Applications of AdS/CFT in Quark Gluon Plasma

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CHAPTER 1

INTRODUCTION

For many years, people have attempted to develop an ultimate theory that would explain the fundamental structure of matter and the very basic mechanisms of nature. One promising candidate is string theory. Born in the late 1960s as a theory that was expected to describe the strong interaction in hadrons, string theory had to accept the fact that another theory, known as QCD (Quantum Chromodynamics), correctly describes the strong nuclear force and the properties of hadrons. A new face of string theory arose in 1974 when John Schwarz and Joel Scherk proposed an interpretation of the spin-two massless particle in the spectrum of string theory to describe the quantum of gravity, namely graviton. Ever since string theory has received great attention of many scientists, not only from high-energy physicists, but also from various other fields of study and so a journey to the ultimate theory has taken a new direction.

String theory today is a forefront in the world of scientific research. It does not only requires knowledge of other fields and sophisticated tools in mathematics but at some level it also tries to solve some puzzles in physics by providing a new approach to the problems. Nevertheless, string theory still lacks of experimental evidences. The natural length scale of the theory is thought to be at the order of Plank scale $\sim 10^{19}\ GeV$, out of reach of any current or future machines built for experiment. The energy scale at which string theoretic effects become relevant is very large compared to the energy scale of well established theory of particle physics namely the Standard Model (electroweak scale $\sim 246\ GeV$ and QCD scale $\sim 300\ MeV$).

In string theory the fundamental objects are one-dimensional objects called strings instead of points-like as in the usual quantum field theory.

String theory is characterized by one parameter α', the string tension, which is also related to the length of strings $l_s = \sqrt{\alpha'}$. These strings have to be embedded into 26-dimensional space-time (without supersymmetry) or 10-dimensional space-time(with supersymmetry) in order to be consistent. The 26-dimensional strings is called bosonic string theory and 10-dimensional strings is called superstring theory [1].

There are two types of strings one can consider. First is closed strings where the two ends of the string meet and form a loop. The second is open strings where the end points of string are confined to subsurfaces in space time (hypersurfaces) called branes. The spectrum of those type of strings are quite different, for example the open string has a massless spin-one gauge field while the closed string has a massless spin-two graviton.

One of the most important developments in string theory is the AdS/CFT correspondence. It is based on holographic principal which states that the description of a volume of space can be thought of as encoded on a boundary of that region. This correspondence encodes a way of using string theory to perform non-perturbative calculation in gauge theory which is still a complicated problem.

The best known example of the correspondence is between weakly coupled gravity theory with AdS (Anti de Sitter) as space-time background and strongly coupled gauge theory with conformal symmetry in one lower dimension. Subsequently people have tried to extend this correspondence to non-conformal gauge theory since the Standard Model itself is not a conformal theory. This extension affects the space-time background where the gravity theory lives in. With this attempt now the correspondence is widely known as gauge/gravity correspondence[1]. Unfortunately there is still no version of the correspondence which realizes Standard Model or even pure QCD.

Nevertheless, the last several years we have seen a considerable success in the application of the AdS/CFT correspondence [2–4] to the study of real world strongly coupled systems, in particular the QGP(Quark Gluon Plasma). The (succesful) application hinges on the belief that the QGP of QCD is thought to be qualitatively very similar to the plasma of $\mathcal{N} = 4$ super Yang–Mills theory at finite temperature, which is dual to string theory in an AdS black hole spacetime. The analysis of scattering amplitudes in the AdS black hole background led to the universal viscosity bound [34], which played an important role in understanding the physics of the elliptic flow observed at RHIC. On the other hand, the study of the physics of trailing strings in the AdS spacetime explained the dissipative and diffusive physics of a quark moving through a field theory plasma, such as the diffusion coefficient and transverse momentum broadening [35, 38–41, 53–55]. The relation between the hydrodynamics of the field theory plasma and the bulk black hole dynamics was first revealed

[1]It is from the massless spectrum of open and closed strings that AdS/CFT correspondence gets another name gauge/gravity correspondence.

in [56, 97] (see also [63]).

This thesis is based on works in [19,20] and ongoing work in [21]. The works describe some applications of AdS/CFT correspondence in QGP. In the following sections we give a brief introduction to string theory; an inside view of QGP; the basic and technical tools of AdS/CFT correspondence; and the outline of this thesis.

1.1 Quark Gluon Plasma

The low-energy properties of the strong interactions are governed by a chiral symmetry. The QCD Lagrangian possesses an $SU(3)_L \times SU(3)_R \times U(1)_V$ symmetry in chiral limit ($m_u, m_d, m_s \to 0$). At the current status, we do not know how to solve QCD in low-energy as the standard perturbation theory can not be applied for energy below QCD scale $\sim 300\ MeV$. Below this scale quarks are in a confined phase and bound to form what is called hadron. In this state quarks can not be separated from each others since the QCD coupling constants are large. Therefore perturbation theory can not be used and we need to work with non-perturbative calculation. As we increase the energy above QCD scale, the QCD coupling constants decrease and the quarks are slowly separated from the hadron. At some point there will be a phase transition to a deconfined phase where the quarks are deconfined from the hadron form and can be identified individually. In this phase the perturbation theory works very well especially at infinite energy where quarks do not interact with each others and it is known as asymptotically freedom.

Perturbative aspects of QCD have been tested to a few percents. In contrast, non-perturbative aspects of QCD have barely been tested. Recent development in gravity/gauge correspondence has revived the hope that the strongly coupled regime of QCD can be reformulated as a solvable string theory.

QGP is a phase of QCD which exists at extremely high temperature and/or density. The QGP contains quarks and gluons, just as normal matter(hadron) does. Unlike hadrons where quarks are confined, in the QGP these mesons and baryons lose their identities and dissolve into a fluid of quarks and gluons. Quarks in QGP are deconfined and make a much larger total mass compared to the corresponding hadron mass. The QGP is believed to have existed during the first 20 or 30 microseconds after the universe came into existence in the Big Bang.

A plasma is matter in which electric charges are screened due to the presence of other mobile charges. Likewise, the colour charge of the quarks and gluons in QGP are screened. There are also dissimilarities due to the fact that the colour charge is non-abelian, whereas the electric charge in a normal plasma is abelian.

