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The lead zeppelin : a force sensor without a handle

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Chapter 4

Flying the Lead Zeppelin

In this chapter we present the experimental results that we obtained. To begin with, we will show some of the first results that were acquired with an older version of the experimental setup, which helped us develop many of the improvements of the experiment described in chapter 2.

We then continue to show the results obtained with the experimental setup of chapter 2. We will find out how the resonance frequencies of the Lead Zeppelin depend on the levitation current, and what the quality factors are of the resonances. Subsequently, we discuss the severity of external vibrations in our measurements. We end by analysing which measured resonances are the Zeppelin's natural frequencies, and which are caused by non-linear mixing amongst themselves and between them and the external vibrations.

All the measurements presented in this chapter have been performed with circular levitation coils. As we noticed while performing the experiments, we might want to change to elliptical levitation coils: combined with an aspherical Zeppelin, namely, this will push the rotational resonances to higher frequencies, and make the x and y translational resonance more distinct. We will come back to this in the discussion, chapter 5.

Several Lead Zeppelins are used in the results that are about to be shown. To keep track of which particle was used in which experiment, we will refer to them mostly by their estimated mass, but also by other distinctive qualities; see table 4.1. See figure 2.2 for a photo of the particles.

Location in figure	Mass (mg)	Size (µm)	Description
Bottom right inset	1.4	670×570	*Chunky*
Top right corner, main panel	1.6	650	*Almost spherical*
Bottom left corner, main panel	2	870×660	*Aspherical*
Top right inset	0.8	520×500	*Almost spherical*

Table 4.1: The Lead Zeppelins used in our experiments, and some properties by which we will refer to them. Based on the photos of figure 2.2.

4.1 Liftoff

Early results were obtained with a chunky Zeppelin (scraped off of a Lead block, but not torched) of size $\sim 670 \times 570$ µm, $m \approx 1.4$ mg, see the bottom right inset in figure 2.2. At the time of these measurements, the PCS was not yet installed, and the pickup coil was an old transformer chip of 44 closely packed, flat spiral, Niobium windings (figure 2.3).

We did frequency sweeps and measured the SQUID signal with a lockin amplifier as described in section 2.5. The result of this can be seen in figure 4.1, where we show for three different levitation currents $I_{lev} = 0.65,\ 0.70,\ 0.75$ A the measured lockin amplitude R, phase Θ and a phase plot (R, Θ). We notice that as I_{lev} is increased, the resonance frequency also increases; this is indeed

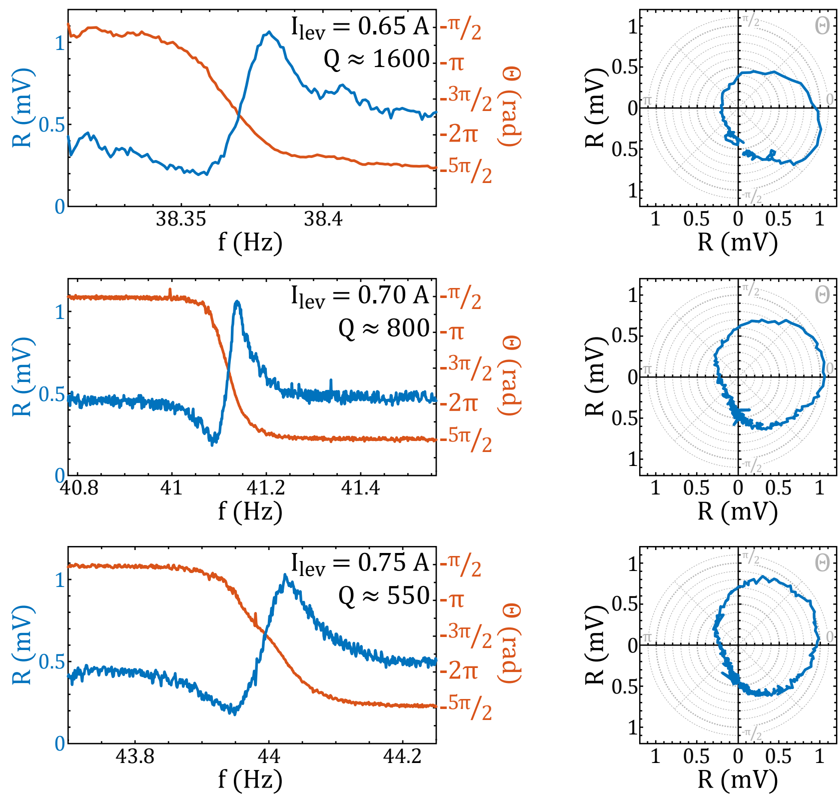

Figure 4.1: Lockin measurements of a chunky 1.4 mg Zeppelin at $I_{lev} = 0.65,\ 0.70,\ 0.75$ A, with the settings $V_{exc} = 100$ mV, $\Delta f = 1$ mHz, $\Delta t = 1$ s, $\tau_{LI} = 0.3$ s. We see similar behaviour to the simulation of figure 2.11. The lockin amplitude output R first goes down, then up and settles to its off-resonant value again (if we were patient enough), while the phase Θ drops by 2π. The phase plots are displaced circles. The Q factors indicated in the figures were obtained with ringdown measurements.

