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A flavour of family symmetries in a family of flavour models

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Chapter 1

Introduction

Di, coeptis [...]
adspirate meis primaeque ab origine mundi
ad mea perpetuum deducite tempora carmen.

Gods, inspire my attempts
and spin out a continuous song
from the first origin of the world into my times.

Ovid [1], Metamorphoses I 2-4.

We live in exciting times. The Large Hadron Collider at CERN is working better than anyone expected; it is collecting data at world-record holding center-of-mass energy and ever growing luminosity (a measure of the number of interactions per unit time). The Tevatron accelerator at Fermilab closed down at the end of the fiscal year 2011. In the last months of its service it worked at top-luminosity to collect as much data as possible before its shut-down. This data is currently being investigated and may contain a treasure of information.

On the neutrino front, the discovery of neutrino oscillations is now a little over ten years old and neutrino physics has entered the precision era. There is hope that in the next decade, all mixing angles, including the only recently measured angle θ_{13} , are known to great precision; that the absolute mass scale and the related hierarchy can be set and that neutrinoless double beta decay may be observed. In September 2011, the OPERA experiment reported superluminal velocities for neutrinos traveling from CERN to Gran Sasso, adding a new mystery to the neutrinos' character.

The oldest light in the universe, the cosmic microwave background, is measured more and more precisely by the WMAP and Planck satellites, thereby delivering a wealth of data also to the particle physics community.

Between all this experimental force, it is up to the theorist community to give frameworks in which the new results can be interpreted. In the first section of this introductory chapter, we describe the Standard Model, that has been the leading theory of particle physics for the last forty years. Still, there are reasons to expect that the Standard Model should be extended. These are described in the next three sections. One of these extensions is the addition of family symmetries (also called flavour symmetries). This is the main topic of this thesis. In this chapter, we also come across the neutrino seesaw, dark matter, supersymmetry and gauge unification, that are other important elements of some chapters of this thesis.

1.1 The Standard Model

The leading theory in particle physics is, with good reason, called the Standard Model (SM). The SM was developed in the 60s and describes all known elementary particles and their interactions with unprecedented precision. The Standard model consists of matter particles, forces and related force carriers and a sector that should give mass to it all, the Higgs sector.

On the matter side, the Standard Model describes quarks and leptons, that are ordered in generations, as shown in figure 1.1. Ordinary matter is made of first generation matter. Atomic nuclei are

u up quark	c charm quark	t top quark
d down quark	s strange quark	b bottom quark
e electron	μ muon	τ tau lepton
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino

Figure 1.1: *The matter content of the SM.*

ultimately build of up and down quarks: two ups and one down form a proton and two downs and one up form a neutron; a nucleus can be seen as a specific combination of protons and neutrons. Around the nucleus circle the electrons to form an atom. Electrons together with the rarely interacting neutrinos are called leptons. See also figure 1.2.

Experimentalists found that the spectrum of elementary particles is much more interesting than just the first generation particles that are needed to build up atomic matter. In 1936-7, a new particle was discovered. Eventually, it became known as muon and was identified as a heavier brother of the electron. This is remarkable. A priori, there is no reason that a new particle should resemble any 'older' particle, but the muon does. It has all its properties in common with the electron: same electric charge, same spin, also invisible to the strong nuclear force, etcetera. Only the mass is different; for the rest, it is a perfect copy.

It was found that this relationship is not unique to the electron and the muon: all first generation particles have exactly two heavier brothers. Next to the electron and the muon, there is the even heavier tau particle. The down quark is accompanied by a strange and a bottom quark and next to the up quark are a charm and a top quark. Also neutrinos come in three types, generally dubbed electron-, muon- and tau-neutrino.

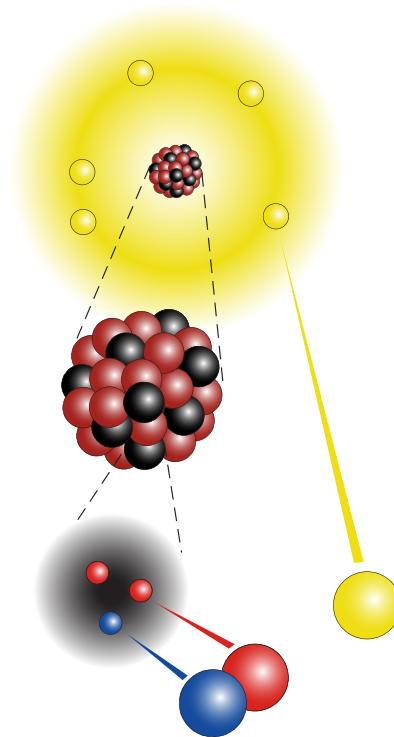


Figure 1.2: Normal matter is made of atoms, that are on their turn build up from up quarks (blue), down quarks (red) and electrons (yellow)[Schematic view – not to scale]

The quarks and leptons of the Standard Model interact via exactly three elementary forces. The first one is the strong nuclear force that only works on the particles in the nucleus, the quarks. The other two are called the weak force and the hyperforce and they are together called the electroweak interactions. At ‘low’ energies, this gives rise to electromagnetism. According to the theory, there is a force carrier (or gauge boson) related to each of these forces. For electromagnetism, this is the well-known photon, or light-quantum. Apart from the photon, the electroweak interactions give rise to W and Z bosons. Gluons, lastly, mediate the strong nuclear force.

A force carrier only couples to a particle that is charged under the appropriate charge. In the language of gauge theory, in which the Standard Model is written, this translates to being in a non-trivial representation of the related gauge group. The relation between the group theory of symmetry groups and the physics of forces and couplings that is behind this is one of the best examples of the power of mathematics in physics.

The interactions of the Standard Model are graphically depicted in figure 1.3. There, it can be seen that also some gauge bosons couple to each other.

The Standard Model as described so far explains all known elementary particles and all their interactions in great detail, but with two important shortcomings: if the above theory were all, all elementary particles would be exactly massless and the electromagnetic force and the weak nuclear force, that combine at higher energies would not separate. Both problems can be solved with the introduction of an extra field to the theory, as explained by Brout, Englert, Guralnik, Hagen, Kibble and Higgs and eventually called after the latter. The Higgs fields can give the required dynamics to break electroweak symmetry and thereby give mass to the W and Z bosons. The quark and lepton masses follow from the so-called Yukawa couplings to the Higgs field, that gives a mass term below the electroweak scale. Addition of more than one Higgs field is in principle possible, but with only one copy we have the most simple complete theory. We refer to this theory as ‘the Standard Model’ and to theories with more than one Higgs as extensions of it.

The addition of the Higgs fields leads to the prediction of the existence of an extra particle, called the Higgs boson. The Higgs has not been found yet and the search for this particle was one of the main motivations to construct the LHC — although this argument was also used for its predecessor, the large electron-positron collider (LEP). Many physicists believe that the LHC will indeed find the Higgs and this would then ‘complete’ the Standard Model.