The QGP can be created by heating high density matter up to a temperature of $190\ MeV$ per particle. To produce such high energy, two heavy particles

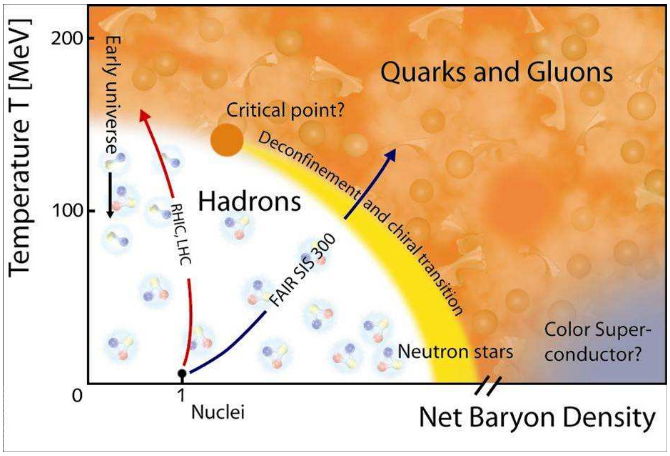

Figure 1.1: QCD phase diagram [17].

are accelerated to ultrarelativistic speeds and slammed into each other. They largely pass through each other, but a significant fraction collides, melts, and "explodes" into a hot fireball. Once created, this fireball expands under its own pressure, and cool while expanding. By carefully studying this flow, experimentalists hope to test the theory.

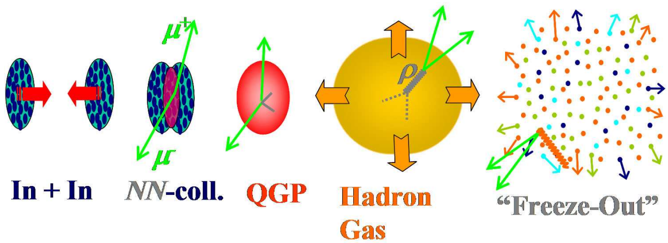

Figure 1.2: Creation process of QGP [18].

As conventional thermodynamic characteristics, the resulting QGP is largely controlled by the equation of state relating the P (pressure) and T (temperature). The equation of state is an important input for the flow equations. The mean free path of quarks and gluons can be computed using perturbation theory as well as string theory. There are indications that the mean free time of quarks and gluons in the QGP may be comparable to the average interparticle spacing: hence the QGP is a liquid as far as its flow properties go. It has been found recently that some mesons built from heavy quarks (such as the charm quark) do not dissolve until the temperature reaches about $350\ MeV$. This has led to speculation that many other kinds of bound states may exist in the plasma. Some static properties of the plasma (similar to the Debye screening length) constrain the excitation spectrum.

Unfortunately, the aspects or properties of QGP which are easiest to compute are not always the ones which are the easiest to probe in experiments. Hence, it is still a difficult task to declare the existence of QGP in the experiments such as in RHIC or LHC. The important classes of experimental observations are:

- Single particle spectra
- Strangeness production
- Photon and muon rates
- Elliptic flow
- Jet quenching
- Fluctuations
- Hanbury-Brown and Twiss effect
- Bose-Einstein correlations.

In general QGP can be weakly or strongly coupled. However, there are a couple of indications that strongly coupled QGP has been created in heavy ion collision experiments at RHIC(and expected stronger signals from the ongoing LHC) with the energy around $200\,GeV$ per nucleon [103]. So far the main theoretical tools to explore the theory of the QGP is lattice gauge theory. One of the properties of QGP computed by lattice gauge theory is the transition temperature in which the latest simulation yields approximately $190\ MeV$ [8]. Surprisingly, with a few steps and an input from the lightest ρ-meson, an AdS/CFT computation shows that the transition temperature is around [16] $191\ MeV$ which is close to the lattice result. In this thesis, we will use AdS/CFT correspondence to work on photon production [19], fluctuation [20], and elliptic flow [21] of the corresponding strongly coupled QGP.

1.2 D-branes

In addition to strings, string theory contains soliton-like “membranes” of various internal dimension called Dirichlet branes(D-branes) which are defined in a very simple way in string perturbation theory [43]. In ten dimensional string theory, a Dp-brane is a $p+1$ dimensional hyperplane living in $9+1$ dimensional space-time to which the ends of open strings are confined. It is charged under a $p+1$-form gauge potential which is part of the massless closed string modes.

The world-volume action of Dp-brane is the so-called DBI (Dirac-Born-Infeld) action. In a flat background it consists of a gauge field A_α and $9-p$ scalars Φ^i and some fermionic fields, with $\alpha = 0, \cdots, p$ and $i = p+1, \cdots, 9$. In static gauge the bosonic part of DBI action is given by

$$S_{DBI} = -T_{Dp} \int d^{p+1}\sigma \sqrt{-\det\left(\eta_{\alpha\beta} + 4\pi^2\alpha'^2 \partial_\alpha \Phi^i \partial_\beta \Phi^i + 2\pi\alpha' F_{\alpha\beta}\right)}, \tag{1.1}$$

where we have rewritten the coordinates that are orthogonal to Dp-brane as scalar fields Φ^i and with $\eta_{\alpha\beta}$ is the flat metric in Dp-brane world-volume and

$$T_{Dp} = \frac{1}{g_s (2\pi)^p (\alpha')^{(p+1)/2}} \tag{1.2}$$

is the tension of Dp-brane. Including background fields(graviton $g_{\mu\nu}$, dilaton ϕ, and the two-form field $B_{\mu\nu}$) takes the following form[2]

$$S_{DBI} = -T_{Dp} \int d^{p+1}\sigma \, e^{-\phi} \sqrt{-\det\left(g_{\alpha\beta} + B_{\alpha\beta} + 4\pi^2\alpha'^2 \partial_\alpha \Phi^i \partial_\beta \Phi^i + 2\pi\alpha' F_{\alpha\beta}\right)}, \tag{1.3}$$

where $g_{\alpha\beta}$ and $B_{\alpha\beta}$ are the pullbacks of $g_{\mu\nu}$ and $B_{\mu\nu}$, with $\mu, \nu = 0, \ldots, 9$. E.g.

$$g_{\alpha\beta} = g_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta}. \tag{1.4}$$

1.2.1 Non-abelian gauge theory on D3-branes

The previous DBI action has an abelian $U(1)$ gauge symmetry. For non-abelian case, the symmetry is enhanced non-abelian gauge symmetry for example with $U(N)$ gauge group by considering N parallel Dp-branes sitting at one point. The fields content are now represented by hermitian $N \times N$ matrices

$$A_\alpha = \sum_n A_\alpha^n T_n, \qquad \Phi^i = \sum_n \Phi^{i,n} T_n, \tag{1.5}$$

with $n = 1, \ldots, N^2$ and T_n are hermitian $N \times N$ matrices satisfying $\mathrm{Tr}\,(T_n T_m) = N\delta_{mn}$. We also define

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha + i\left[A_\alpha, A_\beta\right], \qquad D_\alpha \Phi^i = \partial_\alpha \Phi^i + i\left[A_\alpha, \Phi^i\right]. \tag{1.6}$$

[2]The presence of Φ^i terms must be read carefully since it overlaps with the transverse components of $g_{\alpha\beta}$. We add this terms just to show the explicit dependent of scalar fields Φ^i.

