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## **A flavour of family symmetries in a family of flavour models**

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## Chapter 6

# Summary, conclusions and outlook

La science, mon garçon, est faite d'erreurs, mais d'erreurs qu'il est bon de commettre, car elles mènent peu à peu à la vérité.

Science, my boy, is built upon errors; but upon errors that are good to be made, as step by step they lead to the truth.

*Jules Verne [165], Voyage au centre de la Terre*

One of the oldest and most important questions humanity has asked itself is “What is it made of?” Every time this question seemed answered in a particular field, people started looking for structures in the answer. When structures were found, the next challenge was to understand them from a deeper principle. Particle physics and in particular its flavour branch is no exception to this rule.

Our current world view is that all normal matter is made of atoms. Atoms consist of a shell of electrons surrounding a nucleus. The electrons are thought to be elementary particles; the nucleus is not, but digging deep enough, one finds supposedly-elementary up and down quarks. This is not the whole story; exotic matter can be observed in the cosmic rays and at particle colliders. This gives two new leptons and a whole zoo of new mesons and baryons, but the quark model structures this in terms of just four new quarks. Together with three neutrinos, this is the complete matter content of the Standard Model as recapitulated in figure 6.1.

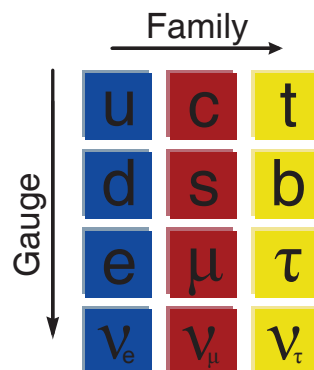


Figure 6.1: The matter particles of the Standard Model and two arrows that might suggest symmetry relations.

## Structure in similarity

The vertical arrow in figure 6.1 generically depicts the three gauge interactions of the Standard Model. These structure the allowed interactions between the elementary particles. In a sense, their representation structure even dictates which particles are to appear in the figure. The weak nuclear force for instance has a doublet as fundamental representation and indeed the electron and the electron neutrino as well as the up and down quark come in pairs in figure 6.1.

The horizontal arrow in the figure corresponds to the fact that particles that are standing next to each other are very similar. The up, charm and top quark have exactly the same quantum numbers of colour, weak isospin and hypercharge and the same holds for all other rows. This is a structure that is easily observed in the answer to the question what matter is made of, but an explanation for the structure is lacking in the conventional Standard Model. The existence of three families is considered an experimental fact, but not connected to a deeper symmetry principle to explain it.

The fundamental assumption of this thesis is that this is unsatisfactory. A theory in which the family structure can be explained is to be preferred over one in which it cannot. A suggestion to the explanation is already given in figure 6.1. There might be a force – or, more general, a symmetry relation – working not in the vertical, but in the horizontal direction. If a horizontal symmetry group with a three-dimensional irreducible representation is assumed, the three-family structure of the Standard Model results.

## Structure in difference 1: the masses

A horizontal symmetry, often also referred to as a flavour or family symmetry, can thus provide and explain structure in the similarities of the particles of the three generations. The particles are indeed identical in their coupling to the gauge sector, but the mass sector is very different. It is an interesting question if there are also structures here. The answer is that there might, both in the masses of the fermions and in the mixings between flavour and mass eigenstates.

Firstly, we observe that the masses of the quarks and (charged) leptons are very hierarchical, with the mass gaps between particles of the same type of comparable order on a logarithmic scale. In section 2.3.1 we observed that the mass ratios of the muon to the tau lepton and of the electron to the muon are respectively  $\lambda^2$  and  $\lambda^3$ , with  $\lambda$  numerically equal to 0.2 and possibly related to the Cabibbo angle. According to Froggatt and Nielsen, this may not be a numerical coincidence, but related to the charges of the different generations under the Froggatt-Nielsen family symmetry. Again, we see that family symmetries can be used to motivate observed structures.

