



Universiteit
Leiden
The Netherlands

The lead zeppelin : a force sensor without a handle

Waarde, B. van

Citation

Waarde, B. van. (2016, November 2). *The lead zeppelin : a force sensor without a handle*. *Casimir PhD Series*. Retrieved from <https://hdl.handle.net/1887/43816>

Version: Not Applicable (or Unknown)

License: [Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden](#)

Downloaded from: <https://hdl.handle.net/1887/43816>

Note: To cite this publication please use the final published version (if applicable).

Cover Page



Universiteit Leiden



The handle <http://hdl.handle.net/1887/43816> holds various files of this Leiden University dissertation

Author: Waarde, Bob van

Title: The lead zeppelin : a force sensor without a handle

Issue Date: 2016-11-02

The Lead Zeppelin

a Force Sensor without a Handle

PROEFSCHRIFT

TER VERKRIJGING VAN
DE GRAAD VAN DOCTOR AAN DE UNIVERSITEIT LEIDEN,
OP GEZAG VAN RECTOR MAGNIFICUS
PROF. MR. C.J.J.M. STOLKER,
VOLGENS BESLUIT VAN HET COLLEGE VOOR PROMOTIES
TE VERDEDIGEN OP WOENSDAG 2 NOVEMBER 2016
KLOKKE 10.00 UUR

DOOR

Bob van Waarde

GEBOREN TE HAARLEM, NEDERLAND
IN 1986

Promotor: Prof. dr. ir. T. H. Oosterkamp

Promotiecommissie: Prof. dr. M. Aspelmeyer (VCQ, Wenen, Oostenrijk)
Dr. O. W. B. Benningshof (QuTech, Delft)
Dr. M. P. Allan
Prof. dr. D. Bouwmeester
Prof. dr. E. R. Eliel
Prof. dr. J. M. van Ruitenbeek

Casimir PhD Series, Delft-Leiden 2016-28

ISBN 978-90-8593-272-7

An electronic version of this thesis can be found at openaccess.leidenuniv.nl

The work described in this thesis was performed at the Kamerlingh Onnes Laboratory, Universiteit Leiden, Niels Bohrweg 2, 2333 CA, Leiden.

This work is part of the research programme of the Foundation for Fundamental Research on Matter (FOM), which is part of the Netherlands Organisation for Scientific Research (NWO).

Dit werk maakt deel uit van het onderzoeksprogramma van de Stichting voor Fundamenteel Onderzoek der Materie (FOM), die deel uit maakt van de Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO).

Cover art by ILLUSTRARIE — www.illustrarie.weebly.com

Adapted from © LED ZEPPELIN

Every effort has been made to trace the copyright owners of the original artwork, and to obtain their permission for reproduction and adaptation. If you believe you are a copyright owner, and we have not requested your permission, we will be pleased to make the necessary arrangements.



Universiteit
Leiden



Casimir
research school

NWO

Contents

1	Introduction	1
2	Designing the Lead Zeppelin	5
2.1	Basic Idea	5
2.2	Experimental Setup	6
2.2.1	Ingredients	6
2.2.2	Fabrication of Zeppelins	7
2.2.3	Fabrication of Coils	7
2.2.4	SQUID Read-out	10
2.3	The Principle of Levitation	11
2.4	The Finite Sized Lead Zeppelin	13
2.4.1	Coupling the Particle Motion to the SQUID	13
2.4.2	Force Balance, Equation of Motion	17
2.4.3	Equilibrium Position and the Limits of Levitation	20
2.4.4	Duffing Oscillator	21
2.5	Lockin Measurement	23
2.6	Damping Mechanisms	27
2.6.1	Background Gas	27
2.6.2	Eddy Currents	27
2.7	The Sensitivity of the Lead Zeppelin	29
2.7.1	Basics of Fourier Transform Measurements	29
2.7.2	Detection Noise	34
2.7.3	Thermal Motion	34
2.7.4	Gravitational Experiments	35
2.8	External Vibrations	36
2.9	Measures Against Electromagnetic Noise	40
3	A Magnetic Persistent Current Switch at milliKelvin Temperatures	45
3.1	Introduction	45
3.2	Design and Fabrication	46
3.3	Results and Discussion	48
3.4	Summary	51

3.5	Acknowledgements	51
4	Flying the Lead Zeppelin	53
4.1	Liftoff	54
4.2	Levitation Current versus Zeppelin Frequency	56
4.3	Quality Factors	58
4.4	External Vibrations	60
4.5	Duffing Behaviour	63
4.6	Naming Resonances	64
5	Discussion and Outlook	69
	Appendix	77
A	Force Exerted by Excitation Coil	77
B	Flux Reduction due to PCS	79
B.1	Effect on Impedance	79
B.2	Effect on Efficiency of Excitation Coil	80
B.3	Effect on Detection by Pickup Coil	82
	References	89
	Samenvatting	91
	List of Publications	95
	Curriculum Vitae	97
	Acknowledgements	99

Chapter 1

Introduction

The PhD dissertation that you are holding, describes the work that has been done over the last 4 years on a project called the Lead Zeppelin. Lead, a superconducting metal, is the material of choice in the measurements that have been performed. These involve the Zeppelin-part of the name: making a small piece of Lead float by means of magnetic levitation.

The inspiration for doing the Lead Zeppelin experiment came, in our case, from the Magnetic Resonance Force Microscopy (MRFM) community. In MRFM, a micrometer-sized ferromagnet attached to the end of a very soft cantilever is used to measure the tiny magnetic force between it and (nuclear) spins in a sample of interest. Much like MRI in hospitals, but on a much smaller scale: a striking result has been the measurement of the magnetic interaction between the ferromagnet and as few as ~ 500 nuclear spins[1]. One of the main goals of these experiments is to image complex molecules (like proteins or viruses) in their natural environment, in 3D and with atomic resolution.

The key to doing such sensitive experiments, is to obtain as low a force noise on the force sensor as possible. A low force noise requires, among other things, a low damping. State-of-the-art MRFM has shown a damping coefficient of $\gamma \sim 10^{-15} \frac{\text{N}}{\text{m/s}}$, and a force noise of $2 \times 10^{-18} \frac{\text{N}}{\sqrt{\text{Hz}}}$ [2].

When thermally limited, the force noise is $\sqrt{4k_B T \gamma}$ as stated by the fluctuation-dissipation theorem[3]. To lower the force noise, then, one has two options. One can decrease the temperature: indeed, a force noise of $0.8 \times 10^{-18} \frac{\text{N}}{\sqrt{\text{Hz}}}$ has been documented by using a force sensor with a higher damping of $\gamma \sim 10^{-13} \frac{\text{N}}{\text{m/s}}$ at a low temperature of 28 mK[4]. Or, one can push γ way down: for instance in [5] an optically levitated glass nanosphere with a damping of $\gamma \sim 10^{-20} \frac{\text{N}}{\text{m/s}}$ has been reported with a force noise $S_F^{1/2} = 2 \times 10^{-20} \frac{\text{N}}{\sqrt{\text{Hz}}}$ at room temperature.

In principal, the main source of damping in MRFM experiments comes from clamping losses. Therefore, to get to an even lower γ it makes sense to get rid of the cantilever; without clamping losses the damping could plummet to unprecedented levels, and the force noise along with it. The Lead Zeppelin aims

to do just that: instead of a cantilever with a small ferromagnet attached to its end, we instead magnetically levitate a small superconductor.

Another important motivation to do the Lead Zeppelin experiment is the investigation of the quantum mechanical behaviour of heavy objects. Namely, it is evident that quantum mechanics breaks down for big objects like chairs and cats[6] and cups of coffee, while it is known to be an excellent theory to describe small objects like quarks and electrons. The superposition principle in quantum mechanics, or, as it is phrased in popular media, ‘being in two places at the same time’, is something that electrons do constantly, yet we never observe this for the large objects we encounter in everyday life. Somewhere along the way from small to big, *something* goes awry[7–9].