The generalization of DBI action with $U(N)$ gauge symmetry is rather complicated but the leading order action of the non-abelian DBI action reduces to Yang-Mills theory

$$S = -\frac{T_{Dp}(2\pi\alpha')^2}{4}\int d^{p1}\sigma e^{-\phi}\mathrm{Tr}\left[F_{\alpha\beta}F^{\alpha\beta} + 2D_\alpha\Phi^i D_\alpha\Phi^i + [\Phi^i,\Phi^j]^2\right]. \quad (1.7)$$

The analysis of loop diagrams in the Yang-Mills theory shows that perturbative calculation is valid when

$$g_{YM}^2 N \sim g_s N \ll 1. \quad (1.8)$$

In the case of N parallel $D3$-branes, the low energy effective action at leading order is $\mathcal{N} = 4\, U(N)$ supersymmetric Yang-Mills theory.

1.3 p-Branes

The p-branes are defined to be classical solutions to supergravity field equations. They also carry a charge under an antisymmetric tensor field $A_{\mu_1\cdots\mu_{p+1}}$ since the low energy effective action of type IIA/B superstring theory is supergravity action. The p-brane solutions are also solutions of the full closed string theory. In string theory p-brane corresponds to Dp-brane and they are believed to be two different descriptions of the same object, and we shall from here on call them by the same name.

One of the example of p-branes is the $D3$-branes solution in type IIB supergravity with the following action [44]:

$$S = \frac{1}{(2\pi)^7 l_s^8}\int d^{10}x\sqrt{-g}\left(e^{-2\phi}\left(R + 4(\nabla\phi)^2\right) - \frac{2}{5!}F_{(5)}^2\right), \quad (1.9)$$

where $F_{(5)}$ is a self-dual five-form field strength. Here we set the other supergravity fields to zero. The $D3$-branes solution is given by

$$\begin{aligned} ds^2 &= \frac{1}{\sqrt{H}}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \sqrt{H}(dr^2 + r^2 d\Omega_5), \\ F_{(5)} &= (1+*)dtdx_1dx_2dx_3dH^{-1}, \qquad e^{-2\phi} = g_s^{-2}, \\ H &= 1 + \frac{L^4}{r^4}, \qquad L^4 \equiv 4\pi g_s {\alpha'}^2 N, \end{aligned} \quad (1.10)$$

with $*$ is the Hodge dual operator and parameter N is the flux of five-form Ramond-Ramond field strength on S^5,

$$N = \int_{S^5} *F_{(5)}. \quad (1.11)$$

This solution corresponds to N parallel $D3$-branes at the center $r = 0$. This solution is called extremal solution which saturates the BPS bound (inequality between the mass M and the charge Q of the black hole). One can also check that this solution has zero Hawking temperature.

This supergravity approximation is valid if the curvature of the geometry is small compared to the string scale, $L \gg l_s$. In order to suppress the string loop corrections, the effective string coupling e^{ϕ} needs to be small and in the case of $D3$-branes the string coupling is constant with $g_s < 1$. So, the $D3$-branes solution is valid when $1 \ll g_s N < N$.

1.4 AdS/CFT correspondence

A long time ago 't Hooft proposed a generalization of the $SU(3)$ gauge group of QCD to $SU(N)$ and computed the Feynman graph [7]. In the limit where N is large while keeping $g_{YM}^2 N$ fixed, each graph is weighted by a topological factor N^{λ} where λ is the Euler characteristic of the graph. These factors also appear in calculation of closed string partition function, if we identify $1/N$ as the string coupling constant, with N^2 for spheres (tree level diagrams), N^0 for tori (one-loop diagrams), etc. Another interesting point is that since the closed string coupling constant is of order N^{-1}, in the large N limit, the string theory is weakly coupled.

From the viewpoint of string theory in the background of N parallel $D3$-branes sitting together, the relevant parts of low energy effective action are the brane action and the bulk action. The low energy effective action of the brane action is just the pure four-dimensional $\mathcal{N} = 4$ $U(N)$ gauge theory and it is known to be conformal field theory while the bulk action is described by supergravity action moves freely at long distance.

On the other hand the low energy limit of the background solution (1.10) has two kinds of low energy excitations. The first excitations are massless particles propagating in the bulk region and the other is any kind of excitations that close to $r = 0$. These two types of excitations decouple from each other because of the present of large gravitational potential. Therefore, there are two low energy theories that live in different region, one is a free bulk supergravity and the other one is living near horizon of the geometry which is $AdS_5 \times S^5$,

$$ds^2 = \frac{r^2}{L^2}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \frac{L^2}{r^2}dr^2 + L^2 d\Omega_5^2. \tag{1.12}$$

This metric can be obtained physically by taking low energy limit $\alpha' \to 0$ but at the same time we keep the energies to be fixed in string units. It then requires that r/α' is fixed or another way is taking $r \to 0$ besides the supergravity approximation limit that we had before, $g_s N \gg 1$.

These two different observations of strings under $D3$-branes background lead Maldacena to conjecture that $\mathcal{N} = 4$ $U(N)$ supersymmetric-Yang-Mills theory in four-dimensional space-time is dual to type IIB superstring theory on $AdS_5 \times S^5$ [2].

The solution (1.10) is not the only solution of the type IIB supergravity action (1.9). There is natural generalization to a non-extremal called Black 3-branes solution with non-zero Hawking temperature. This solution is non-extremal because it does not saturate the BPS bound. Taking the same decoupling limit as we did for the extremal $D3$-brane solution, the near-extremal of Black 3-branes solution is given by

$$ds^2 = L^4 \left[u^2 \left(-hdt^2 + dx_1^2 + dx_2^2 + dx_3^2 \right) + \frac{1}{hu^2} du^2 + d\Omega_5^2 \right]$$
$$h = 1 - \frac{(\pi T_H)^4}{u^4}, \qquad u^2 = \frac{r^2}{L^4}, \tag{1.13}$$

with T_H is the Hawking temperature. The dual gauge theory interpretation of this solution is a field theory at finite temperature. The Hawking temperature is interpreted as temperature on the gauge theory side.

1.4.1 GKPW procedure and holographic renormalization

Having correspondence between two different theories, we still need to know how they are connected precisely. Gubser, Klebanov, Polyakov, and Witten proposed that the string partition function is equal to generating function of correlation functions in the field theory [3, 4][3],

$$\left\langle e^{\int d^4x \, \phi_0(\vec{x}) \mathcal{O}(\vec{x})} \right\rangle_{CFT} = \mathcal{Z}_{string} \left[\phi(\vec{x}, z = 0) = \phi_0(\vec{x}) \right], \tag{1.14}$$

where $\phi(\vec{x}, z)$ is any fields of string theory with boundary condition at the boundary of AdS, $z = 0$, is $\phi(\vec{x}, z = 0) = \phi_0(\vec{x})$ interpreted as the source for operator $\mathcal{O}(\vec{x})$ in conformal field theory. The correlators in gauge theory can be computed by taking derivatives of (1.14) with respect to the source $\phi_0(\vec{x})$ where each derivative will bring down an operator $\mathcal{O}(\vec{x})$ in the conformal field theory.

