The lead zeppelin : a force sensor without a handle

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Appendix

A Force Exerted by Excitation Coil

In this appendix we show that the excitation coil in our experimental setup can be used to excite the Lead Zeppelin in all 3 spatial directions — not just the z-direction. To do so, we expand the levitating force of section 2.4.2, equation (2.28), as follows:

$$\begin{aligned}\mathbf{F}_{tot} &= (k_x\varepsilon_x + \alpha_x\varepsilon_x I_{exc})\hat{\mathbf{x}} + (k_y\varepsilon_y + \alpha_y\varepsilon_y I_{exc})\hat{\mathbf{y}} \\ &\quad + (k_z\varepsilon_z + \alpha_z\varepsilon_z I_{exc} + \kappa I_{exc})\hat{\mathbf{z}}. \end{aligned} \tag{A.1}$$

We already calculated the spring constants $k_{x,y,z}$ and κ. The coefficients $\alpha_{x,y,z}$ are given, in full glory, by:

$$\begin{aligned}\alpha_x = \alpha_y &= \frac{1}{2}K_I\ (R_{lev}^2(2R_{exc}^2(R_{exc}^2 + 3R_{exc}R_{lev} + R_{lev}^2) \\ &\quad + (4R_{exc}^2 + 6R_{exc}R_{lev} - 3R_{lev}^2)z_{exc}^2 + 2z_{exc}^4) \\ &\quad + (R_{exc}^2(-3R_{exc}^2 + 6R_{exc}R_{lev} + 4R_{lev}^2) \\ &\quad - 6(R_{exc}^2 - R_{exc}R_{lev} + R_{lev}^2)z_{exc}^2 - 3z_{exc}^4)z_{lev}^2 \\ &\quad + (2R_{exc}^2 - 3z_{exc}^2)z_{lev}^4), \\ \alpha_z &= -K_I\ (R_{lev}^2(2R_{exc}^2(R_{exc}^2 - 3R_{exc}R_{lev} + R_{lev}^2) \\ &\quad + (4R_{exc}^2 - 6R_{exc}R_{lev} - 3R_{lev}^2)z_{exc}^2 + 2z_{exc}^4) \\ &\quad - (R_{exc}^2(3R_{exc}^2 + 6R_{exc}R_{lev} - 4R_{lev}^2) \\ &\quad + 6(R_{exc}^2 + R_{exc}R_{lev} + R_{lev}^2)z_{exc}^2 + 3z_{exc}^4)z_{lev}^2 \\ &\quad + (2R_{exc}^2 - 3z_{exc}^2)z_{lev}^4), \end{aligned} \tag{A.2}$$

with

$$K_I = \frac{3\mu_0}{2\pi}\frac{L_xL_yL_zR_{exc}R_{lev}z_{exc}z_{lev}}{(R_{exc}^2 + z_{exc}^2)^{7/2}(R_{lev}^2 + z_{lev}^2)^{7/2}}I_{lev}. \tag{A.3}$$

When we fill in the experimental parameters as found in table 2.1 and further the values $I_{lev} = 1.0 \times N_{lev}$ A, $L_x = 300$ µm, $L_y = 400$ µm, $L_z = 500$ µm, then

$$\alpha_x = \alpha_y = 9.75 \times 10^{-6}\ \frac{\mathrm{N}}{\mathrm{m{\cdot}A}}, \quad \alpha_z = 7.57 \times 10^{-6}\ \frac{\mathrm{N}}{\mathrm{m{\cdot}A}}. \tag{A.4}$$

We see that the $\alpha_{x,y,z}$ can indeed be safely ignored: they are multiplied by $\varepsilon_{x,y,z}$, which are at best about a micron, as well as the excitation current I_{exc} to get their force on the Zeppelin. However, we also note that because the coefficients $\alpha_{x,y,z}$ are non-zero, for positions of the Zeppelin away from the origin, we can in principle use the excitation coil to excite the motion of the Lead Zeppelin in all directions.

B Flux Reduction due to PCS

In this appendix we discuss some of the effects of the PCS on the Lead Zeppelin experiment. When the levitation coils are put in the persistent mode, there are some issues that need our reconsideration. Firstly, since the levitation coils are no longer connected to a current source of infinite impedance, but rather have zero resistance now, we can wonder whether the impedance of the excitation coil as met by the function generator has changed significantly. The excitation coil is, after all, close to either levitation coil: if the mutual inductance is large enough, this could mean that we can send down less power through the excitation coil than without the PCS. We will see, though, that at the low frequencies that we use, this makes not much of a difference.

