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Anomalous diffusion of Dirac fermions

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Anomalous diffusion of Dirac fermions

PROEFSCHRIFT

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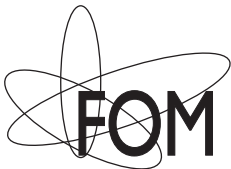
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The cover shows a snapshot from a simulation of anomalous diffusion on a Sierpiński lattice (Chapter 2). The blue and red dots correspond, respectively, to occupied and empty sites. Particles enter the system via the bottom left corner and leave it via the bottom right corner. One can see how obstacles (black triangles) hinder the transport on all length scales.

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