Galois representations of elliptic curves and abelian entanglements

Brau Avila, Julio

Author: Brau Avila, Julio

Title: Galois representations of elliptic curves and abelian entanglements

Issue Date: 2015-12-01

Stellingen

behorende bij het proefschrift

Galois Representations of elliptic curves and abelian entanglements

door **Julio Brau Avila**

1. Let $a, b \in \mathbb{Q}$ be rational numbers such that $E_{a,b} : Y^2 = X^3 + aX + b$ defines an elliptic curve that does not have complex multiplication over $\overline{\mathbb{Q}}$. Then there exists a deterministic algorithm which, given as inputs such a and b, determines the image of the Galois representation $\rho_{E_{a,b}}$ attached to the torsion points of $E_{a,b}$.

2. Let $E/\mathbb{Q}$ be a Serre curve. Let D be the discriminant of $\mathbb{Q}(\sqrt{\Delta})$, where Δ is the discriminant of any Weierstrass model of E over $\mathbb{Q}$, and let C_E be the density (conditional on GRH) of primes p such that the group $\tilde{E}(\mathbb{F}_p)$ is cyclic. Then

$$C_E = \mathfrak{C}_E \prod_{\ell} \left(1 - \frac{1}{(\ell^2 - 1)(\ell^2 - \ell)}\right)$$

where the entanglement correction factor $\mathfrak{C}_E$ is given by

$$\mathfrak{C}_E = \begin{cases} 1 & \text{if } D \equiv 0 \pmod 4 \\ 1 + \prod_{\ell \mid 2D} \dfrac{-1}{(\ell^2 - 1)(\ell^2 - \ell) - 1} & \text{if } D \equiv 1 \pmod 4 \end{cases}$$

3. There exists a modular curve $X'(6)$ of level 6 defined over $\mathbb{Q}$ whose $\mathbb{Q}$-rational points correspond to j-invariants of ellip-

tic curves E over $\mathbb{Q}$ that satisfy $\mathbb{Q}(E[2]) \subset \mathbb{Q}(E[3])$, hence do not have abelian entanglements.

4. The modular curve $X'(6)$ completes a set $\mathcal{X}$ of modular curves such that, for any elliptic curve E over $\mathbb{Q}$ we have that

$$E \text{ is not a Serre curve} \iff j(E) \in \bigcup_{X \in \mathcal{X}} j(X(\mathbb{Q})).$$

5. Let $E/\mathbb{Q}$ be the elliptic curve given by $Y^2+XY+Y = X^3 - X^2 - X - 14$. Let $K = \mathbb{Q}(\zeta_3)$ and $L_m = K(\sqrt[3]{m})$. Then there are infinitely many cube-free m such that rk $E/L_m = 0$.

   (J.Brau. Selmer groups of elliptic curves in degree p extensions, preprint. arxiv: 1401.3304, 2014.)

6. Suppose that G is a normal subgroup of $G_1 \times \cdots G_n$ such that the projection maps $\pi_i : G \to G_i$ are surjective for all i. Then the quotient $(G_1 \times \cdots \times G_n)/G$ is abelian.

7. Let $E/\mathbb{Q}$ be the elliptic curve given by Weierstrass equation $Y^2 + Y = X^3 - X^2 - 10X - 20$. Then we expect $\tilde{E}(\mathbb{F}_p)$ to be cyclic for around 61% of primes p.

8. Let E be a non-CM elliptic curve over $\mathbb{Q}$ and let S be the finite set of primes ℓ for which the representation $\rho_{E,\ell}$ is not surjective. Define

$$\mathcal{T} := \{2,3\} \cup S \cup \{\ell \ : \ \ell \mid N_E\},$$
$$m := \prod_{\ell \in \mathcal{T}} \ell.$$

   Then the integer m splits ρ_E, that is,

$$G = G_m \times \prod_{\ell \nmid m} \mathrm{GL}_2(\mathbb{Z}_\ell).$$