While at present it is not known for sure what exactly is responsible, a very likely candidate is gravity. Indeed, the time-coordinate, which is needed to describe the time-evolution of quantum states through the Schrödinger equation, becomes ill-defined when we put a heavy object in a superposition of positions: the curvature of spacetime by the presence of mass, which is what gravity is[10], is different depending on the position of the heavy object. So if it is in some fancy quantum superposition of positions, the notion of time-evolution breaks down[11–13]. It is therefore argued that the amount of mass of an object plays a pivotal role in the destruction of unitarity in quantum mechanics. If gravity has anything to do with it, there will be a mass-dependent time-scale in which the collapse of the wavefunction occurs[14]. Also, if gravity is the culprit, then a heavy quantum state will decohere even without the act of a measurement. Simultaneously it explains the measurement problem of ‘ordinary’ quantum states, in realizing that a measurement apparatus involves adding a (large) mass to the situation.

A theory of Quantum Gravity is yet to be written down, so we turn to experiments to guide the way. In a variety of realizations, the goal to do quantum mechanics with heavy objects is pursued[5, 15–23].

In the case of the Lead Zeppelin, we want it first and foremost to be a very sensitive force sensor. If we are sensitive enough to detect tiny gravitational forces, which is in itself a very cool thing to be able to do, we can start thinking about doing quantum measurements, which requires freezing out thermal phonons. As we will see in the rest of this dissertation, the sensitivity to tiny forces comes at the prize that unwanted forces must be kept from intruding into the experiment. In particular vibrations must be kept to a very low level, as they will set the Lead Zeppelin in motion very easily.

The outline of this thesis is as follows. In chapter 2 we will give an overview of the design considerations of the Lead Zeppelin experiment, that are necessary to do a successful measurement. One such design feature that proved important is the Persistent Current Switch (PCS) with which we decouple external noise sources from reaching the experiment to a great extent. We describe our PCS in chapter 3. When all considerations of chapter 2 were implemented, several experiments

were performed, which we present in chapter 4. Finally, in chapter 5 we discuss what can (or should) still be improved, and some of the ideas that we had whilst doing the experiments, that will make our desired future measurements more and more feasible.

Chapter 2

Designing the Lead Zeppelin

2.1 Basic Idea

The Lead Zeppelin experiment consists of a (sub-)millimeter sized superconducting particle that is levitated in a potential well created by a suitable magnetic field profile. It was proposed theoretically by Romero-Isart et al.[20]. Due to the Meissner effect[24], the superconducting particle has a lower energy in low magnetic field than in higher magnetic fields. Therefore, a magnetic field that varies in space can serve as a trapping potential.

Our superconductor of choice is Lead. Because of Lead's relatively high critical temperature, measurements at 4 K are possible. Furthermore, Lead has quite a low melting point, which allows us to use a torch to fabricate (near) spherical particles.

Also, because Lead is a Type I superconductor, there is no pinned flux to worry about. Early experiments, that were performed by placing Lead and Niobium particles between two permanent magnets in a liquid Helium cryostat with optical access, showed that the Lead particles levitated at 4 K, whereas the Niobium particles did not. Niobium, being a Type II superconductor, likely trapped flux and subsequently did not levitate. (With the introduction of an adjustable levitating magnetic field by using coils of current-carrying wire, measurements with a Niobium Zeppelin could be feasible; however, we have not performed such measurements.)

In the following sections of this chapter, we describe the design of the experiment and requirements necessary for a successful measurement. We begin with the heart of the experiment: the various coils and their dimensions, as well as their fabrication and the fabrication of Lead Zeppelins. Then follows some theory: starting with a quick estimation of parameters to give us a feeling of what's going on, in the subsequent section we go into more detail. Next, we treat two notable sources for damping of the motion of the Lead Zeppelin. After addressing some relevant Fourier analysis, we then consider the feasibility of

measurements of the thermal motion of the Lead Zeppelin and the use of the Zeppelin as a probe for small gravitational forces. We discuss the external vibrations coupling into the experiment and end by reviewing the measures we took to fight off electromagnetic noise.

2.2 Experimental Setup

2.2.1 Ingredients

To do the Lead Zeppelin experiment, we need a number of building blocks. First of all, we need levitation coils that provide the confining magnetic potential for the levitating superconductor. Then, we need a way to read out the motion: we do this by measuring flux changes in a pickup coil connected to a SQUID. We also add an excitation coil with which to control the Zeppelin. Since we work with superconductors, we need to work at sufficiently low temperatures. We perform our experiments on a vacuum dipstick inserted in a Dewar of liquid Helium. A schematic overview of the experimental setup is given in figure 2.1.

To make the experiment work it is important to consider the sensitivity of the system to noise. The flux change in the detection coil of the experiment is

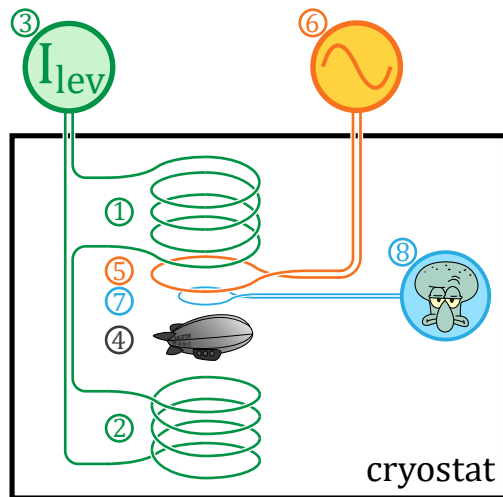


Figure 2.1: The basic Lead Zeppelin experimental setup. We see the **upper (1)** and **lower (2)** levitation coils, wound with opposite polarity, that provide the levitating magnetic field by running a current I_{lev} through them with a **current source (3)**. Placed between the levitation coils is the Lead Zeppelin (4). To control the motion of the Lead Zeppelin, we add an **excitation coil (5)** to the setup, connected to a **function generator (6)**. We read out the motion of the Lead Zeppelin with a **pickup coil (7)** and a **SQUID (8)**. The experiments are performed at low temperatures, typically at 4 K, indicated by the black box.

measured using a SQUID, one of the most sensitive flux sensors available. Because the SQUID is so sensitive it is necessary to shield it from external, unwanted magnetic fields. Very importantly, the current noise in the levitation coils should be very low, as described in section 2.9. We further use the Persistent Current Switch (PCS) described in chapter 3 to reduce the effect of the fluctuations in the current sent by our current source from the experiment.

It is equally important to pay attention to the vibrations in the system. The levitated Lead Zeppelin is a very soft mechanical resonator with a spring constant of $k \approx 10^{-2} \times I_{lev} \frac{\text{N}}{\text{m}}$, I_{lev} in Ampères (section 2.4.2). Its resonance frequencies typically lie between 0.5 Hz and 50 Hz. The measures taken to mitigate external vibrations are described in section 2.8.

In addition to placing a PCS between the levitation coils and the current source, we place line filters between the current source and the leads going to the experiment. We find that without filters, we cannot operate the SQUID while leaving the current source to the levitation coil connected to the experiment, even when the PCS is short circuited in the persistent mode. See also section 2.9. For the same reason, the function generator that we connect to the excitation coil is filtered by using both a line filter and a transformer.

2.2.2 Fabrication of Zeppelins

Early results were obtained with Zeppelins that were simply scraped off of a block of Lead with a knife. These Zeppelins were very chunky; an example can be seen in the bottom right inset in figure 2.2.

