiskundeonderwijs'; IOWO) in 1971, started a new development in mathematics education in primary and secondary education in the Netherlands.
In this study that influence can be found in the design of the student teacher learning environment for the student teachers who participated in the various studies.

The development of the learning environment occurred as a 'design research process' in four stages, with the three components of performing a design project recognisable in each, namely preparing, trying out the design in a group and performing a retrospective analysis (§ 2.7). Focal points for the use of theory in the course (§ 2.6) were used as a

frame of reference for developing and refining particularly the learning environment for the small and large scale studies.
The multimedia character of the learning environments enabled the use of video images of practical situations as objects of learning and inquiry activities for student teachers. In this 'multimedia practice' they could concentrate on teaching situations that had been selected for them both individually and in groups. Assignments for the student teachers' own practice were part of the learning environment. For the two exploratory studies, use was made of the Multimedia Interactive Learning Environment – MILE, the development of which is described in section 3.3 of this thesis. In the small and large scale studies a CD-rom 'The Guide for grade two' (section 4.2.2.2) was used as part of the student teachers' learning environment.

Chapter 3 presents both the exploratory studies within the context of the development of the MILE project.
The first exploratory study (§ 3.5) was set in a learning environment that consisted of ten lessons in grade two on CD-roms, a description of these lessons, the textbooks for the lessons and the first version of the MILE search engine.
The goal of the study was to gain an insight into the character of the learning process of student teachers who explored the diverse content of MILE and in the process, they constructed new knowledge.
The learning and inquiry process of the two student teachers who participated in the study manifested a cyclical process of planning, searching, observing, reflecting and evaluating. In addition, the study showed the levels at which student teachers constructed their knowledge. Relationships between theory and practice were created in the discussions, which were led by the participating teacher educator/researcher, through written reflection on the discussions, and at a later stage based on literature.
Integration of theory and practice occurred particularly at the so-called third and fourth levels of observed construction of knowledge (§ 3.5.5), at moments that student teachers asked themselves questions about observed situations, when they made connections with the literature or when they formulated their own 'local theory.'

In the second exploratory study (§ 3.8) the MILE environment was extended with lessons from several primary school classes, as well as an advanced search engine, which allowed the student teachers to search the lessons and additional materials.
In addition, the two groups of 25 student teachers who participated in the study were provided with a list of 150 theoretical concepts. This list gave them the opportunity to estimate their advance theoretical knowledge at the start of 'The Foundation,' a new course for MILE. The course consisted of ten meetings led by the teacher educator, with group work and individual study after each meeting. The student teachers had a

workbook with learning and investigation assignments for working with MILE.
The purpose of this exploratory study was to inventory 'signals of theory use' shown by student teachers in their reflections on their study of practical situations within MILE.
Connections between theory and practice were made during discussions led by the trainers, during group work, in individual study and in the oral and written reports of assignments the student teachers had to perform.
The research data were obtained through observation of eight student teachers during the collective meetings, through a participatory study of group work and through interviews.
The analysis of the results on the basis of fifteen formulated 'signals of theory use' showed that student teachers only rose above the level of responding in terms of 'practical wisdom' in situations where the teacher educator participated. This led to the conclusion that the student teachers' learning environment would have to be optimized for learning to make the practical knowledge within MILE explicit and to enrich it with theory.

Chapter 4 describes the small scale study.
The learning environment for the small scale study was adapted on the basis of the conclusions from the second exploratory study. The elements that were added to the learning environment, were intended to challenge student teachers to make explicit the theory-loaded practical knowledge that was present in the multimedia practical situations, and to analyze them, to enable the construction of 'theory-enriched practical knowledge.' Examples of such elements were:

- 'The Guide,' a CD-rom, used as learning and private study material, containing practical narratives, with additional theoretical reflections and literature;
- discussions, based among other things on 'theorems' that had been formulated in group work;
- a 'game of concepts,' consisting of four practical situations (video) and for each student teacher six differently colored cards (concepts) and an answer form;
- inquiry on student teacher's own field placement;
- writing 'annotated narratives' and reflective notes.

As in the second exploratory study, the student teachers had a list of theoretical concepts available to them. This functioned not only as an initial assessment, but it played a prominent part for the student teachers in the whole course.

The research question of the small scale study (§ 4.1) was aimed at the way in which and the amount of theory student teachers used when describing practical situations after their course.

The assumption was that the renewed learning environment would enable the student teachers to reason in a diverse way about practical situations, and that there would be demonstrable differences in the depth of their theory use. The study was aimed at mapping that variance and depth. Two groups of six, respectively eight, student teachers voluntarily worked in the learning environment for five one-and-a-half hour meetings, led by two experienced teacher educators. In advance of these meetings some components of the learning environment (The Guide, list of concepts, 'theorems') were tested with four groups of 63 second year student teachers in total. Next, the researcher developed the first version of the learning environment for the fourteen student teachers taking part in the small scale study. After each meeting there was an evaluation, and the researcher made suggestions for the next step based on (video) observations and the input from the teacher educators. The research data from the fourteen student teachers were the source material for describing a case about the learning process of student teacher Anne (§ 4.3).

Considerations based on the results and experiences from the previous studies and from research literature, inspired the development and testing of the *first version of the reflection-analysis instrument*.
In contrast to the collection of fifteen 'signals of theory use' that had been compiled in the second exploratory study, this instrument allowed *comparing* the use of theory by Anne and her fellow student teachers (§ 4.4).
The study showed that all student teachers used theory in their oral and written responses to practical situations. The differences in both *the way* theory was used in the descriptions as the *number of theoretical concepts* in use were relatively large. The difference in the *level* at which student teachers used theoretical concepts in their reflection on practical situations, could be distinguished based on the student teacher's ability to create meaningful connections between theoretical concepts. Reasoning leading to a rise in level of theory use were observed especially during interactions led by the teacher educator and during interviews (§ 4.4.2).

The study of the *relationship between theory use and the level of numeracy*, also a part of the small scale study, strengthened the suspicion that there was a positive relationship between the two variables (§ 4.4.1).
In addition, the output of the anonymous questionnaire the student teachers were handed as an evaluation form after the series of meetings, could be interpreted as an appreciation of theory by the student teachers and of the integrated approach in offering theory and practice in the student teachers' learning environment.

Chapter 5 describes the large scale study of 269 student teachers.
The study was set up partly on the basis of the results of the small scale study.

Specifically, the research questions were refined, and the learning environment and the research instruments were adapted.

The research questions of the large scale study were formulated as follows:

1. In what way do student teachers use theoretical knowledge when they describe practical situations after spending a period in a learning environment that invites the use of theory?
2. What is the theoretical quality of statements made by the student teachers when they describe practical situations?
3. a). Is there a meaningful relationship between the nature and the level of theory use? If so, how is that relationship expressed in the various components of theory use and in various groups of students?
   b). To what extent is there a relationship between the nature or the level of the student teachers' use of theory and their level of numeracy?

The changes in the learning environment for the large scale study mainly involved the initial assessment, the final assessment and the numeracy test. In addition a manual for teacher educators was designed, which contained a detailed description of the student teachers' course, with guidelines for the content and organisation of the meetings, instructions for known to be successful interventions, criteria for good learning questions and hints for stimulating the use of theory and rises in level.
A general characteristic in the development of the curriculum for the student teacher course was the plural embedding of theory (intrinsic, extrinsic; § 2.6.4.) and the desire for a balance between content components, as well as between self-guidance and guidance from the teacher educator, and between the student teachers' teaching practice and the professional practice targeted by the teacher training colleges (§ 2.7.1).

The data analysis of the small scale study also provided new insight into the use of theory by student teachers. It turned out that two dimensions could be distinguished, namely *the nature* and *the level* of theory use. Other than with the 'signals of theory use' that were developed in the second exploratory study, four signals for nature, combined with three categories for level (table 7.1), allowed a consistent and systematic categorisation of theory use.
Using the twelve (4 x 3) categories as a foundation, the reflection-analysis instrument was developed further, validated and assessed for reliability (§ 5.3.6).
Table 7.1 gives an overview of the twelve categories for nature and level of theory use.

Table 7.1 Reflection analysis tool. Brief description of the twelve score combinations, with horizontally the division based on the nature of theory use and vertically the level of theory use

	A **Factual description** facts: who, what, where, how	**B** **Interpreting** For instance opinion or conclusion without foundation	**C** **Explaining** For instance 'explaining why'	**D** **Responding, gearing to** For instance, anticipation, continuation or alternative design, meta-cognitive reactions
Level 1	**A1** Factual description of events without use of theoretical concepts.	**B1** Interpretation of events without use of theoretical concepts.	**C1** Explanation of events without use of theoretical concepts.	**D1** Description, alternative event, continuation or meta-cognition without use of theoretical concepts.
Level 2	**A2** Factual description of events using one or more theoretical concepts without mutual connection.	**B2** Interpretation of events using one or more theoretical concepts without mutual connection.	**C2** Explanation of events using one or more theoretical concepts without mutual connection.	**D2** Description, alternative event, continuation or meta-cognition using one or more theoretical concepts without mutual connection.
Level 3	**A3** Factual description of events using one or more theoretical concepts with a meaningful connection.	**B3** Interpretation of events using one or more theoretical concepts with a meaningful connection.	**C3** Explanation of events using one or more theoretical concepts with a meaningful connection.	**D3** Description, alternative event, continuation or meta-cognition using one or more theoretical concepts with a meaningful connection.

Generally speaking, *the scoring procedure* came down to dividing the student teachers' reflective notes into 'meaningful units,' on average seven per student teachers, and in total 1740 units. For each unit the nature (A-D) and the level (1-3) were determined. Table 7.2 describes two examples of meaningful units to which score combinations have been (A2 respectively C3) assigned.

Table 7.2 Example of setting scores A2 and C3

Score (combination)	Example of the meaningful unit with an explanation of the score
A2	The suitcase with balls that was put down by 'Black Piet' is used by Minke as a reason to count (in a structured way) with the children. The fragment starts at the moment that the balls are snatched away and are put in transparent cylinders. *Explanation: It is a factual reproduction of a situation, in which one theoretical concept (structured counting) is used.*
C3	The class already comes up with 2 x 5 followed by 3 x 5. Because she visualises the five times table for the children, they can also tell a story to accompany a problem. 1 x 5 will be possible to see as 1 tube times 5 balls. She also makes a connection between concrete material and a grid model. At one point Clayton is counting 10 x 5, the teacher confirms this for the class. There is in fact a transition being made here from multiplication by counting to structured multiplication. *Explanation: the whole text has the character of an explanatory description, with the words 'because,' 'also' and 'in fact' functioning among other things as signal words. Seven concepts are used in connection (five times table, visualises, story to accompany a problem, concrete material, grid model, multiplication by counting and structured multiplication).*

The results of the large scale study (§ 5.4) give an insight into the way in which and the amount of connections student teachers make between theory and practice and the degree to which there is a relationship between the nature and the level of theory use and the student teachers' level of numeracy.
It turned out that nearly all student teachers used theory, but that there were large differences in the way in which they used theory and the amount of theory that they used. Table 7.3 shows the average percentages that student teachers scored per category.

Table 7.3 Average percentages categories A1 to D3

Nature / Level	A	B	C	D
1	12	5	12	7
2	8	4	12	5
3	5	3	18	9

What stands out is the high score for 'explaining' (category C) and the fact that the highest, third level of theory use mainly occurs in combination with 'explaining' and 'responding to situations' and hardly with 'factual description' and 'interpreting.'

Seven hypotheses have been formulated for the research questions into the nature and the level of theory use and the relationship between the two dimensions. The hypotheses were largely confirmed by the study (§ 5.4.2 - § 5.4.4).

One of the conclusions was that nearly 80% of the student teachers dominated in one of the four categories in relation to the nature of theory use. One possible explanation of that dominance might be differences in learning or writing styles between student teachers. A similar dominance could be seen in the level of theory use; 76% of the student teachers were dominant at one of the three levels of theory use.

It also turned out that student teachers used theoretical concepts mainly in explaining situations. These theoretical concepts were mostly general pedagogical concepts.
No relationship could be found between the number of *pedagogical content* concepts and the nature of theory use, including 'explaining.' A positive correlation was found between the number of pedagogical content concepts and level 3.
Factual description and interpretation mainly occurred at the *first* and *second level* of theory use. The *third level* of theory use is mainly related to *explaining* teaching situations and *responding to* situations.