The correspondence in (1.14) is strictly speaking only valid for massless scalar field ϕ. For massive case, there is relation between the mass m of the field ϕ and the scaling dimension Δ of the operator $\mathcal{O}$. In AdS_{d+1}, the Klein-Gordon equation for a massive field ϕ in Euclidean signature has two independent solutions that behave as $z^{d-\Delta}$ and z^{Δ} near the boundary $z = 0$, with

$$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + L^4 m^2} \tag{1.15}$$

[3] Here we use coordinate $z \propto \frac{1}{u}$.

which is the largest root of $\Delta(\Delta - d) = L^4 m^2$. In that case, the boundary condition should be changed to $\phi(\vec{x}, \epsilon) = \epsilon^{d-\Delta}\phi_0(\vec{x})$ with $\epsilon \to 0$. A detailed study reveals that one has to regulate the theory by introducing an IR cutoff.

Generically correlators in quantum field theory can contain divergences. Therefore, we need to renormalize the theory in order to have meaningful observables. These divergences must also appear in string theory as the feature of gauge/gravity correspondence. Indeed, there is a relation of divergences between two side of these theories called as UV/IR connection [10]. It connects the ultraviolet effects or UV-divergences in quantum field theory with the infrared effects or IR-divergences in string theory. Eliminating the IR-divergences in string theory should effect on removing the UV-divergences in quantum field theory. The procedure to remove this IR-divergences in string theory is known as holographic renormalization [22, 72].

The background metrics considered in gauge/gravity correspondence are mostly asymptotically AdS. The on-shell action of a bulk field on these metrics near the boundary contains terms that are divergent depending on the scale dimension of the dual operator. In order to remove these divergent terms, we first regulate the on-shell action by cutting the space near the boundary at a point that is very close to the boundary at $z = \epsilon$ where ϵ is a small positive number. The regulated action contain two terms which are divergent and convergent as we take a limit $\epsilon \to 0$,

$$S_{reg}[\epsilon] = S_{reg}^{div}[\epsilon] + S_{reg}^{con}[\epsilon]. \tag{1.16}$$

The divergent part of the regulated action can be removed by a counterterm action[4]

$$S_{ct}[\phi;\epsilon] = -S_{reg}^{div}[\phi_0;\epsilon]. \tag{1.17}$$

Now we obtain a renormalized on-shell action without IR-divergencies which is given by

$$S_{ren}^{on-shell}[\phi] = \lim_{\epsilon\to 0}\left(S_{reg}[\phi;\epsilon] + S_{ct}[\phi;\epsilon]\right). \tag{1.18}$$

A detail application of this holographic renormalization will be discussed in section 3.D.1.

1.4.2 Top-down approach

AdS/CFT gives a tool to study the strongly coupled regime of quantum field theories. In the original proposal of AdS/CFT by Maldacena [2], the $\mathcal{N} = 4$ $U(N)$ supersymmetric Yang-Mills is clearly far from the expected theory

[4]There is a subtlety in writing the counterterm action. The counterterm action must be covariant and be written in terms of the bulk field ϕ instead of the boundary source ϕ_0. The details of how to write the covariant counter term action can be found in [22, 72].

namely QCD. Beside the finite large gauge symmetry group, it has maximum supersymmetry and does not have flavor symmetry. One proposal to break supersymmetry is to consider $D4$-brane system and to compactify one of the spatial direction in $D4$-branes with anti-periodic boundary conditions on fermions breaking the supersymmetry [9]. The flavor symmetry can also be added to the system within the frame work of probe D-branes [36].

One can consider various D-branes configurations and try to get as close as possible to the more realistic models. All these setups have a clear gravity picture where one can write down the low energy effective action on the gravity side. These D-branes constructions are known as “top-down” apporach. Many examples of the “top-down“ approach can be found in [37] and the references therein.

1.4.3 Bottom-up approach

In ”top-down“ approach most of the theories which can be solved using AdS/CFT techniques differ substantially from QCD in particular regarding the lack of asymptotically freedom and the strong coupling in the UV regime. Inspired by holography and using the tools that were reviewed in section 1.4.1, another approach is to start from the known phenomenological models in gauge theory and try to construct the background metric and field content of the gravity side. In this approach, we don’t have a complete picture of the gravity theory but nevertheless we can loosely apply the correspondence between some of the fields in gravity theory, with a background metric and usually non-interacting action, and operators in gauge theory. This type of construction is called ”bottom-up“ approach. Examples of this ”bottom-up“ approach will be discussed in the next section.

1.5 Holographic models of Hadrons

1.5.1 Hard-wall model

A theory has been built starting from QCD and constructing its 5-dimensional holographic dual which differs from other theories, by means of ”top-down“ approach, which are basically trying to deform supersymmetric Yang-Mills theory in order to obtain QCD. This theory from its approach is in the class of Holographic model of Hadrons and in particular known as AdS/QCD model [12, 15].

This model has 4 free parameters which can be fixed by the number of colors N_c, ρ meson mass, π mass, and pion decay constant F_π. Fixing these parameters, the model can predict other low energy hadronics observables within $10\% - 15\%$ accuracy [12]. Furthermore, the properties such as vector meson dominance and QCD sum rules show up naturally in this AdS/QCD model.

The field content of 5D-theory consists of one scalar and two gauge fields. It was engineered to reproduce holographically the dynamics of chiral symmetry breaking in QCD. On the 4D-theory, the relevant operators are one quark condensate operator and two current operators which are the left- and right-handed currents corresponding to the $SU(N_f)_L \times SU(N_f)_R$ chiral flavor symmetry. The global chiral flavor symmetry will correspond to gauge symmetry in the 5D-theory. These operators are important in the chiral dynamics and the relation of their parameters with 5-dimensional fields are described in table 1.1

4D:$\mathcal{O}(x)$	5D:$\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$\frac{2}{z}X^{\alpha\beta}$	0	3	-3

Table 1.1: 4D-Operators/5D-fields of the holographic model

The 5D masses m_5 are determined via the relation [4]

$$(m_5)^2 = (\Delta - p)(\Delta + p - 4), \tag{1.19}$$

where Δ is the dimension of the corresponding p-form operator. The factor $1/z$ in table 1.1 is to give the correct dimension to the operator $\bar{q}q$ with z corresponds to the energy scale of QCD.