To incorporate more symmetry into the experiment, we started fabricating Zeppelins by torching the scraped-off pieces of Lead with a butane gas flame. When the temperature supersedes the melting point of Lead, the particles will form (almost-perfect) spheres, due to the effect of surface tension. For the sake of controlled asymmetry, if desired, the now spherical Zeppelins can be dented slightly. Figure 2.2 shows some of the Zeppelins that were fabricated.

2.2.3 Fabrication of Coils

Figure 2.4 shows the design of the coils that we use. We wind the levitation coils on spindles made from Macor, which is a non-magnetic, easily machinable, ceramic material. The pieces of Macor are held together by a brass clamp, and the whole is inserted into a superconducting Niobium tube.

The coils are made from 100 μm diameter unclad NbTi wire in a Formvar insulation. The levitation coils are each given 275 windings. The excitation coil consists of a single loop. In calculations, we take the center values of the coils as their dimension, as indicated in the figure, and as tabulated in table 2.1.

To read out the motion of the Lead Zeppelin, we use a superconducting pickup coil. Early results were obtained with the chip of figure 2.3 (left). We find (see section 4.1) that this pickup coil distorts the Zeppelin motion because of its

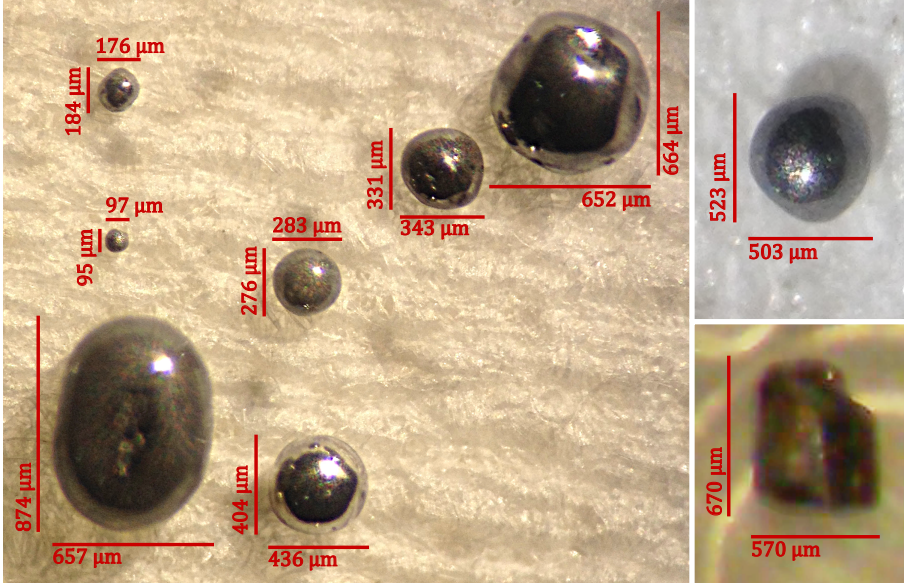


Figure 2.2: Some Zeppelins that were made by torching small chunks of Lead. Some are made slightly aspherical by denting them. The sizes that are indicated next to the Zeppelins are correct to $\sim 10\%$. The two Zeppelins in the top right and bottom left corners of the main panel, as well as the one in the top right inset, have been used in experiments described in this thesis. The bottom right panel shows a more crude Zeppelin that has not been torched, which was used in early experiments.

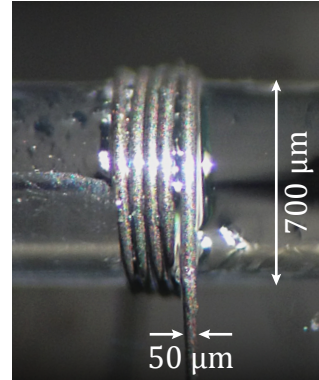
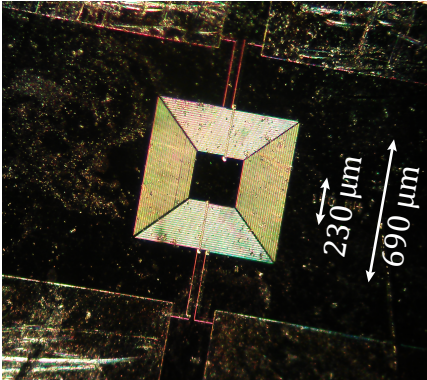


Figure 2.3: Left: an on-chip pickup coil used in early experiments. It is a transformer chip of 2×22 Niobium windings. The two coils of the transformer are wirebonded in series for use as a pickup coil. Right: a self-wound pickup coil during fabrication. It is wound from $50 \mu\text{m}$ diameter Niobium wire with Formvar insulation, around a nylon fishing wire of $700 \mu\text{m}$ diameter. We fixate the coil with some superglue.



Figure 2.4: (a) The design for the coils and the clamp that holds them. The dimensions in millimeters are indicated in the figure. In light blue the pieces of Macor onto which we wind the coils: the two deep, 2 mm wide slots carry the levitation coils; the middle, shallow groove carries the excitation coil. In gold the brass clamp that holds everything together. The dotted lines indicate the chamber in which the Zeppelin is free to vibrate, and where the pickup coil is placed. (b) Zoomed in to the coils. Indicated are the levitation coils, the pickup coil, the excitation coil and their dimensions, the $z = 0$ line and the Lead Zeppelin. (c) A photo of the actual clamp and coils. We wound the levitation coils using 100 μm diameter NbTi wire with a 13 μm Formvar insulating layer and were able to put $N_{lev} = 275$ windings on each Macor coil holder. The $\sim 650 \mu\text{m}$ diameter Zeppelin from figure 2.2 can be seen next to the coils. A green piece of printed circuit board can be seen near the upper levitation coil, behind which the excitation coil is situated. The excitation coil is a single loop made from the same wire as the levitation coils. The piece of PCB is a remnant of an older version of the experiment. Also visible is the Lead capillary that shields the pickup coil wires to the SQUID.

R_{lev}	R_{exc}	R_p
2.5 mm	3.05 mm	375 μm
z_{lev}	z_{exc}	z_p
2.25 mm	200 μm	500 μm
N_{lev}	N_{exc}	N_p
275	1	5

Table 2.1: The dimensions of important components of the experiment, see figure 2.4.

superconducting repulsion of magnetic fields. To make this effect smaller, we wound the pickup coil of figure 2.3 (right).

All cold connections are made by either spot welding or wirebonding. This ensures superconductivity to a very large extent: e.g. as we find out in chapter 3, spot welded connections have a resistance of less than 3.3 p Ω . This means that there is no heating at the cold parts of the experiment, and that there is no loss of signal between the pickup coil and the SQUID.

2.2.4 SQUID Read-out

The motion of the Lead Zeppelin is read-out by a Superconducting QUantum Interference Device (SQUID). A SQUID is a superconducting loop interrupted by two Josephson junctions. The flux in the SQUID is quantized in chunks of the magnetic flux quantum Φ_0 .

$$\Phi_0 = \frac{h}{2e} = 2.067833 \dots \times 10^{-15} \text{ Wb}. \quad (2.1)$$

The addition of the Josephson junctions yields a flux-dependent potential difference $V(\Phi)$ due to the Josephson effect[25].

