The lead zeppelin : a force sensor without a handle

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Chapter 5

Discussion and Outlook

We have shown that we can make a superconducting particle levitate by means of a suitable magnetic field, and read out its motion. There are, naturally, improvements that can be made which will turn the Lead Zeppelin into a force sensor that can compete with the state-of-the-art, e.g. starting from being able to detect the thermal motion, towards a gravitational force measurement, and ultimately to a quantum measurement. We want to discuss some of the improvements here, that we thought of while doing the experiments — a few of which have even already been realized, awaiting testing.

Vibration Isolation

There are some ideas on how we can improve our vibration isolation. As mentioned in section 2.8, the new measurement hall of Leiden University has vibration isolated islands with a displacement noise of $\sim 2 \times 10^{-10}\ \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$ at 10 Hz[36]. Compared to a typical laboratory movement of $\sim 10^{-8}\ \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$, this is an improvement of 2 orders of magnitude. If we were to put a crane on such an island, and hang our Helium Dewar on it with bungee cords, the Dewar motion should drop by 2 orders of magnitude as compared to now, figure 2.13.

This is still neglecting the boiling noise coming from the evaporating Helium. This contribution to the displacement noise of the Lead Zeppelin is difficult to calculate, as it depends on many factors like the exact geometry of the Helium Dewar and dipstick, as well as the boiling rate (which depends on the geometry, the amount of liquid Helium left in the Dewar, the heat input, ...). To know how bad this is, really one should measure it; we have not done this, so we are not sure whether it is something to worry about. Then again, when we repeat our measurements on a vibration isolated island, and the Zeppelin motion isn't diminished, it was probably the boiling noise that caused the major source of trouble.

Another idea then is to damp external motion directly at the experiment.

Rather than screwing the experimental chamber tightly to the dipstick, as in figure 2.16, we could instead suspend it from soft springs, see figure 5.1, which could be placed on a dipstick, but also in a cryostat. When we succeed in getting the resonance frequency to be (much) lower than a Zeppelin resonance ω_0, we can diminish the dipstick motion reaching the Zeppelin; as explained in section 2.8, the transfer function that takes external motion to Zeppelin motion then is $H(\omega) = \left(\frac{\omega_0}{\omega}\right)^2$. In fact, when we assume that the boiling noise from evaporating Helium is negligible in at least the z-direction, which seems to make sense, this frame acts as a second vibration isolation stage on motion from the laboratory ceiling, which is then filtered to 4^{th} order.

The frame from figure 5.1 has a resonance frequency of a few Hz. The Zeppelins of this thesis have resonances of around 10 Hz, and these can be pushed upwards as we explain below. Unfortunately, the thermalization of the experiment in this frame, without spoiling its ability to damp external vibrations, is difficult and no successful experiment has been performed yet.

Sensitivity

When the externally driven Zeppelin motion gets smaller, at some point we will want to lower the SQUID noise and increase the coupling to the pickup coil.

We have tried already to get the Zeppelin closer to the pickup coil by introducing an asymmetry in the levitation currents, i.e. $I_{up} \neq I_{down}$. This way, one can manoeuvre the Zeppelin to a position where $\frac{d\Phi}{dz}$ is larger. However, this led to unintuitive results in our earliest experiments without a PCS (section 4.1), and hasn't (yet) resulted in a successful measurement after instalment of a PCS at each levitation coil (section 4.2).

Similarly, placing the pickup coil at a more optimal position can increase the signal a great deal. As is clear from e.g. figure 2.7, a lot can be won from this simple modification.

One can also use a SQUID with a lower noise level. Our SQUIDs remained at best at about 35 $\frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$, and more tyically around 100 $\frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$. This number is far higher than what is possible with a SQUID: the commercial Quantum Design SQUIDs that we use should be able to reach 5 $\frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$ at 4 K down to below 1 Hz[26], and SQUIDs with still better figures do in fact exist.

Higher Resonance Frequencies

A higher resonance frequency makes many envisioned experiments easier. At the moment most importantly, the Dewar motion becomes less of a nuisance.

Ideally, we simply crank up the levitation currents; not only does this increase the resonance frequencies, but also will the Zeppelin hover closer to the z = 0 point, which means less non-linear (Duffing) behaviour as well as a higher

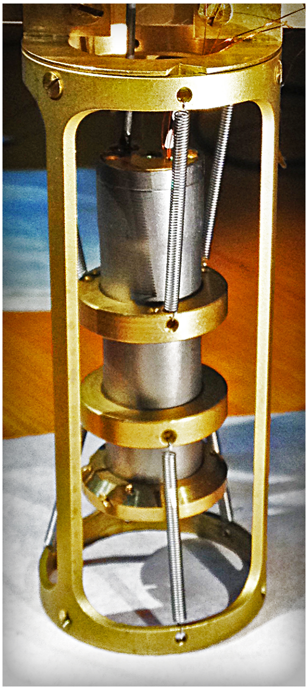

Figure 5.1: By suspending the experiment (housed in the Nb tube) in a frame with some soft springs, external vibrations are mitigated.

coupling to the pickup coil. Of course, we cannot just increase the levitation currents unlimitedly, because we cross the critical magnetic field quickly (section 4.2). However, there are easy adjustments that we can make that do enable this.