I_{lev} (A)	f_{res} (Hz)	Q	γ (10^{-7} $\frac{\mathrm{N}}{\mathrm{m/s}}$)
0.65	38.37	1600	2.1
0.70	41.12	800	4.5
0.75	44.00	550	7.0

Table 4.2: The resonance frequencies and quality factors of the Zeppelin for three levitation currents, as seen in figure 4.1, acquired by ringdown measurements. The damping factors are calculated as $\gamma = \frac{m\omega_{res}}{Q}$, with $m = 1.4$ mg.

the Lead Zeppelin that we are measuring.

Due to a combination of signal attenuating factors, the Zeppelin motion was higher than the noise floor only when driven by the excitation coil.

The graphs are phenomenologically the same to what we simulated in section 2.5. R has a dip before its peak, Θ drops by 2π across the resonance and the phase plot is a moved circle.

We measure the Qs of the resonances with ringdown measurements: we ring up the Zeppelin with the function generator and measure the decay in time of the signal at resonance with the lockin. Fitting an exponential to this yields the decay time τ_{zep} and subsequently the quality factor $Q = \pi f_{res}\tau_{zep}$. We added the measured quality factors to figure 4.1.

As we increase I_{lev}, the quality factors go down. This could be due to the fact that we are close to the critical magnetic field of Lead at these levitation currents; as we increase the levitation current, a larger part of the Zeppelin becomes a normal conductor. Because of the expulsion of magnetic fields by the superconducting pickup coil, which distorts the levitating magnetic field, we do not know the exact shape of the levitating magnetic field, and therefore which fraction of the Zeppelin sits in too high a field. However, at $I_{lev} = 0.8$ A there is no Zeppelin response at all anymore, indicating that we must have been indeed very close to the critical field.

When we take a look at what the SQUID measures when we send a reference (off-resonant) signal through the excitation coil and calculate what we should have measured, we find a big discrepancy: the measured signal is a (conservative) factor of $\sim 10^7$ too low! Possibly a wire from the pickup coil to the SQUID has (partly) snapped, or some of the superconducting wiring is not superconducting, or a wirebond is oxidized. In any case, rather remarkably, this is not all bad. Without this extreme attenuation of signal, the noise coming from the current source, which is not reduced by a PCS (because we didn't have one at the time) would be far too much for the SQUID to handle and we would not have been able to measure anything at all (section 2.9).

In an attempt to make the signal bigger, we tried to get the Lead Zeppelin closer to the pickup coil by increasing the lower levitation current and simultaneously decreasing the upper one; that is, instead of $I_{up} = I_{down} = I_{lev}$, we did $(I_{up}, I_{down}) \rightarrow (I_{up} - \Delta I, I_{down} + \Delta I)$, with ΔI typically some multiple of 10 mA. This has the effect of moving the equilibrium position more towards the pickup coil. What we found is that, as the Zeppelin is brought closer to the pickup coil, the signal strength remains about the same, but the resonance frequency goes down. We believe that this is due to the Meissner effect of the pickup coil: being effectively a superconducting slab with a small hole in the middle, it distorts the levitating magnetic field. As the Zeppelin gets closer to the pickup coil, the magnetic field gets weaker, thus giving rise to a lower resonance frequency.

To reduce this effect, we wound the new coil visible on the right in figure 2.3. Also, by making this new coil from sturdy Niobium wire, and by going straight to the input coil of the SQUID, we prevent the 10^7 mismatch from occurring.

4.2 Levitation Current versus Zeppelin Frequency

We installed a Persistent Current Switch in the experiment, and placed the new pickup coil. At first, we put each levitation coil in its own, separate persistent mode. We wanted to keep the possibility open to charge each coil with a different current, thereby having control over the equilibrium position of the Lead Zeppelin. However, we were unable to find any Zeppelin resonances. We suspect that this is due to the effect of flux reduction of the Zeppelin flux as explained in appendix B, despite the fact that this should only make for a factor of $\sim 5 - 10$ difference in signal. We then put the levitation coils in series, in anti-Helmholtz configuration (that is, with opposite polarity), controlled by one PCS.

We fabricated the Lead Zeppelin which is roughly spherical with diameter 650 µm, $m = 1.6$ mg, of figure 2.2's top right corner. We then swept the levitation current in the range $I_{lev} = 200 \rightarrow 450$ mA in steps of 10 mA. At each levitation current, we measured the SQUID signal 10 consecutive times for 10 seconds, calculated the Power Spectral Density (PSD) of each 10 s time trace, and took the (quadratic) average of the resulting 10 PSDs. The result is shown in figure 4.2.

We look whether among the many peaks in the PSD we find moving peaks, as they are tell-tale signs of a Zeppelin resonance. There is indeed movement of some peaks, two of which have been indicated with coloured lines in the figure. What is immediately clear is that the peaks are not moving linearly with I_{lev}, which is what we predicted in section 2.4.2. However, we remember figure 2.8: at the lower levitation currents, we are in the non-linear part of the force curve. It is then not at all strange that we see a deviation from linearity also in the change of resonance frequencies.

Furthermore, we notice that peaks start to emerge from $I_{lev} = 250$ mA upwards. Below this levitation current, the gravitational force is still stronger than the levitation force.