We use a 2-stage SQUID readout, see figure 2.5. We have a ‘sensor SQUID’, to which we feed the flux from the Zeppelin by connecting the pickup coil to the input coil of the SQUID. Our sensor SQUID is a commercially available Quantum

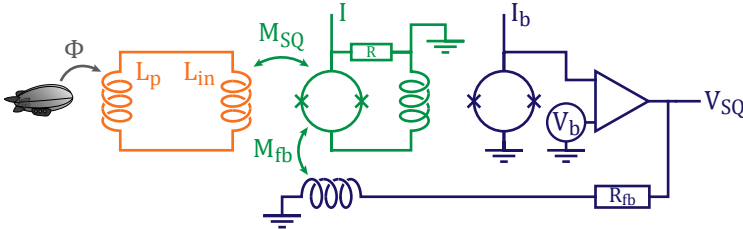


Figure 2.5: The SQUID read-out of the Zeppelin motion. The flux change caused by the Zeppelin in the pickup coil L_p is led to the input coil L_{in} of a sensor SQUID. The potential difference across this SQUID is then fed to an amplifier SQUID. In Flux Locked Loop mode, we keep the flux in the sensor SQUID constant with a feedback coil. We measure V_{SQ} , which is directly linked to the position of the Lead Zeppelin.

Design SQUID that can reach a noise level of $5 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$ at 4.2 K[26]. Further, in order not to be limited by room-temperature pre-amplifier noise, we use an ‘amplifier SQUID’ as a low-noise pre-amplifier of $V(\Phi)$. With a resistor R we feed $V(\Phi)$ to the input coil of the amplifier SQUID.

The potential $V(\Phi)$ is dependent on the bias current I through the SQUID. We tune I such that the slope of the potential $\frac{dV(\Phi)}{d\Phi}$ is largest, which means that we are very sensitive to flux changes. We do feedback on the flux in the SQUID with a feedback coil: by keeping the flux in the SQUID loop constant, we stay at large $\frac{dV(\Phi)}{d\Phi}$ and stay very sensitive. Ultimately, V_{SQ} as indicated in figure 2.5 is the signal that we use in this thesis to characterize the position of the Lead Zeppelin.

2.3 The Principle of Levitation

The Lead Zeppelin levitates because of the Meissner effect. In a magnetic field $\mathbf{B}$, a small spherical superconducting particle will expel all magnetic field and will have a magnetisation $\mathbf{M} = -\mu_0\mathbf{B}$, with $\mu_0 = 4\pi \times 10^{-7} \frac{\text{N}}{\text{A}^2}$ the magnetic permeability of free space. It will have a magnetic moment $\mathbf{m} = V\mathbf{M}$, with V the volume of the particle. The energy of the system is $E = -\mathbf{m} \cdot \mathbf{B} = \frac{V}{\mu_0} |\mathbf{B}|^2$. We assume for now that the magnetic field is constant over the size of the particle. Even though this is not completely accurate in our case, it does quickly sketch a fairly decent picture of what’s going on. In section 2.4, we will dive deeper into the theory.

The magnetic field profile between the two levitation coils, which are put in an anti-Helmholtz configuration, is shown in figure 2.6. It shows a zero in magnetic field in the center of symmetry of the coils. Near this zero the magnetic field is well approximated by its first order Taylor expansion

$$\mathbf{B} = \frac{\partial B_r}{\partial r} r \hat{\mathbf{r}} + \frac{\partial B_z}{\partial z} z \hat{\mathbf{z}} \equiv G_r r \hat{\mathbf{r}} + G_z z \hat{\mathbf{z}}, \quad (2.2)$$

where the magnetic field gradients $G_{r,z}$ will depend on the actual configuration of the levitation coils. Substituting the magnetic field of the anti-Helmholtz configuration into the expression for the energy of the particle results in a harmonic potential given by

$$E = \frac{V}{\mu_0} (G_r^2 r^2 + G_z^2 z^2). \quad (2.3)$$

The force on the particle due to the anti-Helmholtz magnetic field is given by $\mathbf{F}_{\text{aHH}} = -\nabla E = -2 \frac{V}{\mu_0} (G_r^2 r \hat{\mathbf{r}} + G_z^2 z \hat{\mathbf{z}})$. Simultaneously, the gravitational force acting on it is given by $\mathbf{F}_g = -m_{zep} g \hat{\mathbf{z}}$. Since $\mathbf{F}_{\text{aHH}} + \mathbf{F}_g = \mathbf{0}$ in equilibrium, we can solve for the equilibrium height z_{eq} at which the particle levitates:

$$z_{eq} = -\frac{\mu_0 m_{zep} g}{2V G_z^2} = -\frac{\mu_0 \rho_{zep} g}{2G_z^2}. \quad (2.4)$$

When we fill in G_z in units of $\frac{\text{T}}{\text{m}}$, this reduces to

$$z_{eq} = -\frac{70}{G_z^2} \text{ mm}, \quad (2.5)$$

The force $\mathbf{F}_{aHH}$ also immediately yields the spring constants $k_{r,z} = 2 \frac{V}{\mu_0} G_{r,z}^2$. This in turn gives the resonance frequencies

$$f_{r,z} = \frac{1}{2\pi} \sqrt{\frac{k_{r,z}}{m_{zep}}} = \frac{1}{2\pi} \sqrt{\frac{2G_{r,z}^2}{\mu_0 \rho_{zep}}} = 1.9 \times |G_{r,z}| \text{ Hz}, \quad (2.6)$$

with $G_{r,z}$ in $\frac{\text{T}}{\text{m}}$. Note that to this order, $f_{r,z}$ and z_{eq} are not determined by the size of the particle, but only by the magnetic field gradients $G_{r,z}$ and the density of the Lead Zeppelin $\rho_{zep} = \rho_{Pb} = 11.34 \times 10^3 \frac{\text{kg}}{\text{m}^3}$.

One could envision using other superconductors than Lead. For instance Niobium, which is a Type II superconductor, but which has a higher critical temperature than Lead, or Aluminium, which has a lower critical field and temperature, but which is much lighter than Lead. The densities of Niobium and Aluminium are $\rho_{Nb} = 8.57 \times 10^3 \frac{\text{kg}}{\text{m}^3}$ and $\rho_{Al} = 2.70 \times 10^3 \frac{\text{kg}}{\text{m}^3}$. Therefore

$$f_{r,z} = 2.2 \times |G_{r,z}| \text{ Hz}, \quad z_{eq} = -\frac{53}{G_z^2} \text{ mm}, \quad \text{Niobium}, \quad (2.7)$$

and

$$f_{r,z} = 3.9 \times |G_{r,z}| \text{ Hz}, \quad z_{eq} = -\frac{17}{G_z^2} \text{ mm}, \quad \text{Aluminium}, \quad (2.8)$$

where again we fill in $G_{r,z}$ in units of $\frac{\text{T}}{\text{m}}$.

To get a feeling of what to expect experimentally, we can extract some numbers from the simple picture sketched above. We begin with the on-axis magnetic field produced by a loop of wire of radius R carrying a current I , which is given by

$$B = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}. \quad (2.9)$$

Its derivative to z is

$$G_z = -\frac{3\mu_0 I}{2} \frac{R^2 z}{(R^2 + z^2)^{5/2}}. \quad (2.10)$$

For two anti-Helmholtz coils of radius $R_{lev} = 2.5 \text{ mm}$, positions $z_{lev} = 2.25 \text{ mm}$, current $I_{lev} = 1.0 \text{ A}$ and number of windings $N = 275$ the combined gradient evaluated at $z = 0$ is

$$G_z = -3\mu_0 N I_{lev} \frac{R_{lev}^2 z_{lev}}{(R_{lev}^2 + z_{lev}^2)^{5/2}} = -34 \frac{\text{T}}{\text{m}}. \quad (2.11)$$

It then follows that

$$f_z = 65 \text{ Hz}, \quad z_{eq} = -61 \text{ } \mu\text{m}. \quad (2.12)$$

Likewise $f_z = 75 \text{ Hz}$, $z_{eq} = -46 \text{ } \mu\text{m}$ for Niobium and $f_z = 133 \text{ Hz}$, $z_{eq} = -15 \text{ } \mu\text{m}$ for Aluminium.