## Structure in difference 2: the mixings

The second part where one can look for structures in the fermion masses is in the mixing of flavour eigenstates to mass eigenstates. The data shown in figure 6.2 give rise to several models constructed in the main body of this thesis. These models invoke some new physics, at least containing a horizontal symmetry, to reproduce the pie charts of figure 6.2, either by predicting exactly these values or by producing a probability distribution for the mixing angles peaked at or close to the values corresponding to those in the figure. The fact that the models correctly ‘postdict’ the mixing angles obviously isn’t enough to immediately declare them the new Standard Model. All models also make a number of predictions that can be tested by experiments in the near or more distant future and lastly, there is the difficult issue of the ‘aesthetical value’ of models. We try to make a final balance in the remainder of this chapter.



Figure 6.2: Pie charts showing the flavour content of the three quark mass eigenstates (left) and neutrino mass eigenstates (right). Quark data taken from [6] and the errors on these data are very small. Neutrino data correspond to average values of the elements of the mixing matrix from [7]. The errors here are large, corresponding to tens of percents in some cases.

## The beauty and the beast

The astuicle that chapter 4 of this thesis is based on, was originally called ‘Discrete Flavour Symmetries in GUTs: the Beauty and the Beast’. After a rightful comment of the referee that he or she did not consider this a very appropriate title for a scientific publication, the title was changed. Still, the concept of beauties and beasts, positive and negative messages about beyond-the-Standard-Model models, is an important theme in the article as well as in the articles on which the other chapters of this thesis are based. In that fashion, this summarizing and concluding chapter can continue in a ‘beauty-like’ or a ‘beast-like’ way.

## The beauty

The beautiful conclusion of the thesis would be that the field of flavour model building is very rich. There are many ideas around, making the field very versatile and flexible.

The application of family symmetries to flavour mixing was first popularized approximately ten years ago when the mixing deferred from neutrino oscillations seemed to be in accordance with the bimaximal mixing pattern, although there were large experimental errors.

$$\theta_{12}^l = \theta_{23}^l = \pi/4; \theta_{13}^l = 0, \quad \text{or} \\ \sin^2 \theta_{12}^l = \sin^2 \theta_{23}^l = 1/2; \sin^2 \theta_{13}^l = 0.$$

This pattern is probably the third simplest mixing pattern one can think of. Only ‘no mixing’ and ‘maximal mixing in just one sector’ are simpler. It is indeed a very remarkable fact that the data were in accordance with such a simple pattern and it was worth researching if there is a deeper reason for this. This turned out not to be the case; the agreement between early data and the bimaximal pattern was accidental.

A larger data set allowed a more precise determination of the solar mixing angle  $\theta_{12}^l$  and it was found that it is in fact not ‘maximal’, but seems to settle near another special value; the one corresponding to  $\sin^2 \theta_{12}^l = 1/3$ . The focus on bimaximal mixing pattern was swiftly changed for this tribimaximal one. One way to realize tribimaximal mixing is using  $A_4$  as a horizontal symmetry group as discussed in section 2.4.  $A_4$  is one of the simplest groups available in the mathematical literature. It only has 12 elements, which makes it the smallest group to have a three-dimensional representation. The ability of  $A_4$  to generate tribimaximal mixing is another elegant example of the force of group theory in physics. As elaborated upon in chapter 3, an accidental symmetry is necessary when  $A_4$  reproduces tribimaximal mixing. The group  $S_4$  on the other hand, is almost as simple as  $A_4$ ; it can reproduce tribimaximal mixing as well as bimaximal mixing without resort to accidental symmetries.

The bimaximal and the tribimaximal mixing differ in the values of the solar mixing angle, but they agree on the two other mixing angles. The atmospheric angle  $\theta_{23}^l$  should be  $45^\circ$  and the reactor angle  $0^\circ$ . The first prediction has so far stood the test of time and is in accordance with the current best fits at  $1\sigma$  or less. It is interesting to note that the two different global fits given in chapter 2 present central values that deviate from this value in opposite directions.