Above $I_{lev} = 410$ mA, the signal begins to disappear. We attribute this to the fact that at this levitation current, the levitating magnetic field strength starts to become similar to the critical magnetic field of Lead at 4.2 K. We calculated in equation (2.41) that

$$B_{c,\mathrm{Pb}}(4.2\ \mathrm{K}) = 13.9\ \mathrm{mT}. \tag{4.1}$$

The levitating magnetic field at the underside of the Zeppelin, $R_{zep} = \frac{1}{2} \times 650$ µm, for $I_{lev} = 410$ mA is

$$B_{lev} = G_z(410 \text{ mA}) \times (-R_{zep} + z_{eq}) = 11.5 \text{ mT}, \tag{4.2}$$

for the typical numbers $G_z(1 \text{ A}) = -34 \frac{\text{T}}{\text{m}}$ (eq. (2.11)), and $z_{eq} = -0.5$ mm (figure 2.9).

Of the many peaks in the PSD of the Zeppelin motion, some are not coming from the Zeppelin. As we are sensitive to any motion between the coils and the Zeppelin, a vibration of e.g. the dipstick might be picked up. Also, notoriously, 50 Hz is visible. Importantly, none of these non-Zeppelin resonances will move to a different frequency when we change the levitation current. Of course, making matters more complicated, the non-Zeppelin motions probably mix with the Zeppelin motion, leading to peaks at the sum and difference frequencies. As the Zeppelin resonances shift, so do the sum and difference frequencies. We go into more detail regarding this in section 4.6.

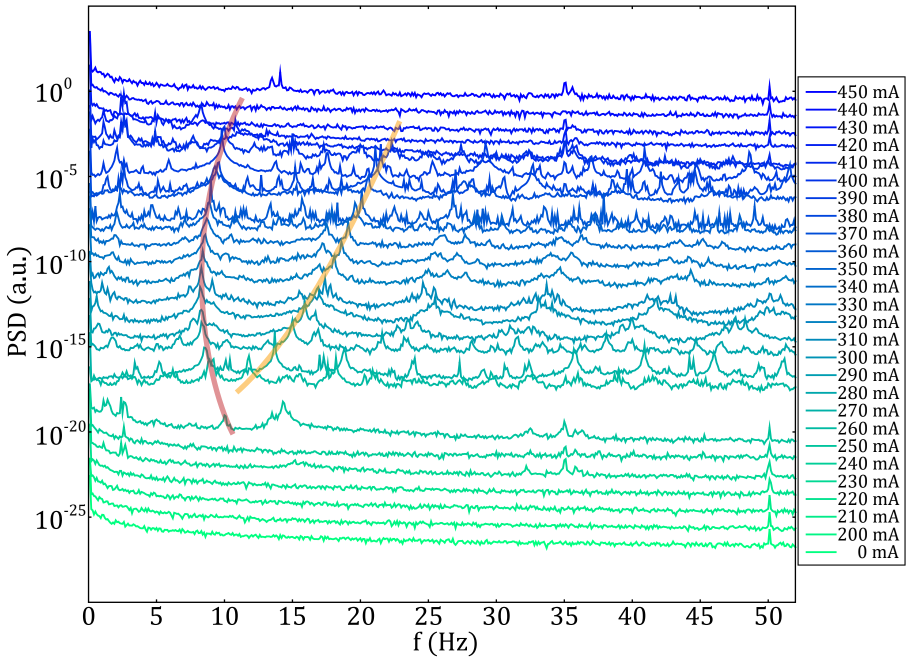

Figure 4.2: The PSDs ($\Delta f = 0.1$ Hz) of the Zeppelin signal for different levitation currents. In this measurement, the Zeppelin is roughly spherical of 650 µm diameter and mass $m = 1.6$ mg. The PSDs have been pulled apart to make the movement of resonances better visible. The coloured lines are meant as a guide to the eye, indicating the shift of resonances as the levitation current is changed.

4.3 Quality Factors

When we give the Lead Zeppelin a kick, its resonances will gain higher amplitudes: we observe an increase of peak heights in the PSD of the signal. The decay in time back to their rest-values is governed by the quality factor of the resonance.

We administer kicks to the levitating Zeppelin in two ways. Inevitably, whenever we change the levitation current, the levitating force on the Zeppelin suddenly changes, which changes the equilibrium position, which thus constitutes a kick. More crudely, we can actually give a (small) kick to the hanging Helium Dewar in which the experiment is put, which has the effect of shaking the levitation coils around the Zeppelin. Such a kick is delivered by gently tapping the dipstick with e.g. a screwdriver handle or a rubber hammer.

We should remark here that we cannot measure the Zeppelin motion whilst the levitation coils are being charged; if they are not in the persistent mode, the input of noise is too big for the SQUID to handle. It is for this reason that we revert to physically kicking the Dewar to get the Zeppelin in motion, such that we need not tamper with the levitation currents every time.

When the Zeppelin hovers in the non-linear part of the levitation force curve (figure 2.8), or when we kick it hard enough to drive it into the non-linear regime, we can expect that the peaks in the PSD will be coming from the 6 modes of the Lead Zeppelin (3 translational and 3 rotational) and mixing terms between these modes. Also, some will come from the experimental surroundings' resonances, and mixing terms can appear between those and the Lead Zeppelin. We study this in more detail in section 4.6.