The levitating field $\mathbf{B}$ scales linearly with the current I_{lev} sent through the levitation coils. Consequently, the spring constants $k_{r,z} \propto I_{lev}^2$ and the resonance frequencies $f_{r,z} \propto I_{lev}$. This will serve as our ‘smoking gun’: as we change the levitation current I_{lev} , the resonances should change along with it.

For a Zeppelin of size $V = (500 \text{ } \mu\text{m})^3$ the assumption that the magnetic field is constant everywhere in the particle is not valid with coils the size of ours. In the next section, we present a more detailed analysis of the levitating Lead Zeppelin.

2.4 The Finite Sized Lead Zeppelin

In the previous section, we calculated the levitation parameters of the Lead Zeppelin experiment while assuming the Lead Zeppelin to be infinitesimally small. In this section, we find out what happens when we don’t neglect the finite size of the Lead Zeppelin.

We begin by calculating the flux in the pickup coil and SQUID as a consequence of there being a levitating superconductor between the levitation coils, as a function of position and levitation current. We then calculate the spring constants and resonance frequencies in the three spatial directions, taking the size of the Lead Zeppelin into account to 1st order. This calculation also yields the excitation force exerted by the excitation coil on the Lead Zeppelin, which means that we have all the relevant parameters in the equation of motion. After that, we calculate the equilibrium position due to the effect of gravity, and we find out the limits of levitation. The equilibrium position helps us understand that the Lead Zeppelin will behave as a Duffing oscillator at low levitation currents, and that if the Zeppelin is too large the magnetic field may not be smaller than the critical field everywhere inside the Zeppelin.

2.4.1 Coupling the Particle Motion to the SQUID

In order to calculate what the flux in the pickup coil is as a function of the position of the Lead Zeppelin in the levitating magnetic field, we approximate the Zeppelin as being cylindrically shaped. We use the equation derived in [27] that relates the surface current density $\mathbf{j}_S$ on the surface S of a superconductor of arbitrary shape in a magnetic field $\frac{1}{\mu_0} \mathbf{B}_{lev} = \mathbf{H}_{lev}$:

$$\mathbf{j}_S = 2 \hat{\mathbf{n}} \times \mathbf{H}_{lev}, \quad (2.13)$$

with $\hat{\mathbf{n}}$ the unit vector normal to S . We further use that the flux Φ in the pickup coil is $\Phi = MI$ and that the mutual inductance between two coaxial, circular loops of current with radii $R_{1,2}$ a distance l apart is

$$M(R_1, R_2, l) = \frac{\mu_0}{2} \int_0^{2\pi} \frac{R_1 R_2 \cos \xi}{(R_1^2 + R_2^2 + l^2 - 2R_1 R_2 \cos \xi)^{1/2}} d\xi, \quad (2.14)$$

which follows from the Neumann formula for the mutual inductance. The end result is a function $\Phi(z_{zep})$ that calculates the flux in the pickup coil from the position z_{zep} of the Zeppelin.

We shall only look at movement in the z -direction here, leaving movement in r as an exercise for the future. This would require using a more complete expression for the mutual inductance, where the two loops no longer remain on the same axis.

We model the Lead Zeppelin as a cylindrical superconductor with radius R_{zep} and thickness h_{zep} , that is placed longitudinally to the z -axis of a cylindrical coordinate system in which $z = z_{zep}$ is the location of the superconductor's center point. We place it in the experimental setup as shown in figure 2.4. There are two levitation coils on either side of the superconductor in an anti-Helmholtz configuration; one coil of N_{up} windings above the superconductor at $z = z_{up}$, given a current I_{up} , and another with N_{down} windings below the superconductor at $z = z_{down}$ given a current I_{down} . Both coils have a radius R_{lev} . At $z = z_p$ we place a pick-up coil of radius R_p , that is used to pick up the flux Φ induced in it by the surface currents on the Zeppelin.

According to the Biot-Savart law, a single infinitesimally thin circular loop of wire of radius R , carrying a current I , that is positioned coaxially to the z -axis of a cylindrical coordinate system (r, z, φ) at a height $z = z_{loop}$, induces a magnetic field of:

$$\begin{aligned} H_r(r, z) &= \frac{IR}{4\pi} \int_0^{2\pi} \frac{(z - z_{loop}) \sin \xi}{(R^2 + r^2 + (z - z_{loop})^2 - 2rR \sin \xi)^{3/2}} d\xi, \\ H_z(r, z) &= \frac{IR}{4\pi} \int_0^{2\pi} \frac{R - r \sin \xi}{(R^2 + r^2 + (z - z_{loop})^2 - 2rR \sin \xi)^{3/2}} d\xi, \\ H_\varphi &= 0. \end{aligned} \quad (2.15)$$

The total field produced by the two levitation coils will be taken to be the sum of many such thin loops: in the above we replace $R \rightarrow R_{up,down}$, $z_{loop} \rightarrow z_{up,down}$ and $I \rightarrow N_{up,down} I_{up,down}$ for the two coils and then we add their contributions together, $\mathbf{H}_{lev} = \mathbf{H}_{up} + \mathbf{H}_{down}$. Also, from here on we employ the symmetries $R_{up} = R_{down} \equiv R_{lev}$, $z_{up} = -z_{down} \equiv z_{lev}$, $I_{up} = -I_{down} \equiv I_{lev}$ and $N_{up} = N_{down} \equiv N_{lev}$ that reflect the current experimental setup of figure 2.4. For the record: we are taking identical R_{lev} and z_{lev} for each of the 275 windings that make up a levitation coil.

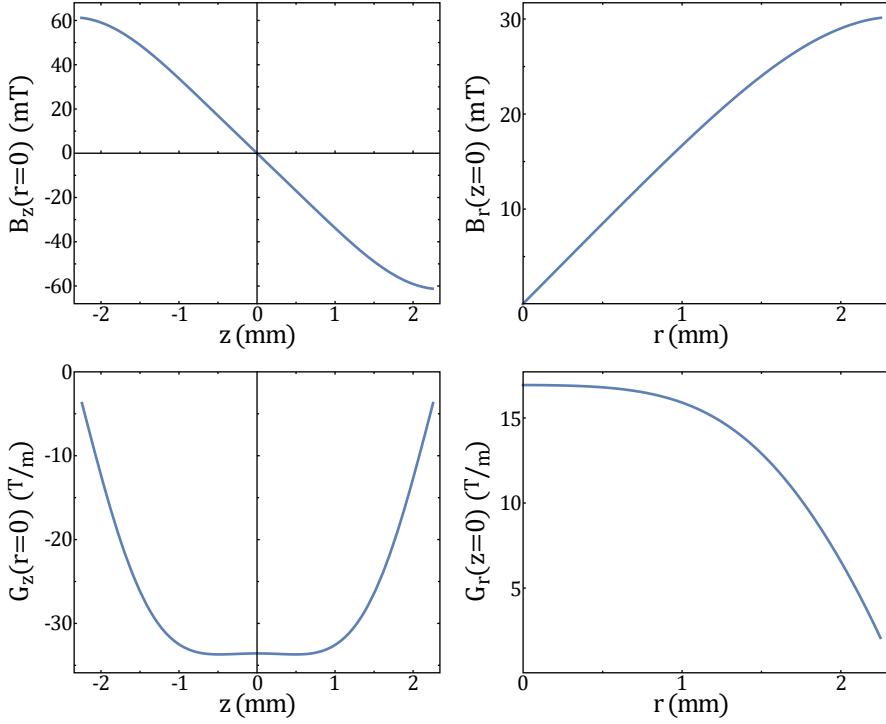


Figure 2.6: The levitating magnetic fields $B_{r,z}$ at $I_{lev} = 1$ A and the gradients $G_{r,z}$ ($B_{r,z} \propto I_{lev}$).

