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Vibrations in Materials with Granularity

Zorana Zeravcic

Vibrations in Materials with Granularity

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Mojim makiću, Bebiju i Andreju

More is Different.

P. W. ANDERSON

CONTENTS

1	Introduction	1
1.1	Vibrations in Classical Solids	2
1.1.1	Dynamical matrix	2
1.1.2	Vibrational spectrum	3
1.1.3	Disorder	5
1.1.4	Localization	5
1.2	Vibrational modes in granular matter near Jamming	6
1.2.1	Jamming idea	7
1.2.2	Density of States	8
1.2.3	Maxwell Rigidity Criterion	9
1.2.4	Packings	11
1.2.5	Anomalous scalings	12
1.3	Vibrational modes and response to driving in bubble clusters	16
1.4	This Thesis	18
2	Excitations of Ellipsoid Packings	21
2.1	Introduction	22
2.2	The Maxwell stability argument for spheroids and the occurrence of zero modes	23
2.3	Interaction potential: the Gay-Berne potential	24
2.4	Preparation of the packings and elimination of rattlers	25
2.5	Equations of motion and dynamical matrix	26
2.5.1	Equations of motion	27
2.5.2	Linearization of the equations of motion	29
2.5.3	Dynamical matrix	30
2.5.4	Units and rescaling of the frequencies	32
2.6	Analysis	32
2.6.1	Continuous change of the average contact number Z and volume fraction ϕ	32
2.6.2	Harmonic potential	33
2.6.3	Hertzian potential	39
2.6.4	Participation ratio	40
2.6.5	Scaling of the relevant frequencies	41

2.6.6	Absence of elastic modes in our systems	44
2.7	Conclusions and outlook	44
3	Localization in Granular Packings	47
3.1	Introduction	48
3.1.1	Reminder on localization	49
3.1.2	Types of disorder	49
3.1.3	Outline	50
3.2	Method	50
3.3	Granular packings and $D(\omega)$	51
3.4	Measuring the localization length ξ	52
3.4.1	Anisotropy and spread	52
3.4.2	Frequency bin-averaged localization length $\bar{\xi}(\omega)$	54
3.4.3	Quasi-localized low-frequency modes at high pressure	56
3.4.4	Distribution of large ξ	57
3.4.5	Level spacing statistics	58
3.5	Implications from Random Matrix Theory	58
3.5.1	Distribution of large ξ 's revisited	63
3.6	Exploring the method	65
3.6.1	$1d$: disordered chain	65
3.6.2	$2d$: disordered hexagonal lattice	67
3.6.3	Percolation clusters	69
3.7	Conclusions	73
4	Collective oscillations in bubble clouds	75
4.1	Introduction	76
4.2	Qualitative discussion of the physical ingredients and competing effects and main results	79
4.2.1	Single bubble properties: resonance frequency, damping and Q-factor	79
4.2.2	Bubble-bubble interactions	80
4.2.3	The Density of States (DOS)	80
4.2.4	Excitation field	80
4.2.5	The effect of viscous damping and number of bubbles on collective dynamics	81
4.2.6	Polydispersity	81
4.2.7	Influence of geometry and geometric disorder	82
4.2.8	Effect of dimensionality	82
4.2.9	Localization	83
4.3	Oscillations of the bubbles	83
4.3.1	Extended Rayleigh-Plesset equation with driving	83
4.3.2	Spectrum of the system	84
4.3.3	Response of the system to harmonic driving	86
4.3.4	Localization of vibrations	87

4.4	Results: Monodisperse bubbles on a lattice	87
4.4.1	Undriven system	87
4.4.2	Driven system	90
4.5	Results: Polydisperse bubbles on a lattice	94
4.5.1	Weak disorder	94
4.5.2	Strong disorder	99
4.5.3	Exponential vs. power-law localization	105
4.5.4	Polydisperse bubbles on a random underlying structure	106
4.6	Outlook on experimental verification	107
Bibliography		111
Samenvatting		119
Publications		125
Curriculum vitae		127
Acknowledgements		129