Secondly, any flux introduced to the levitation coils could be very efficiently counteracted, as the levitation coils are keeping their flux constant whilst in the persistent mode. This effect will reduce the flux coming from the excitation coil, leading to a less efficient drive of the Zeppelin, and it might also affect the flux produced by the moving Zeppelin, meaning that there is less signal to be measured. We will see that when the levitation coils are each put separately in the persistent mode, this effect is quite severe. When we place the levitation coils in series with opposing polarity (anti-Helmholtz), and put them in the persistent mode together, this effect is small.

B.1 Effect on Impedance

We hook up a function generator to the excitation coil and call the combination of the function generator's 50 Ω impedance and the wires to the excitation coil R_{fg}. The wires typically have a resistance of 10 Ω, making it a total of $R_{fg} = 60$ Ω. When only one of the levitation coils is present, having a mutual inductance M to the excitation coil, and accounting for the possibility it is in the normal state with resistance R_{ns}, the total impedance Z that the function generator sees follows from Kirchhoff's Laws and can be written as

$$Z = R_{fg} + i\omega L_{exc} + k^2 L_{exc} \frac{i\omega - \omega_{ns}}{1 + \left(\frac{\omega_{ns}}{\omega}\right)^2}. \tag{B.1}$$

Here, $L_{exc} = 0.2$ µH is the inductance of the excitation coil, $\omega_{ns} = \frac{R_{ns}}{L_{lev}}$, with $L_{lev} = 220$ µH the inductance of the levitation coil, and k comes from $M = k\sqrt{L_{exc} L_{lev}}$, the mutual inductance between the levitation coil and the excitation coil.

The effect that the levitation coil has on the impedance, is very small. When $R_{ns} = 0$ Ω, the full effect is governed by k^2; however, it starts to be noticeable only at a frequency $\omega = \frac{R_{fg}}{L_{exc}} = 2\pi \times 48$ MHz. For our purposes, this is completely negligible.

When $R_{ns} = 1$ Ω, $\omega_{ns} = 2\pi \times 723$ Hz. This is also outside our region of interest.

B.2 Effect on Efficiency of Excitation Coil

The question remains what the net flux is that the excitation coil produces when the levitation coils are in the persistent mode. In the persistent mode, any coil obeys flux conservation: if externally some flux is offered, it will produce a countercurrent such that the total flux through the coil remains the same.

Let's begin with the on-axis magnetic field $B(z)$ of a circular current loop of radius R in the (x, y)-plane centered at the origin::

$$B(z) = \frac{\mu_0 I}{2R} \times \frac{R^3}{(z^2 + R^2)^{3/2}}. \tag{B.2}$$

In the above form, one can recognize the $B(0)$-term which is multiplied by a dimensionless distance correction factor. Let us assume that the flux decreases in the same way:

$$\Phi(z) = \Phi(0) \times \frac{R^3}{(z^2 + R^2)^{3/2}} \equiv \Phi \times P_{\text{dis}}(z, R). \tag{B.3}$$

The flux in another circular loop of radius L, coaxial to the first, some distance s away, can be calculated with the above formula combined with an area correction factor. Namely, when $R > L$ we say, feeling comfortable in assuming that the field produced by the first, bigger loop is roughly constant across the area of the second, smaller loop:

$$\Phi_L = B_R(s) A_L = \Phi_R(s) \frac{A_L}{A_R}. \tag{B.4}$$

The flux has been further diminished by the ratio of the areas of the two loops.

In the case that $L > R$, something similar is going on. Consider that $\Phi = MI$: the flux through the larger loop due to a current I in the smaller loop, is exactly the same as the flux in the smaller loop due to the same current I in the larger loop. In other words, the flux that is 'lost' on the way is the same in both cases.

The flux is therefore diminished by the ratio of the area of the smallest loop to the largest loop:

$$P_{\text{area}}(R, L) \equiv \begin{cases} \left(\frac{L}{R}\right)^2, & R > L, \\ \left(\frac{R}{L}\right)^2, & L > R. \end{cases} \tag{B.5}$$

And, combining:

$$\Phi_L = \Phi_R \ P_{\text{dis}}(s, R) \ P_{\text{area}}(R, L). \tag{B.6}$$

In the case of the excitation coil and the levitation coils, we simply need to make the right combinations of P_{dis} and P_{area}. We place the center of the setup