The Standard Model passed many experimental tests over the last decades and it was always able to perfectly fit the experimental outcomes. Still there are reasons to believe that the Standard Model, after the discovery of a single Higgs particle, is not the whole story. The arguments for this claim fall in three groups. Firstly, there are experimental/observational reasons to believe that the Standard Model is incomplete. Interestingly enough, arguments come mostly from cosmological and astrophysical sources. Secondly, there are theoretical reasons to believe that the Standard Model is ‘unnatural’, meaning that although it can explain many experimental results very well, this requires very special values of the parameters of the theory. Thirdly, the Standard Model fails to give a microscopic theory of gravity. When we study places in the universe like black holes or times like the Big Bang, we need both the physics of the very small and of the very heavy. Unfortunately, the Standard Model of particle physics and the leading theory of gravity, Einstein’s general relativity, are not compatible and a new theory of quantum gravity is needed.

We discuss the first two challenges to the Standard Model in some detail in the rest of this chapter, each time suggesting solutions. As we will see, these are often the addition of extra particles and/or symmetries to the Standard Model. The search for a quantum theory of gravity has been the holy grail for high energy physicists for the last decades. Interesting theories, like string theory and loop quantum gravity exist, but a conclusive answer has not yet been given. We do not study quantum gravity in this thesis, although we refer to the scale at which quantum gravity is thought to exist — the Planck scale — quite often.

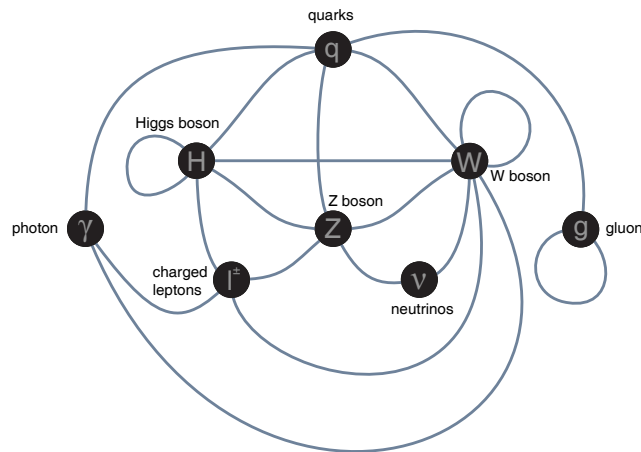


Figure 1.3: *Schematic representation of the interactions of the Standard Model*

1.2 Reasons why the Standard Model is incomplete

The Standard Model has successfully explained a multitude of experimental data, meaning that it well predicted the behaviour of elementary particles in Earthbound colliders. However, particle physics also has a strong connection with astrophysics and cosmology. This shows itself in the fact that the Earth is constantly bombarded with particles from the Sun and many other sources in the sky; it shows itself in the fact that the particle content of the universe steers its fate of contraction or expansion and it shows itself in the fact some of the early phases of the universe, such as inflation and nucleosynthesis can be described extremely well by particle physics.

Indeed, many of the elementary particles were first discovered as elements of the cosmic radiation and detected in bubble and cloud chambers long before they could be recreated and observed in colliders on Earth. However, there are cases where the Standard Model falls short to describe what we observe in the cosmos. We describe neutrino oscillations, dark matter and the baryon asymmetry of the universe in the next sections.

1.2.1 Neutrino Oscillations

The Sun shines very brightly. Indeed, if we could capture the energy it radiates on Earth for just one hour, this would be enough to satisfy humanity's energy consumption for a full year. The energy needed for this enormous amount of radiation is created in the center of the Sun, where protons are fused into helium nuclei in one of the most energetic reactions imaginable. In this process, energy is not only produced in the form of photons, or light, but also as (kinetic) energy of particles called neutrinos, that we have seen above of companions to the charged leptons.

Neutrinos were postulated in 1930 to explain properties of radioactive materials and were eventually observed in 1956. Their name is well-chosen. They are electrically neutral and the Italian suffix *-ino* refers to something that is small. In particle language, that translates to having a small cross section, a measure of how easily a particle interacts. Above, it was mentioned that neutrinos interact rarely. This is an understatement: if we send a beam of neutrinos through a bar of lead, the bar would have to be *half a lightyear* long in order to let half of the neutrinos interact with it. For comparison, for visible light, this would be only a few atomic distances and even for the most energetic X-rays, it would be a few millimetres. Still, no matter how rarely, neutrinos sometimes do interact with matter. This is key to one of the most outstanding theorist-experimentalist collaborations ever.

We know the amount of radiation that the Sun produces very well and we also know how the origin of this radiation is also the origin of many neutrinos. We can therefore calculate how many neutrinos should reach the Earth every second. If we also know how many, or rather how few, of these interact with a certain detector material, we can predict a detection rate. This was done by Ray Davis and John Bahcall, using an ingenious set up in the old Homestake gold mine. They indeed found evidence for reactions that were initiated by solar neutrinos, but it was a factor three less than they expected. Of course, first their solar model and experimental efficiency were challenged, but eventually the result stood. In 2002, Ray Davis received the Nobel Prize in physics.

Above, it was mentioned that neutrinos, like the other charged leptons and the quarks, come in three 'generations'. The electron-neutrino is defined as the neutrino that reacts together with an electron under the weak nuclear force and analogous for the muon-neutrino and the tau-neutrino. The neutrinos produced in the Sun are of the electron type, as their creation is related to transmutation of normal, first generation matter. The experiment of Davis and Bahcall was only sensitive to electron neutrinos. It could not measure muon- or tau-neutrinos. The conclusion of their experiment is thus that of the electron-neutrinos produced in the Sun, only a third arrives at Earth as an electron-neutrino.

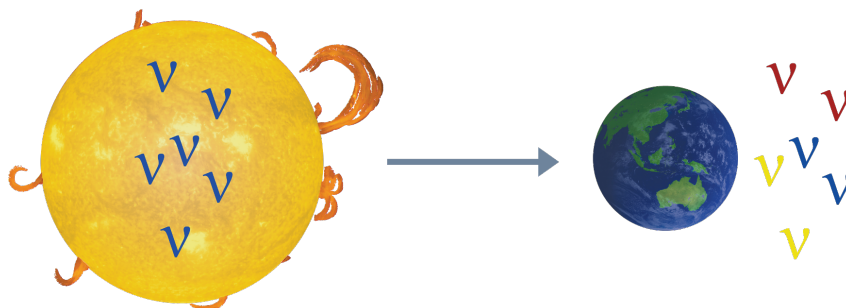


Figure 1.4: Schematic view of neutrino oscillations. In the centre of the Sun all neutrinos are of the blue electron-type; at Earth all three flavours are equally present.