A significant positive correlation has been found between the nature of theory use 'explaining' and the *level of numeracy*, as well as between the level of numeracy and the *third level* of theory use.

Concerning the relationship between the use of theory and the variable *prior education*, the group of student teachers with 'mbo without mathematics' as their prior education stands out. This group of mainly third year student teachers holds an in all respects negative position. This is the case for both the number of theoretical concepts used and the nature and level of theory use. These student teachers' reflections mainly manifest as factual description and interpreting at the lowest level.

In a more general sense, a meaningful relationship existed between the nature and the level of theory use. At the higher levels less *factual description* (category A) and *interpretation* (category B) emerged. On the other hand, *explaining* (category C) appeared to occur more at the highest level. For *responding to* situations (category D) no strong relationship between nature and level was found (§ 5.4.4).

Chapter 6 describes the general conclusions, limitations, suggestions for future research and implications for teacher training colleges. In addition, a local theory for (the learning of) integrating theory and practice is discussed.
Based on the four sub-studies, the following *general conclusion* can be formulated (§ 6.2):

- The use of theory in practical situations could be established unambiguously through the use of the reflection-analysis instrument by determining the *nature* and the *level* of theory use by student teachers.

- The nature is shown in four types of theory use: factual description, interpreting, explaining and 'responding to' situations. The three levels have been defined based on the degree to which theoretical concepts are used meaningfully.
- Nearly all student teachers used theory in the final assessment of their course, and a large number of student teachers dominated on one component for the nature and level of theory use.
- The nature of theory use by student teachers manifested mainly in '*explaining*' situations. The average percentages of the three levels were roughly similar.
- It turned out to be the case that the higher their prior education, the less student teachers used '*factual description*' and '*interpretation*,' and the more they '*explained*.' The student teachers with mbo without mathematics described significantly more *factually* and *explained* less. Student teachers with vwo with mathematics *explained* more.
- The student teachers used significantly more *general* pedagogical concepts in explaining and significantly less in factual descriptions of situations.
- *No relationship* was found between the nature of theory use and the number of *pedagogical content* concepts.
- The first level of theory use occurred mainly with first year student teachers, while level 3 mainly occurred with second and third year student teachers, and those with a higher level of prior education.
- Student teachers who used more theoretical concepts, reflected at a higher level and vice versa. A significant *positive relationship* was found between the number of *pedagogical content* concepts and *level 3* in the final assessment and *no relationship* between the same variables in the *initial assessment.*
- A meaningful relationship was found between nature and level of theory use. The characteristics of factual description and interpreting for the nature of theory use mostly occurred at the first and second levels, while explaining and – to a lesser degree – responding to situations were mostly connected with the third level of theory use.
- Reasoning leading to a *rise in level* of theory use were observed especially during the interactions led by the teacher educator and during interviews. In a few instances that rise in level could be interpreted as 'vertical didacticizing.'
- A strong relationship was found between the category 'explaining' for the nature of theory use and numeracy, and slightly less strong between level 3 of theory use and numeracy. Another strong relationship was found between the level of numeracy and the level of prior education.
- Student teachers who participated in the study appreciated the learning environment aimed at integrating theory and practice.

The results of the study and the analysis of the student teachers' activities aimed at the use of theory, provided the basis for reflection in relation to a *local theory for (learning) integrating theory and practice by student teachers* (§ 6.3).
The core of the theory is that student teachers learn to integrate theory and practice in a learning environment that invites the use of theory. The process of learning to integrate is supported by teaching materials and targeted interventions by the teacher educator. One aim of that support is finding a connection to student teachers' existing knowledge network, as well as stimulating a rise in level for reasoning about practical situations. The level character of the theory is expressed in the nature and the level of theory use.
The process of learning to integrate theory and practice leads increasingly to the gaining of 'theory-enriched practical knowledge.'
The local theory as described, consists of three main components, namely the *formulated concepts* of theory, practice and the relationship between theory and practice, *the theoretical knowledge base of the learning environment* for student teachers and the *guidelines for teacher educators* to support student teachers' learning and development processes. The theory-laden practice narratives, the multifunctional lists of concepts, the varied and practice-oriented activities for the student teachers, the input of the teacher educator and the reflection-analysis instrument are essential components for the elaboration of that theory in the curriculum of the teacher training course.

To a certain extent this study was limited by choices that were made (§ 6.4). One example is the context in which the study occured. It was not their own teaching practice that was at the centre of the study, but 'practice' for the student teachers consisted mainly of practice situations that were represented in multimedia form. Despite all the advantages of the multimedia practice, the question remains whether situations from the student teachers' own practice as an object of discussion and reflection would not lead to a better insight into making relations between theory and practice. It is particularly the real practice of teaching where student teachers can become aware of theory as a necessary instrument for reflection on their thinking and actions, with as its goal understanding and adequate response to situations.
A second example of limitations of this study was the collection of data for the large scale study. The nature of this collection, mainly consisting of reflective notes, may possibly have limited the insight into some aspects of the use of theory.

Further research is necessary (§ 6.5), partly to counter the limitations mentioned above. Short term research may focus on the field placement of (student) teachers. Long term research is desirable to achieve an insight into the use of theory – consciously or subconsciously – of beginning and experienced teachers in daily practice and the effect it has on the quality of teaching.

Considering the various approaches from which the data would have to be analyzed, it would be desirable for such a study to be led by an interdisciplinary research team of pedagogues and specialists in the pedagogy of mathematics and language.

The reflection-analysis instrument that was developed in this study, offers the opportunity to analyze reflections systematically and in detail to other areas of teacher training than mathematics and pedagogy alone. The instrument can support teacher educators and student teachers in assessing or judging the quality of theory-enriched practical knowledge. The instrument may also be simplified by limiting it to the vertical dimension for the description of the level of theory use.
A combination of small and large scale studies is recommended for any further research. The use of various types of data or data sources from both studies can lead to deeper, coherent analyses, which will, as a result of the possibility to have more nuances within the data system, be more consistent and cogent than the analysis of data from individual studies. The rise in levels of theory use of student teachers in this study is an example of this.

In addition this study provides possible directions for the design of the curriculum for teacher education (§ 6.6). Primarily, this study shows that multimedia in student teachers' learning environment may serve a useful purpose, particularly where learning to integrate theory and practice by student teachers is concerned. A multimedia learning environment offers student teachers the chance to concentrate on studying practical knowledge, outside the pressure and complexity of their own practice class. The 'communally experienced' practice can be observed and studied individually or with a team.
Multimedia, for instance video images that could be recorded by student teachers or teacher educators themselves, can be used within teacher training in four stages of learning and research activities.

The conclusion was drawn within all (sub)studies of this study that *the input of the teacher educator is crucial* for the quality of student teacher activities. Like no other the teacher educator is able to make (hidden) practical knowledge explicit and to enrich it with theory. For example, rises in level in the reasoning of student teachers were observed almost exclusively in discussions that were led by the teacher trainer or in interviews with the researcher.
The *multifunctional lists of concepts* was useful to both student teachers and their educators. During the whole of the course, the lists supported awareness of the progress in student teacher learning processes.

The reflection-analysis instrument can be used to assess or evaluate the use of theory by student teachers.

A second direction for the teacher training curriculum is derived from the tendency, found in the study, of student teachers to use general pedagogical concepts more than pedagogical content concepts. This underlines the importance of *more attention to* the use of *domain-specific instruction theory* and particularly the importance of giving meaning to general pedagogical concepts within the context of a domain-specific subject.

A third direction concerns attention to student teachers with 'mbo without mathematics' as their prior education as well as for student teachers with prior education 'vwo with mathematics.' For different reasons, the results of this study point out the need for extra pedagogical measures for both groups of student teachers.

Finally, the anonymous questionnaires show that the student teachers appreciated the learning environment in which they had the opportunity to gain their practical knowledge, and that they appreciated in particular the theory that had been integrated into these practical situations as a support of their own practice. They believe that the learning environment makes it clear that you need theory, and that theory helps in understanding practice and guiding your students.

8 Samenvatting

Dit onderzoek richt zich op de theorie-praktijkproblematiek in de opleiding voor leraren basisonderwijs, toegespitst op het vak rekenen-wiskunde en didactiek.
Het gaat daarbij vooral om de vraag op welke wijze studenten theorie en praktijk kunnen integreren en hoe de inrichting van hun leeromgeving daaraan kan bijdragen. Er is nog weinig bekend omtrent de kennisverwerving door studenten van de lerarenopleiding en al evenmin over de wijze waarop studenten theoretische kennis en praktijksituaties met elkaar in verband brengen, beide cruciale componenten van het leren onderwijzen.

Hoofdstuk 1 van het proefschrift geeft een korte beschrijving van achtergrond, context, probleemstelling, onderzoeksvragen, de relevantie en het karakter van het onderzoek.
Het doel van het onderzoek is inzicht te krijgen in de wijze waarop studenten theorie en praktijk verbinden en in welke mate en op welk niveau zij in staat zijn theorie te gebruiken in onderwijssituaties.
Het onderzoek bestond uit vier deelonderzoeken, te weten twee exploratieve onderzoeken, een kleinschalig onderzoek en een grootschalig onderzoek, elk met hun eigen functie. Telkens leverde de opbrengst van het voorgaande onderzoek de middelen voor het volgende onderzoek, met meer verfijnde onderzoeksvragen en een meer adequaat ontwerp van de leeromgeving voor de studenten die aan het onderzoek deelnamen. Het multimediale karakter van de leeromgeving maakte het mogelijk het gebruik van theorie te optimaliseren.
De eerste drie onderzoeken verschaften gaandeweg nieuw inzicht in het theoriegebruik. Dat inzicht werd in het kleinschalige onderzoek benoemd als de *aard* en het *niveau* van theoriegebruik door studenten. De categorisering van de aard en het niveau leverde het gereedschap voor het ontwikkelen van een reflectie-analyse instrument ten behoeve van het grootschalige onderzoek.

Hoofdstuk 2 beschrijft het theoretisch kader van het onderzoek.
De noodzaak van het leren integreren van theorie en praktijk door aanstaande leraren wordt allerwegen onderkend. Toch is er nog weinig bekend over het karakter van dat integratieproces. Al vele jaren leeft in opleidingskringen het besef dat het voorschrijvend ‘overdragen’ van theorie niet leidt tot de gewenste integratie en bovendien dat de theorie vaak onvoldoende is afgestemd op de realiteit en de complexiteit van het handelen in de praktijk.
Opleidingen verschillen – ook mondiaal – sterk in de manier waarop zij in hun curricula de integratie van theorie en praktijk proberen vorm te geven. Van de richtingen die daarin kunnen worden onderscheiden, ligt de laatste jaren het accent op de zogenaamde ‘reflective practice’-oriëntatie en op de ‘development of professional knowledge’-oriëntatie (§ 2.2).

De laatste richting lijkt veelbelovend, vooral door de aandacht voor het gelijktijdig en geïntegreerd gebruik van theoretische kennis en praktijkkennis door aanstaande leraren. Dat biedt echter nog geen antwoord op de vraag op welke wijze de integratie van de verschillende elementen van de kennisbasis van aanstaande leraren tot stand komt en hoe die integratie kan worden bevorderd. Een barrière bij het zoeken naar een opleidingsdidactische aanpak is het gemis aan eenduidigheid over de concepten theorie en praktijk. Er is een verscheidenheid aan opvattingen in de onderzoeksliteratuur. In § 2.3 en § 2.4 van dit proefschrift wordt gepoogd die verscheidenheid in kaart te brengen en de voor het onderzoek noodzakelijke keuzen te verantwoorden. In dat keuzeproces speelt het concept 'praktijkkennis' (teacher practical knowledge) een essentiële rol (§ 2.3.4). Dit geldt in het bijzonder voor het daarvan afgeleide concept 'met theorie verrijkte praktijkkennis,' dat in de loop van dit onderzoek is ontwikkeld. Onderstaand citaat toont 'met theorie verrijkte praktijkkennis' van studente Anne als zij reflecteert op de uitkomsten van het onderzoek naar vermenigvuldigstrategieën dat zij in haar stagepraktijk heeft uitgevoerd (§ 4.3.6). Zij beschrijft de praktijk met behulp van theoretische begrippen (vermenigvuldigstrategieën, memoriseren, automatiseren, steunsommen) en gebruikt die begrippen betekenisvol en in onderlinge samenhang.