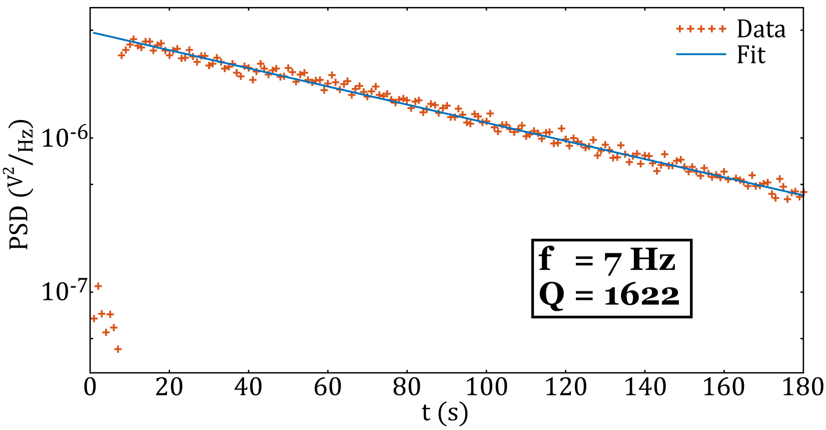

Figure 4.3: The peak height of the PSD component at 7 Hz in time when we apply a kick to the Dewar, with the 2 mg Zeppelin levitating and $I_{lev} = 350$ mA. We fit an exponential decay function $Ae^{-t/\tau}$, and calculate the quality factor $Q = \pi f_{res}\tau = 1622$. It takes a few minutes for the Zeppelin to relax to its equilibrium again.

2 mg Zeppelin			
f (Hz)	τ (s)	Q	γ ($10^{-8}\frac{\text{N}}{\text{m/s}}$)
$I_{lev} = 300$ mA			
8	89	2217	4.5
17	43	2295	9.2
23	45	3198	9.0
26	68	5481	5.9
31	41	3924	9.8
40	67	8308	6.0
48	38	5619	10.6
$I_{lev} = 350$ mA			
7	74	1622	5.4
8	83	2068	4.8
18	81	4560	5.0
19	78	4665	5.1
33	53	5464	7.6
$I_{lev} = 400$ mA			
1.9	170	1013	2.4
3.7	66	765	6.1
5.6	54	952	7.4
8.7	190	5191	2.1
16.1	69	3504	5.8
17.0	61	3246	6.6
18.0	145	8177	2.8
18.9	80	4753	5.0
19.8	54	3368	7.4
20.2	78	4946	5.1
20.7	173	11270	2.3
22.6	96	6788	4.2
24.8	47	3700	8.4
25.2	44	3463	9.1
27.0	78	6611	5.1
28.9	150	13628	2.7
29.4	90	8322	4.4
30.8	80	7640	5.1
37.6	79	9334	5.1

0.8 mg Zeppelin			
f (Hz)	τ (s)	Q	γ ($10^{-8}\frac{\text{N}}{\text{m/s}}$)
$I_{lev} = 340$ mA			
8.0	112	2806	1.4
8.44	109	2901	1.5
8.6	115	3112	1.4
9.0	105	2972	1.5
11.9	36	1338	4.5
16.88	58	3089	2.7
17.06	51	2710	3.2
18.2	113	6479	1.4
19.0	113	6728	1.4
20.4	52	3349	3.1
25.4	37	2958	4.3
26.7	47	3964	3.4
29.0	51	4629	3.1
35.1	37	4120	4.3
36.4	61	6917	2.6
$I_{lev} = 350$ mA			
17.0	65	3494	2.5
18.0	56	3180	2.9
18.7	81	4731	2.0
19.6	41	2536	3.9

Table 4.3: The exponential decay times τ, quality factors Q and damping coefficients γ for several resonances of the Lead Zeppelin, for two different Zeppelins (left the aspherical 2 mg Zeppelin; right the almost spherical 0.8 mg Zeppelin) and some levitation currents. The τs are obtained by giving the Helium Dewar a small kick, and fitting an exponential decay function to the ringdown of the emergent peaks in the PSD (see figure 4.3). Subsequently, we calculate $Q = \pi f_{res}\tau$ and $\gamma = \frac{2m}{\tau}$. These results are gathered from several data sets with differing frequency resolutions.

We fabricated the aspherical Lead Zeppelin that can be seen in the bottom left corner of figure 2.2. Based on its dimensions, we estimate it to have a mass $m = 2$ mg. We gave it a small kick, at $I_{lev} = 350$ mA, and recorded the PSD of the SQUID signal in time. Figure 4.3 shows the height of the peak at $f_{res} = 7$ Hz from this Zeppelin at this levitation current decreasing in time. We have fitted an exponential to it, which yields the ring-down time τ and consequently the quality factor $Q = \pi f_{res}\tau$. In this case, we find a Q of about 1600. Table 4.3 shows more quality factors obtained in this way.

The damping coefficient $\gamma = \frac{2m}{\tau}$ is a frequency independent comparator between different Zeppelins and levitation currents. It also sets the thermal force noise on the Zeppelin motion through $S_{F,Th} = 4k_BT\gamma$, which ultimately is the limiting force noise in a gravitational experiment (section 2.7.4), although this is not yet our concern as long as the external vibrations are limiting us. We added the damping coefficients to table 4.3. We see that for the resonances of the 2 mg Zeppelin, the damping is mostly around $\sim 5 - 10 \times 10^{-8}$ $\frac{\mathrm{N}}{\mathrm{m/s}}$, except for 5 of them at $I_{lev} = 400$ mA who have $\gamma = 2.1 - 2.8 \times 10^{-8}$ $\frac{\mathrm{N}}{\mathrm{m/s}}$. As we will detail in section 4.6, the resonances with higher damping can be considered non-linear mixing terms of these modes, and will decay more quickly.