Comparable results were obtained using neutrinos that are produced when a cosmic-ray particle collides high in the atmosphere and produces a muon and a muon-neutrino. Also of these muon-neutrinos, only a limited fraction reaches the Earth as a muon-neutrino. Other experiments use neutrinos from nuclear reactors or specific collider processes and they all find the same result: only a limited fraction of the neutrinos produced is observed later as a neutrino of the same species. They also found that it is possible to detect neutrinos of a species that is not the one that is produced.

The above observations can be explained if we assume that neutrinos oscillate. This means that if a neutrino is produced as a neutrino of a certain species, it can become a neutrino of a different species while flying through space.

Neutrino oscillations can be explained using simple techniques from quantum mechanics, but only if one fundamental assumption is satisfied: neutrinos should have mass. In the Standard Model, neutrinos do not have mass. Mass terms in the Standard Model are discussed in more detail in chapter 2. The crucial point is this: in order to obtain a mass from interactions with the (standard) Higgs field, a particle should appear both in ‘lefthanded’ and ‘righthanded’ form. Only the lefthanded neutrino interacts via the weak nuclear force and as can be seen in figure 1.3, this is the *only* interaction that neutrinos have. This means that all observations of neutrinos are of those of the lefthanded type. When the Standard Model was constructed, it was deemed enough to include only lefthanded neutrinos and leave out the righthanded neutrinos. An automatic consequence was that neutrinos were supposed to be massless, but that was in good agreement with all experimental data at that time.

As the above discussion suggests, the problem of the massless neutrinos can be solved in a straightforward way. If righthanded neutrinos are added to the theory, the left- and righthanded neutrino can interact with the Higgs field and acquire a mass term. In chapter 2, we will see that this is indeed a possibility. We will also see that the addition of righthanded neutrinos to the theory also gives rise to a different possibility to make neutrinos massive. This mechanism is called the type-I seesaw and it also explains why neutrinos, even if they are massive, are much lighter than all other known particles. We also discuss two other seesaw theories (type-II and III), that give rise to lefthanded neutrino mass without a righthanded neutrino.

In conclusion, the observation of neutrino oscillations proves that the Standard Model is incomplete, as it allows only massless neutrinos that cannot oscillate. There are a number of suggested remedies for this problem and all of them involve beyond the Standard Model physics.

1.2.2 Dark Matter

The Sun accounts for 99.9% of the mass in the solar system. When we look out in the night sky, we see many stars and not much more. It is therefore intuitive to assume that most of the matter in the cosmos is in stars. This turns out to be incorrect. Interstellar and intergalactic gas clouds make up more than three times more mass than there is in stars, but as they barely emit any light, they are much harder to see.

The real surprise comes when we make an energy balance of the universe. Then we see that only about one sixth of all matter in the universe can be in the form of normal atomic matter. The majority of the matter in the universe occurs in the form of ‘dark matter’¹. It is called ‘dark’ because it does not interact with light in any way; invisible matter would be an even better name.

In terms of the Standard Model, not interacting with light means not coupling to the photon. A glimpse at figure 1.3 shows that there are a few particles that do not couple to the photon. However, the Z- boson, the Higgs boson and combinations of gluons (they cannot occur on their own), are all unstable and decay to particles that do couple to the photon. The only candidates left are the (normal lefthanded) neutrinos, but also these eventually get discarded. This has to do with the fact

¹An even larger part of the energy content of the universe is in the form of ‘dark energy’. It is even more mysterious than dark matter, but we do not discuss it any further in this thesis as it might as well be related to new aspects of gravity as to new aspects of particle physics.

that even if neutrinos have mass, this mass is too low to give them the right dark matter properties. In particular, neutrinos do not cluster enough, behaviour that was observed for the unknown dark matter particles.

We conclude that the Standard Model does not have a credible candidate for the dark matter. Again, the remedy is rather straightforward. Just add *any* particle to the theory that is stable and does not interact with the photon. The problem is that physicists generally don't like the *ad hoc* addition of a particle to the theory. It would be better if the dark matter candidate naturally occurs in a beyond the Standard Model theory. Indeed some of the extensions of the Standard Model discussed in sec 1.3 automatically give a dark matter candidate. In chapter 5, we discuss the appearance of a dark matter candidate after a flavour symmetry gets broken.

1.2.3 Matter-antimatter Asymmetry of the Universe

When we discussed the matter content of the Standard Model at the beginning of this chapter, we left out an important detail. We mentioned that the electron has two brothers, the muon and the tau lepton that are much like it, but only have a different mass. There is actually a particle that looks even more like the electron, called the positron. It has all the properties of the electron, only its charge is exactly the opposite. The electron has charge $-e$, so the positron has charge $+e$. All particles of the Standard Model have these almost-identical twins and together they are called antimatter. When a particle and its corresponding antiparticle meet, they cannot peacefully coexist. Instead, they annihilate each other in a burst of pure energy.

Now if the Earth were made partly of matter and partly of antimatter, that would be quite inconvenient, as matter-antimatter annihilating explosions would occur with catastrophic consequences. Fortunately, this does not happen outside Dan Brown novels. The Earth is completely made of matter and so is the solar system and most probably the whole observable universe. Obviously, the (observable) universe has a preference to be in a matter state rather than in an antimatter state. Even matter is quite scarcely present in the universe though. If we count the number of matter particles and the number of light particles (photons), the photons win by a factor in the order of a billion.

Now comes the crucial point. There are many reasons to believe that the universe went through a phase of so-called inflation almost directly after the Big Bang. Theory demands that directly after this inflationary era, there were equal amounts of antimatter particles, matter particles and photons. This is clearly not what we observe today: now there are no antimatter particles, few matter particles and many photons.

The annihilation relations referred to above, provide part of the solution. In later stages of the universe, matter and antimatter particles met and annihilated. This certainly explains why there is much more light than matter, but it fails to explain why there is more matter than antimatter or even why there is still any matter at all. Matter and antimatter could have completely annihilated each other until nothing was left of either.

So the story can be this: directly after inflation, there were equal numbers of matter, antimatter and light particles. Then some reactions occurred that slightly shifted the balance. For every billion antimatter particles, there were a billion +1 matter particles (and still about a billion photons). Now the annihilation reactions set in. All billion antimatter particles annihilated with a billion of the matter particles, but a single matter particle survived. This matter particle, however, is dwarfed in a sea of billions of photons. This scenario is schematically represented in figure 1.5.