> De verschillende strategieën blijken toch vrij ingewikkeld te zijn voor sommige rekenaars. Ze hebben moeite met het kiezen van de juiste steunsom of de juiste grootte van de som (...). Strategieën zijn nodig om de sommen te automatiseren en memoriseren. Andersom kun je ook zeggen dat gememoriseerde sommen nodig zijn bij de strategieën. Denk aan de steunsommen. Door middel van sommen die ze al weten kunnen ze andere sommen uitrekenen. Strategieën, automatiseren en memoriseren zijn onlosmakelijk met elkaar verbonden. Het gebruik van strategieën komt niet alleen voor bij het vermenigvuldigen, maar bij alle andere onderdelen van het reken- en wiskundeonderwijs. Een andere reden om strategieën aan te bieden is de controle mogelijkheid. Je komt in de praktijk kinderen tegen die bepaalde sommen verkeerd hebben gememoriseerd. Door sommen uit te rekenen m.b.v. de strategieën kun je de antwoorden controleren. Mits de strategieën goed gebruikt worden, wat voor zwakkere rekenaars soms moeilijk is.

De historie van de Nederlandse opleiding voor leraren basisonderwijs in het vak rekenen-wiskunde en didactiek, in het bijzonder die van ná 1970, laat zien hoe de verhouding tussen theorie en praktijk zich in deze opleiding heeft ontwikkeld (§ 2.5). Met de oprichting van het Instituut voor ontwikkeling van het wiskundeonderwijs (IOWO) in 1971, startte in Nederland een nieuwe ontwikkeling van het reken- en wiskundeonderwijs in het primair en voortgezet onderwijs. Die ontwikkeling had grote invloed op de verwante lerarenopleidingen.
In dit onderzoek is deze invloed zichtbaar in het ontwerp van de leeromgevingen van studenten die aan de verschillende onderzoeken deelnamen.

De ontwikkeling van de leeromgevingen voltrok zich als een 'design research proces' in vier fasen, in elk waarvan de drie componenten van het uitvoeren van een designproject herkend kunnen worden, namelijk voorbereiden, uitproberen in een groep en het uitvoeren van een retrospectieve analyse (§ 2.7). Aandachtspunten voor het gebruik van theorie in de opleiding (§ 2.6) fungeerden als referentiekader voor het ontwikkelen en verfijnen van met name de leeromgeving voor het klein- en grootschalige onderzoek.
Het multimediale karakter van de leeromgevingen maakte het mogelijk om videobeelden van praktijksituaties in te zetten als object van leer- en onderzoeksactiviteiten voor studenten. In deze 'multimediale praktijk' konden zij zich individueel, maar ook gezamenlijk, concentreren op voor hen geselecteerde onderwijssituaties. Opdrachten voor de eigen stagepraktijk maakten deel uit van de leeromgeving. Ten behoeve van de twee exploratieve onderzoeken is de Multimediale Interactieve Leeromgeving – MILE – ingezet, waarvan de ontwikkeling beschreven is in § 3.3 van dit proefschrift. In het klein- en grootschalige onderzoek werd de CD-rom 'Gids voor rekenen/wiskunde' (§ 4.2.2.2) gebruikt als onderdeel van de leeromgeving voor studenten.

Hoofdstuk 3 presenteert de beide exploratieve onderzoeken in de context van de ontwikkeling van het MILE-project.

Het eerste exploratieve onderzoek (§ 3.5) speelde zich af in een leeromgeving die bestond uit tien lessen van groep 4 op CD-roms, een beschrijving van die lessen, de bij de lessen behorende werkboeken en de eerste versie van de zoekmachine van MILE.
Doel van het onderzoek was inzicht te krijgen in het karakter van het leerproces van studenten die in MILE op onderzoek gingen en in de opbrengst van hun leerproces in termen van kennisconstructie.
Het leer- en onderzoeksproces van de twee aan het onderzoek deelnemende studenten manifesteerde zich als een cyclisch proces van plannen, zoeken, observeren, reflecteren en evalueren. Het onderzoek gaf verder een beeld van de niveaus waarop de studenten hun kennis construeerden. Relaties tussen theorie en praktijkrelaties werden gelegd in de discussies onder leiding van de participerende opleider-onderzoeker, door schriftelijke reflectie op de discussies en in een later stadium op basis van vakliteratuur.
Met name op het als zodanig benoemde derde en vierde niveau van de geconstateerde kennisconstructie (§ 3.5.5), was er sprake van integreren van theorie en praktijk, op momenten dat studenten zichzelf vragen stelden over geobserveerde situaties, wanneer zij verband legden met de literatuur of wanneer zij een eigen 'lokale theorie' formuleerden.

In het tweede exploratieve onderzoek (§ 3.8) werd de MILE-omgeving uitgebreid met lessen uit diverse groepen van de basisschool en een geavanceerde zoekmachine waarmee studenten de lessen en additionele materialen konden doorzoeken.

De twee groepen van 25 studenten die aan het onderzoek deelnamen kregen verder de beschikking over een lijst met 150 theoretische begrippen. Die gaf hen de mogelijkheid om hun theoretische voorkennis in te schatten bij het starten van 'Het Fundament,' een nieuwe leergang voor MILE. De leergang bestond uit tien bijeenkomsten van twee uur onder leiding van de opleider met na elke bijeenkomst groepswerk en zelfstudie. De studenten hadden de beschikking over een werkboek met leer- en onderzoeksopdrachten voor het werken met MILE.
Dit exploratieve onderzoek beoogde het inventariseren van 'signalen van theoriegebruik' die studenten toonden in hun reflecties op hun studie van praktijksituaties in MILE.
Theorie-praktijkrelaties werden gelegd tijdens discussies onder leiding van de opleiders, in het groepswerk, tijdens zelfstudie en in de mondelinge en schriftelijke verslaggeving van opdrachten die studenten moesten uitvoeren.
De onderzoeksdata werden verkregen door observatie van acht studenten tijdens de gezamenlijke bijeenkomsten, door een participerende studie van groepswerk en door interviews.
Uit de analyse van de resultaten aan de hand van vijftien geformuleerde 'signalen van theoriegebruik' bleek dat studenten alleen in de situaties waarin de opleider participeerde, uitstegen boven het niveau van reageren in termen van 'practical wisdom'. Dat leidde tot de conclusie dat de leeromgeving van de studenten geoptimaliseerd zou moeten worden met betrekking tot de mogelijkheid om de in MILE aanwezige praktijkkennis te leren expliciteren en theoretisch te verrijken.

Hoofdstuk 4 beschrijft het kleinschalige onderzoek.
De leeromgeving van het kleinschalige onderzoek werd aangepast op basis van de conclusies uit het tweede exploratieve onderzoek. De elementen die aan de leeromgeving werden toegevoegd, waren bedoeld om de studenten uit te lokken de in de multimediale praktijksituaties aanwezige theoriegeladen praktijkkennis te expliciteren en te analyseren, teneinde constructie van 'met theorie verrijkte praktijkkennis' mogelijk te maken. Voorbeelden van dergelijke elementen waren:

- de CD-rom 'Gids voor rekenen/wiskunde' (§ 4.2.2.2), ingezet als leer- en zelfstudiemateriaal met praktijkverhalen, voorzien van theoretische reflecties en literatuur;
- discussies, onder andere op basis van in groepswerk geformuleerde stellingen;
- een 'begrippenspel,' bestaande uit vier praktijksituaties (video) en voor iedere student zes verschillend gekleurde kaartjes (begrippen) en een antwoordformulier;
- onderzoek in de eigen stagepraktijk;
- het schrijven van 'geannoteerde verhalen' en reflectieve notities.

Evenals in het tweede exploratieve onderzoek hadden de studenten de beschikking over

een lijst met theoretische begrippen. Die kreeg nu echter niet alleen een functie als beginpeiling, maar speelde in de gehele leergang van de studenten een prominente rol.

De onderzoeksvraag van het kleinschalige onderzoek (§ 4.1) richtte zich op de manier waarop en de mate waarin studenten theorie gebruikten als zij praktijksituaties beschreven na afloop van de leergang die zij volgden.
De veronderstelling was dat de vernieuwde leeromgeving de studenten in staat zou stellen gevarieerd te redeneren over praktijksituaties en dat er aanwijsbare verschillen zouden zijn in de diepgang van hun theoriegebruik. Het onderzoek was erop gericht die gevarieerdheid en diepgang in kaart te brengen. Twee groepen van zes, respectievelijk acht studenten gingen op vrijwillige basis onder leiding van twee ervaren opleiders in de leeromgeving aan de slag gedurende vijf bijeenkomsten van anderhalf uur. Voorafgaand aan die bijeenkomsten werden enkele componenten van de leeromgeving (CD-rom, begrippenlijst, stellingen) uitgeprobeerd met vier groepen van in totaal 63 tweedejaarsstudenten. Vervolgens ontwikkelde de onderzoeker een eerste versie van de leergang voor de veertien aan het kleinschalige onderzoek deelnemende studenten. Na elke bijeenkomst vond een nabespreking plaats en deed de onderzoeker voorstellen voor het vervolg op basis van (video-)observaties en de inbreng van de opleiders. De onderzoeksdata van de veertien studenten vormden het bronnenmateriaal voor het beschrijven van een casus over het leerproces van studente Anne (§ 4.3).

Overwegingen op basis van de resultaten en ervaringen van de voorgaande onderzoeken en van onderzoeksliteratuur, inspireerden tot het ontwikkelen en uitproberen van de *eerste versie van het reflectie-analyse instrument.*
Anders dan met de de verzameling van vijftien 'signalen van theoriegebruik' die in het tweede exploratieve onderzoek tot stand was gekomen, werd het met dit instrument mogelijk het gebruik van theorie door Anne en haar medestudenten te *vergelijken* (§ 4.4).
Het onderzoek wees uit dat alle studenten theorie gebruikten in hun mondelinge en schriftelijke reacties op praktijksituaties. De verschillen in zowel *de manier waarop* theorie gebruikt werd in de beschrijvingen als de inzet van *het aantal theoretische begrippen* waren relatief groot. Het verschil in *niveau* waarop studenten theoretische begrippen gebruikten bij hun reflectie op praktijksituaties, kon onderscheiden worden op basis van het vermogen van studenten om betekenisvolle relaties tussen theoretisch begrippen te leggen. Vooral tijdens de interactie onder leiding van de opleider en tijdens interviews werden redeneringen geobserveerd die leidden tot niveauverhoging in het gebruik van theorie (§ 4.4.2).

Het onderzoek naar *de relatie tussen het theoriegebruik en het niveau van gecijferdheid*, ook onderdeel van het kleinschalige onderzoek, versterkte het vermoeden van een positief verband tussen beide variabelen (§ 4.4.1).

Verder kon de opbrengst van de anonieme vragenlijst, aan de studenten voorgelegd als evaluatieformulier na afloop van de serie bijeenkomsten, geïnterpreteerd worden als een appreciatie van theorie door studenten en van de geïntegreerde aanpak in het aanbod van theorie en praktijk in de leeromgeving van de studenten.

Hoofdstuk 5 beschrijft het grootschalige onderzoek onder 269 studenten.
Het onderzoek werd mede op basis van de resultaten van het kleinschalige onderzoek ingericht. In het bijzonder werden de onderzoeksvragen verfijnd en de leeromgeving en het onderzoeksinstrumentarium aangepast.

De onderzoeksvragen van het grootschalige onderzoek zijn als volgt geformuleerd:

1. Op welke wijze gebruiken aanstaande leraren theoretische kennis wanneer zij praktijksituaties beschrijven na afloop van een periode die zij hebben doorgebracht in een leeromgeving die uitnodigt tot het gebruik van theorie?
2. Wat is het theoretische gehalte van uitspraken van aanstaande leraren wanneer zij praktijksituaties beschrijven?
3. a. Is er een betekenisvol verband tussen de aard en het niveau van theoriegebruik? Zo ja, hoe komt die relatie tot uitdrukking in de verschillende componenten van het theoriegebruik en bij verschillende groepen studenten?
   b. In hoeverre is er verband tussen de aard en het niveau van theoriegebruik en de eigen vaardigheid c.q. gecijferdheid van studenten?