The prediction  $\theta_{13}^l = 0^\circ$  for the reactor mixing angle seems not to have held. Over the last years,

non-zero  $\theta_{13}^l$  was first presented as a hint in global fits of all neutrino oscillation observables. For a long time the best-fit value of the angle was positive, although a vanishing angle was still allowed at the  $2$  to  $3\sigma$  level. In the summer of 2011, observations of the T2K collaboration have finally swung the balance to  $\theta_{13}^l > 0^\circ$ . The next big step from an experimental point of view will be the independent confirmation or refutation of this measurement. As mentioned in the start of chapter 3, the falsification of the hypothesis of exact tribimaximal mixing opens up a number of directions in the space of model building. We investigated many of these ways in the different chapters of this thesis.

In chapter 3 itself we systematically scanned a large number of mixing patterns that naturally arise when horizontal symmetry groups and residual symmetries are selected. We found that the bimaximal and tribimaximal patterns are elements of a large, but finite, list of naturally obtained mixing structures. Four of these structures,  $M_1 - M_4$ , are particularly appealing, as they naturally predict non-zero reactor mixing angle, while the other two neutrino angles are also close to the current best fits on the data. They are different from the predictions of tribimaximal mixing. There are thus three unique predictions that will be tested when more precise fits of the mixing angles become available.

Chapter 4 discusses a set up in which the mixing angles are significantly different at leading order in the number of flavons present in the Lagrangian from the eventual all-order result. In this set up, indeed one of the predictions is a non-zero reactor mixing angle. The main message of the chapter is that the combination of the  $S_4$  family symmetry (with bimaximal mixing at leading order) and the Pati-Salam grand unified theory (with quark-lepton complementarity) is indeed feasible. The combination of flavour and GUT symmetry enabled a double stroke of unification, making the model attractive from an aesthetical point of view. A number of predictions is made by the model; these include the observation of neutrinoless double beta decay in a particular range and the existence of doubly charged scalars at a few TeV. In a more distinct future, new experimental techniques might be able to scan the complete gauge structure, but at the moment the related energy scales are still very far out of sight.

From the point of view of predicting neutrino mixing angles, chapter 5 presents a less ambitious set up. In this chapter, we discuss a situation in which there are no separate flavons and correspondingly there is no separate scale of the flavour symmetry breaking. The flavour symmetry is realized via non-trivially transforming Higgs bosons and all the flavour symmetry processes take place at the electroweak scale. The fact that these Higgses provide only one direction in flavour space ('just one flavon') means that the mixing angles in quark and lepton sector cannot be uniquely determined. Patterns such as the tribimaximal mixing are obtained by finetuning some parameters, although significantly less than in the Standard Model. When the tuning is not exact, a pattern close to, but not identical to tribimaximal naturally comes out and this is indeed what the current data seem to point at. The strongest point of the set up of chapter 5 is that the predictions are testable at the LHC. We have given elaborate lists of the Higgs bosons and their masses in each of the vacua considered. Furthermore, pre-LHC data can be used to significantly reduce the parameters space available in each of the vacua as well as to test specific models in which details of the fermionic content are given.

## The beast

On the other hand, a beastly conclusion is possible. This focusses on the point that application of the 'hierarchy-problem argument' might not be very fit in the context of flavour symmetries.

We recall that in chapter 1 the reasons to go beyond the Standard Model were separated into two groups. There is observational evidence that the Standard Model is incomplete and there are theoretical or aesthetical reasons to expand it. The first reasons pose a strong obligation to the theorists to extend the Standard Model. As shown by e.g. Shaposhnikov and alluded to in section 1.2.4, this does not need much new physics. Just the addition of three righthanded neutrinos with finetuned masses does the job. The crucial point is that this finetuning is much disliked in the

theoretical physics community. New physics can be assumed in order to prevent a theory from being finetuned. Axions to fight the strong-CP problem and supersymmetry to fight the Higgs-mass hierarchy problem are examples given in section 1.3. The motivation for flavour symmetries falls into the same category.