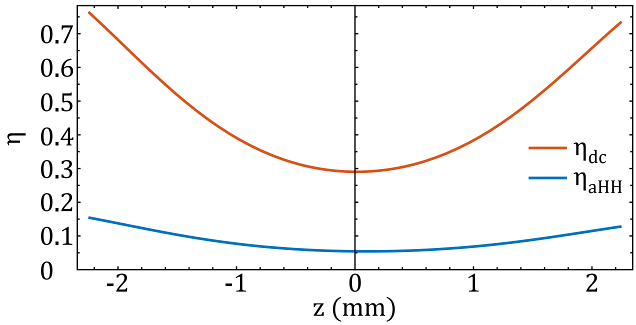

Figure B.1: The loss factors η as a function of z, $z = 0$ being the center of the experimental setup as in figure 2.4, for the two cases that the levitation coils are decoupled from each other and each connected to their own PCS (η_{dc}), and when they are coupled in an anti-Helmholtz configuration to a single PCS (η_{aHH}). The excitation coil is placed at $z_{exc} = 0.2$ mm, and we further used the other experimental parameters of table 2.1. These loss factors tell us how much more difficult it is to drive the Lead Zeppelin due to the levitation coils being in the persistent mode. The coupled loss factor η_{aHH} goes to zero for all z when the excitation coil is placed at $z_{exc} = 0$.

at $z = 0$, such that the flux Φ of an excitation coil of radius R_{exc} at position z_{exc} is given as a function of z as

$$\Phi_{exc}(z) = \Phi P_{\mathrm{dis}}(z - z_{exc}, R_{exc}). \tag{B.7}$$

When the levitation coils are decoupled and both in their own separate persistent mode, the flux that reaches either of them is perfectly counteracted. For the upper levitation coil, the counteracting flux as a function of z is:

$$\Phi_{up} = -\Phi P_{\mathrm{dis}}(z_{up} - z_{exc}, R_{exc}) P_{\mathrm{area}}(R_{exc}, R_{up}) P_{\mathrm{dis}}(z - z_{up}, R_{up}). \tag{B.8}$$

The same goes for $\Phi_{down}(z)$, with (up $\leftrightarrow$ down).

The effective excitation flux is the sum of the three contributions:

$$\Phi_{\mathrm{eff,dc}}(z) = \Phi_{exc}(z) + \Phi_{up}(z) + \Phi_{down}(z). \tag{B.9}$$

The loss factor $\eta_{\mathrm{dc}}(z)$ describes the amount of flux 'lost' compared to what was going to be lost anyway already

$$\eta_{\mathrm{dc}}(z) = 1 - \frac{\Phi_{\mathrm{eff,dc}}(z)}{\Phi_{exc}(z)}. \tag{B.10}$$

We will ignore higher order reduction effects: e.g. the counteraction of the flux of the upper levitation coil by the lower levitation coil is small and is not taken into account.

When the levitation coils are coupled and in anti-Helmholtz configuration, the flux that reaches the upper levitation coil minus that which reaches the lower levitation coil is what is counteracted. We place absolute signs around the difference of the two fluxes, which is the easiest way to get all the signs right.

$$\begin{aligned}\Phi_{up,\mathrm{aHH}}(z) = & -\Phi\Big| P_{\mathrm{dis}}(z_{up} - z_{exc}) P_{\mathrm{area}}(R_{exc}, R_{up}) \\ & -P_{\mathrm{dis}}(z_{down} - z_{exc}) P_{\mathrm{area}}(R_{exc}, R_{down})\Big| P_{\mathrm{dis}}(z - z_{up}, R_{up})\end{aligned} \tag{B.11}$$

Again, for $\Phi_{down}(z)$ we do (up $\leftrightarrow$ down). The effective excitation flux is this time

$$\Phi_{\mathrm{eff,aHH}}(z) = \Phi_{exc}(z) + \Phi_{up,\mathrm{aHH}}(z) + \Phi_{down,\mathrm{aHH}}(z), \tag{B.12}$$

and the loss factor

$$\eta_{\mathrm{aHH}}(z) = 1 - \frac{\Phi_{\mathrm{eff,aHH}}(z)}{\Phi_{exc}(z)}. \tag{B.13}$$

The two loss factors are shown in figure B.1. At a typical Zeppelin equilibrium position $z_{eq} = -700$ µm, we find the values

$$\eta_{\mathrm{dc}}(z_{eq}) = 34\ \%, \quad \eta_{\mathrm{aHH}}(z_{eq}) = 6.6\ \%. \tag{B.14}$$

Decoupling the levitation coils means that we can drive the Lead Zeppelin significantly less efficiently. Coupling them means that there is also a reduction, but it is a lot smaller.

B.3 Effect on Detection by Pickup Coil

Now, the loss factors of the flux picked up by the pickup coil due to the Lead Zeppelin will follow the same basic rules. The changing flux expulsion of the Lead Zeppelin as it moves through the levitating magnetic field, will be counteracted by the levitation coils.

When the levitation coils are coupled to a single PCS, and when the Lead Zeppelin has its equilibrium position at $z_{eq} = 0$, due to symmetry there is no flux reduction. As soon as the equilibrium position is away from the origin, a reduction will kick in. Decoupling the coils with two separate PCSs will mean that there is always a reduction, no matter where the Zeppelin has its equilibrium. We therefore choose to couple the levitation coils to minimize a loss of signal.