The crucial question is of course what are the 'some reactions' of the previous paragraph. Clearly, they treat matter and antimatter on different footing. In technical terms, this is called violating charge-parity symmetry, or CP. Actually, there are some reactions in the Standard Model that violate CP and these were very important in the history of particle physics. After the discovery of CP violation in the decay of subatomic particles called kaons in the 1960s, Makoto Kobayashi and Toshihide Maskawa showed in 1973 that the Standard Model can account for this CP violation, but only if there are at least three generations of quarks and leptons. At that time, only two generations

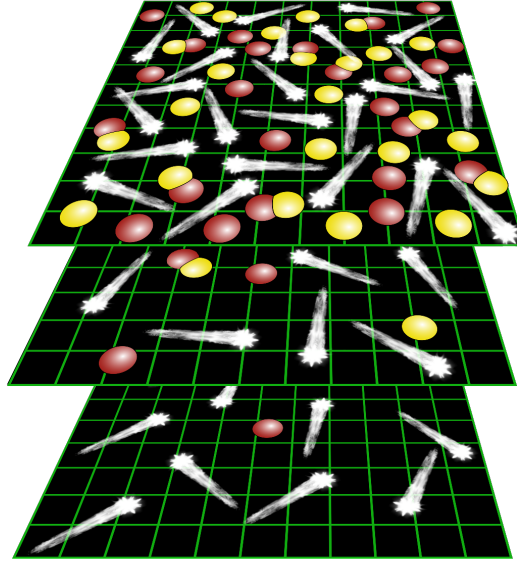


Figure 1.5: Schematic view of matter (red), antimatter (yellow) and photons (white) at different stages of the history of the universe.

were observed, so their argument was really the prediction of a third generation. Particles of the third generation were indeed discovered soon thereafter: the tau lepton in 1975 and the bottom quark in 1977, with the top quark and the tau neutrino following in 1995 and 2000. Kobayashi and Maskawa received the Nobel Prize in Physics in 2008.

A detailed analysis of the CP violation in the Standard Model shows however that this is *not enough* to explain the cosmic overabundance of matter over antimatter. To explain this, new physics is needed. The solution to this problem might be related to the solutions of other challenges for the Standard Model. For instance, if the problem of neutrino masses is indeed solved by the introduction of righthanded neutrinos, the physics of these particles in the early universe might give the matter-antimatter asymmetry by a process known as leptogenesis. And if the Standard Model is extended to a supersymmetric Standard Model, there are new ways to generate the asymmetry at the moment when the Higgs field makes the electroweak symmetry break as described above.

1.2.4 Minimal extensions of the Standard Model

From the above sections, it is clear that cosmological observations prove that the Standard Model is incomplete. It should be extended with at least a number of new fields and operators such that it is able to generate neutrino masses; that it has a dark matter candidate and that it can explain that the universe is matter-antimatter asymmetric.

Recently, a very minimal model was proposed by M. Shaposhnikov and collaborators, dubbed the neutrino minimal Standard Model or ν MSM [2]. In this model, only three righthanded neutrinos are added to the Standard Model. These open the possibility for (normal) neutrino mass terms. Of the new righthanded neutrinos, one is much lighter than the others and it is stable and can serve as the dark matter. The two others are much heavier and unstable. They decayed early in the universe and did so in a way that shifted the matter-antimatter balance. It is technically very challenging to make these decays generate *enough* asymmetry. A number of points in parameter space have been found however, where the two heaviest neutrinos are almost of identical mass and the balance shift is large enough.

Extensions of the Standard Model like the one by Shaposhnikov *et al* are very appealing from the

point of view of minimality. They explain all the *hard* criticism on the Standard Model, all the points where we are certain that it lags behind. However, it fails to address many *why*-questions. Why are the parameters the way they are. An example is the tiny mass gap between the two heavy righthanded neutrinos. Why is it this small? Only to provide the right amount of matter-antimatter asymmetry? That would be a justification *post hoc*. Or is there a physical reason why this difference is naturally of that order?

In the next section, we will see that questions like these also plague the (normal) Standard Model. We will see that these *theoretical* reasons to extend the Standard Model generally call for more symmetry.

1.3 Theoretical reasons to extend the Standard Model

In this section we study a number of phenomena that can in principle be explained by the Standard Model, but only if some of its parameters are very large, very small or finetuned to very specific values. These are not *hard* problems of the Standard Model; in principle the parameters can be ‘just like this’. But it is also possible – and from a theoretical perspective this is preferable – that there is an underlying theory that can explain these particular values.

We can compare this to a person playing poker and getting four-of-a-kind aces many times in a row. In principle, there is a finite probability of a long list of hands with four aces, but any casino or opponent would investigate this player with great scrutiny. Cards in his sleeve are a much more probable explanation than great luck.

In this section, we discuss the strong CP problem, the Higgs hierarchy problem and the gauge unification possibility. Candidate solutions for these problems are respectively the Peccei-Quinn symmetry; an extended space-time symmetry, called supersymmetry and an extended and unified gauge symmetry. The last two symmetries in this list are important ingredients of chapter 4 of this thesis. In the next section, we discuss the so-called flavour problem. This problem, and its candidate solution, flavour symmetries are the main topic of this thesis.

1.3.1 The strong CP problem

In section 1.2.3, we discussed the fact that matter and antimatter behave differently under the laws of particle physics, as the Standard Model violates CP. In principle, CP violation can be related to two of the three forces of section 1.1, both the weak and the strong nuclear force. However, all CP violation observed so far comes from the weak sector. The CP violation in the strong interactions is thus either very small or vanishing.

In the Standard Model, CP violation in the strong sector is parameterized by a parameter called the theta angle. In principle, θ can take any value between 0 and 2π , which is the natural range for an angle. Measurements so far have not observed any non-zero value of the angle, but have restricted it to be very small: $\theta < 10^{-10}$. Now it might well be that θ ‘just happens’ to have a small value of, for instance, 5×10^{-11} . However, one is tempted to believe that θ might be *exactly* zero. In 1977, Roberto Peccei and Helen Quinn postulated a theory that predicts the theta-angle to be zero in a natural way. In this theory, θ is no longer a constant value (i.e. a parameter), but actually a dynamical field that is charged under a new symmetry, the Peccei-Quinn symmetry. An extra advantage is the presence of a dark matter candidate, called the axion.

In this thesis we do not discuss the Peccei-Quinn mechanism in more detail, but the framework it uses is very typical. Firstly, we noticed that there is an element of the Standard Model that cannot be explained in a natural way. Secondly, we gave an explanation using new fields and new symmetries. Lastly, we found that this solution also helps to solve other problems of the Standard Model, in this case the dark matter problem.

1.3.2 The hierarchy problem

The next theoretical challenge to the Standard Model is related to the mass of the Higgs boson. Unfortunately, the Higgs mass is the last great unknown of the Standard Model. Indirect measurements constrain the Higgs, if it exists, to be lighter than approximately 200 GeV [3]². A theoretical argument, the perturbativity bound, gives the same value.

Naively, this value of the Higgs mass – or in fact any other value – would not pose a problem. We would just need to assume a parameter of the right size. The problem is, that masses in particle physics are not just given by parameters. The masses given by the *bare parameters* can be significantly changed by the effects of virtual particles. These can be thought of as being created from the vacuum for a very short while before disappearing again in this vacuum in a process that is in line with the uncertainty principle between energy and time.

To see the effect of virtual particles on the Higgs mass, we study a Higgs boson propagating through the empty vacuum. This can be graphically represented by a line running from left to right as in figure 1.6. The line is dashed to indicate that the particle propagating is a so-called boson, a particle for which the spin quantum number is an integer, zero in this case.