De aanpassingen van de leeromgeving ten behoeve van het grootschalige onderzoek betroffen vooral de beginpeiling, de eindpeiling en de peiling gecijferdheid. Verder werd een handboek voor opleiders ontworpen, met een gedetailleerde beschrijving van de leergang voor studenten, voorzien van richtlijnen voor de inhoud en organisatie van de bijeenkomsten, aanwijzingen voor succesvol gebleken interventies, criteria voor goede leervragen en tips voor het stimuleren van theoriegebruik en niveauverhoging.
Algemeen kenmerk van de curriculumontwikkeling voor de leergang van de studenten was de meervoudige inbedding van theorie (intrinsiek, extrinsiek; § 2.6.4.) en het streven naar een balans tussen inhoudelijke componenten, alsmede tussen zelfsturing en begeleiding door de opleider en tussen de stagepraktijk van de studenten en de gewenste professionele praktijk van de opleidingen (§ 2.7.1).

De data-analyse van het kleinschalige onderzoek verschafte verder nieuw inzicht in het theoriegebruik van studenten. Het bleek dat twee dimensies onderscheiden konden worden, namelijk *de aard* en *het niveau* van theoriegebruik. Anders dan de ‘signalen van theoriegebruik’ die in het tweede exploratieve onderzoek werden ontwikkeld, kon met vier categorieën voor de aard, gecombineerd met drie categorieën voor het niveau (tabel 8.1), het theoriegebruik eenduidig en systematisch worden bepaald.
Met de twaalf (4 x 3) categorieën als fundament, werd het reflectie-analyse instrument

verder ontwikkeld, gevalideerd en op betrouwbaarheid getoetst (§ 5.3.6).
Tabel 8.1 geeft een overzicht van de twaalf categorieën voor de aard en het niveau van theoriegebruik.

Tabel 8.1 Reflectie-analyse instrument. Beknopte beschrijving van de twaalf score-combinaties, met horizontaal de indeling in aard van theoriegebruik en verticaal het niveau van theoriegebruik

	A **Feitelijk weergeven** feiten: wie, wat, waar, hoe.	**B** **Interpreteren** o.a. opinie of conclusie zonder onderbouwing.	**C** **Verklaren** o.a. 'uitleg van het waarom'.	**D** **Inspelen op** o.a. anticiperen, vervolg of alternatief ontwerpen, metacognitieve reacties.
Niveau 1	**A1** Feitelijke beschrijving van gebeurtenissen zonder gebruik van theoretische begrippen.	**B1** Interpretatie van gebeurtenissen zonder gebruik van theoretische begrippen.	**C1** Verklaring van gebeurtenissen zonder gebruik van theoretische begrippen.	**D1** Beschrijving alternatieve gebeurtenis, vervolg of metacognitie zonder gebruik van theoretische begrippen.
Niveau 2	**A2** Feitelijke beschrijving van gebeurtenissen met gebruik van één of meer theoretische begrippen, zonder onderlinge samenhang.	**B2** Interpretatie van gebeurtenissen met gebruik van één of meer theoretische begrippen, zonder onderlinge samenhang.	**C2** Verklaring van gebeurtenissen met gebruik van één of meer theoretische begrippen, zonder onderlinge samenhang.	**D2** Beschrijving alternatieve gebeurtenis, vervolg of metacognitie met gebruik van één of meer theoretische begrippen zonder onderlinge samenhang.
Niveau 3	**A3** Feitelijke beschrijving van gebeurtenissen met gebruik van twee of meer theoretische begrippen in betekenisvolle samenhang.	**B3** Interpretatie van gebeurtenissen met gebruik van twee of meer theoretische begrippen in betekenisvolle samenhang.	**C3** Verklaring van gebeurtenissen met gebruik van twee of meer theoretische begrippen in betekenisvolle samenhang.	**D3** Beschrijving alternatieve gebeurtenis, vervolg of metacognitie met gebruik van twee of meer theoretische begrippen in betekenisvolle samenhang.

De scoreprocedure kwam er in grote lijnen op neer, dat de reflectieve notities van de studenten in 'betekenisvolle eenheden' werden verdeeld, gemiddeld zeven per student, in totaal 1740 eenheden. Per eenheid werd de aard (A-D) en het niveau (1-3) vastgesteld.

In tabel 8.2 zijn twee voorbeelden beschreven van betekenisvolle eenheden waaraan een scorecombinatie (A2, respectievelijk C3) is toegekend.

Tabel 8.2 Voorbeeld van toekenning scores A2 en C3

Score (combinatie)	**Voorbeeld van betekenisvolle eenheid met toelichting bij de score**
A2	De koffer met ballen die door zwarte piet is neergezet, gebruikt Minke als aanleiding om met de kinderen (gestructureerd) te gaan tellen. Het fragment begint op het moment dat de ballen uit de koffer worden gehaald en in kokers worden gestopt. *Toelichting: Het is de feitelijke weergave van een situatie waarbij één theoretisch vakdidactisch begrip (gestructureerd tellen)wordt gebruikt.*
C3	Uit de klas komt al 2 x 5 en vervolgens 3 x 5. Doordat ze de tafel van vijf visualiseert voor de kinderen kunnen zij ook een verhaal bij een som vertellen. 1 x 5 zal om te zetten zijn in 1 koker keer 5 ballen. Tevens maakt ze een verbinding tussen concreet materiaal en een roostermodel. Clayton telt op een gegeven moment 10 x 5, de juf bevestigt dit naar de klas toe. Er wordt hier eigenlijk een overgang van het tellend vermenigvuldigen naar het structurerend vermenigvuldigen gemaakt. *Toelichting: De gehele tekst ademt het karakter van een verklarende beschrijving, de woorden 'doordat,' 'tevens' en 'eigenlijk' fungeren o.a. als signaalwoorden. Een zevental begrippen wordt in samenhang gebruikt (tafel van vijf, visualiseert, verhaal bij som, concreet materiaal, roostermodel, tellend vermenigvuldigen en structurerend vermeningvuldigen).*

De resultaten van het grootschalige onderzoek (§ 5.4) geven inzicht in de manier waarop en de mate waarin pabostudenten verband leggen tussen theorie en praktijk en in hoeverre er een verband bestaat tussen de aard en het niveau van theoriegebruik en het niveau van gecijferdheid van studenten.
Het bleek dat vrijwel alle studenten theorie gebruikten, maar dat er grote verschillen waren in de manier waarop en de mate waarin dat gebeurde. Tabel 8.3 geeft de gemiddelde percentages weer die de studenten per categorie scoorden.

Tabel 8.3 Gemiddelde percentages categorieën A1 tot en met D3

Aard / **Niveau**	**A**	**B**	**C**	**D**
1	12	5	12	7
2	8	4	12	5
3	5	3	18	9

Wat opvalt is de hoge score van ‘verklaren’ (categorie C) en het feit dat het hoogste, derde niveau van theoriegebruik vooral voorkomt bij ‘verklaren’ en ‘inspelen op situaties’ en nauwelijks bij ‘feitelijk weergeven’ en ‘interpreteren’.

Er zijn zeven hypothesen gesteld bij de onderzoeksvragen naar de aard en het niveau van theoriegebruik en naar het verband tussen die beide dimensies. De hypothesen zijn grotendeels bevestigd door het onderzoek.
Er werd onder meer vastgesteld dat bijna 80% van de studenten domineerde op één van de vier categorieën met betrekking tot de aard van theoriegebruik. Een mogelijke verklaring voor die dominantie is het verschil in leer- of schrijfstijl tussen studenten. Ook voor het niveau van theoriegebruik manifesteerde zich een soortgelijke dominantie; 76% van de studenten domineerde op één van de drie niveaus van theoriegebruik.

Verder bleek dat studenten theoretische begrippen vooral gebruikten bij het verklaren van situaties. Dat waren dan voornamelijk algemeen-didactische begrippen.
Tussen het aantal *vakdidactische* begrippen en de aard van theoriegebruik is geen verband geconstateerd, ook niet voor ‘verklaren’. Er is wel een positieve correlatie vastgesteld tussen het aantal vakdidactische begrippen en niveau drie.
Feitelijk weergeven en *interpreteren* speelde zich vooral af op het *eerste en tweede niveau* van theoriegebruik. *Het derde niveau* van theoriegebruik is voornamelijk gerelateerd aan *verklaren* van onderwijssituaties en *inspelen op* situaties.

Er is een significant positieve correlatie geconstateerd tussen de aard van theoriegebruik ‘*verklaren*’ en *het niveau van gecijferdheid*, evenals tussen het niveau van gecijferdheid en het *derde niveau* van theoriegebruik.

Wat het verband tussen het theoriegebruik en de variable *vooropleiding* betreft, valt vooral de groep studenten met vooropleiding ‘mbo zonder wiskunde’ op. Deze groep van voornamelijk derdejaarsstudenten neemt een in alle opzichten significant negatieve uitzonderingspositie in. Dat geldt voor zowel het aantal theoretische begrippen dat werd gebruikt als voor de aard en het niveau van theoriegebruik. Het reflecteren van deze studenten manifesteerde zich voornamelijk als feitelijk weergeven en interpreteren op het laagste niveau.

In meer algemene zin werd er een betekenisvol verband tussen de aard en het niveau van theoriegebruik aangetoond. Er wordt minder *feitelijk weergegeven* (categorie A) en *geïnterpreteerd* (categorie B) naar mate het niveau hoger wordt. Bij *verklaren* (categorie C) is het omgekeerd: er wordt meer verklaard naar mate het niveau hoger wordt. Voor *inspelen* op situaties (categorie D) is geen sterk verband tussen aard en niveau geconstateerd (§ 5.4.4).

Hoofdstuk 6 beschrijft de algemene conclusies, de beperkingen, suggesties voor vervolgonderzoek en implicaties voor de lerarenopleiding. Verder wordt er een lokale theorie voor het (leren) integreren van theorie en praktijk besproken. Op basis van de vier deelonderzoeken, kunnen de volgende *algemene conclusies* worden geformuleerd (§ 6.2):

- Het gebruik van theorie in praktijksituaties kon met behulp van het reflectie-analyse instrument eenduidig worden bepaald door het vaststellen van de *aard* en het *niveau* van theoriegebruik door studenten.
- De aard is zichtbaar in vier soorten van theoriegebruik: feitelijk weergeven, interpreteren, verklaren en 'inspelen op' situaties. De drie niveaus zijn gedefiniëerd naar de mate waarin theoretische begrippen betekenisvol worden gebruikt.
- Nagenoeg alle studenten gebruikten theorie in de eindpeiling van de leergang die zij volgden en een groot aantal studenten domineerde op één van de componenten voor de aard en het niveau van theoriegebruik.
- De aard van het theoriegebruik van studenten manifesteerde zich vooral in het '*verklaren*' van situaties. De gemiddelde percentages van de drie niveaus waren ongeveer gelijk.
- Het bleek dat studenten minder 'feitelijk weergaven' en 'interpreteerden' en meer 'verklaarden' naarmate hun vooropleiding hoger was. Voor de studenten met vooropleiding mbo zonder wiskunde gold dat zij significant *meer feitelijk weergaven* en *minder verklaarden*, voor de studenten met vooropleiding vwo met wiskunde dat zij *meer verklaarden*.
- De studenten gebruikten significant meer *algemeen* didactische begrippen bij het verklaren en significant minder bij het feitelijk weergeven van situaties.
- Er is *geen verband* geconstateerd tussen de aard van theoriegebruik en het aantal gebruikte *vakdidactische* begrippen.
- Het eerste niveau van theoriegebruik kwam vooral voor bij eerstejaarsstudenten, terwijl niveau drie zich vooral manifesteerde bij tweede- en derdejaarsstudenten of bij studenten met een hogere vooropleiding.
- Studenten die meer theoretische begrippen gebruikten, reflecteerden op een hoger niveau en omgekeerd. Er werd een significant *positief verband* geconstateerd tussen het aantal *vakdidactische* begrippen en *niveau drie* in de *eindpeiling* en *geen verband* tussen diezelfde variabelen in de *beginpeiling*.
- Er is een betekenisvol verband vastgesteld tussen de aard en het niveau van theoriegebruik. De karakteristieken feitelijk weergeven en interpreteren voor de aard van het theoriegebruik speelden zich vooral af op het eerste en tweede niveau van theoriegebruik, terwijl verklaren en – in mindere mate – inspelen op situaties, vooral gerelateerd waren aan het derde niveau van theoriegebruik.
- Vooral tijdens de interactie onder leiding van de opleider en tijdens interviews

werden redeneringen geobserveerd die leidden tot *niveauverhoging* in het gebruik van theorie. In enkele gevallen kon die niveauverhoging geïnterpreteerd worden als verticaal didactiseren.
- Er is een sterk verband geconstateerd tussen de categorie 'verklaren' voor de aard van het theoriegebruik en gecijferdheid en in mindere mate tussen niveau drie van theoriegebruik en gecijferdheid. Ook is een sterk verband vastgesteld tussen het niveau van gecijferdheid en het niveau van de vooropleiding.
- De aan het onderzoek deelnemende studenten appreciëerden de op integratie van theorie en praktijk gerichte leeromgeving die hen werd aangeboden.