For instance, we can use a smaller Zeppelin. Indeed, let us calculate how high a levitation current we can allow for when we use a smaller Zeppelin of, say, $R_{zep} = 5$ µm. The critical field at 4.2 K for Lead, equation (2.41), is $B_{c,\mathrm{Pb}}(4.2\text{ K}) = 13.9$ mT. When we assume the Zeppelin to float at $z = 0$, which for smaller particles and higher levitation currents becomes true rapidly (figure 2.9), we find that in our setup the maximum levitation current is, using equation (4.2),

$$I_{lev,c} = 81\text{ A}. \tag{5.1}$$

From this levitation current onwards, the critical magnetic field is exceeded at the edge of the Zeppelin. At this levitation current, the resonance frequencies are (equation (2.36))

$$f_{x,y} = 1.4\text{ kHz}, \quad f_z = 2.2\text{ kHz}. \tag{5.2}$$

Of course, for smaller and smaller particles, levitation may be more difficult to achieve, due to the Van der Waals force that will make the particle stick to the surface of the Macor holder in which we put it. At this point, one would have to implement some sort of launching device that can fling the Zeppelin towards the zero point of magnetic field; for instance a short voltage burst on a piezo element on which one places the Zeppelin. Also, one must be careful not to cross the critical current density of the superconducting wiring, nor that of the spot welded wire joints.

For even smaller particles, on the order of the glass nanospheres that are used in optical levitation experiments (radii $20 - 500$ nm, e.g. [5, 19]), say $R_{zep} = 100$ nm, we would have $I_{lev,c} = 4$ kA and $f_{x,y} = 67$ kHz, $f_z = 110$ kHz. Here, it will be even more challenging to get the Zeppelin off the surface and into the zero point of magnetic field.

The resonance frequencies can also be increased when we make a Zeppelin out of another superconducting material than Lead. As was hinted to in section 2.3, one could go for a Niobium Zeppelin or an Aluminium Zeppelin. Because of the smaller densities of these materials compared to Lead ($\rho_{\mathrm{Pb}} = 11.34 \times 10^3$ $\frac{\mathrm{kg}}{\mathrm{m}^3}$ versus $\rho_{\mathrm{Nb}} = 8.57 \times 10^3$ $\frac{\mathrm{kg}}{\mathrm{m}^3}$ and $\rho_{\mathrm{Al}} = 2.70 \times 10^3$ $\frac{\mathrm{kg}}{\mathrm{m}^3}$), they will have higher resonance frequencies and they will levitate closer to $z = 0$, already at the same levitation currents.

The downsides of Aluminium are its low critical temperature $T_{c,\mathrm{Al}} = 1.18$ K and critical field $B_{c,\mathrm{Al}}(T = 0\ \mathrm{K}) = 10.5$ mT[50, p. 224]. This means that measurements in liquid Helium are out of the question and that the increase in resonance frequency as a consequence of having a smaller density is partially lost on a lower maximum levitation current.

A Niobium Zeppelin is not all that less dense than Lead, and spherical Zeppelins won't be made in-house owing to the high melting temperature of Niobium (although such particles can be bought), but it has the big advantages of higher critical temperature $T_{c,\mathrm{Nb}} = 9.25$ K and critical fields $B_{c1,\mathrm{Nb}}(T = 0\ \mathrm{K}) = 174$ mT, $B_{c2,\mathrm{Nb}}(T = 0\ \mathrm{K}) = 400$ mT[49]. Even if one wants to stay below $B_{c1,\mathrm{Nb}}$, e.g. to avoid vortex dynamics, then still much higher levitation currents can be used than with Lead. As an example: at 4.2 K we find that $B_{c,\mathrm{Pb}} = 13.9$ mT and $B_{c1,\mathrm{Nb}} = 138$ mT; an order of magnitude difference.

An idea we had to get the rotational modes to higher frequencies, is to wind elliptical levitation coils, rather than circular ones, combined with a Zeppelin that is (very) aspherical. This will help to clamp rotational modes more firmly. The rotational frequencies about the x- and y-axis are probably increased most effectively by the amount that the Zeppelin deviates from being spherical. The rotation about the z-axis we can help a little further by introducing the rotational asymmetry in the levitation coils. At the same time, the translations in x and y will be decoupled, which will help us identify which peak belongs to which resonance. We have fabricated such coils, but no successful experiments were performed yet.