In figure 2.6 we plot the magnetic field components of $\mu_0 \mathbf{H}_{lev} = \mathbf{B}_{lev}$ and the gradients, at $I_{lev} = 1$ A for the standard experimental setup parameters as listed in table 2.1.

With these expressions for the levitating magnetic field, we calculate the surface current density and consequently the flux in the pickup coil. To this end, let us decompose the superconducting cylinder into its side, top and bottom surfaces. Beginning with the side surface, consider an infinitesimally thin circular loop of radius R_{zep} and height dz . For the side surface, $\hat{\mathbf{n}} = \hat{\mathbf{r}}$ so

$$\mathbf{j}_{S,side} = 2H_z \hat{\boldsymbol{\phi}}, \quad (2.16)$$

and since $\Phi = MI$ the flux in the pick-up coil caused by the thin loop is

$$d\Phi_{side} = M j_{S,side} dz = 2M H_z dz. \quad (2.17)$$

The total contribution from the superconductor's side surface to the flux is the

sum of many of such thin loops:

$$\Phi_{side} = 2 \int_{z_{zep} - \frac{1}{2}h_{zep}}^{z_{zep} + \frac{1}{2}h_{zep}} M(R_p, R_{zep}, z_p - z) H_z(R_{zep}, z) dz. \quad (2.18)$$

For the top surface, consider an infinitely thin circular loop of inner radius r and outer radius $r + dr$. Here, $\hat{\mathbf{n}} \times \mathbf{H}_{lev} = -H_r \hat{\boldsymbol{\phi}}$ and the contribution from the thin loop to the flux is $d\Phi_{top} = M j_{S,top} dr$. Summing:

$$\Phi_{top} = -2 \int_0^{R_{zep}} M(R_p, r, z_p - (z_{zep} + \frac{1}{2}h_{zep})) H_r(r, z_{zep} + \frac{1}{2}h_{zep}) dr. \quad (2.19)$$

Similarly, for the bottom surface we calculate

$$\Phi_{bottom} = 2 \int_0^{R_{zep}} M(R_p, r, z_p - (z_{zep} - \frac{1}{2}h_{zep})) H_r(r, z_{zep} - \frac{1}{2}h_{zep}) dr. \quad (2.20)$$

The total flux through the pick-up coil due to the surface currents in the superconducting cylinder induced by the levitation coils is the sum of these three contributions:

$$\Phi = \Phi_{side} + \Phi_{top} + \Phi_{bottom}. \quad (2.21)$$

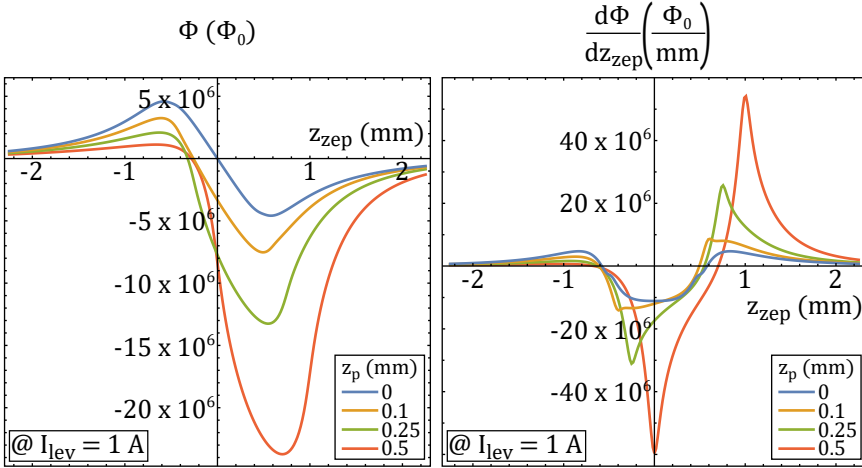


Figure 2.7: Left: the flux in the pickup coil Φ as a function of the position of the Zeppelin z_{zep} in units of Φ_0 ; right: $\frac{d\Phi}{dz_{zep}}$ as a function of z_{zep} . Calculated for some locations of the pickup coil z_p , $I_{lev} = 1 \text{ A}$ ($\Phi \propto I_{lev}$), the experimental parameters of table 2.1, and a cylindrical Zeppelin of dimensions $R_{zep} = 325 \text{ } \mu\text{m}$, $h_{zep} = 1 \text{ mm}$. We see that the flux change is largest when the Zeppelin enters and leaves the pickup coil.

Figure 2.7 shows the flux in the pickup coil and its derivative as a function of the position of the Zeppelin, for a few locations of the pickup coil z_p . It is calculated for the standard values of the setup of table 2.1, and $I_{lev} = 1$ A, $R_{zep} = 325$ μm and $h_{zep} = 1$ mm. We see that the flux change is biggest at the moments that the Zeppelin enters or leaves the pickup coil. The graphs of figure 2.7 have the symmetry $\Phi(z_{zep}, z_p) = \Phi(-z_{zep}, -z_p)$, i.e. when the pickup coil is moved from z_p to $-z_p$, the graphs are mirrored in the Φ -axis.

As depicted in figure 2.5, we connect the pickup coil to the input coil of a SQUID to measure this flux. The measured flux in the SQUID Φ_{SQ} follows from Kirchhoff's Laws:

$$\Phi_{SQ} = \Phi \frac{M_{SQ}}{L_{in} + L_p + \frac{R}{i\omega}} = \frac{\Phi}{220}. \quad (2.22)$$

With $L_p \approx 0.2$ μH the inductance of the pickup coil, $L_{in} = 2$ μH the inductance of the input coil of the SQUID, $M_{SQ} = 10$ nH the mutual inductance from L_{in} to the SQUID loop and R the resistance between L_p and L_{in} which is zero for a superconducting connection.

For future reference, we calculate Φ_{SQ} for some realistic values. In our case $z_p = 0.5$ mm, so at $z_{zep} = -0.7$ mm, figure 2.7 tells us that at $I_{lev} = 1$ A, the flux in the pickup coil is $\Phi = 1.1 \times 10^6 \Phi_0$ and $\frac{d\Phi}{dz_{zep}} = 1.9 \times 10^5 \frac{\Phi_0}{\text{mm}}$. At the more representative levitation current $I_{lev} = 0.35$ A, the flux in the SQUID Φ_{SQ} then is:

$$\Phi_{SQ} = 1750 \Phi_0, \quad \frac{d\Phi_{SQ}}{dz_{zep}} = 302 \frac{\Phi_0}{\text{mm}}. \quad (2.23)$$

For more signal, it would be better to replace the pickup coil to a more optimal position: it could increase $\frac{d\Phi}{dz_{zep}}$ greatly, especially for low equilibrium positions of the Zeppelin around $z_{eq} = -0.5$ mm.

2.4.2 Force Balance, Equation of Motion

The equation of motion of a damped, driven, harmonic oscillator is given by

$$m\ddot{x}(t) + \gamma\dot{x}(t) + kx(t) = F_{exc}(t). \quad (2.24)$$

In the case of the Lead Zeppelin, we can say to good approximation that the spring constant is provided by the levitation coils $k \propto I_{lev}^2$, and that the excitation coil provides the drive $F_{exc}(t) \propto I_{exc}(t)$.