De resultaten van het onderzoek en de analyse van de op theoriegebruik gerichte activiteiten van studenten, vormden de basis voor de gedachtevorming met betrekking tot een *lokale theorie voor het (leren) integreren van theorie en praktijk door aanstaande leraren* (§ 6.3).
Kern van de theorie is dat studenten theorie en praktijk leren integreren in een leeromgeving die uitnodigt tot het gebruik van theorie. Het proces van leren integreren wordt ondersteund met onderwijsmiddelen en doelgerichte interventies van de opleider. Die ondersteuning richt zich onder meer op het zoeken naar aansluiting bij het bestaande kennisnetwerk van studenten en op het stimuleren van niveauverhoging in het redeneren over praktijksituaties. Het niveaukarakter van de theorie komt tot uiting in de aard en het niveau van theoriegebruik.
Het proces van leren integreren van theorie en praktijk leidt in toenemende mate tot het verwerven van 'met theorie verrijkte praktijkkennis'.
De lokale theorie zoals beschreven, bestaat uit drie hoofdcomponenten, namelijk de *geformuleerde concepten* van theorie, praktijk en de relatie tussen theorie en praktijk, *de theoretische kennisbasis van de leeromgeving* voor studenten en de *richtlijnen voor opleiders* ten behoeve van het ondersteunen van leer- en ontwikkelingsprocessen van studenten. De theoriegeladen praktijkverhalen, de multifunctionele begrippenlijsten, de gevarieerde en op de praktijk gerichte activiteiten voor de studenten, de inbreng van de opleider en het reflectie-analyse instrument zijn essentiële componenten voor de uitwerking van die theorie in het curriculum van de opleiding.

Dit onderzoek werd tot op zekere hoogte beperkt door omstandigheden of door keuzen die zijn gemaakt (§ 6.4). Een voorbeeld daarvan is de context waarin het onderzoek plaats vond. Niet de eigen stagepraktijk stond centraal in het onderzoek, maar 'de praktijk' bestond voor de studenten voornamelijk uit praktijksituaties die in multimediale vorm waren weergegeven. Ondanks alle voordelen van de multimediaal weergegeven praktijk is het de vraag of situaties uit de eigen stagepraktijk als object van discussie en reflectie niet tot een beter inzicht in het leggen van relaties tussen theorie

en praktijk leiden. Het is vooral de reële onderwijspraktijk waar studenten zich bewust kunnen worden van theorie als noodzakelijk instrument voor reflectie op hun denken en handelen, met als doel het begrijpen en adequaat inspelen op situaties.
Een tweede voorbeeld van beperkingen van dit onderzoek vormde de dataverzameling van het grootschalige onderzoek. De aard van deze verzameling, bestaande uit reflectieve notities, heeft mogelijkerwijs het zicht beperkt op enkele aspecten van theoriegebruik.

Vervolgonderzoek is noodzakelijk (§ 6.5), onder andere om de hiervoor genoemde beperkingen op te heffen. Onderzoek op korte termijn kan zich richten op de stagewerkplek van (aanstaande) leraren. Langetermijnonderzoek is gewenst om inzicht te verkrijgen in het theoriegebruik – bewust of onbewust – van beginnende en ervaren leraren in de praktijk van alledag en de invloed daarvan op de kwaliteit van het onderwijs.
Gezien de verschillende invalshoeken van waaruit de data moeten worden geanalyseerd, zou het wenselijk zijn dat zo'n onderzoek wordt geleid door een interdisciplinair onderzoeksteam van onderwijskundigen en vakdidactici op het gebied van rekenen-wiskunde en taal.

Het analyse instrument dat in dit onderzoek is ontwikkeld, biedt ook voor andere opleidingsvakgebieden dan rekenen-wiskunde en didactiek de mogelijkheid om reflecties systematisch en genuanceerd te analyseren. Het instrument kan opleiders en studenten ondersteuning bieden bij het peilen of beoordelen van de kwaliteit van de met theorie verijkte praktijkkennis. Eventueel kan het instrument vereenvoudigd worden door het te beperken tot de verticale dimensie waarin het niveau van theoriegebruik beschreven is.
Ook voor het vervolgonderzoek is een combinatie van klein- en grootschalig onderzoek aan te bevelen. Het gebruik van verschillende soorten gegevens of gegevensbronnen uit de beide onderzoeken kan leiden tot meer diepgaande, samenhangende analyses, die door de mogelijkheid van nuancering binnen de systematiek van data meer consistent zijn en meer overtuigingskracht hebben dan de data-analyses van de afzonderlijke onderzoeken. De niveauverhogingen die zich bij studenten voordeden in dit onderzoek zijn daar voorbeelden van.

Verder geeft dit onderzoek aanwijzingen voor de inrichting van het curriculum van de lerarenopleiding (§ 6.6).
In de eerste plaats wordt in dit onderzoek aangetoond dat multimedia in de leeromgeving van studenten nuttige functies kunnen vervullen, in het bijzonder wat betreft het leren integreren van theorie en praktijk door studenten. Een multimediaal ingerichte leeromgeving biedt studenten de mogelijkheid om zich te concentreren op de studie van de praktijkkennis, buiten de druk en complexiteit van de eigen stagegroep.

Individueel of in teamverband kan de 'gemeenschappelijk ervaren' praktijk geobserveerd en bestudeerd worden.
Voor het curriculum van de opleiding kunnen multimedia, bijvoorbeeld al of niet door studenten of opleiders zelf opgenomen videobeelden, ingezet worden in vier fasen van leer- en onderzoeksactiviteiten.

In alle (deel)onderzoeken van deze studie werd geconstateerd dat *de inbreng van de opleider cruciaal* is voor de kwaliteit van de studentenactiviteiten. De opleider is als geen ander in staat de al of niet verborgen praktijkkennis te expliciteren en die theoretisch te verrijken. Niveauverhoging in het redeneren van studenten over praktijksituaties zijn bijvoorbeeld vrijwel alleen maar waargenomen in de discussies onder leiding van de opleider of tijdens interviews met de onderzoeker.
De multifunctionele begrippenlijsten bleken studenten en hun opleiders houvast te geven. Ze vervulden tijdens de gehele leergang ondersteuning bij het bewustmaken van de voortgang in de leerprocessen van studenten.

Het reflectie-analyse instrument kan worden ingezet ten behoeve van het peilen of beoordelen van het theoriegebruik door studenten.

Een tweede aanwijzing voor het curriculum van de lerarenopleiding is afgeleid van de in het onderzoek geconstateerde neiging van studenten om eerder algemeen-didactische begrippen te gebruiken dan vakdidactische begrippen. Het onderstreept het belang van *ruimere aandacht* voor het gebruik van *vakspecifieke kennis en vakdidactische theorie* en bovendien het belang van betekenisverlening aan algemeen-didactische begrippen in een vakspecifieke context.

Een derde aanwijzing betreft de aandacht voor studenten met vooropleiding 'mbo zonder wiskunde' en studenten met vooropleiding 'vwo met wiskunde'. Om redenen van verschillende aard wijzen de resultaten van dit onderzoek op de noodzaak van extra opleidingsdidactische maatregelen voor deze beide groepen studenten (§ 6.6).

Uit de anonieme vragenlijsten die zijn afgenomen blijkt ten slotte, dat de studenten de leeromgeving waarin ze die praktijkkennis konden verwerven appreciëerden en dat zij de aangeboden, in de praktijksituaties geïntegreerde theorie vooral waardeerden als ondersteuning van hun stagepraktijk. Ze zijn van mening dat de leeromgeving laat zien dat je theorie nodig hebt en dat theorie helpt bij het begrijpen van de praktijk en bij het begeleiden van leerlingen.

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Appendices part I

Appendix 1 Fifteen signals of use of theory by student teachers

The tool

Each of the next fifteen signals of use of theory has been coupled with an example. The examples can be considered as representative cases of a theory, with references to sources of the theory cited.

1. While observing practical situations, student teachers can refer to the theory that comes to mind.
   Example: student teacher points to a teacher who interprets the product of 2 x 5 and, in doing so, employs the rectangle model (Treffers & De Moor, 1990, p. 75).
2. Theory is used to explain (as a means to understand) what occurred in the practical situation observed.
   Example: student teacher explains the method employed by the pupil who is using base ten material as a working model (Gravemeijer, 1994, p. 57).
3. The student reflects the intention of the teacher or pupil(s) with the help of theory.
   Example: student teacher points out the 'mirroring technique' applied by the teacher to help the pupil (Van Eerde, 1996, p. 143).
4. The student teacher substantiates an idea arising from observing a practical situation.
   Example: student teacher explains the process used by the teacher concerning the transition from context to model, based on an idea about the teacher's opinion of contexts (Treffers et. al., 1989, p. 16).
5. The theory generates new practical questions.
   Example: student teacher wonders at which level (stage) of learning multiplication the pupils are (Goffree, 1994, p. 280).
6. Theory generates new questions about the student teachers' individual notions, ideas and opinions.
   Example: in referring to the theory of the next zone of development, the student teacher wonders whether she is approaching her pupils (during fieldwork) at the appropriate level (Verschaffel, 1995, p. 154; Van Hiele, 1973, p. 101).
7. The student teacher can theoretically underscore his personal beliefs about an actual practice situation.
   Example: student teacher explains her opinion about a positive working environment that according to her is created by the teacher and based on classroom environment theory (Marx, De Vries, Veenman & Sleegers, 1995, p. 62; Lampert & Loewenberg Ball, 1998, p. 123).

8. The student teacher estimates the practical knowledge of the teacher and identifies its theoretical elements.
   Example: student teacher describes the practical knowledge (of process shortening) that, according to him, motivates the teacher to employ certain actions (Gravemeijer, 1994, p. 58).
9. The student teacher reaches certain conclusions from his observations based on theoretical considerations.
   Example: student teacher reaches the conclusion that group work and beginning with repeated counting better fit the foreknowledge and experience of the children (Simons, 1999, p. 579; Van den Heuvel-Panhuizen et al., 1998, p. 60).
10. Making connections between practical situations in MILE and own fieldwork experiences with the help of theory.
    Example: student teacher establishes similarities between approaching a pupil in MILE and a pupil in his/her own practical training group (Goffree, 1994, p. 211).
11. (Re)considering points of view and actions on the basis of theory.
    Example: student teacher revises her opinion about a pupil's approach to multiplication, basing it on a fellow student's reflections on the theory behind the strategy employed (Nelissen, 1987; Van den Heuvel-Panhuizen et al., 2000, p. 47).
12. Constructive analysis (= adapting given teaching material) that is underpinned with theory.
    Example: student teacher adjusts a given course by incorporating contexts that provoke 'didactic conflicts' (Van den Brink, 1989, p. 203).
13. The student teacher shows his appreciation of theory.
    Example: student teacher expresses her appreciation of theory when she is able to explain the solution strategy employed by a pupil (Lampert & Loewenberg Ball, 1998, p. 70).
14. Realizing the usefulness of theory as a tool for reflecting on actual practice ('reflection on action').
    Example: in a logbook, student teacher describes his modified views on theory in favour of RME (Schön, 1983, p. 278; Korthagen, 1993; Verloop, 1995, p. 137).
15. Developing a personal theory to underpin his interpretation (creation) of a practical situation.
    Example: student teacher develops his/her own theory about open and closed questions (Boekaerts & Simons 1993, p. 208; Von Glasersfeld, 1995, p. 192).