Finetuning in the fermion sector can appear in the masses or the mixings of the particles. The hierarchy in the fermion masses was already discussed above as well as the potentially related Froggatt–Nielsen symmetry. The crucial point is that the fermion mass hierarchy provides a rather clear signal. This is less the case in the fermion mixings. The three pie-charts on the right hand side of figure 6.2 were calculated with the central values of the allowed ranges for each of the nine mixing matrix elements. These figures show the suggestion of near-tribimaximal mixing with small  $\theta_{13}^l$  effects. The assumption is that if the data favour such a special mixing pattern, we should see the appearance of this pattern as a finetuning and try to explain it.

The problem with this reasoning is that the data do not point at the patterns as strongly as perhaps suggested by figure 6.2. Indeed, the central values may provide an interesting signal, but those central values are surrounded by large error bands. This already revealed itself in the fact that in the past years there were ‘phase transitions’ between different paradigms, going from the bimaximal to the tribimaximal to the ‘near-tribimaximal with non-zero reactor angle’ paradigm. All those patterns were allowed within the 2 or  $3\sigma$  bounds at the time of their popularity. Within those bounds, also other mixing schemes exist that do not show any pattern and thus do not call for a symmetry explanation. This is graphically illustrated in figure 6.3.

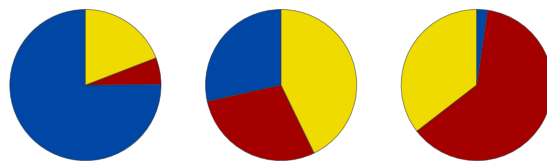


Figure 6.3: *Pie charts showing a possibility for the flavour content of the three neutrino mass eigenstates. Instead of taking the average values given in [7] we have taken other values in the  $3\sigma$  intervals. The figures resemble neither the tribimaximal nor any other simple pattern much.*

With this in mind, flavour symmetric model building can be seen as preemptive model building. In the last few years, the tribimaximal mixing pattern was allowed by the data, but not firmly pointed at. The models were constructed to anticipate that the data might settle at exact tribimaximal mixing that is in need of an explanation of the apparent fine-tuning.

The data did not settle at exact tribimaximal mixing. Instead evidence for non-zero reactor mixing angle was found. ‘Vanilla’ tribimaximal mixing models cannot explain this and in this thesis, suggestions for more elaborate flavours are given. These however, are all still in the framework of flavour symmetries. The question is how well one can still defend this framework.

Let us imagine a world that has never been through the ‘tribimaximal paradigm’ era. In this world, one day after the establishment of neutrino oscillations, an ingenious experimentalist builds a machine that immediately measures all three mixing angles with a precision that in our world was only reached in the summer of 2011. She sends this information to her theorist colleagues and asks if they see anything in the data. The theorists will not come up with tribimaximal mixing: it is already excluded by the data; it is also unlikely that they will go in one step from nothing to ‘(tri-) bimaximal mixing with significant corrections’, as in the model of chapter 4. The motivation to defend this from a finetuning perspective is simply lacking.

It is interesting to ask whether the hypothetical theorists would come up with groups such as  $\Delta(96)$  and  $\Delta(384)$  of chapter 3 to address the mixing patterns. Indeed, in our world, where we have been through the ‘tribimaximal paradigm’ phase, it can be defended that these groups are not much more

complicated than  $A_4$  and  $S_4$  and indeed are generated by the same basic principles. In the other world, assuming a very complicated group to explain a mixing pattern that on first sight looks quite innocent might be a step too far. The same holds for the assumption of an  $A_4$ -triplet of Higgs fields as discussed in chapter 5.

If the beastly conclusion is true, flavour symmetries may not be ‘the truth’, but in discussing them, we may step by step have moved closer to understanding Majorana neutrinos, dark matter, multi-Higgs models and many other aspects of the physics of the exciting times we live in.