For a driving force $F_{exc}(t) = F_0 \cos(\omega t)$ at frequency $\omega = 2\pi f$ (a phase could be added, but we will omit this for now) the solution to the equation of motion can be split in a steady state solution and a transient solution. For large enough t , the transient solution will have died out leaving only the steady state solution. The steady state solution is

$$x(t) = x_{\text{ampl}}(\omega) \cos(\omega t - \theta(\omega)), \quad (2.25)$$

with

$$x_{\text{ampl}}(\omega) = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2}}, \quad (2.26)$$

and

$$\theta(\omega) = \arctan\left(\frac{\omega\omega_0}{Q(\omega^2 - \omega_0^2)}\right). \quad (2.27)$$

The Q -factor is related to the damping coefficient as $Q = \frac{\omega_0 m}{\gamma}$ and $\omega_0 = 2\pi f_0 = \sqrt{\frac{k}{m}}$.

To calculate how exactly k and F_0 depend on the system parameters, we will begin by calculating the force on the Lead Zeppelin in a magnetic field $\mathbf{B}_{ext}$ using the formula derived in [27]

$$\mathbf{F} = \frac{2}{\mu_0} \oint dS [\mathbf{B}_{ext}(\hat{\mathbf{n}}_{zep} \cdot \mathbf{B}_{ext}) - \hat{\mathbf{n}}_{zep} |\mathbf{B}_{ext}|^2], \quad (2.28)$$

where the integral covers the whole of the Lead Zeppelin's surface S and where $\hat{\mathbf{n}}_{zep}$ is the unit vector normal to S . The magnetic field $\mathbf{B}_{ext}$ will be taken to be the sum of the magnetic fields provided by the two levitation coils and the excitation coil

$$\mathbf{B}_{ext} = \mathbf{B}_{lev} + \mathbf{B}_{exc}. \quad (2.29)$$

The magnetic fields will be assumed to be induced by sums of circular current loops, equation (2.15).

We go from the cylindrical coordinates of the previous section back to standard cartesian coordinates and give the Zeppelin dimensions $L_{x,y,z}$. We no longer assume the Zeppelin to be a small cylinder, as in the approximation we are about to make, this no longer makes any difference. What we gain is that we can include dynamics in the x - and y -directions.

In order for the integral to be calculable in a time span short enough to be practical, we shall restrict ourselves to three 1st order Taylor expansions: in the dimensions of the Lead Zeppelin $L_{x,y,z}$, the movement away from the origin $(x, y, z) = (\varepsilon_x, \varepsilon_y, \varepsilon_z)$ and the excitation current I_{exc} . The end result then looks like this:

$$\mathbf{F}_{tot} = k_x \varepsilon_x \hat{\mathbf{x}} + k_y \varepsilon_y \hat{\mathbf{y}} + (k_z \varepsilon_z + \kappa I_{exc}) \hat{\mathbf{z}}. \quad (2.30)$$

The parameter κ takes the excitation current to the excitation force $F_{exc} = \kappa I_{exc}$.

To first order, the excitation coil can be used only to excite the Lead Zeppelin in the z -direction, due to the symmetry of the setup. The excitation coil can in fact be used to excite the other directions as well, which is derived in appendix

A, but this is a higher order effect: the Lead Zeppelin has to be somewhere other than $x = 0$ or $y = 0$ for this to be possible.

It should be noted that the effect of flux reduction by the levitation coils in the persistent mode, as explained in appendix B, is not taken into account here; i.e. the flux produced by the excitation coil is counteracted, causing a less efficient excitation, and the flux produced by the motion of the Zeppelin is counteracted, causing a small reduction of the picked up flux in the pickup coil.

The spring constants are

$$k_x = k_y = K (5R_{lev}^2 - 3z_{lev}^2), \quad k_z = K (2R_{lev}^2 + 6z_{lev}^2), \quad (2.31)$$

with

$$K = \frac{3\mu_0}{2\pi} \frac{L_x L_y L_z R_{lev}^2 z_{lev}^2}{(R_{lev}^2 + z_{lev}^2)^5} I_{lev}^2. \quad (2.32)$$

The resonance frequencies are $f_{x,y,z} = \frac{1}{2\pi} \sqrt{\frac{k_{x,y,z}}{m_{zep}}}$. Like we found in section 2.3, here too we find the resonance frequencies to be proportional to I_{lev} and to be independent of the dimensions of the Zeppelin.

The coefficient κ is

$$\begin{aligned} \kappa = & -K_I \frac{1}{z_{exc}} (R_{exc}^2 + z_{exc}^2) (R_{lev}^2 + z_{lev}^2) \\ & \times (R_{lev} (R_{exc}^3 + (R_{exc} + R_{lev}) z_{exc}^2) + z_{lev}^2 z_{exc}^2), \end{aligned} \quad (2.33)$$

with

$$K_I = \frac{3\mu_0}{2\pi} \frac{L_x L_y L_z R_{exc} R_{lev} z_{exc} z_{lev}}{(R_{exc}^2 + z_{exc}^2)^{7/2} (R_{lev}^2 + z_{lev}^2)^{7/2}} I_{lev}. \quad (2.34)$$

We note that $\kappa \propto I_{lev}$; in other words, the amount by which we can excite the Lead Zeppelin depends not only on the strength of the excitation I_{exc} , but also on the strength of the magnetic levitation potential.

For a Zeppelin of dimensions $L_x = 300 \mu\text{m}$, $L_y = 400 \mu\text{m}$, $L_z = 500 \mu\text{m}$, therefore having a mass $m = 0.68 \text{ mg}$, and at $I_{lev} = 1 \text{ A}$, we find

$$k_x = k_y = 7.5 \times 10^{-3} \frac{\text{N}}{\text{m}}, \quad k_z = 20 \times 10^{-3} \frac{\text{N}}{\text{m}}, \quad (2.35)$$

leading to

$$f_x = f_y = 16.7 \text{ Hz}, \quad f_z = 27.2 \text{ Hz}, \quad (2.36)$$

and

$$\kappa = -1.06 \times 10^{-7} \frac{\text{N}}{\text{A}}. \quad (2.37)$$

2.4.3 Equilibrium Position and the Limits of Levitation

For the levitating force F on the Lead Zeppelin over the whole range of z , we favour the formula for spherical superconductors derived in [28] over equation (2.28), because it doesn't require Taylor expanding. (It is, however, harder to include an r -dependence, for which reason we did not use it earlier.)

Let the Zeppelin be a sphere of radius R_{zep} located at $z = 0$. When there is a circular loop of current I_{lev} with radius R_{lev} at a height z_{lev} , coaxial to the z -axis, the force on the Zeppelin due to the current loop is

$$\mathbf{F} = \alpha I_{lev}^2 \hat{\mathbf{z}}, \quad (2.38)$$

where

$$\begin{aligned} \alpha = & -\mu_0 R_{zep} S_{lev}^3 (S_{lev}^2 - R_{zep}^2) \cos \theta_{lev} \sin^2 \theta_{lev} \\ & \times \int_0^\pi \frac{d\xi \cos \xi}{(R_{zep}^4 + S_{lev}^4 - 2R_{zep}^2 S_{lev}^2 (\cos^2 \theta_{lev} + \sin^2 \theta_{lev} \cos \xi))^{3/2}}, \end{aligned} \quad (2.39)$$

in which

$$S_{lev} = \sqrt{R_{lev}^2 + z_{lev}^2}, \quad \theta_{lev} = \arctan \left(\frac{R_{lev}}{z_{lev}} \right). \quad (2.40)$$

In our case there are two such current loops at $\pm z_{lev}$ of N_{lev} windings. Including a z -dependence by doing $z_{lev} \rightarrow z_{lev} - z_{zep}$, filling in the experimental setup parameters of table 2.1, and using $R_{zep} = 325 \mu\text{m}$, yields the levitating force F on the Zeppelin for $-z_{lev} \leq z_{zep} \leq z_{lev}$; we show a plot of F in figure 2.8.