Appendix 2A The development and try-out of the 'Concept list' (short version)

The detailed development and background of the list of concepts has been described in an extended Dutch version (appendix 2B) on the added CD-rom.

In both the small scale and large scale studies a 'Concept list' (see model next page) has been filled in by the students at both the start and the end of the course. There are small differences between the initial and final lists. Section 2.2 of the appendix 2B describes the development of the idea that launched the design of the list of concepts, with an initial description of its functions. The development of the first design is sketched, with an example of the first version (section 2.3). This first version has been tried out with four groups of second year students with a total of 63 students. The yield of that trial is described in detail in section 2.4.

The final two sections of appendix 2B describe the changes in function and content of the list of concepts for both the small scale study (section 2.5) and the large scale study (section 2.6).

This is followed by – a part of – the final version of the final list of concepts used in the final assessment of the large scale study, after which a quotation from the teacher educators' manual is given, containing the guideline for the introduction of the list of concepts at the start of the course.

Model of the 'Concept list' in the final assessment of the large scale study

Name student:
Class:
Name Pabo:

The concepts given below are key concepts from the teaching method of learning to multiply. You have already filled in the list at the start of the course to indicate which concepts did or did not have a meaning for you and for which concepts you believed you knew a teaching narrative. Now, at the end of the course, the question is to indicate which concepts have become better known to you as a result of the course, and have gained enough meaning that you can relate a teaching situation or a teaching narrative which will explain – to others as well – these concepts.
Use the list you filled in at the start of the course as a comparison.
In the list below, check off the concept if the answer is 'yes,' if not leave that line blank. Circle one of the four categories in the third column. Do not work too fast and be conscientious; this is not a test, but a determination of where you stand.

Concept	**This concept has become <u>more familiar</u> for me. I can relate a teaching narrative in which this concept has meaning/becomes clear.** **Check the line below for yes, or leave it blank**	**The narrative is from:** **1=own teaching practice** **2=The Guide** **3=magazine/book** **4=instructions/ discussions at Pabo** **Circle (possibly more categories for each concept)**	**Explanation (optional)**
1. adaptive teaching		1 2 3 4	
2. anchor points		1 2 3 4	
3. automation		1 2 3 4	
4. ...		1 2 3 4	
...		1 2 3 4	
57. visualizing		1 2 3 4	
58. working memory		1 2 3 4	
59. rich problems		1 2 3 4	

Procedure for using the 'concept list'

The following procedure has been discussed during the teacher educators' training day.

The list of concepts:

- Hand out the initial questionnaires and let the student teachers read the text from appendix 1 (p. 33 of the manual). Then read the following text to them: "*This is a self-assessment, intended to gain a first impression of the topic of the course, how much you already know and what you can still learn. For me (as a teacher and researcher) it gives an insight into your starting situation so that I can take it into account in discussions and supervizing.*" It must be especially clear to the students that this is a self-assessment; by pretending to know more than you actually do or by filling it in inaccurately you are putting yourself at a disadvantage.
- Experience has taught that the concept 'teaching narrative' requires a clear explanation. Give the following example of a teaching narrative for the concept 'starting error' (which is not part of this list). "*Tom is 6 years old and is in grade 3. In his arithmetic notebook he has the problem 5 + 3 = 7. The teacher ask him how he knows that 5 plus 3 is 7. 'I use my fingers,' Tom says and he counts on his fingers 'five, six, seven.' Tom makes a starting error, he should have started with 6, not with 5.*" Remind the students again that they must consider well before placing checkmarks or encircling a choice.
- Go through the headers of the list one more time with the students. It may happen that a student does know what a concept means, but has no teaching narrative for it. Also, more than one category can be circled in the final column for each concept.
- Have the students complete the list and hand it in, make copies (the originals are for the researcher) and give the students back a copy; they can use it to make notes during the course if they like.

(Quotation from the teacher educators' manual, page 15; see also appendix 22 on CD-rom)

Appendix 3A Try-out of 'The Theorem' (short version)

The detailed development and background of this try-out has been described in an extended Dutch version of this appendix on the added CD-rom (see appendix 3B).

To prepare for the small and large scale studies the activity 'The Theorem' has been tried out with 63 second year students, in the same series of meetings as the try-out with the lists of concepts.

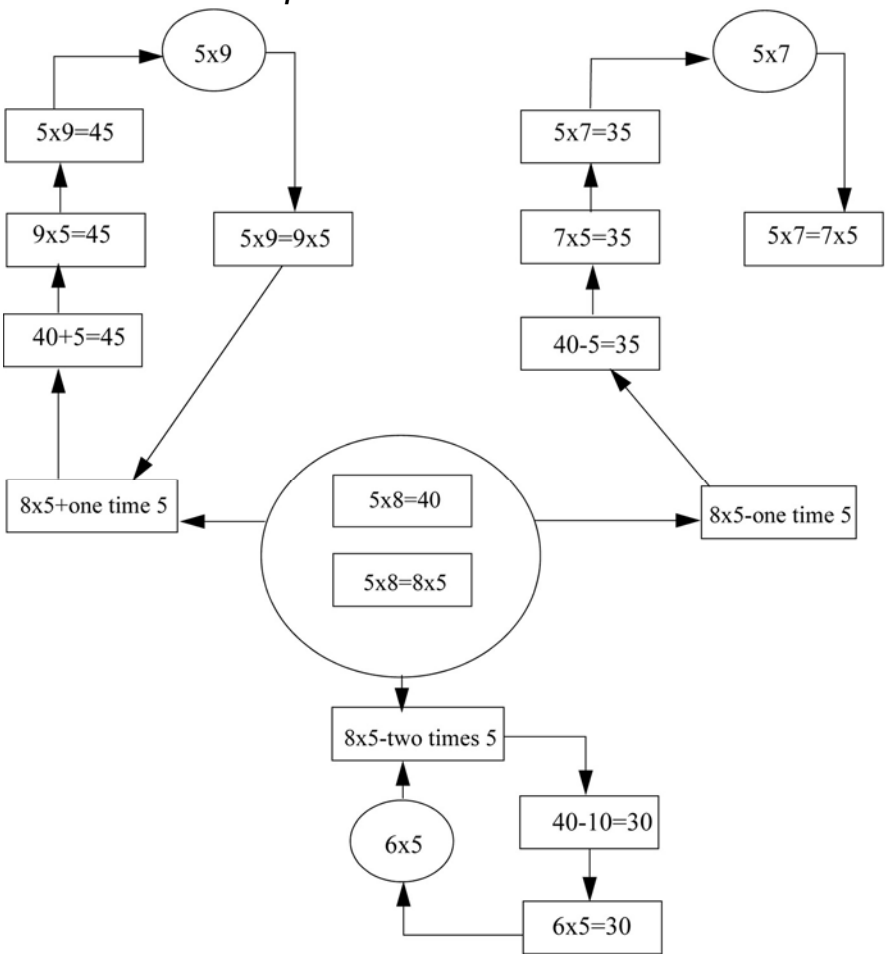
The goal for the student teachers was to gain 'theory-enriched practical knowledge' on the subject of multiplication. 'Designing, defending and refuting a theorem' teaches to defend one's opinions and allows the active acquisition of knowledge (Loughran, 2002).

The goal for the researcher was to find out which variant of that activity – having the students formulate their own theorems or having the teacher educator present a 'ready-made' theorem – would lead to the best result in terms of use of theory.

Appended is a description of the students' assignment, titled 'Designing, defending and refuting a theorem' included in the extended version of this appendix along with the yield of and conclusion drawn from the try-out (appendix 3B, section 3.3).

Appendix 4 Cognitive network of student, constructed by Anne

From Anne's research report: student's times-table network for 5x8



My [= Anne's; w.o.] interpretation.

This part of the times-table network is a bit more complicated than the previous part. You can see that 5 x 8 = 8 x 5 = 40 is the centre that Donna takes as her starting point for these three problems. Again, Donna starts with the reversing strategy and then uses the anchor point 5 x 8 = 8 x 5 = 40. When she finds out that she is dealing with a problem from the five-times table, she knows she can use 8 x 5 = 40. The next steps she takes are jumps of five. Ahead or back on the number line. Or put differently: she uses the strategy 'one time 5 more or less' or even 'two times 5 more or less.' She herself [Donna; w.o.] calls them jumps.

Appendix 5 Two of Anne's teaching narratives for theoretical concepts

If I know that one, I also know the other one

Teaching narrative for the concept 'anchor point'

Today, grade 2 is introduced to a new table, the eight-times table. The teacher wants to use cars with trailers as the context. A car with a trailer has eight wheels. This can be seen clearly in the visual material. De question is how many tires the garage needs for a certain number of cars and trailers. The teacher wants the children to find a solution for the multiplications they do not yet know with the aid of the multiplications they do know.

After the break the children enter the classroom. On their desk is a tray with all kinds of cars with trailers. On the teacher's desk there is also a toy car with trailer. The teacher starts telling a story about a garage where they have to replace all the tires on this car. He asks the children how many new tires the mechanic will need. The teacher asks both for the answers and for the approach the children took. After that, he discusses which times problems are suitable; 2 x 4, 4 x 2, 1 x 8.

Now the teacher tells them which table they will look at. He has written the eight-times table on the blackboard without the answers. Now he asks the children which answers they know already. They know 1 x 8. 2 x 8 is simple as well; 8 + 8. Mark knows another problem; 5 x 8, because he already knows the five-times table very well. "You just reverse it," he tells the teacher, "it becomes 8 x 5 and that is 40."

After all the problems the children know have been filled in, there are a few left. They do not know 6 x 8 and 9 x 8. The teacher says the children should be able to find the answers. "Try to look at the problems you do know. Then you can also do these." The children get to work. Lisa tells what she did: "I know 10 x 8, then 9 x 8 is a jump of 8 back. That is 80 - 8. 8 + 2 = 10 so 72." Esther can calculate 6 x 8. "We already knew 5 x 8 together. That is 40, 40 + 8 is 48." Together they have completed the eight-times table, and they now continue working on several different assignments.

I do it like this… and I do it differently

Teaching narrative for the concept 'strategy'

The teacher is sitting at the instruction table together with a group of four children. The others are working independently on a task. The teacher wants to get an impression of how the children solve a multiplication. This has been looked at with the whole group before, but she is curious which strategies the children use by themselves. She uses the six-times table for this; this table has not yet been treated in class. The 1, 2, 3, 4, 5 and 10 times tables have. She uses the context of six large biscuits in a box. She asks a

number of different questions of the children, like “How many biscuits do I have if I buy four boxes?” The children can work out the answer on their own. Afterwards they explain how they did it. Chris says: “1 box is 6 and then another box is 6 + 6 is 12. Four boxes is 12 + 12 = 24.” Hanneke starts: “1 box is 6, plus 6 is 12, plus 12 is 18, plus 6 is… 18 + 2 is 20, 20 plus 4 is 24.” “Hey,” says Chris, “I do it like this…” to which Hanneke replies proudly “… and I do it differently.” “Did you use another way, Henk and Marjolein?” the teacher asks. “Marjolein, how did you do it?” “Like Hanneke, jumps of six.” “And you, Henk?” Henk says he just knew in his head with the problems. The teacher helps him by asking what his first step was. “First I knew that the boxes is twelve. Then I knew 4 boxes is 4 x 6. The reverse problem is 6 x 4 and I already knew that. It is 24.”

After the discussion the teacher lists the strategies the children have used for herself. Chris doubled 2 x 6, Hanneke and Marjolein started with 1 x 6 and took jumps of 6 to get to 4 x 6 and Henk used the reverse rule. Henk turned the problem 4 x 6 into the problem 6 x 4 from the four-times table which he already knew well. So the children use doubling, shortened counting and reversing.

Appendix 6 Anne's reflective note for 'the suitcase full of balls'

The children are given the arithmetic problem of the week. This is clearly linked to current events. The children are offered a realistic situation; a suitcase which is filled with balls and a letter from Saint Nicholas. This fits into realistic mathematics education really well. It assumes that by having the children work within realistic situation they will understand and apply problems better.