The simplest possible metric for this AdS/QCD model is a slice of the AdS metric

$$ds^2 = \frac{1}{z^2}(-dz^2 + dx^\mu dx_\mu), \;\; 0 < z \leq z_m. \tag{1.20}$$

As we mentioned before, the fifth coordinate z corresponds to the energy scale [13] with momentum transfer $Q \sim 1/z$. With this metric, we neglect the running of the QCD gauge coupling in a window of scales until an IR(infrared) scale $Q_m \sim 1/z_m$ where the 4-dimensional theory is confining and at this scale the AdS space is cut-off by introducing an IR cutoff or "infrared brane"(IR-brane) in the metric at $z = z_m$ and imposing certain boundary conditions on the fields at $z = z_m$. Therefore this model is called hard-wall model. In addition, an UV cutoff can be provided to $z = \epsilon$ with $\epsilon \ll 1$.

5D action

The action of the 5D-theory is given by [12]

$$S = \int d^5x\sqrt{g}\,Tr\left\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}, \tag{1.21}$$

where

$$D_\mu X = \partial_\mu X - iA_{L\mu}X + iXA_{R\mu}, \qquad X = X_0\, e^{2i\pi^a t^a} \tag{1.22}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu], \qquad A_{L,R} = A^a_{L,R} t^a, \tag{1.23}$$

where X_0 is the background field and π^a are the $N_f^2 - 1$ pion fields.

At the IR-brane, we must impose some gauge invariant boundary conditions and the simplest choice is $(F_L)_{z\mu} = (F_R)_{z\mu} = 0$. We also fix the gauge $A_z = 0$ in which the boundary conditions now become Neumann. The classical solution to X is determined in such it satisfies the UV boundary condition $(2/\epsilon)X(\epsilon) = M$ and the IR boundary condition where the quarks condensate:

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3, \tag{1.24}$$

where matrix M and Σ are the quark mass and the quark condensate respectively act as input parameters. Assume the mass and quark condensate matrixs to be the following; $M = m.\mathbb{I}$ and $\Sigma = \sigma.\mathbb{I}$, with m and σ are constants.

As we can see this hard-wall model has four free parameters: m, σ, z_m, and g_5. The gauge coupling g_5 can be fixed by comparing the holographic computation with the QCD OPE(Operator Product Expansion) [14] for the product of two currents, where the current corresponds to vector defined as $V = (A_L + A_R)/2$, which gives us

$$g_5^2 = \frac{12\pi^2}{N_c}, \tag{1.25}$$

with N_c is the number of gauge fields.

1.5.2 Soft-wall model

The hard-wall model successfully describes the spectrum of the lowest energy hadrons, however it is unable to explain the linear spectrum of excited hadrons, $m_n^2 \sim n$. Instead, it shows that the masses of excited hadron grow as $m_n \sim n^2$. An improvement was made to the hard-wall model by considering a smooth rather than a hard cutoff in the 5D-theory of AdS/QCD model. This model is called soft-wall model specifically the IR-brane is now replaced by a smooth dilaton profile up to $z = +\infty$.

5D action

The 5D-action, is that of the hard-wall model plus a non-dynamical dilaton, with $U(1)$ gauge symmetry is [15]

$$S = \int d^5x e^{-\Phi}\sqrt{g}\left\{-|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}, \tag{1.26}$$

where g_5^2 is given by (1.25) and the background fields for metric and dilaton are

$$ds^2 = e^{2A(z)}\left(dz^2 + dx^\mu dx_\mu\right), \tag{1.27}$$

$$\Phi = \Phi(z). \tag{1.28}$$

Solutions to these background fields are obtained by considering the spectrum of radial ρ excitations in such a way $\Phi(z) - A(z) \sim z^2$ for large z. Another consideration should be taken into account is the conformal symmetry in the UV near the boundary which is $\Phi(z) - A(z) \sim \ln z$ for small z. The simplest example solution for the background fields (1.27) and (1.28) is $\Phi(z) - A(z) = z^2 + \ln z$. Indeed this solution gives a nice formula for the mass spectrum of ρ mesons in the units of the lowest ρ spectrum [15]:

$$m_n^2 = 4(n+1). \tag{1.29}$$

1.6 Thermal field theory

QGP is considered as a finite temperature system. Therefore we need AdS/CFT correspondence in which the gravity theory possesses the characteristic features of finite temperature system of the corresponding gauge theory. First, let's briefly review the characteristic features of a finite temperature system from field theory perspective.

In finite temperature system of field theory, time is a complex variable with imaginary part is periodic. The period of the imaginary part is $\beta = 1/T$ which is the inverse of temperature. Physics can be studied using imaginary or real time-formalisms. In this thesis we will only discuss the real-time formalism which is more interesting in particular if we want to study the system that slightly deviates from the equilibrium.

In real-time formalism, time t is allowed to be a complex variable with aforementioned periodicity in its imaginary part. The path $\mathcal{C}$ in complex t-plane is taken such that the imaginary part of t is decreasing, as we increase the parameter of the path ϑ, in order to have a well defined propagator. The time-ordering $\mathcal{T}_\mathcal{C}$ is generalized to the complex t-plane along this path $\mathcal{C}$, $t = t(\vartheta)$ (with large value of ϑ is later than small value of ϑ). We also generalize δ- and θ-functions in terms of the path $\mathcal{C}$.

Thermal Green's functions $G_\beta(x_1, \cdots, x_n)$ of an operator O are defined by [25]

$$G_\beta(x_1, \cdots, x_n) = \frac{1}{\mathrm{Tr}\, e^{-\beta H}} \mathrm{Tr}\left[e^{-\beta H} \mathcal{T}_\mathcal{C}\left(O(x_1)\cdots O(x_n)\right)\right], \tag{1.30}$$

where H is the Hamiltonian operator. In terms of the generating functional $Z[j]$, the thermal Green's function can be written as

$$G_\beta(x_1, \cdots, x_n) = \frac{1}{i^n Z_\mathcal{C}[\beta, j]} \frac{\delta^n Z_\mathcal{C}[\beta, j]}{\delta j(x_1)\cdots \delta j(x_n)}\bigg|_{j=0}, \tag{1.31}$$

with

$$Z_{\mathcal{C}}[\beta, j] = \mathrm{Tr}\left[e^{-\beta H}\mathcal{T}_{\mathcal{C}} \exp\left(i\int_{\mathcal{C}} d^4x\, j(x)O(x)\right)\right]. \tag{1.32}$$

In the form of a path integral, the generating functional is given by

$$Z_{\mathcal{C}}[\beta, j] = \int \mathcal{D}O \exp\left(i\int_{\mathcal{C}} d^4x\,(\mathcal{L}(x) + j(x)O(x))\right), \tag{1.33}$$

where $\mathcal{L}(x)$ is the Lagrangian density and $j(x)$ is the source for field $O(x)$. Note that we have a boundary condition[5] $O(t, \vec{x}) = O(t - i\beta, \vec{x})$.

