



Universiteit
Leiden
The Netherlands

Monotonicity and Boundedness in general Runge-Kutta methods

Ferracina, L.

Citation

Ferracina, L. (2005, September 6). *Monotonicity and Boundedness in general Runge-Kutta methods*. Retrieved from <https://hdl.handle.net/1887/3295>

Version: Corrected Publisher's Version

License: [Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden](#)

Downloaded from: <https://hdl.handle.net/1887/3295>

Note: To cite this publication please use the final published version (if applicable).

Monotonicity and Boundedness in general Runge-Kutta methods

Proefschrift

ter verkrijging van
de graad van Doctor aan de Universiteit Leiden,
op gezag van de Rector Magnificus Dr. D.D. Breimer,
hoogleraar in de faculteit der Wiskunde en
Natuurwetenschappen en die der Geneeskunde,
volgens besluit van het College voor Promoties
te verdedigen op dinsdag 6 september 2005
klokke 15.15 uur

door

Luca Ferracina

geboren te Vicenza, Italië
in 1973

Samenstelling van de promotiecommissie:

promotors: Prof.dr. M.N. Spijker
Prof.dr. J.G. Verwer (UvA/CWI)

referent: Dr. W. Hundsdorfer (CWI)

overige leden: Prof.dr. G. van Dijk
Prof.dr.ir. L.A. Peletier
Prof.dr. S.M. Verduyn Lunel

*Alla mia famiglia:
papà, mamma, Marco, Anna.*

**Monotonicity and Boundedness in
general Runge-Kutta methods**

THOMAS STIELTJES INSTITUTE
FOR MATHEMATICS



Preface

This thesis consists of an introduction and four papers which appeared (or were submitted for publication) in scientific journals. The introduction has been written with the intention to be understandable also for the reader who is not specialized in the field. The papers, which are listed below, are essentially self-contained, and each of them may be read independently of the others.

FERRACINA L., SPIJKER M.N. (2004): Stepsize restrictions for the total-variation-diminishing property in general Runge-Kutta methods, *SIAM J. Numer. Anal.* **42**, 1073–1093.

FERRACINA L., SPIJKER M.N. (2005): An extension and analysis of the Shu-Osher representation of Runge-Kutta methods, *Math. Comp.* **249**, 201–219.

FERRACINA L., SPIJKER M.N. (2005): Computing optimal monotonicity-preserving Runge-Kutta methods, submitted for publication, report Mathematical Institute, Leiden University, MI 2005-07.

FERRACINA L., SPIJKER M.N. (2005): Stepsize restrictions for total-variation-boundedness in general Runge-Kutta procedures, *Appl. Numer. Math.* **53**, 265–279.

Contents

Introduction	1
1 Monotonicity for Runge-Kutta methods	1
2 A numerical illustration	3
3 Guaranteeing the monotonicity property: reviewing some literature	6
4 The limitation of the approach in the literature	7
4.1 Stepsize restriction guaranteeing monotonicity for general Runge-Kutta methods	7
4.2 Optimal Shu-Osher representations	8
4.3 Computing optimal monotonic Runge-Kutta methods . . .	8
4.4 Boundedness for general Runge-Kutta methods	9
5 Scope of this thesis	10
Bibliography	11
I Stepsize restrictions for the total-variation-dimishing property in general Runge-Kutta methods	15
1 Introduction	16
1.1 The purpose of the paper	16
1.2 Outline of the rest of the paper	18
2 A general theory for monotonic Runge-Kutta processes	19
2.1 Stepsize-coefficients for monotonicity in a general context .	19
2.2 Irreducible Runge-Kutta schemes and the quantity $R(A, b)$	21
2.3 Formulation of our main theorem	23
3 The application of our main theorem to the questions raised in Sub- section 1.1	24
3.1 The equivalence of process (1.3) to method (2.2)	24
3.2 The total-variation-diminishing property of process (3.1) . .	25
3.3 The strong-stability-preserving property of process (3.1) . .	26
3.4 Illustrations to the Theorems 3.2 and 3.6	27
4 Optimal Runge-Kutta methods	28
4.1 Preliminaries	28
4.2 Optimal methods in the class $E_{m,p}$	28
4.3 An algorithm for computing $R(A, b)$, for methods of class $E_{m,p}$	30

4.4	Final remarks	31
5	Kraaijevanger's theory and our proof of Theorem 2.5	32
5.1	A theorem of Kraaijevanger on contractivity	32
5.2	The proof of Theorem 2.5	34
	Bibliography	40

II An extension and analysis of the Shu-Osher representation of Runge-Kutta methods 43

1	Introduction	44
1.1	The purpose of the paper	44
1.2	Outline of the rest of the paper	47
2	An extension, of the Shu-Osher approach, to arbitrary Runge-Kutta methods	49
2.1	A generalization of the Shu-Osher process (1.8)	49
2.2	A generalization of the Shu-Osher Theorem 1.1	50
2.3	Proving Theorem 2.2	52
3	Maximizing the coefficient $c(A, b, L)$	54
3.1	Irreducible Runge-Kutta schemes and the quantity $R(A, b)$	54
3.2	The special parameter matrix L^*	56
3.3	Proving Theorem 3.4	57
4	Applications and illustrations of the Theorems 2.2 and 3.4	59
4.1	Applications to general Runge-Kutta methods	59
4.2	Applications to explicit Runge-Kutta methods	60
4.3	Illustrations to the Theorems 3.4 and 4.3	62
	Bibliography	63

III Computing optimal monotonicity-preserving Runge-Kutta methods 67

1	Introduction	68
1.1	Monotonic Runge-Kutta processes	68
1.2	The Shu-Osher representation	69
1.3	A numerical procedure used by Ruuth & Spiteri	72
1.4	Outline of the rest of the paper	72
2	An extension and analysis of the Shu-Osher representation	73
2.1	A generalization of Theorem 1.1	73
2.2	The maximal size of $c(L, M)$	75
2.3	Proof of Theorems 2.5, 2.6	77
3	Generalizing and improving Ruuth & Spiteri's procedure	79
4	Illustrating our General Procedure III in a search for some optimal singly-diagonally-implicit Runge-Kutta methods	81
5	A numerical illustration	83
6	Conjectures, open questions and final remarks	85
	Bibliography	86

IV	Stepsize restrictions for total-variation-boundedness in general Runge-Kutta procedures	89
1	Introduction	90
1.1	The purpose of the paper	90
1.2	Outline of the rest of the paper	93
2	Kraaijevanger's coefficient and the TVD property	94
2.1	Irreducible Runge-Kutta methods and the coefficient $R(A, b)$	94
2.2	Stepsize restrictions from the literature for the TVD property	95
3	TVB Runge-Kutta processes	96
3.1	Preliminaries	96
3.2	Formulation and proof of the main result	97
4	Applications and illustrations of Theorem 3.2 and Lemma 3.6	100
4.1	TVB preserving Runge-Kutta methods	100
4.2	Two examples	101
4.3	A special semi-discretization given by Shu (1987)	102
5	The proof of Lemma 3.6	102
	Bibliography	106
	Samenvatting (Summary in Dutch)	109
	Curriculum Vitæ	111