We repeated the 'kick-and-ringdown' measurements with a slightly smaller Zeppelin, of almost spherical dimensions 520 × 500 µm. We estimate it to have a mass $m = 0.8$ mg (table 4.1). We added the obtained quality factors and damping coefficients to table 4.3.

If the dominating cause of damping comes from eddy currents in the surrounding non-superconducting metal, one would expect to see an increase in damping for increasing levitation current. However, for both particles this is not what we observe. The damping is lower for the smaller particle, which is an indication that at present, it is the surrounding Helium gas that provides the leading cause of damping. Indeed, when we use equation (2.57) in reverse, we can find out what the pressure must have been at these γs, assuming it is the ambient Helium that's causing the damping. For the 2 mg particle, we fill in $R_{zep} = 380$ µm (based on figure 2.2) and $\gamma = 3 \times 10^{-8}$ $\frac{\mathrm{N}}{\mathrm{m/s}}$ to find that $P = 2.7 \times 10^{-2}$ mbar. The smaller 0.8 mg particle, with $R_{zep} = 250$ µm and $\gamma = 2 \times 10^{-8}$ $\frac{\mathrm{N}}{\mathrm{m/s}}$, has $P = 4.1 \times 10^{-2}$ mbar. These values lie nicely in the pressure range that we did the experiments in, which was kept between $10^{-2} - 10^{-1}$ mbar for thermalization of the experiment.

4.4 External Vibrations

In our experimental setup, we find that vibrations from external sources couple to the Lead Zeppelin noticeably. Having suspended the Helium Dewar from the ceiling as discussed in section 2.8 helps a great deal, in that for instance the opening and closing of doors in the laboratory doesn't affect the experiment,

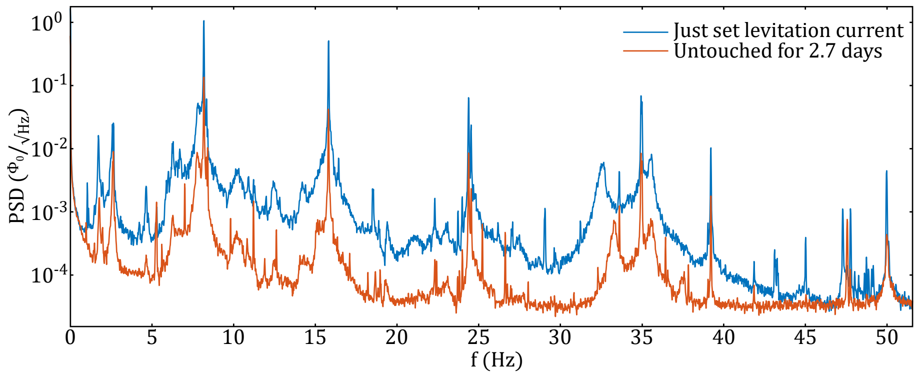

Figure 4.4: In blue the PSD of the SQUID signal after just having set the levitation current to $I_{lev} = 350$ mA. The Zeppelin signal is then very big, also off resonance. In orange, the PSD of the SQUID signal after having left the experiment untouched for 2.7 days. The signal is diminished, but it has by no means sunk into the SQUID noise. The PSDs are the average of 20 consecutive 50 s time traces ($\Delta f = 0.02$ Hz). The Zeppelin used here is the 2 mg aspherical particle.

nor does the act of sitting down in a chair; but the motion of the laboratory ceiling still couples measurably to the Dewar through the bungee cords and, subsequently, measurably to the Lead Zeppelin.

Figure 4.4 shows the difference in the PSDs of the Zeppelin signal between that when it has just gotten a jolt due to a change in levitation current, and that after having left it untouched for a long time. The signal shortly after the jolt shows higher peaks on Zeppelin resonances, but also the off-resonant signal is increased. This could be due to the non-linear nature of the levitating force, see figure 2.10. We also see that after waiting a long time, the signal has decayed somewhat, but has not nearly died out yet. In fact, we know from e.g. figure 4.3 that, depending on Q, already after a few minutes will the peaks have settled down to their rest values.

When the Zeppelin were driven solely thermally, its resonances would reach peak heights typically of around $S_{x,Th}^{1/2} \sim 1.18 \times 10^{-9}\ \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$ at 10 Hz (eq. (2.93)), or $S_{\Phi,Th}^{1/2} = 3.5 \times 10^{-4}\ \frac{\Phi_0}{\sqrt{\mathrm{Hz}}}$. A glance at figure 4.4 reveals that currently the Zeppelin moves quite a bit more than that — about 3 orders of magnitude.

Having taken special care to shield electromagnetic noise from coupling into the levitation coils, we do not believe that this could be driving the Lead Zeppelin to these heights (section 2.9). Instead, it is much more likely that there are external vibrations, shaking about the levitation coils around the levitated Zeppelin, that cause this excitation.