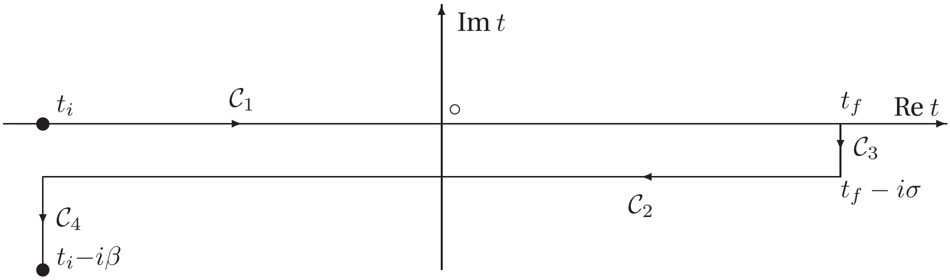

Figure 1.3: The modified Schwinger-Keldysh time contour

The path $\mathcal{C}$ can be taken in various ways. One common version is drawn in Figure 1.3 where the path $\mathcal{C}$(also the fields and operators) is divided into four segments $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3$, and $\mathcal{C}_4$. The first path $\mathcal{C}_1$ starts from t_i and ends at t_f. This is where the physical field O_1 that we observe lives. It is continued by $\mathcal{C}_3$ which makes a vertical turn from t_f to $t_f - i\sigma$, with σ is arbitrary between 0 to β. The next path is $\mathcal{C}_2$ which is parallel to path $\mathcal{C}_1$ but takes the opposite direction from $t_f - i\sigma$ to $t_i - i\sigma$. The field O_2 lives in $\mathcal{C}_2$ acts as a "ghost" field which contribute only to the internal line of the thermal Green's function. Lastly, the path $\mathcal{C}_4$ takes another vertical turn starts from $t_i - i\sigma$ and ends at $t_i - i\beta$. With this division of the path $\mathcal{C}$, the generating functional consists of four Lagrangian densities correspond to different segments.

In general, parameter σ can be chosen arbitrarily. One of the example is $\sigma = \beta/2$ which was studied in [45]. If $(t_i = -t_f) \to +\infty$, the generating functional can be factorized in to two parts

$$Z_{\mathcal{C}} = Z_{\mathcal{C}_{12}} Z_{\mathcal{C}_{34}}, \tag{1.34}$$

[5]Here we assume field $O(x)$ to be bosonic. For fermionic case, the boundary condition is anti-periodic in the direction of imaginary component of t.

where $\mathcal{C}_{ij} = \mathcal{C}_i \cup \mathcal{C}_j$. Therefore, we can effectively work with just generating functional $Z_{\mathcal{C}_{12}}$. The action for $Z_{\mathcal{C}_{12}}$ is the sum of contributions from the two parts of the path,

$$S = \int_{t_i}^{t_f} dt\, L(t) - \int_{t_i}^{t_f} dt\, L\left(t - i\frac{\beta}{2}\right), \tag{1.35}$$

where

$$L(t) = \int d\vec{x}\, \mathcal{L}[O(t,\vec{x})]. \tag{1.36}$$

So, the generating functional $Z_{\mathcal{C}_{12}}$ is

$$Z_{\mathcal{C}_{12}}[j_1, j_2] = \int \mathcal{D}\phi\, \exp\left(iS + i\int_{t_i}^{t_f} dt \int d\vec{x}\, j_1(x) O_1(x) - i\int_{t_i}^{t_f} dt \int d\vec{x}\, j_2(x) O_2(x) \right), \tag{1.37}$$

where $j_1(j_2)$ is the physical("ghost") source living in $\mathcal{C}_1(\mathcal{C}_2)$ and for the fields living in path $\mathcal{C}_2$ the time is understood to be $t \equiv t - i\beta/2$. From now on the parameter β will be implicit.

Now we can take second variations of $Z_{\mathcal{C}_{12}}$ with respect to the source j_1 or j_2 and obtain the Schwinger-Keldysh propagators from the free action,

$$iD_{ab}(x-y) = \frac{1}{i^2} \frac{\delta^2 \ln Z_{\mathcal{C}_{12}}[j_1, j_2]}{\delta j_a(x)\, \delta j_b(y)}\bigg|_{j_a = j_b = 0} = i \begin{pmatrix} D_{11} & -D_{12} \\ -D_{21} & D_{22} \end{pmatrix}, \tag{1.38}$$

with $a, b = 1, 2$ and

$$\begin{aligned} iD_{11}(t,\vec{x}) &= \langle \mathcal{T} O_1(t,\vec{x}) O_1(0) \rangle, & iD_{12}(t,\vec{x}) &= \langle O_2(0) O_1(t,\vec{x}) \rangle, \\ iD_{21}(t,\vec{x}) &= \langle O_2(t,\vec{x}) O_1(0) \rangle, & iD_{22}(t,\vec{x}) &= \left\langle \overline{\mathcal{T}} O_2(t,\vec{x}) O_2(0) \right\rangle. \end{aligned} \tag{1.39}$$

where $\overline{\mathcal{T}}$ denotes reversed time ordering in path $\mathcal{C}_2$, and

$$O_1(t,\vec{x}) = e^{iHt - i\vec{P}\cdot\vec{x}} O(0) e^{-iHt + i\vec{P}\cdot\vec{x}}, \tag{1.40a}$$
$$O_2(t,\vec{x}) = e^{iH(t-i\beta/2) - i\vec{P}\cdot\vec{x}} O(0) e^{-iH(t-i\beta/2) + i\vec{P}\cdot\vec{x}}. \tag{1.40b}$$

These Schwinger-Keldysh propagators are related to the retarded and advanced Green's functions, which are defined as

$$iG^{\text{Ret}}(x-y) = \theta(x^0 - y^0) \langle [O(x),\, O(y)] \rangle, \tag{1.41a}$$
$$iG^{\text{Adv}}(x-y) = \theta(y^0 - x^0) \langle [O(y),\, O(x)] \rangle. \tag{1.41b}$$

In momentum space, defined by

$$G(k) = \int dx\, e^{-ik\cdot x} G(x), \tag{1.42}$$

we can show that

$$G^{\text{Adv}}(k) = G^{\text{Ret}*}(k). \tag{1.43}$$

Furthermore, we can rewrite the Schwinger-Keldysh propagators in terms of retarded Green's function as below (for bosons):

$$\begin{aligned} D_{11}(k) &= \text{Re}\, G^{\text{Ret}}(k) + i \coth \frac{\omega}{2T} \,\text{Im}\, G^{\text{Ret}}(k), \\ D_{22}(k) &= -\text{Re}\, G^{\text{Ret}}(k) + i \coth \frac{\omega}{2T} \,\text{Im}\, G^{\text{Ret}}(k), \\ D_{12}(k) &= D_{21}(k) = \frac{2ie^{-\frac{\beta}{2}\omega}}{1-e^{-\beta\omega}} \,\text{Im}\, G^{\text{Ret}}(k). \end{aligned} \tag{1.44}$$

with $\omega \equiv k^0$.