A larger degree of involvements is evoked, which stimulates the learning process. It is clear from the sound level that the children join in enthusiastically. The teacher plays into this well by looking what is inside the suitcase. She devotes a lot of attention and time to the introduction of the problem, so that it leaves a lasting impression. By referring back to the instruction in the letter to use the balls for arithmetic, the teacher uses a good bridge to get to the real work.

She creates the problem through having to guess and not count one by one. How do you know how many balls there are exactly. She does not let the children give their own solutions. It is very good to actually do this, but there is a risk of missing the target (the five-times table). The tubes that are shown give the solution for the problem in a very natural way, as there is a boy who immediately mentions the problems in the table.

Here she leaves the children more freedom to solve the problem. One child counts 10, 20, 30, 40, 50. While the other counts with jumps of 5. By discussing these different ways with the whole group, the different strategies are shown. Here the strategy of shortened counting is applied in two different ways.

When all the balls have been placed in tubes on the edge of the blackboard, they count the number of tubes with the class. The children are counting the balls with jumps of five. The teacher stops them and repeats her question. I am glad to say she gets back to that later, but now she has to stick to her own step-by-step plan to get to the corresponding table problem. She wants to hear that problem first. You can see that the boy who answers is already familiar with the way of saying it. You can also say that it shows that he understands well what concrete situation goes with a table problem. He knows that 20x5 implies 20 groups, tubes of five.

Next, they count together in the way the class originally started [jumps of 5; w.o.]. You may ask if this is the right way. If you are working from the children's point of view, it would be good to first follow the children, the problem can be formulated after. The only thing is that the emphasis will be on shortened counting as an activity, rather than a strategy.

If you want to present shortened counting as a solution strategy for multiplication you will have to start with the times problem. You will then use shortened counting to solve it, counting with jumps of 5. Most likely this was her goal and she achieves it. You avoid the

children knowing the answer immediately, which gets the strategy across less well.
The context has been set up really well. Only she could have made the problem even bigger and livelier by making it a real problem. For example, Saint Nicholas wants to give a tennis ball to each child in the area. He wants to know how many tennis balls he has in his suitcase. He wants to know if he has enough. He needs 75. Does he have enough? Will he have any left? This gives the problem even more meaning.

Appendix 7 Characteristics concept map

Students	1=Anne	2	3	4	5	6	7	8
Number of concepts (max. 10)	10	10	9	8	8	10	7	7
Number of relations	9 implications	14 implications, 1 bi-impl.	7 implications	7 impl	9 impl, 2 bi-impl	5 impl	5 bi-impl	3 impl, 8 bi-impl.
Number of explanations	5	9	7	3	13	0	2	11
Other characteristics of ordering	Places all concepts in four successive, logical steps with the realistic method as a start and, cognitive network and memorising at the end. Underpins her order.	Successive ordered placement starting with structure ('Structure fits everywhere'); but also: 'everything goes under cognitive network' Sees anchor point and making concrete as examples of strategies.	Has made a circle with the realistic method and cognitive network at the centre. For 'context' she uses the 'brownies' as the take-off point for the circle.	Sees cognitive network as the core (calls it that too); all arrows start there.	Starts with the realistic method; Bi-impl conter and struct-mental model. Does give meaning, but no logical structuring.	Starts with the realistic method, followed by 'structure' and 'context'. Followed by a 'loose' middle part. Finishes with network.	Group of three resp. four concepts: 'have to do with presenting problems' resp. 'have to do with the way children think'.	She gives meaning to concepts and the relations between them, but not in terms of placing them within a method.

Appendix 8 Key questions for the final interview

Lists of concepts
Which concepts did you find out more about?
What additional things did you learn? (example/story)
Are there concepts of which you think now that you were wrong at the beginning of the course to say that you either did or did not know them?

Final assessment: the suitcase full of balls
Where does this lesson fit into the method/learning trajectory for multiplication?
What would be your next step with/after this lesson, with these children?

Concept map
Can you explain the structure you used?
Can you give short examples for the 'if---then' arrows?
(if you do this as a teacher then...)
Which of these ten concepts do you think belong to the 'suitcase lesson'?

Numeracy test
How did you do? What did you think? Give some thought to that (evoke inquiry).

Evaluation
What did you find difficult in the course?
The questionnaire mentions the concept theory a few times. What do you think of when you hear the word theory? What do you think of theory?
Do you think there is theory in this course?
What do you think is an example of theory that should be part of learning to multiply?

Appendix 14 Descriptive statistics of questionnaire large scale study (*n* = 256)

	N	Min.	Max.	Mean	Std. deviation
1. I found this course difficult/simple	252	1	5	3,17	0,80
2. The course is not/very useful for my teaching practice	256	1	5	3,59	1,03
3. As a result of the course I know nothing/everything about learning to multiply	253	1	5	3,65	0,69
4. This course offered no/a lot of theory	254	1	5	3,59	0,82
5. I did not/did know the theory that was offered	249	1	5	2,80	0,83
6. After this course I do not/do have a better understanding of practice	256	1	5	4,02	0,81
7. The course is boring/challenging	256	1	5	3,31	1,10
8. The course is vague/concrete	251	1	5	3,47	1,09
9. The course is theoretical/practical	252	1	5	3,47	0,92
10. The course does not/does make you understand students' behaviour better	255	1	5	3,80	0,97
11. The course does not help/helps me to supervize the students in my practice class better	254	1	5	3,61	1,13
12. I can not see/see the point of theory after this course	254	1	5	3,84	0,82
13. This course does not/does make it clear that you need theory	251	1	5	3,95	0,82
14. Theory and practice are far apart/have been integrated	248	2	5	4,22	0,75
Valid N (listwise)	229				

Notes

[1] 'Constructive coaching' (Bakker et al., 2008) can be considered as a way of coaching that teaching strategies matches with learning strategies (Vermunt & Verloop, 1999), for example by using the principle of the zône of proximal development (Vygotskij, 1978).

[2] In the same publication, (e.g., 1991, p. 100) Freudenthal shows the relationship between theory and practice in mathematics education by intertwining observing, reflecting, mathematizing and didactizing (cf. Oonk, 2005).

[3] In his publication 'Didactical Phenomenology of Mathematical Structures,' Freudenthal (1983) lays a theoretical foundation for 'realistic teaching' of mathematics. In chapter 4, after a phenomenological reflection on number theories through history, he sketches the building blocks for a didactical phenomenology of numbers and operations with numbers. Characteristic is his view, that in teaching one should not so much try to find realizations that start from the number, but that one has to look for phenomena that necessitate the mental object 'number'. Number is a 'thinking thing' that, according to Freudenthal, students get a grip of through offering 'multiple embodiment' in various situations. In the 1970s this was a view that went against the dominant view of (isolated) development of concept.

[4] Wiskobas stands for 'Wiskunde op de basisschool' (mathematics in primary school). At the time (1971-1981) the Wiskobasteam had, as a part of the IOWO-team ('Institute for the development of mathematics education,' the precursor of the Freudenthal Institute), the task of developing and implementing mathematics education in primary school.

[5] The ideas of the followers of associative psychology were mechanistic and atomistic in their approach. According to them knowledge was caused by one or more sensory experiences. By repeating mental experience over time, sensory information formed connections, was the idea. The Brit John Locke with his 'Association of ideas' (1690) is seen as the founder of associationism.

[6] The Babylonians (ca. 3000 BC) have left clay tablets which among other things contained the tables of multiplication from 1 x 1 up to 59 x 59 from their positional, sexagesimal (base sixty) system. Egyptian writings (papyrus Rhind, ca. 2000 BC) show us multiplication tables that show they calculated partly by heart, particularly through handy doubling and halving; they did not just do this for whole numbers, but also for fractions and decimal numbers. Probably the natural development of multiplication, including the accompanying (mathematical) development of language – the so-called 'practical character' of multiplication – gave no cause to take up the development of a mathematical foundation for the numerical system. That foundation was in fact not laid until about two thousand years later, by Euclid (ca. 300 BC).

[7] Freudenthal says (1984a, p. 122): Multiplication is at first repeated addition, and this repeated addition can be structured very efficiently by pair collection within the rectangular model –

product within set theory, partly to calculate amounts as products. However, this model is insufficient. Not insufficient mathematically (...). But insufficient didactically, because a mathematically obvious restructuring does by no means have to occur within learning processes – either spontaneous or encouraged – and, if it does occur, does not have to be conscious enough to be made explicit and be available.

[8] That the criticism did exist, can be seen among others in publications from the wellknown Dutch pedagogue Ligthart (1859-1916) and from the researchers Brownell and Chazal (1935). Ligthart felt that the then-current approach to education – and not just of mathematics – had deteriorated to lifeless imitating, copying of reasoning and memorization. *'Learning through experience, learning by doing, learning with empathy,'* was Ligtharts' credo (De Jong, 1996, pp. 282-284). He stood for learning in a physical and mental interaction between child, environment and teacher, allowing the child to actively acquire the new material. The influence of American pedagogue and philosopher Dewey (1859-1952) can be recognized in these ideas. De Jong writes that Ligthart learned about Dewey's work in 1908 through the book 'Méthodes Americaines d'education générale et technique' by Belgian author Omer Buyse, and recognized his own ideas in Deweys work, sometimes in great detail. There was only one of Dewey's axioms he disagreed with: the recapitulation theory, according to which students would have to relive events from history to become interested in current culture. Ligthart did not believe in this idea. It was better for didactical reasons to take a starting point as close as possible to the environment of the child, rather than to go back two thousand years. Brownell and Chazal studied different ways of adding and subtracting. They concluded that 'drill activities' have little effect if not preceded by understanding of what has to be learned.

[9] Lankford, 1974; Erlwanger, 1975 and Codd, 1981.

[10] He supports the view of Lesh and Landau (1983) that the clinical interview gives a more complete view of the development of mathematical notions and processes in children and does not agree with some researchers who claim that children are on the whole unwilling to relate their thoughts (Ter Heege, 1986, p. 31).

[11] Among others, Ter Heege (1986) refers to the work of Ebbinghaus – with his influential publication 'Über das gedächtnis' from 1885, in which he gives much attention to the laws of association, Bartlett (1932), who makes a distinction between reconstruction and reproduction, and knowledge, and Van Parreren (1964), on among other things functional and maneuverable knowledge. Furthermore, he cites the researchers Brownell and Chazal, who conclude that 'drill activities' have little effect if not preceded by understanding, Thorntons (1978) on applying mental strategies on their own by children and Baroody (1985) in relation to the dynamic cognitive network.

[12] TAL means Tussendoelen Annex Leerlijnen (A Learning-Teaching Trajectory with

Intermediate Attainment Targets). The publication is the output of the TAL-project, initiated by the National Department of Education, Culture and Science. It has been executed by a group of thirteen experts of the Freudenthal Instituut in cooperation with the Netherlands Institute for Curriculum Development (SLO) and the National Centers for School Improvement (CED).

[13] According to Ter Heege (1986, p. 110), the division between reproduction and reconstruction comes from the psychologist Bartlett. Treffers & De Moor (1990, pp. 72, 87) refer to Baroody (1985, pp. 83-98) for that. Baroody uses these concepts when he discusses dynamic (knowledge) networks.

[14] In the TAL-brochure (Van den Heuvel-Panhuizen et al., 1998, p. 63) an example that gives off such a signal is included. It shows how the table of eight can be reconstructed and subsequently reproduced in a process of shortening and memorizing. An analogous example is given in De Proeve (Treffers & De Moor, 1990, p. 76) which in its turn is derived from the theory of Ter Heege (1985). Next the example from the TAL brochure (p. 63).

'Table of eight'

1 x 8 to be known [weetje]

2 x 8 to be known ('the double' 8 + 8); or through switching (8 x 2)

3 x 8 through 2 x 8 + 8 ('one time more'); or through switching (8 x 3)

4 x 8 double of 2 x 8, or 5 x 8 - 8 ('one time less'); or through switching

5 x 8 half of 10 x 8 = 80; or through switching

6 x 8 through 5 x 8 ('one time more'); or through switching; or doubling (3 x 8 + 3 x 8)

7 x 8 through 5 x 8 + 2 x 8, of 6 x 8 + 8 ('one time more')

8 x 8 various, will be a 'known' quickly

9 x 8 10 x 8 - 8 ('one time less'); or through switching

10 x 8 'known'

12 x 8 an inquiry problem…

In many of the above cases, access through other tables is possible through the commutative property: 3 x 8 through 8 x 3 if that is already known; 4 x 8 through 8 x 4, and so on.