That being said, there are a number of potential candidates. Evidently, as discussed, the Helium Dewar suspended from the laboratory ceiling has a certain motion, which will couple into the experiment. Likewise, the dipstick on which the experiment is mounted, will be moving; it moves according to the movement of the Dewar, but also the boil-off of Helium will cause it to fluctuate.

From figure 4.4, we can estimate how much Zeppelin movement there remains after leaving it untouched for a long time. Equation (2.81) relates the RMS movement $x_{\rm RMS}$ to the PSD peak at a frequency f_p

$$x_{\rm RMS} = \sqrt{\frac{2}{T_m\ {\rm NPBW}} \sum_{n\sim f_p} S_n}, \tag{4.3}$$

with in this case $T_m = 50$ s and NPBW = 1.5 (we used a Hann window). We let the sum extend over 5 frequency bins, of which the middle one coincides with the highest measured PSD value. Using $\frac{d\Phi_{SQ}}{dz} = 3.0 \times 10^5\ \frac{\Phi_0}{\rm m}$, and pretending that all peaks are z-resonances, we can convert the measured S_Φ to S_x. We calculate the Zeppelin movement for a bunch of prominent peaks. Also, we calculate the effective temperature of the motion with equation (2.88), namely

$$T_{\rm eff} = \frac{m\omega_0^2}{k_B}\ x_{\rm RMS}^2, \tag{4.4}$$

where $m = 2$ mg. The results are listed in table 4.4. At all peaks the motion of the Zeppelin is very large and its effective temperature is very high, compared to a thermal excitation, where for $T = 4.2$ K at 10 Hz we have $x_{\rm RMS} = 86$ pm.

Note that we have been assuming that all the motions are in the z-direction: we did not calculate $\frac{d\Phi}{dr}$ in section 2.4.1, although we do know that it has to be smaller than $\frac{d\Phi}{dz}$ on account of the levitating magnetic field being smaller when moving in the r-direction as opposed to in the z-direction (see figure 2.6). If a motion is not in the z-direction, then, its calculated value of S_x is larger, and, subsequently, so are the values for $x_{\rm RMS}$ and $T_{\rm eff}$.

f (Hz)	$x_{\rm RMS}$ (nm)	$T_{\rm eff}$ (10^6 K)
2.64	22	0.020
8.18	339	44
15.8	107	16
24.38	23	1.9
24.54	6.0	0.12
34.94	21	3.0
35.0	16	1.7
39.22	4.2	0.16
47.56	2.3	0.068

Table 4.4: The movement of the 2 mg Zeppelin, taken from the 'Untouched for 2.7 days' data of figure 4.4. We assume all motions to be along the z-axis to get $x_{\rm RMS}$. These numbers are much higher than a thermal excitation $x_{\rm RMS} = 86$ pm at 10 Hz and at 4.2 K, which is also reflected by the effective temperatures of the motions.

4.5 Duffing Behaviour

Figure 4.5 shows two time traces of the Zeppelin signal. The top one shows the 'long-term' behaviour of the spherical 1.6 mg Zeppelin at $I_{lev} = 400$ mA, a good while after setting the levitation current. Notice the 'square' shape of the signal, which shows the non-linearity of the Lead Zeppelin. The signal goes on like this forever. It does not decay, nor does it change appreciably otherwise, for days on end: this motion is driven, and most likely by the Dewar.

The bottom time trace is measured right after having set the levitation current to $I_{lev} = 300$ mA with the aspherical 2 mg Zeppelin. We attenuated the signal 10× here, to keep our measurement devices happy. We see that the movement of the Zeppelin is a lot bigger here, and there is more chaos. More chaos, as well as mixing between modes, is expected because we are at a lower levitation current, with a less symmetrical Zeppelin and at a larger excitation. When left to decay, this signal becomes similar to the top figure.

These types of time signals are what one would also observe when dealing with a (driven) Duffing oscillator (section 2.4.4). The top time trace resembles a weakly driven Duffing oscillator, and the bottom time trace a strongly driven Duffing oscillator.