Instead of ‘just’ propagating, the Higgs boson can do more interesting things, like budding of a second (virtual) Higgs particle and then immediately reabsorbing it (figure 1.7) or temporarily splitting in a fermion-antifermion pair. Fermions are particles whose spin quantum number is a half-integer and they are represented by continuous lines. The top quark couples most strongly to the Higgs (and is thus the heaviest) of all known fermions, so a top-antitop loop is most probable and this is shown in figure 1.8.



Figure 1.6:
The Higgs propagator.

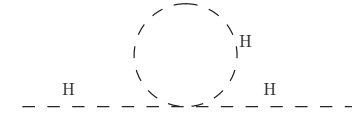


Figure 1.7:
The Higgs propagator with a Higgs loop.

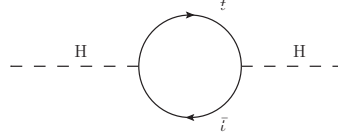


Figure 1.8:
The Higgs propagator with a top loop.

The diagrams 1.7 and 1.8 effect the Higgs mass squared: the Higgs loop in a positive way and the top-antitop loop in a negative way. The question is how large these corrections are. This is related to how much energy the particles in the loop can have. Even if the energy of the Higgs boson that is propagating is fixed, the energy of the short-lived particles in the loop can be much larger. Naively, the energy of the particles in the loop can go up to infinity.

Allowing virtual particles with infinite energies gives unsolvable problems for the theory. This can be circumvented by declaring that the virtual particles should be allowed to have energies at least as high as the next scale of new physics. Until this scale, the Standard Model is a good description of nature and we know what should be the physics of particles (including virtual particles) with these

²The gigaelectronvolt or GeV is a unit for energy or mass; according to Einstein's $E = mc^2$ these are equivalent. The mass of a proton is approximately 1 GeV and most processes in particle physics find place at an energy scale up to a few hundred GeV.

energies. Above this scale, we do not know what the physics is and we also do not know what is the effect of virtual particles of these energies.

The fundamental scale of the physics of the Standard Model is the scale where electroweak symmetry breaking takes place as alluded to in section 1.1. The other fundamental scale of particle physics is the scale of quantum gravity, the Planck scale introduced at the end of section 1.1. We do not know the details of the new theory that emerges at the Planck scale, but we can estimate the scale itself by an analysis of the fundamental constants of gravitation (Newton's constant G), relativity (the speed of light c) and quantum physics (the Planck constant h). Max Planck did this calculation immediately after the postulation of his constant and the scale he found turned out to be much larger than the (later discovered) electroweak scale. The Planck scale is 1.2×10^{19} GeV, while the electroweak scale is a mere 246 GeV.

Unless there is another scale of new physics between the electroweak scale and the Planck scale, we conclude that the virtual particles in the loops of figures 1.7 and 1.8 can have energies up to the Planck scale. Even if this energy is not infinite, it is very large and it leads to enormous corrections to the Higgs mass. To find a Higgs in the range where it should be, the bare parameter should not be around the Higgs mass squared, but around the Planck mass squared and it should be almost perfectly equal to the corrections. The difference between the original parameter and the corrections should be 10^{-34} times smaller than either of these. This is a strong finetuning and the chance that it happens 'naturally' is as small as a poker playing showing four aces nine times in a row.

There are two loopholes in the argument above and they serve as candidate solutions to the hierarchy (or finetuning) problem. The first focusses on the claim that the scale of quantum gravity is as large as 10^{19} GeV. A crucial assumption made here is that the known laws of gravity are valid at all intermediate scales. More specifically, the assumption is that gravity always spreads out over only three space dimensions. If there are extra dimensions, the scale of quantum gravity can be lower and the finetuning is less severe.

A second solution is related to the fact that there are both a positive and a negative correction to the Higgs mass squared. If these corrections cancel each other in a natural way, the finetuning problem is also evaded. This is the case if there is both a loop with a boson and one with a fermion that have related couplings to the Higgs field. This is not the case for the Higgs and the top of figures 1.7 and 1.8, but we can introduce a Higgs-like fermion ('Higgsino') and a top-like boson ('stop'). These give rise to diagrams 1.9 and 1.10, that cancel exactly to respectively 1.7 and 1.8.

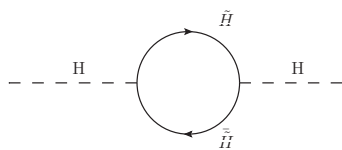


Figure 1.9:
The Higgs propagator with a Higgsino loop.

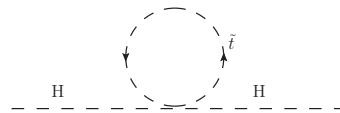


Figure 1.10:
The Higgs propagator with a stop loop.

The symmetry that gives a boson for every fermion in the theory and vice versa, is called supersymmetry. The superpartner of a Standard Model fermion is called a sfermion and its name is given by putting an s (of 'supersymmetric') in front of the name of the particle, for instance a stop as a partner to the top quark. The names of the supersymmetric partners of the Standard Model bosons are formed by adding '-ino' to the original names, giving photinos, Winos, Zinos, gluinos and Higgsinos.

If supersymmetry were exact, fermions and sfermions would have identical masses. This is not what we observe: for none of the Standard Model fermions, there is a boson with the same mass. The same holds for the known bosons. Obviously, supersymmetry cannot be an exact symmetry of nature, but it should be broken. The superpartners of the known particles are much heavier than their Standard Model counterparts. Still, if supersymmetry is to solve the finetuning problem, the gap may not be too large. The prediction is that if superpartners exist, their masses should be in the range of the LHC

and they should be discovered there. As of fall 2011 no signs of supersymmetry have been found (see e.g. [4] and [5]). This significantly constrains the parameter space for certain supersymmetric implementations, but by no means rules out supersymmetry at the LHC scale.

We conclude that supersymmetry can solve the finetuning problem related to the Higgs mass, but this comes with a price. We need to add quite a large new (broken) symmetry to the theory and the existence of many new particles is needed. The good news is that one of these particles typically can serve as the dark matter candidate and that supersymmetry enables the possibility of gauge unification, as described in the next section. Supersymmetry is an important ingredient of the models in chapters 2 and 4. The models described in chapter 5 are non-supersymmetric.

1.3.3 Gauge unification

The Standard Model is a consistent theory of three of the four forces of nature, as described in section 1.1. All of these forces couple to some of the matter particles as shown in figure 1.3 and they do so with their own strength. At small distances, the hierarchy of the forces is such that the strong nuclear force is the strongest, followed by the weak nuclear force. The electromagnetic force is relatively feeble.

Much like the Higgs mass discussed in the previous section, the coupling constant of a force is not just a parameter, but there are strong effects from virtual particles. To study these, we consider an electron somewhere in the vacuum.