A signal regarding influences from earlier theories on learning and memory can be found in 'De Proeve,' in a reference to Van Parreren and to (via) Ter Heege's work. The TAL brochure contains no references to these theories, only a general comment on the necessity to integrate the content component with cognitive, social and affective-emotional development (p. 75).

[15] We consider practice as a situation, (learning) environment or domain with materials, tools and actors in which professional actions occur, that is to say adequate action based on (practical) knowledge.

[16] Here the concept of paradigm is interpreted according to the views of Kuhn, namely the paradigm of a scientific community (Kuhn, 1970, a.o. p. 210). This to distinguish it from the

meaning Freudenthal gives it: the paradigm as a representative example of a phenomenon, a concept or a theory (Freudenthal, 1984b, p. 102; Goffree en Dolk, 1995, p. 114).

[17] Ter Heege talks in various terms about making the connection between practice and theory (among others in 1986, p. 5, 170). On page 5 he says: "There is a didactical gap [between theory and practice] that will be bridged with the development of learning materials for elementary multiplications."

[18] Much is being written and said about the complexity of learning and teaching mathematics. Lampert (2001) makes the concept of 'complexity' concrete and debatable.

[19] Boersma and Looy describe a practical theory as the knowledge that describes actions in specific practices and provides guidance for those actions (Boersma and Looy, 1997, p. 16). [Note that these researchers designate theory as knowledge].

[20] For example the environment Ter Heege created: a challenging, open discussion situation about multiplication with student Johan.

[21] Heard from A. Treffers at HKRWO (Historische Kring Reken- en Wiskundeonderwijs – Historical Circle Mathematics Educations) Symposium VIII, May 25, 2002 in Utrecht.

[22] Verschaffel uses the adjective 'powerful' in the meaning of 'efficient,' in the sense that a strong learning environment will lead to efficient acquisition of knowledge and skills.

[23] As from 06-06-2003 the MILE project (meanwhile in version 4) is executed under government of 'The association MILE2' (Den Hertog, 2006).

[24] Oonk, W. Verhalen van reken-wiskundeonderwijs in groep 4 [Telling stories about grade 2], This book functioned as a sourcebook for helping student teachers to find an appropriate research question.

[25] De Wereld in Getallen ['The World in Numbers,' one of the current Dutch teachers guides and textbooks].

[26] By the Dumb August approach the teacher pretends to be quasi-stupid in order to evoke interaction in Class.

[27] Some teachers use free of charge packing materials as egg boxes for grouping on base ten.

[28] During this so-called 'game of concepts' the students discussed, under the direction of the teacher educator, the question of whether there was a demonstrable connection between six given theoretical concepts (context, informal procedure, thinking model, anchorpoint, structure and strategy) and four practical situations. The game element was that each student attempted to defend his or her choice of a concept – written on a colored card – in the plenary session.

[29] The group work was partly stimulated by thinking of, formulating and discussing a 'theorem' based on joint observation of a practical situation. There is an assumption that such activities lead to 'ownership' of knowledge that is directly linked to one's own experience (Loughran, 2002; see also appendix 3AB).

[30] For instance: "Try to recall what you were thinking" and "Say 'stop' when you want to react."

[31] This is a selection of ten from the 59 concepts that occurred in the course. The selection has been made by the researcher based on his assessment regarding an optimal data yield (selection criteria: theoretical 'load' of the concepts, coherence and use in the meetings).

[32] Anne is a fictive name.

[33] The reckon reck is a variation on the traditional abacus and is used in realistic mathematics education (Heuvel-Panhuizen, van den (red.), 2001).

[34] In the zero-version, there were five categories, but upon consideration 'prediction' was included with response.

[35] As well as his own interpretation of Van Hiele's (1973) division in levels for thinking respectively reasoning in mathematics, Freudenthal also formulated levels in use of language, with someone's choice of language being an expression of that person's level of thinking. An example of the latter is the indication at different levels of the location of a thing or a person by describing the location (active, demonstrative use of language), by using concepts of orientation such as left, right, front, back (active or fact-establishing relative use of language), or by using coordinates (fact-establishing, functional use of language) (see also Van Dormolen 1982, p. 148).

[36] This comparison is based on data from the CBS [Central Statistics Bureau] and the Ministry of Education (OCW).

[37] Drs. K. Olofsen, co-author of the publication '*Gecijferdheid'* (Faes et al., 1992); the other judge was drs. K. Tjon Soei Sjoe.

[38] This concerns the reflective note for the final assessment. For the initial assessment the description for a situation was considered as a meaningful unit.

[39] For instance, words such as 'furthermore' or 'also' are often indications that a sentence should be added to a preceding sentence or paragraph.

[40] Indicator words are usually marks (connecting words, lexical signals) of coherent relations in a text (Pander Maat, 2002); they may give an indication for the type of description being used. It depends on the meaning within the given context whether such a word actually gives an indication and to what degree it does so.

[41] Reflective, contemplative descriptions by the students are often accompanied by a rise in level for the use of theory.

[42] A statement that is seen as unlikely is a statement by a student about a practical situation that is judged to be almost impossible by the expert who judges it, such as a student stating that the teacher 'apparently feels that the number line is not useful for what is being taught in this lesson,' while there is no indication that the teacher holds this opinion.

[43] Dr. R. Keijzer and the researcher.

[44] The score list (appendix 16) contains general pedagogical and pedagogical content concepts.

Students also often use certain concepts more than once in their reflection. For that reason a distinction has been made between the total number of general pedagogical and pedagogical content concepts and the number of *different* general pedagogical and pedagogical content concepts. The general total has also been determined for both groups.

[45] From our experiences with the student teachers in the small scale study, we know that 'characteristic dominance' of theory use in relation to one of the four categories exists.

[46] In general, over the last years many mathematics teacher educators in the Netherlands are concerned about – probably related – phenomena such as workload, the decreasing amount of contact time and the decreasing attention to mathematics education in the curricula of their teacher training college (Keijzer & Van Os, 2002).

Curriculum vitae

Wil Oonk (1940) ging na zijn voltooiing van de HBS-B opleiding in 1957 naar de Kweekschool voor Onderwijzers en behaalde daar in 1959 en 1960 de akten voor onderwijzer, respectievelijk volledig bevoegd onderwijzer. Na de vervulling van zijn militaire dienstplicht in 1962, ging hij werken in het lager onderwijs in Enschede en Muiderberg en was leraar wiskunde aan het Woltjer Gymnasium te Amsterdam. In avondstudie behaalde hij de akten Wiskunde MO-A en MO-B.

Zijn betrokkenheid bij de nieuwe ontwikkelingen in het reken-wiskundeonderwijs voor de basisschool leidde in 1971 tot zijn benoeming als docent aan de reguliere en Montessori dag- en avondopleiding van de Gemeentelijke Pedagogische Academie te Amsterdam, voorganger van de tegenwoordige Pabo van de Hogeschool van Amsterdam. Hij werkte daar als opleider, als leidinggevende van de vakgroep wiskunde & didactiek en als cursusleider van de part-time lerarenopleiding Wiskunde L.O. In die hoedanigheden was hij lid van diverse landelijke ontwikkel- en adviesgroepen en examencommissies, onder andere de ontwikkelgroep lerarenopleidingen (OGLO), de veldadviescommissie Wiskunde & Informatica (VALO) van de SLO en de staatsexamencommissie Wiskunde L.O. Ook werkte hij mee aan de ontwikkeling van het standaardwerk 'De Proeve' voor de opleiding rekenen-wiskunde & didactiek op de Pabo. Verder gaf hij mede leiding aan de ontwikkeling en uitvoering van studiedagen voor pabodocenten wiskunde & didactiek en was betrokken bij internationale projecten.

Van 1996 tot 2001 was hij als lid van het landelijk kernteam MILE gedetacheerd bij het Freudenthal Instituut, waar hij na zijn pensionering is blijven werken als gast-onderzoeker.

In het najaar van 2003 verbleef hij op uitnodiging van de School of Education (Universiteit van Michigan) gedurende vier maanden in Ann Arbor. Hij werkte daar als opleider en participeerde in het ontwikkel- en onderzoeksteam van Deborah Ball.

Wil Oonk is auteur van publicaties op het gebied van rekenen-wiskunde & didactiek.

Tegenwoordig geeft hij mede leiding aan de landelijke 'Kerngroep Opleiders' voor dit vakgebied en aan een project voor de vernieuwing van het reken-wiskundeonderwijs in Suriname. Verder is hij redactielid van het 'Tijdschrift voor nascholing en onderzoek van het reken-wiskundeonderwijs' en eindredacteur van een uitgave i.o. voor rekenen-wiskunde & didactiek op de Pabo.

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Een eerste woord van dank verdient eigenlijk de pedagoog en filosoof Dewey (1859-1952). Zijn werk koos ik in 1960 als onderwerp van mijn examenwerkstuk voor de 'akte volledig bevoegd onderwijzer', gefascineerd als ik was door zijn vooruitstrevende ideeën over het bij elkaar brengen van theorie en praktijk in het onderwijs.
Enkele decennia later had ik het voorrecht kennis te mogen maken met het op integratie van theorie en praktijk gerichte curriculum van de School of Education van de universiteit van Michigan. In het bijzonder inspireerde het werk van de hoogleraren Magdalene Lampert en Deborah Ball mijn collega's en mij tot het ontwikkelen van MILE voor de Pabo's in Nederland. Die bron van inspiratie werkte nog in een ander opzicht door. Op één van de studiereizen naar Ann Arbor ontstond het idee voor dit onderzoek. Ik ben Magdalene en Deborah zeer erkentelijk voor hun gastvrijheid tijdens mijn bezoeken aan Ann Arbor en de collegiale en persoonlijke wijze waarop zij hun expertise met mij wilden delen.
In de eerste fase van mijn onderzoek was het Mileteam – met Maarten Dolk, Willem Faes, ons helaas veel te vroeg ontvallen, Fred Goffree, Han Hermsen en later Jaap den Hertog en Chris Rauws – een klankbord voor mijn ideevorming. Dat gold in het bijzonder voor Fred Goffree in zijn functie als begeleider van het onderzoek.
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In totaal hebben 398 studenten aan de vier verschillende onderzoeken deelgenomen. Bijzondere vertegenwoordigers van die groep zijn Dieneke Blikslager en Hayet de Bont, die als ware pioniers de spits hebben afgebeten.
Vanaf die eerste momenten tot en met het vierde en laatste onderzoek konden studenten profiteren van de expertise van de 'Mile-leraren' Minke Westveer en Willie van Ouwerkerk.
De studenten vormden de doelgroep van mijn onderzoek. Maar zonder de betrokkenheid en de belangeloze inzet van hun opleiders was het onderzoek niet mogelijk geweest. Zij moesten de onderzoeksbijeenkomsten inpassen in de bestaande programma's en bovendien bracht het onderzoek veel extra werk met zich mee. Mijn dank gaat uit naar de volgende opleiders en hun Pabo's, c.q. Hogescholen: Frits Barth (Stenden Hogeschool, Leeuwarden), Hanneke Beemer (Fontys Hogeschool, Eindhoven), Jos van den Bergh en Frans Van Mulken (Avans Hogeschool, Breda), Nico den Besten (Hogeschool Driestar Educatief, Gouda), Mat Bos en Mark Sanders (Hogeschool de Kempel, Helmond), Gert Gelderblom (Gereformeerde Hogeschool, Zwolle), Riny

Kollöffel (Hogeschool van Amsterdam), Ad Peijnenburg en Eric Ansems (Fontys Hogeschool, Den Bosch), Jan Haarsma (Chr. Hogeschool Windesheim, Zwolle), An te Selle (Stenden Hogeschool, Meppel), Jan Stapel (Hogeschool INHolland, Dordrecht), Belinda Terlouw (Katholieke Pabo Zwolle) en Marc van Zanten (Hogeschool Edith Stein, Hengelo).
Kenneth Tjon Soei Sjoe (Hogeschool van Amsterdam) liet van meet af aan zijn interesse voor mijn onderzoek blijken door het stellen van indringende vragen; die leidden vrijwel altijd tot interessante en vruchtbare discussies.
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