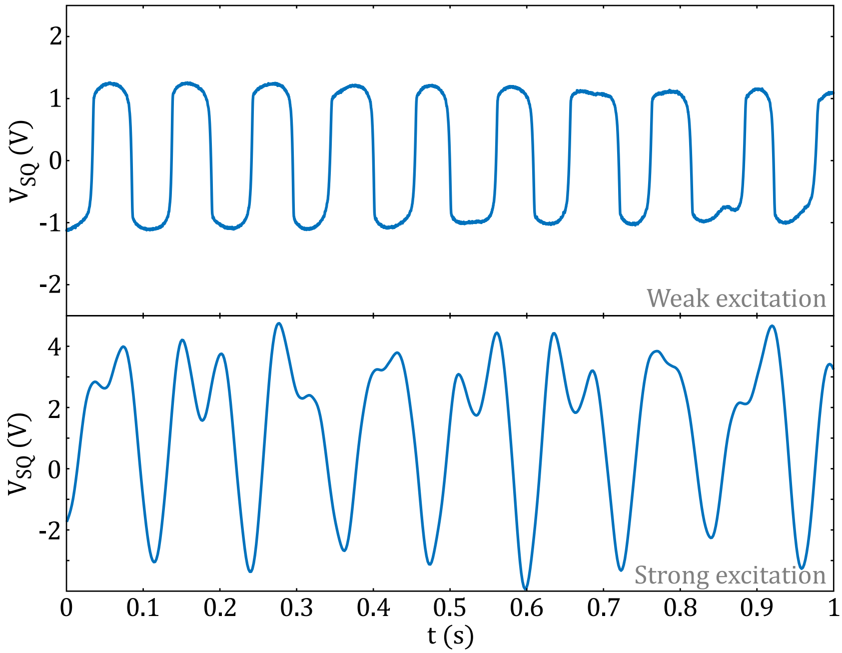

Figure 4.5: Two time traces of the measured SQUID signal. Top: the spherical 1.6 mg Zeppelin, at $I_{lev} = 400$ mA, some time after having set the levitation current. Bottom: the aspherical 2 mg Zeppelin, at $I_{lev} = 300$ mA, shortly after having set the current, attenuated 10×. This is very similar to what one would expect for the non-linear Duffing oscillator at weak (top) and strong (bottom) excitations, see section 2.4.4.

4.6 Naming Resonances

In this section we want to see if we can assign peaks in the PSD to resonances of the Lead Zeppelin. We will be looking for 6 resonances, 3 translational and 3 rotational. Also, there will be mixing between them, because of the non-linearity of the levitating force and because the Zeppelin movement is quite big — especially when we kick it.

When the Zeppelin is a perfect sphere, there are only translational resonances: a spherical particle has no restoring force in its rotational degrees of freedom due to the symmetry of the setup, and when it spins, this does not lead to flux changes in the pickup coil. With an aspherical Zeppelin, there is a restoring force on rotations. The rotations about the x and y axes are readily measurable, and the rotation about the z axis becomes measurable when $r \neq 0$ (a 2$^{\text{nd}}$ order effect). Of

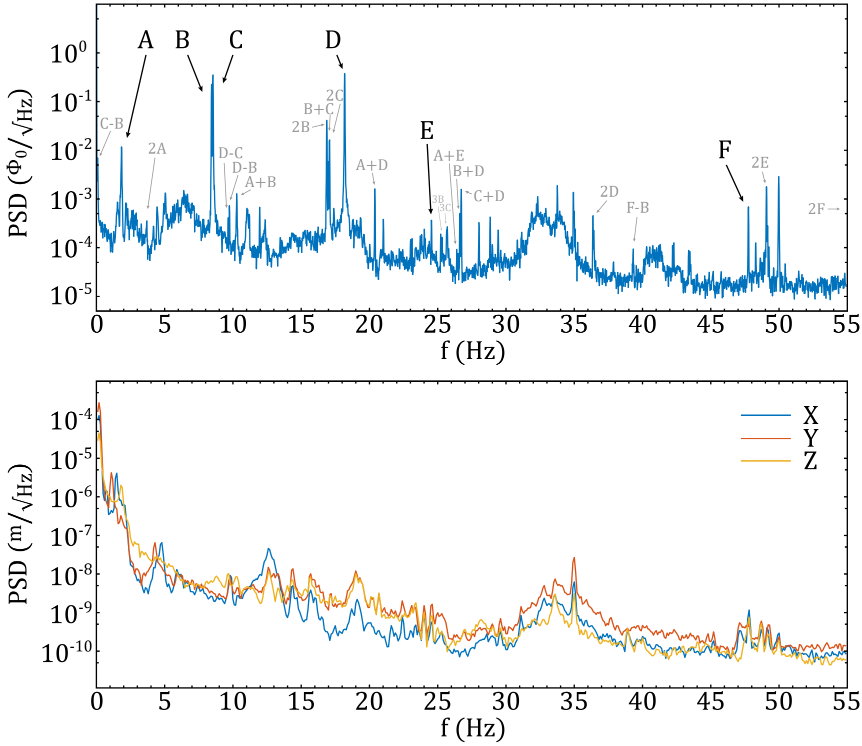

Figure 4.6: Top: the Zeppelin signal of the 0.8 mg spherical Zeppelin at $I_{lev} = 340$ mA after a kick, $\Delta f = 0.02$ Hz. The labelling A−F attempts to understand which peaks belong to the Zeppelin; see table 4.5. Bottom: the geophone measurement of the Dewar motion plotted in the same frequency range for comparison. In both measurements, there is a prominent peak at 35 Hz. Clearly, this Dewar resonance seeps through to the Zeppelin. The bulge around this peak in the geophone measurement, also shows up in the Zeppelin measurement.

#	A	B	C	D	E	F
f (Hz)	1.84	8.44	8.54	18.18	24.54	47.76

±	A	B	C	D	E	F
A	3.7	10.28		20.02	26.4	
B		16.88	16.98	26.62		56.2
C		0.1	17.08	26.72		
D		9.74	9.66	36.38		
E					49.08	
F		39.32				

Table 4.5: Assigning the peaks of figure 4.6 to resonances and their mixing. On the diagonal and above, we fill in the location of a peak that corresponds to the sum of column and row (e.g. 2A, A+B, etc...), below the diagonal the difference (B−A, etc...). A blank entry means there is no corresponding peak in the PSD. All numbers are in Hertz. It is likely that B, C and D are the x, y and z translational resonances, respectively.

course, due to imperfections of the setup and Zeppelin, there will in reality always be a little stiffness to all rotations; e.g. when the levitation coils are not wound perfectly circularly. We expect that for the Zeppelins that we use the restoring force to rotations is quite weak, resulting in low rotational spring constants and therefore low resonance frequencies. (If the rotational spring constants are *really* weak, the Zeppelin could even 'skid through' its (local) potential minimum and start spinning unhindered; in that case it is hard to assign a frequency to the motion.)