Figure 1.11: *An electron and the field created around it.*

The electron is an (electromagnetically) charged particle and it thus creates a strong field around it, represented by the grey lines in figure 1.11. Now if a virtual electron-positron pair is created in the vacuum, this pair will tend to align along the field lines, with the positron pointing towards the electron to partially shield it. The resulting situation is depicted in figure 1.12.

Due to the screening of the positrons, the strength of the field that we observe is less than the strength of the original field. How much less this is, depends on how far we have zoomed in. If we use very energetic photons to probe the electron, we can look deeper into the cloud of electron-positron pairs than when we use less energetic photons. We conclude that to energetic photons, the strength of the field of the electron is greater than it is to less energetic photons. This translates to the electromagnetic coupling constant growing with the energy scale we study it at.

If we study the strong nuclear force in detail, we see that not a screening effect, but an anti-screening effect dominates. The strong force is very strong at low energies, but it becomes less strong when the energy grows. The weak nuclear force starts in between the electromagnetic and the strong force. It becomes weaker as energy grows, but at a smaller pace than the strong nuclear force.

This allows us to pose an interesting question. If we look at higher and higher energy scales, the

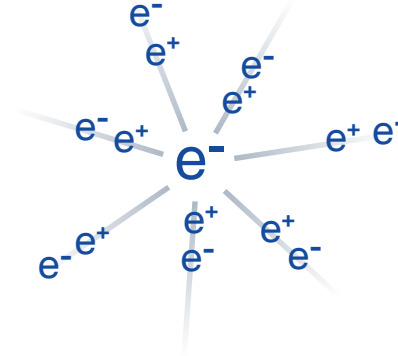


Figure 1.12: An electron and the cloud of aligned positron-electron pairs around it.

coupling constants of the three forces of the Standard Model grow towards each other. Is there an energy scale where all three of them become of comparable strength? In the Standard Model, the answer is ‘no’, as can be seen from figure 1.13. Plotted here is $1/\alpha$ for the three forces, where α is a dimensionless number that is proportional to the coupling constants squared.

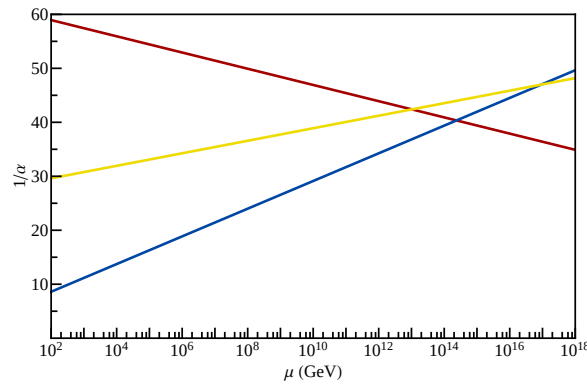


Figure 1.13: The ‘running’ of the coupling constants of the Standard Model as a function of the scale μ they are observed at. Red: hypercharge, that is directly related to electromagnetism; yellow: the weak nuclear force and blue: the strong nuclear force. We observe that the three forces do not meet in one point.

In a supersymmetric extension of the Standard Model, the superpartners of the known bosons and fermions can also contribute to the screening or anti-screening effects. They do so only if we probe the coupling constants at energies larger than their masses. In a theory with superpartners, the lines of the running coupling constants therefore have a kink at the energy scale that corresponds to the masses of the superpartners. This kink bends the running coupling constants in the right direction. In a theory with supersymmetry, it is possible that the three coupling constants of the Standard Model meet in one point. This is shown in figure 1.14. Note that this unification scale is higher than the candidate unification scale in the non-supersymmetric theory. This has to do with the fact that supersymmetric particles partly cancel the effect of their partners. In supersymmetry, the running of the coupling constants is slower, so they meet at a higher energy.

In figure 1.14 the three coupling constants go through one point, but after that they just continue to run and diverge again. If this is the case, there is probably no great importance of the three constants going through one point. There is a second possibility however. After the unification scale, the three forces may actually unite and form one superforce. Theories with this superforce are called Grand Unified Theories (GUTs). In a sense, these theories are much simpler than the Standard Model. The SM really needs three forces; in GUTs there is only one fundamental force, that has three manifestations after it gets broken. Also fermions can often be described more economically in

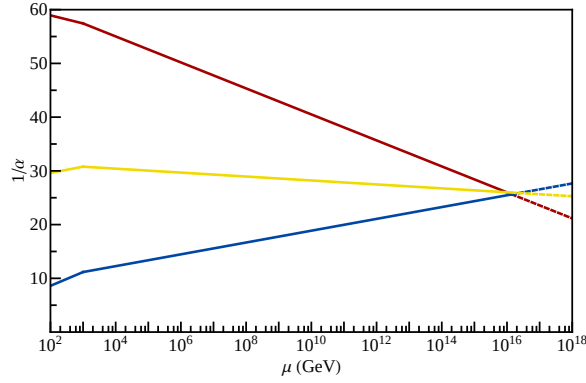


Figure 1.14: *The running of the coupling constants in a supersymmetric extension of the Standard Model. The three coupling constants seem to meet in one point.*

GUTs, as several particles that are unrelated in the SM, may have a common origin in Grand Unified Theories. In chapter 4 we study a particular GUT, the Pati–Salam theory, in more detail.

1.4 Flavour symmetries

There are many free parameters in the Standard Model. We already discussed some of them. The theta parameter of QCD in section 1.3.1; the Higgs mass (that eventually relates to two parameters λ and μ) in section 1.3.2 and the three gauge coupling constants in section 1.3.3. Together this gives six parameters.

By far the most parameters however, are related to the masses of the elementary fermions, that originate from the Yukawa couplings of the quarks and leptons to the Higgs field. In the original Standard Model, where neutrinos are massless, this amounts to 13 parameters. In a theory that includes neutrino masses, this number grows to 20 if neutrinos are particles of the Dirac type (not their own antiparticle) and to 22 if neutrinos are of the Majorana type (their own antiparticle).

When we look at the fermion masses and the way the mass eigenstates combine to form interaction eigenstates, there seems to be some structure. In principle, this apparent structure can just be the result of particular values of the 20 or 22 available free parameters. Some parameters then need to be tuned to ‘special’ values like 1, $1/2$ or $1/3$ to at least a few percent accuracy. Other parameters need to be quite large, with for instance the top quark to electron mass ratio being of the order of 340 000. This is all quite unnatural and unsatisfying from a theoretical physicist’s point of view. We prefer to explain observed structures with physical arguments instead of with resort to coincidence.

The central claim of this thesis is that there may be new physics in the fermion mass sector that helps explaining the observed structures in the data. In this section, we first discuss some of the structures in the fermion sector. Then we sketch how family symmetries can explain these structures. The working of family symmetries is explained in more (mathematical) detail in chapter 2.