Lower Damping, Thermalization

We found that in our experiments the quality factor of the resonances seems to be limited by the ambient Helium gas, which was typically at a pressure between $P = 10^{-2} - 10^{-1}$ mbar, and resulted in damping factors on the order of $\gamma \sim 10^{-8}\ \frac{\mathrm{N}}{\mathrm{m/s}}$, see table 4.3. The obvious thing to do to achieve lower damping is to go to lower pressures. However, in our setup, the thermalization of the various components depends on having enough background gas; e.g. the SQUID modulation becomes worse, and completely disappears below $\sim 8 \times 10^{-3}$ mbar, but also the PCS can no longer be operated.

A better thermalization can be achieved by screwing components tightly onto the dipstick with metal clamps, rather than with a load of Teflon tape, figure 2.16. This would certainly make the SQUID be operable at lower pressures, as well as the components of the PCS and its various coils: we know that all of that works just fine in a dilution refrigerator at 10 mK in a cryogenic vacuum[40]. If need be, a (thin) strip of metallic material could provide cooling to components that still require a better thermalization; for instance, maybe the levitation coils need this once we remove the brass clamp that holds them together.

When thermalization is taken care of, namely, the next spoiler of γ is this brass clamp (section 2.6.2). To get around this, we will replace it with a clamp made from the plastic PEI. Please note that one has to be very careful not to add too much normal metal around the Lead Zeppelin for cooling purposes, as this will set a new stage for eddy currents to take place.

Using smaller particles also helps us here. With the eddy current damping eliminated, the contribution from gaseous Helium is brought down as R_{zep}^2 (eq. (2.57)). The proposed Zeppelin of size $R_{zep} = 100$ nm would result in a $7 - 8$ orders of magnitude improvement of γ over the current Zeppelins, even without changing the pressure.

Future Prospects

Finally, we compare the Lead Zeppelin's force sensing abilities to other techniques around. As far as our measurement scheme is concerned, the detection noise is sufficiently low to detect the thermal motion. The (on-resonance) force detection limit of the Lead Zeppelin is

$$S_{F,lim}^{1/2}(\omega_0) = \frac{m\omega_0^2}{Q}\ S_{x,lim}^{1/2} = 8.7 \times 10^{-16}\ \frac{\mathrm{N}}{\sqrt{\mathrm{Hz}}}, \tag{5.3}$$

for $m = 2$ mg, $Q = 3000$, $\frac{\omega_0}{2\pi} = 10$ Hz and $S_{x,lim}^{1/2} = 3.3 \times 10^{-10}\ \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$ (eq. (2.89)), whereas the thermal force noise at 4.2 K is

$$S_{F,Th}^{1/2} = \sqrt{4k_B T\gamma} = 3.1 \times 10^{-15}\ \frac{\mathrm{N}}{\sqrt{\mathrm{Hz}}}, \tag{5.4}$$

for the same typical numbers and $\gamma = \frac{m\omega_0}{Q}$. However, as we know already from section 2.8, the force noise on the Lead Zeppelin brought in by the motion of the Helium Dewar is much larger, and as a matter of fact presently limits our force detection:

$$S_{F,zep}^{1/2} = m\omega_0^2 \; S_{x,\mathrm{Dewar}}^{1/2} = 7.9 \times 10^{-12} \; \frac{\mathrm{N}}{\sqrt{\mathrm{Hz}}}, \tag{5.5}$$

for a typical Dewar motion at 10 Hz of $S_{x,\mathrm{Dewar}}^{1/2} \sim 10^{-9} \; \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$, figure 2.13, and where we used $S_{x,zep}^{1/2} = H(\omega) \; S_{x,\mathrm{Dewar}}^{1/2} = Q \; S_{x,\mathrm{Dewar}}^{1/2}$ on resonance, eq. (2.103).

To cross from the vibrationally limited domain, eq. (5.5), into the thermally limited domain, eq. (5.4), using a smaller Lead Zeppelin offers a solution: the vibrational force noise scales in the Zeppelin size R_{zep} as $S_{F,zep}^{1/2} \propto m \propto R_{zep}^3$ and the thermal force noise as $S_{F,Th}^{1/2} \propto \sqrt{\gamma} \propto R_{zep}$, because when the background Helium provides the damping we have $\gamma \propto R_{zep}^2$. So when we reduce R_{zep} by a factor of 50 (i.e. from $\sim$ 250 µm down to the earlier proposed 5 µm), leaving everything else as it is, the thermal force noise and the vibrational force noise are on a par at $S_{F,zep}^{1/2} = S_{F,Th}^{1/2} = 6 \times 10^{-17} \; \frac{\mathrm{N}}{\sqrt{\mathrm{Hz}}}$. The damping then is $\gamma = 4 \times 10^{-12}$ $\frac{\mathrm{N}}{\mathrm{m/s}}$, and $Q = 3 \times 10^7$. A still smaller particle will be limited by thermal noise, rather than by vibrations.