1.7 Holographic real-time propagator

The gauge/gravity correspondence was originally formulated with Euclidean signature. For some cases, we need to perform computation of Green's functions of gauge theory with Lorentzian signature. While there are subtleties working with Lorentzian signature AdS/CFT correspondence [46–48], one could try to avoid Minkowski formulation of AdS/CFT by working with the Euclidean version. The resulting correlators can be analytically continued to Minkowski space using Wick rotation. Unfortunately this does not always work, in particular for finite temperature gauge theory. Analytic continuation to Minkowski space is possible only when we know the Euclidean correlators for all Matsubara frequencies which are beyond reach.

To see how the problem arises, consider as an example a scalar field ϕ in the AdS_5 black hole background with metric

$$\begin{aligned} ds^2 &= \frac{L^2}{z^2}\left(-h(z)dt^2 + dx^i dx^i + \frac{1}{h(z)}dz^2\right) = g_{\mu\nu}dx^\mu dx^\nu + g_{zz}dz^2, \\ h(z) &= 1 - \frac{z^4}{z_H^4}, \end{aligned} \tag{1.45}$$

where $i = 1, 2, 3$ and z in the range $z_B \leq z \leq z_H$, with the following action

$$S = \int d^4x \int_{z_B}^{z_H} \sqrt{-g}\left(g^{zz}(\partial_z\phi)^2 + g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right). \tag{1.46}$$

The integration is taken between the boundary z_B and the horizon z_H.

The equation of motion for ϕ is given by

$$\frac{1}{\sqrt{-g}}\partial_z\left(\sqrt{-z}g^{zz}\partial_z\phi\right) + g^{\mu\nu}\partial_\mu\partial_\nu\phi - m^2\phi = 0. \tag{1.47}$$

The equation has to be solved with a fixed value at z_B for the solution of the form

$$\phi(z,x)=\int\frac{d^4k}{(2\pi)^4}e^{ik_\mu x^\mu}f_k(z)\phi_0(k), \tag{1.48}$$

with $f_k(z_B)=1$ and $\phi_0(k)$ is identified as the Fourier transform of a source field in the gauge theory. The effective equation of motion for the radial profile $f(z)$ is

$$\frac{1}{\sqrt{-g}}\partial_z\left(\sqrt{-z}g^{zz}\partial_z f_k\right)-(g^{\mu\nu}k_\mu k_\nu+m^2)f_k=0. \tag{1.49}$$

In order to have a unique solution for $f_k(z)$, we need to impose a condition at the horizon z_H. In the Euclidean signature, this can be done by imposing a regularity condition at the horizon z_H. But this is not the case for Lorentzian signature since near the horizon $f_k(z)$ oscillates wildly and has two modes(incoming and outgoing modes). Physical reasoning implies that the incoming modes correspond to the retarded Green's function while the outgoing modes correspond to the advanced Green's function.

Knowing the choices for boundary condition at the horizon does not immediately solve the problem. Suppose we want to compute the retarded Green's function by taking the incoming-wave boundary condition. The on-shell action of (1.46) reduces to

$$\begin{aligned}S&=\int\frac{d^4k}{(2\pi)^4}\phi_0(-k)\mathcal{F}(z,k)\phi_0(k)\Big|_{z_B}^{z_H},\\ \mathcal{F}(z,k)&=\sqrt{-g}g^{zz}f_{-k}(z)\partial_z f_k(z).\end{aligned} \tag{1.50}$$

The retarded Green's function is computed by taking two functional derivatives over ϕ_0 of the on-shell action which give us

$$G^{\text{Ret}}(k)=-\mathcal{F}(z,k)-\mathcal{F}(z,-k)|_{z_B}^{z_H}. \tag{1.51}$$

Using $f_k^*(z)=f_{-k}(z)$, which is also a solution, we can show that the imaginary part of $\mathcal{F}$ is proportional to a conserved flux and such it is independent of z. This means that the retarded Green's function $G^{\text{Ret}}(k)$ is a real function which is not a satisfying result since the retarded Green's function in general is a complex function.

1.7.1 Minkowski prescription I

Son and Starinet gave an ad hoc resolution to this problem of how to compute the Green's function in Minkowski AdS/CFT correspondence consistently [26]. They provided a prescription and various checks on the validity of the formula. The prescription goes as follows:

1. Solve the mode equation (1.48) with two boundary conditions at the boundary z_B and the horizon z_H. First, at the boundary $z = z_B$, $f_k(z_B) = 1$. Second, the asymptotic solution is incoming(outgoing) wave at the horizon for retarded(advanced) Green's function. If we have space-like momenta then the second boundary condition is similar to the Euclidean version which is regular at the horizon.

2. Evaluating at the boundary $z = z_B$, the retarded(advanced) Green's function is given by

$$G(k) = -2\mathcal{F}(z_B, k), \tag{1.52}$$

   where $\mathcal{F}$ is computed from the surface terms of the on-shell action as shown in (1.50).

Despite the success of the prescription, it can only be applied to two point-functions. Extension of the prescription to more than two-point functions was not known. Besides, the lack on the details of how the prescription works left some questions to be answered.

The real-time formulation in finite temperature field theory involves doubling the degree of freedom. At the same time, the full Penrose diagram of asymptotically AdS metric containing a black hole has two boundaries and it was conjectured that there are doubler fields living on the second boundary of the AdS dual. These features were indeed realized by Herzog and Son's formulation of a more rigorous way to compute the Green's function in Minkowski AdS/CFT [42]. Their results originate in studies on black holes thermal radiation by Hawking and Hartle [49], Unruh [50], and Israel [51].

The upshot of their observation is that the gravity action must be modified by adding contribution from region L, where the doubler fields live, as shown in Figure 1.4; time in the region L reverses its direction. The bulk fields in both regions R and L are written in terms of physical and "ghost" sources of the finite temperature field theory, as defined previously, with boundary conditions that at the boundary of R the bulk fields in R becomes the physical sources and at the boundary of L the bulk fields in L becomes the "ghost" sources. At the horizon, the natural boundary conditions are defined so that positive frequency modes are incoming and negative frequency modes are outgoing in region R of the Penrose diagram. With the definition of retarded and advanced Green's functions in [26], as given in (1.52), second functional derivatives of the boundary action on gravity side over the sources yields Schwinger-Keldysh propagators (1.44).

There are a few interesting points in Herzog and Son's formula. It looks natural from gravity side to take the path $\mathcal{C}$ for complex time plane as shown in Figure 1.3 with $\sigma = \beta/2$. Since the Green's functions are obtained by functional derivatives on the gravity action, in principal we can extend the formula to more than two-point functions. A detailed discussion on thermal three-point functions from Minkowski AdS/CFT using this formula can be found in [52].