1.4.1 Structures in the fermion masses

In section 1.1 we introduced the three families of quarks and leptons of the Standard Model. We plot the measured masses of the fermions in figures 1.15 and 1.16 on respectively a linear and a logarithmic scale. In the top figure, we see that the top quark is much heavier than all the other particles, effectively dwarfing their masses.

The logarithmic plot holds more information. Firstly, we notice that the neutrinos are *much* lighter

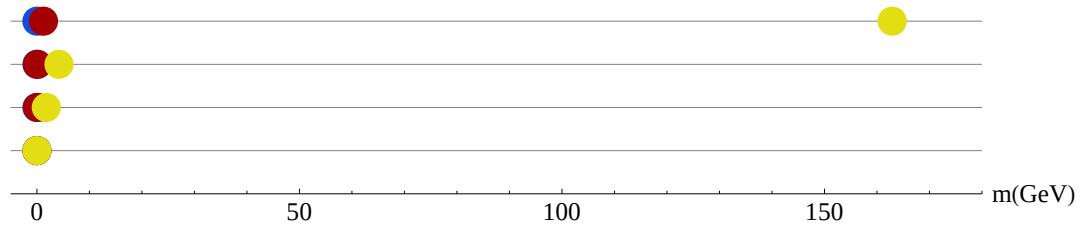


Figure 1.15: The masses of the Standard Model fermions on a linear scale. The top quark appears right on the top line. All other quarks and all leptons appear on top of each other on the left side of the diagram.

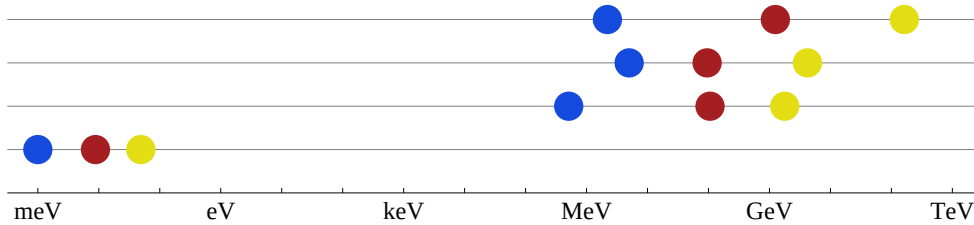


Figure 1.16: The masses of the Standard Model fermions on a logarithmic scale. Top line from right to left: top quark, charm quark and up quark. Second line: bottom quark, strange quark and down quark. Third line: tau lepton, muon and electron. Bottom line: the three neutrinos. Their exact masses are not known at the moment, but from indirect measures, we conclude that their masses are in the milli electronvolt to electronvolt region.

than all the other particles around. It is no wonder that up to 13 years ago, they were thought to be massless. This, combined with the observation that neutrinos are the only electrically neutral particles in the Standard Model, gives rise to the assumption that neutrino mass might come from an entirely other mechanism than quark and charged lepton masses. This mechanism is called the see-saw mechanism and is called such because it works in the same way as the seesaw in a children's playground. The neutrinos are on the seat that goes up and end up with a very small mass. The theory predicts the existence of other particles that sit on the seat that goes down. These particles are identified as new heavy ('righthanded') neutrinos or bosonic or fermion triplets. As much as the neutrinos are extremely light, these particles are very heavy and this might be the reason they are not observed so far, although there is some hope at the Large Hadron Collider. The seesaw and its consequences are discussed in more detail in section 2.1.4.

If we ignore the neutrinos, we can do a second interesting observation. All particles of the second generation are much heavier than all particles of the first generation, while all particles of the third generation are much heavier than all particles of the second generation. There seems to be quite a strict hierarchy between the generations. Actually, when we look at the gaps between the first and second and second and third generation, these are for each type of particle more or less of the same order. This relation is not exactly true; for instance, we see that the electron is slightly too light, while the down quark is a bit too heavy for the relation to hold. The masses of the Standard Model particles vary slightly with the energy scale we observe them at, due to a mechanism with virtual particles, much like what we described for the Higgs boson in section 1.3.2. At the scale of gauge unification, the relation described above holds much better.

Let's zoom in to the up-type quark sector. The mass of the top quark is given by a 'normal' parameter, that is approximately 1, while the charm quark is given by a small parameter and the up quark by a parameter that is small-squared (i.e. $m_c/m_t \approx m_u/m_c \ll 1$). Alternatively, we can describe the charm mass by a parameter that is normal, but now multiplied by an attenuating factor that is approximately the ratio of the top to the charm mass. The up-quark mass is now described by a third normal parameter, and the square of the attenuation factor. Froggatt and Nielsen gave a mechanism that describes the attenuation factors for the charm and up quarks, assuming they are charged under a type of charge that differs per generation. The Froggatt–Nielsen symmetry is thus the first family

symmetry we encounter in this thesis; we study the principles of the mechanism in more detail in section 2.3.1 and the Froggatt–Nielsen symmetry is also an important element of the model of chapter 4.

Evaluating the quark and charged lepton masses at the unification scale, yields two more interesting observations. The first is that at this scale the bottom quark and the tau lepton have the same mass to good accuracy. The second is that the strange quark is quite precisely three times lighter than the muon, known as the Georgi–Jarlskog relation. This might suggest that these masses are not given by unrelated parameters, but (in both cases) by one parameter that effects both masses. In the Pati–Salam GUT, Bottom-tau unification and the Georgi–Jarlskog relation can easily follow from symmetry principles and we use this in the construction of the model of section 4.

1.4.2 Structures in the fermion mixings

In figures 1.15 and 1.16 we plotted the masses of the quarks and leptons. In the weak interactions, not exactly these mass eigenstates react, but rather a very specific combination of them. There is a coupling between the top-quark, a W boson and a quark of the down type. We can define this quark as the interaction eigenstate b' . The b' quark does not have a uniquely defined mass. It is a mixture of the lightest, middle and heaviest eigenstate.

In figure 1.17 we show the mass eigenstates d , s and b that are linear combinations of the down-type quarks that couple to respectively the up, charm and top quark (the interaction eigenstates d' , s' and b').

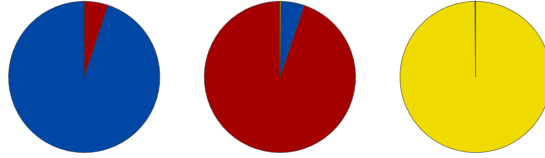


Figure 1.17: Pie charts showing the flavour content of the three quark mass eigenstates. .
Data taken from [6]

The pie-charts of figure 1.17 are almost ‘diagonal’, meaning that the lightest mass eigenstate d is almost entirely the interaction eigenstate d' that couples to the up quark; in the second generation, we have $s \approx s'$ and in the third $b \approx b'$.

A priori, we would expect a mixing where each of the mass eigenstates is a mixture of significant portions of each of the interaction eigenstates. The data on the other hand seem to signal that coupling to the ‘own’ generation is preferred. Indeed in both chapter 4 and section 5.10.4 we describe mechanisms where at the leading order quarks couple diagonally and where the small slivers of red and blue (and the almost invisibly small piece of yellow) in figure 1.17 are given by correcting effects.