In table 5.1 we compare the force noise on the Lead Zeppelin and its damping coefficient to those of the magnet on cantilever of Nichol et al.[2] and that of Vinante et al.[4], as well as to those of the optically levitated nanoparticle of Gieseler et al.[5]. We include in the table the scenario in which we make the Lead Zeppelin 50 times smaller, leaving the rest of the experiment as it has been throughout this thesis. We also add the 'Thermal LZ', explained below, in which we do not change the size of the Zeppelin, but assume that our vibration isolation is improved to the point where it no longer dominates the force noise.

The force noise only has to be so low as to allow for a measurement of the force of interest. In the table we therefore also include the mass m of the force sensors, their resonance frequency f_0 and the gravitational force F_g between them and an oscillating test mass M as in equation (2.96)

$$F_g = 2a\frac{GmM}{r^3}. \tag{5.6}$$

Here, r is the distance between M and m, and a is the amplitude of the oscillation of M. To compare between the different sensors, we fill in the same values as we did in section 2.7.4, $M = 1$ g, $r = 3$ cm and $a = 1.5$ mm (or equivalently $M = 1$ kg, $r = 1$ m and $a = 5$ cm).

Considering that $m \propto R_m^3$ and $M \propto R_M^3$, where $R_{m,M}$ are the sizes of m and M, and further that the distance between them is $r \geqslant R_m + R_M + a$, the gravitational force scales as $F_g \propto \frac{aR_m^3 R_M^3}{(R_m+R_M+a)^3}$, or $F_g \lesssim R_m^3$. In other words, when we are limited by vibrations, as is the case for the Lead Zeppelin, a

System	$S_F^{1/2}$ ($\frac{\mathrm{N}}{\sqrt{\mathrm{Hz}}}$)	γ ($\frac{\mathrm{N}}{\mathrm{m/s}}$)	m (kg)	f_0 (Hz)	F_g (N)
LZ	8×10^{-12}	10^{-8}	10^{-6}	10	10^{-17}
Small LZ†	6×10^{-17}	10^{-12}	10^{-11}	10	10^{-22}
Thermal LZ†	3×10^{-15}	10^{-8}	10^{-6}	10	10^{-17}
MoC 8 K[2]	2×10^{-18}	10^{-15}	10^{-17}	10^{5-6}	10^{-28}
MoC 28 mK†[4]	8×10^{-19}	10^{-13}	10^{-13}	10^{3-4}	10^{-24}
OLN†[5]	2×10^{-20}	10^{-20}	10^{-18}	10^{4-5}	10^{-29}

Table 5.1: The force noise and damping coefficient on the Lead Zeppelin (LZ) compared to state-of-the-art Magnet on Cantilever (MoC) and Optically Levitated Nanoparticle (OLN) force detection. For the 'Small LZ' we reduce the size of the Zeppelin, but we leave the rest of the experiment unaltered. The 'Thermal LZ' shows the case that we install the proposed vibration isolation measures. We add the mass and resonance frequency to the table, and calculate the gravitational interaction with a test mass (see main text). A dagger (†) indicates that the force noise comes from thermal fluctuations.

gravitational measurement does not become any harder or easier when reducing R_m. We are less susceptible to vibrations, which is good, but at the same time we will have a smaller gravitational interaction. When limited by thermal noise, reducing R_m actually makes the gravitational measurement more difficult.

The resonance frequency of the Zeppelin is much lower than the other force sensors, meaning that the gravitational force will be easier to generate. Further, we see that although a gravitational measurement in the current Lead Zeppelin experiment would require very long measurement times T_m, the same is true for the other techniques.

Carrying out the improvements discussed in this chapter, a measurement of the gravitational interaction with a small source mass within reasonable measurement times, should be possible. For instance, when we move our experiment to the new measurement hall of Leiden University, we immediately gain 2 orders of magnitude on $S_F^{1/2}$, due to the lower vibrations present there. When we also install the vibration isolation frame of figure 5.1 (for which we need to get its thermalization in order), the force noise will readily be dominated by thermal fluctuations. See the 'Thermal LZ' in table 5.1. At this point, a gravitational measurement will be possible within a very reasonable measurement time of a few hours, and could be brought down further by e.g. a modest increase of the resonance frequency (less vibrations, f_0^{-4} scaling) combined with going to a lower pressure (lower damping, less thermal noise).