By using elliptical levitation coils, rather than circular ones, in combination with a Zeppelin which is deliberately aspherical (like the 2 mg particle), we ought to be able to bring the rotational resonances to higher frequencies. See also chapter 5.

Furthermore, even though we calculate that the translational resonance frequencies in x and y are equal when using circular coils, regardless of the shape of the Zeppelin, we should keep in mind that this is the case only to 1^{st} order in the Zeppelin dimensions and its movement. With our non-spherical Zeppelins, we can expect to see a decoupling of the x- and y-resonances. Moreover, when the levitation coils are not perfectly circular, there will be a decoupling of x and y even if the Zeppelin were a perfect sphere.

As mentioned earlier, we cannot ignore the motion of the experimental surroundings. The motion of the Dewar and dipstick will seep through to the Zeppelin, and their resonances (should) mix with the Zeppelin's. Figure 4.6 shows the Zeppelin signal for the almost spherical 0.8 mg Zeppelin shortly after a kick (by setting the levitation currents) together with a geophone measurement of the motion of the Dewar for comparison.

If the Zeppelin is a Duffing oscillator, we can expect a whole lot more than just sum and difference frequencies (figure 2.10). Even without mixing, there could be peaks popping up everywhere (e.g. at sub-harmonics), depending on the amount of non-linearity. Here, we will constrain ourselves to look only for the 'easy' mixing terms, in order to be able to say something about the Zeppelin's resonances.

In table 4.5 we have assigned the labels A through F to 6 peaks from figure 4.6. The criteria to this labelling are the narrowness of the peaks (the Zeppelin has a high Q, the Dewar most likely does not), their height, and the presence of peaks at the location of mixing terms ($f_{\rm A} \pm f_{\rm B}$, etc...).

We find that many peaks have a sister at double their frequency, a manifestation of the non-linear nature of the Lead Zeppelin. We include these double frequencies in the table.

A could be a rotational resonance, being the peak lowest in frequency. B, C and D are likely to be the x, y and z translational resonances, respectively, because B and C are very close to one another, and because all simple mixing terms between them show up. Though not listed in table 4.5, B and C also have a triple frequency. E and F are added to the list because they explain some mixing terms. If B, C and D are translational resonances, then A, E and F are either rotational modes of the Zeppelin, or resonances from the experimental setup. A, low in frequency, could well be a rotation. For E and F, quite high in frequency, we find this harder to believe: they may be the rotations around the x and y axes, but they could also simply be Dewar or dipstick resonances. All three peaks A, E and F have quite a low height, which makes sense if it are rotations: a rotation would cause less flux change in the pickup coil than a translation, especially in the case of the near-spherical 520×500 µm Zeppelin.

Using the same analysis, we can turn to the 2 mg Lead Zeppelin for which the location of peaks and their quality was presented in table 4.3. If at $I_{lev} = 400$ mA we single out the 5 peaks with the longest ringdown times, we find that many of the other peaks can be considered mixing terms, because they are located at sum and difference frequencies. Not only are the frequencies related, the ringdown times must also be related. Namely: a resonance at $f_{\rm AB}$ which is due to the mixing of two resonances at $f_{\rm A}$ and $f_{\rm B}$ will have an amplitude $A_{\rm AB}$ which is proportional to the amplitudes $A_{\rm A}$ and $A_{\rm B}$ of its parents:

$$A_{\rm AB} \propto A_{\rm A} A_{\rm B}. \tag{4.5}$$

This implies that when both parent resonances decay in time exponentially, i.e. $A_{\rm A,B} \propto e^{-t/\tau_{\rm A,B}}$, the daughter resonance amplitude does so too with a shorter time constant

$$A_{\rm AB} \propto e^{-t/\tau_{\rm AB}} \propto e^{-t/\tau_{\rm A}} \, e^{-t/\tau_{\rm B}}, \tag{4.6}$$

leading to

$$\frac{1}{\tau_{\rm AB}} = \frac{1}{\tau_{\rm A}} + \frac{1}{\tau_{\rm B}}. \tag{4.7}$$

f (Hz)	Designation	Measured τ (s)
1.9	A	170
8.7	B	190
18.0	C	145
20.7	D	173
28.9	E	150

f (Hz)	Designation	Measured τ (s)	Calculated τ (s)
16.1	C−A	69	78
20.2	E−B	78	84
22.6	D−A	96	86
27.0	E−A	78	80
29.4	B+D	90	91
30.8	A+E	80	80
37.6	B+E	79	84

Table 4.6: We assign the labels A−E to the 5 PSD peaks with the longest ringdown times τ of the 2 mg Zeppelin at I_{lev} = 400 mA from table 4.3. By declaring A−E to be fundamental Zeppelin modes, many of the other peaks can be understood as mixing terms on account of both their frequency, as well as their shorter ringdown times.

In table 4.6 we compare the measured ringdown times of mixing terms to what we calculate with the above formula. We see that the measured and calculated τs indeed agree with each other.