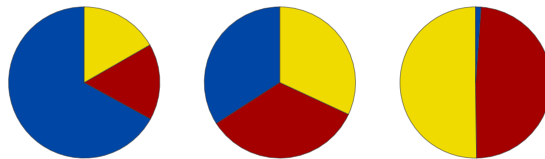


Figure 1.18: Pie charts showing the flavour content of the three neutrino mass eigenstates.
Data are average values from [7]

The situation in the lepton sector is different. The mass eigenstates are mixtures of significant

portions of all three interaction eigenstates as shown in figure 1.18. In the last paragraph we mentioned that this is exactly what could be a priori be expected. However, there is more to it here. A closer look at the diagrams of figure 1.18 shows that the combinations seem to be special. The second mass eigenstate is an almost perfect equal combination of all three interaction eigenstates, while the third mass eigenstate is so for two of the three interaction eigenstates. This mixing pattern is called tribimaximal mixing [8]. It is interesting to note that it can be reproduced by a quite limited amount of new physics as shown in section 2.4. The words ‘almost perfect’ are important here. Recent measurements of the so-called reactor neutrino angle have shown that the thin sliver of blue in the rightmost diagram of 1.18 is unmistakeably there, ruling out the possibility of exact tribimaximal mixing as was in line with the data until very recently.

1.4.3 Models with family symmetries

To solve the numerical coincidence with the θ parameter (the strong CP problem), the Peccei-Quinn symmetry was introduced; supersymmetry can account for the numerical coincidence of the finetuning of the Higgs mass. In the same spirit, there is a symmetry related to the observations of the previous subsections.

The patterns we observed account to striking similarities and striking differences between the three generations. The new symmetry will therefore have to connect these generations. The words family symmetry and flavour symmetry are used interchangeably for this symmetry.

It is interesting to note that flavour symmetries work in a certain way perpendicular to the known gauge symmetries of the Standard Model. The electromagnetic force couples for instance one electron to a photon; the weak nuclear force can couple an electron neutrino to an electron and a W boson and the strong force couples for instance a blue up quark to a red one. Obviously, these three forces treat the particles within a generation very differently, according to their charges under the related symmetry. All this concerns couplings within a generation. The idea of family symmetries is to have very specific couplings between particles of different families and treat the three families different by charging them differently under the new symmetry. This is graphically represented in figure 1.19. The figure also explains why family symmetries are sometimes called horizontal symmetries.

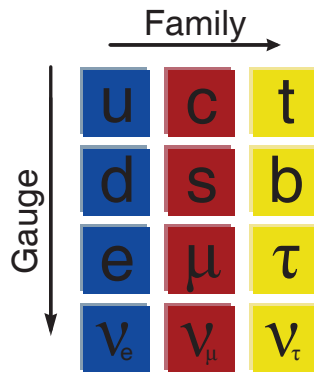


Figure 1.19: A graphical representation of family and gauge symmetries

The introduction of family symmetries concludes the content-part of this chapter. We conclude that the Standard Model provides an excellent description of the physics of elementary particles, but that there are reasons to extend it. One of these extensions is the introduction of flavour symmetries that form the heart of this thesis.

1.5 Outlook of this thesis

In this thesis we study different manifestations of flavour symmetries and their consequences. To be well-prepared for these models and their technical details, chapter 2 is a second introductory chapter. It is more technical than this chapter and contains not only an introduction to relevant concepts, but also to the mathematical formulation. The number of equations grows to 90 compared to zero in this chapter.

After a discussion of fermion masses in one and three families, chapter 2 discusses two well-known family-symmetric models. The model of Froggatt and Nielsen [9] mainly gives an explanation for the patterns found in the fermion masses as discussed in section 1.4.1. The model of Altarelli and Feruglio [10, 11] reproduces the tribimaximal mixing alluded to in section 1.4.2. In both cases the key element of the model is the assumption of an extra symmetry. We end the model sections with a balance: how much information is gained by assuming an extra symmetry and how much extra complications are the price that is paid for it.

The discussion of the Altarelli–Feruglio model contains a lot of information. This is true at a technical level – an example is the method of using driving superfields – but it also shows directions to proceed in. One of the conclusions is that the tribimaximal mixing pattern and in particular the limited possibility to have so-called next-to-leading order corrections to this mixing scheme points to the investigation of different mixing schemes.

Chapter 3 starts with a discussion about where to go now the measurement of non-zero reactor neutrino mixing angle has excluded exact tribimaximal mixing. After that, we consider a large class of candidate flavour symmetries and investigate to which mixing patterns they can give rise to. Tribimaximal mixing is one of them, but it is accompanied by a large family of other mixing patterns. Of particular interest are two mixing patterns that follow from relatively large groups ($\Delta(96)$ and $\Delta(384)$ respectively) that naturally predict the third neutrino mixing angle to be non-zero. This feature is not shared by other familiar groups and exactly what the most recent data point at.

Chapter 4 interprets the hint from the Altarelli–Feruglio model in a different way. This chapter describes a model in detail, in which the tribimaximal mixing pattern is replaced by the so-called bimaximal one. In first approximation this describes the data far worse than the tribimaximal case. The advantage lies in the fact that in the bimaximal case, corrections are possible and indeed needed.

The model of chapter 4 is thus a model of bimaximal mixing. The other important characteristic of the model is that it combines a flavour symmetry with a grand unified symmetry, the Pati–Salam GUT to be precise. We discuss the consequences of the combination of the two types of symmetries and show that in the interplay between the symmetries significant interference appears. The model can explain a wealth of data, but it becomes quite baroque. The Higgs sector is strongly extended and effects from renormalization group flow exclude much of the initially preferred parameter space.

Chapter 5 is inspired by another lesson learned from the Altarelli–Feruglio model. Supersymmetry and extra dimensions are no essential elements of family symmetric models, but at least one of them is needed for so-called flavon alignment in a model with two or more directions in flavour space. Both supersymmetry and extra dimensions are very interesting and well-motivated types of new physics, but it is important to see if flavour symmetry models can be constructed independent of them. The obvious way out, is to consider cases where just one direction in flavour space appears. Chapter 5 discusses a particular realization in which a multitude of Higgs fields transforms under a family symmetry. We give the scalar potential if there are three Higgs fields that transform as a triplet under A_4 , the most popular family symmetry and continue to find all its minima. The scale of new physics in this set up is the electroweak breaking scale. The model thus provides predictions testable at the LHC and precision measurements can already constrain it. We finish the chapter by an elaborate discussion of these tests and applications to various implementations.

Chapter 6 does what a last chapter usually does. It summarizes and concludes the thesis. In this chapter we make a large balance of the positive and negative things about flavour symmetries and flavour symmetric model building seen in this thesis.