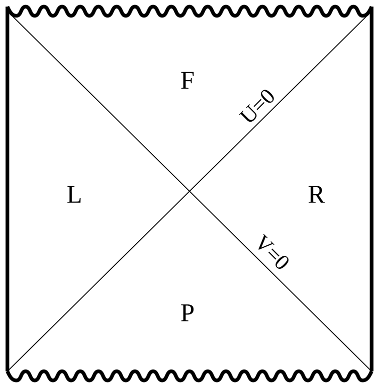

Figure 1.4: The full Penrose diagram for asymptotically AdS metric with a black hole solution. U and V are the Kruskal coordinates.

1.7.2 Minkowski prescription II

Although Herzog and Son's formula gives the correct Schwinger-Keldysh propagators of the finite temperature system, there are still some unsatisfactory issues in the procedure. The computation they did in [42] did not include the boundary contribution from timelike infinity to the on-shell action which is non vanishing in general. Furthermore, it depends entirely on the retarded and advanced Green's function that are still conjectured in Minkowski AdS/CFT.

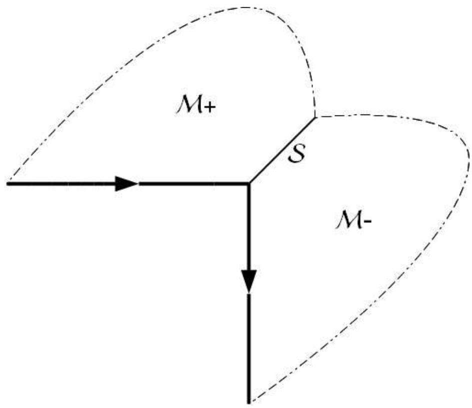

Figure 1.5: Holographic path. The bold lines with arrow are the segments of complex time path of the field theory at the boundary.

Skenderis and van Rees subsequently showed how to overcome these issues and developed a fully holographic prescription [73, 74]. The idea is try to construct a bulk manifold $\mathcal{M}_\mathcal{C}$ from a given complex time path $\mathcal{C}$ in finite temperature field theory at the boundary. It involves gluing different manifolds for each segments of the path $\mathcal{C}$. The paths that live in the real part of time correspond to Lorentzian solutions and ones live in the imaginary part of time correspond to Euclidean solutions.

When two segments of the complex time path $\mathcal{C}$ intersect at a point, the point is extended to a hypersurface $\mathcal{S}$ in the bulk. The time signature of the metric changes at this hypersurface corresponds to this intersection point. In this hypersurface $\mathcal{S}$, we need to impose two matching conditions:

1. Continuity of the field ϕ across $\mathcal{S}$:

$$\phi_-(\mathcal{S}) = \phi_+(\mathcal{S}). \tag{1.53}$$

2. If the two bulk manifolds $\mathcal{M}_-$ and $\mathcal{M}_+$ (correspond to two segments of the path) intersect at a boundary $\mathcal{S}$ (corresponds to the intersection point between two segments of the path) then we also impose continuity of the momentum conjugate π^ϕ across $\mathcal{S}$:

$$\pi^\phi_-(\mathcal{S}) = \eta\, \pi^\phi_+(\mathcal{S}), \tag{1.54}$$

   where $\pi^\phi_\pm$ is the conjugate momentum in $\mathcal{M}_\pm$. We set $\eta = -i$ if $\mathcal{M}_-$ is Euclidean and $\mathcal{M}_+$ is Lorenzian; and $\eta = 1$ if both $\mathcal{M}_-$ and $\mathcal{M}_+$ are Lorentzian (this is the case with $\sigma = 0$ in Figure 1.3).

A more detailed application of Skenderis and Van Rees's holographic prescription can be found in section 3.D.3.

1.8 Outline

In chapter 2, we will apply gauge/gravity correspondence on photon and dilepton production in QGP. We start with definition of spectral density function in momentum space $\chi(K)$ which is proportional to the photon and dilepton production rates. Having determined the observables in gauge theory or QGP, We write down the relevant $5D$-action in gravity theory with AdS black hole background metric. In particular we consider the AdS/QCD soft-wall model, as discussed in 1.5.2, with non-trivial dilaton background. Using holographic real-time prescription 1.7.1, we compute the spectral density function by means of solving the equations of motion of the dual fields numerically and also analytically for low- and high-frequency.

Using a semiclassical approach to gauge/gravity duality, we describe in chapter 3 Brownian motion of a quark in strongly coupled plasma, for example QGP, from string theory perspective. We first define properties of Brownian motion given by a generalized Langevin equation. In this case, the random force R is assumed to be Gaussian and the Langevin equation is linear in momentum. The gravity description is given by the motion of a string under some black hole background metric where the ends of the string are stretching from boundary to horizon. The end of the string at the boundary is interpreted as an external quark behaves as Brownian particle moving in a heat bath, which are represented by the black hole background metric. Explicitly, we consider the background metrics which are non-rotating BTZ black hole for neutral plasma and STU black holes for charged plasma.

The two- and four-point functions of the random force R can be computed by taking derivatives of the dual coordinate in the expansion of the Nambu-Goto action. In computing the correlation functions, we use the holographic real-time prescription 1.7.2 together with holographic renormalization 1.4.1 to removed the UV divergences while the IR divergences are removed by introducing a cutoff near the horizon. Most of the calculations are done in low frequency limit, $\omega \to 0$. This way we can find the impedance $\mu(\omega)$ from the two-point functions which eventually gives us the friction coefficient in non-relativistic limit. With a simple model of random force profile, a time scale can be extracted from the two- and four-point functions which is defined as mean-free path time t_{mfp}.

The last chapter 4 is an attempt to study anisotropic effects in QGP from semiclassical string point of view as we already used in chapter 3. Here, we argue that the anisotropic in QGP can be encoded in rotating black hole solutions. This chapter mainly discusses about how to compute the drag force with a given background metric. We first consider a $4D$ AdS-Schwarzschild black hole and compute the drag force of the great circle solution at the equatorial plane with linear ansatz and then generalize the drag force computation for non-equatorial case.

As one example of rotating black hole solutions, we look at $4D$ Kerr-AdS black hole in Boyer-Lindquist coordinates. A simple drag force computation will be the equatorial great circle solution with linear ansatz. The non-equatorial solutions in general are very difficult. For a simple case, we consider drag forces at the leading order for small angular momentum a and velocity ω of the Kerr-AdS black hole. We use a map of coordinates transformation from Kerr-AdS coordinates to Boyer-Lindquist coordinates to derive an ansatz for "static" string solution in Boyer-Lindquist coordinates. The solution is written in terms of the static thermal rest mass quark m_{rest} and the temperature of plasma T. We also plot the drag forces for different values of angular momentum a and parameter M_T.