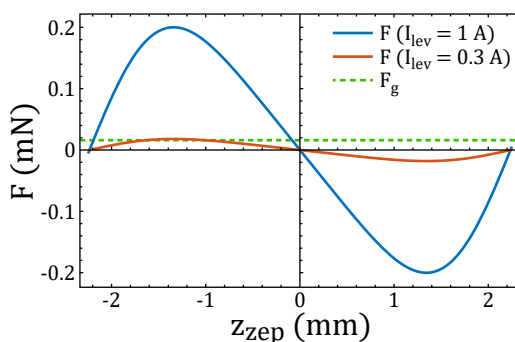


Figure 2.8: The levitating force F versus the position of the Lead Zeppelin z_{zep} , when it is a spherical particle of $650 \mu\text{m}$ diameter ($m = 1.6 \text{ mg}$). For comparison, also the gravitational force $F_g = mg = 16 \mu\text{N}$ is plotted. At $I_{lev} = 1 \text{ A}$, F_g pulls the Zeppelin away from $z = 0$ by only a small amount; we're still in the linear part of F . At $I_{lev} = 0.3 \text{ A}$, gravity pulls the Zeppelin off-center so much, that we are deep in the non-linear part of the levitating force.

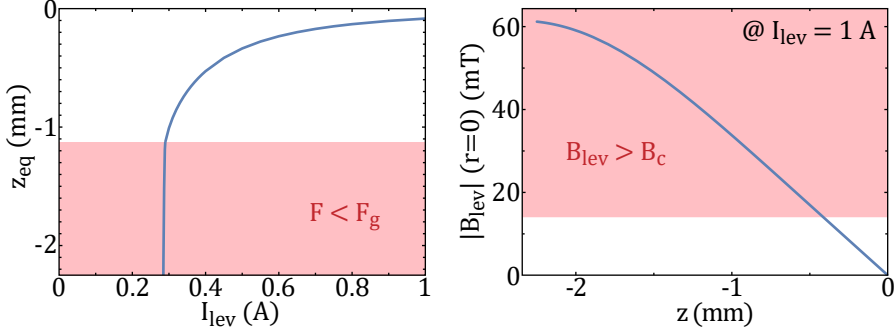


Figure 2.9: Left: the equilibrium position z_{eq} where the Zeppelin levitates as a function of levitation current I_{lev} , for a spherical particle of 650 μm diameter ($m = 1.6$ mg). Right: the levitating magnetic field B_{lev} at $I_{lev} = 1$ A as a function of Zeppelin position z . In red we indicate where the Zeppelin does not levitate: at too low levitation currents the force of gravity is stronger than the levitating force, and at too high levitation currents the levitating magnetic field is higher than the critical magnetic field of Lead at $T = 4.2$ K.

Due to gravity, the Zeppelin will be pulled downwards. For high enough levitation currents, e.g. 1 A, the Lead Zeppelin will hover in the linear regime of the force curve. However, at lower levitation currents the levitating force gets weaker and gravity will make the Zeppelin hover in the non-linear part.

To get the equilibrium position z_{eq} at a certain I_{lev} , we equate the levitation force to the gravitational force, $F(z_{eq}) = mg$, and solve for z_{eq} . The result of doing this can be seen in figure 2.9, where we plot z_{eq} as a function of I_{lev} . In this particular calculation, the Zeppelin doesn't levitate below $I_{lev} \approx 0.29$ A.

In figure 2.9 we also plot the strength of the levitating magnetic field. In red, we indicate in both plots the limit of levitation: the levitating force must be strong enough to overcome the force of gravity, but the levitating field B_{lev} must remain smaller than the critical field $B_{c,Pb}$ of Lead. We calculate $B_{c,Pb}$ according to

$$B_{c,Pb}(T) = \left(1 - \frac{T}{T_{c,Pb}}\right)^2 B_{c,Pb}(T = 0 \text{ K}) = 13.9 \text{ mT @ } 4.2 \text{ K}, \quad (2.41)$$

where $T_{c,Pb} = 7.2$ K and $B_{c,Pb}(T = 0 \text{ K}) = 80$ mT[29].

2.4.4 Duffing Oscillator

The force as plotted in figure 2.8 shows that for big deviations from $z = 0$, the curve departs from linear behaviour and acquires a z^3 component. We can therefore expect that when the Zeppelin moves in this non-linear part of the curve, it will behave as a Duffing oscillator.

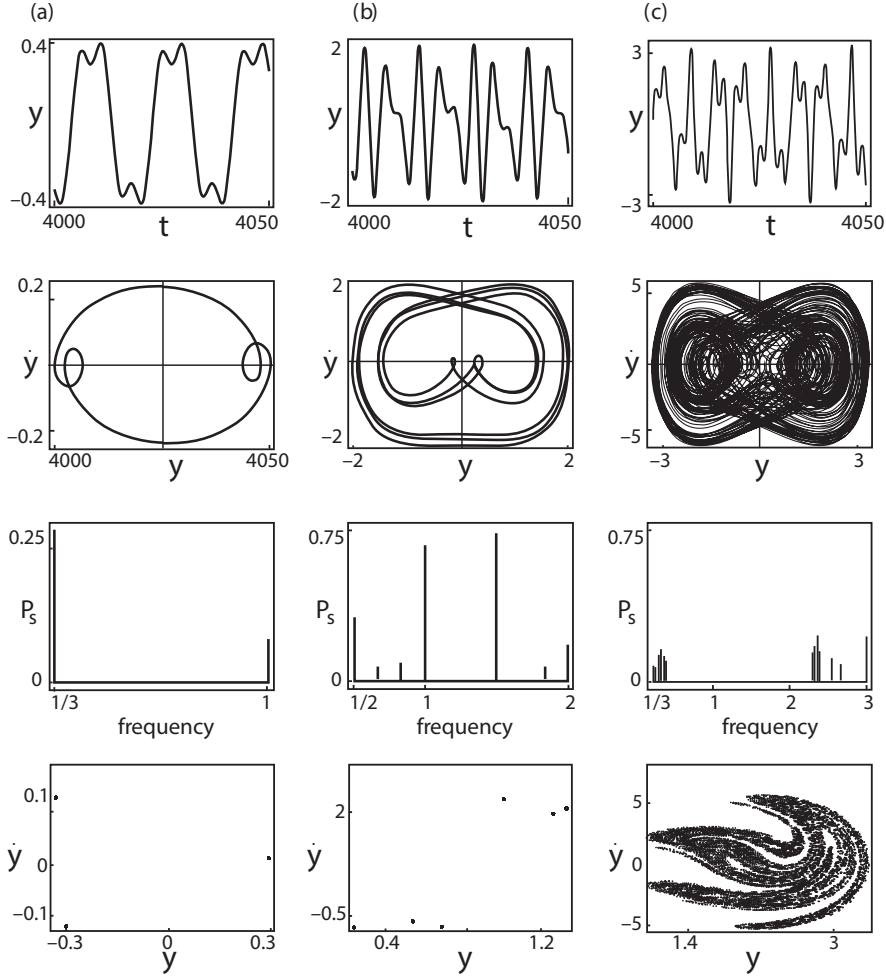


Figure 2.10: Reprinted with permission from [30, fig. 5.13], © John Wiley & Sons, Ltd. Some solutions to the Duffing equation (2.42), for $\omega_d = 1 \frac{\text{rad}}{\text{s}}$, $\frac{\delta}{m} = 1 \frac{\text{N}}{\text{m}^3}$, $\frac{\gamma}{m} = 0.02 \text{ s}^{-1}$ and zero spring constant. Left to right: $\frac{F_0}{m} = 0.1, 0.9, 0.7 \frac{\text{m}}{\text{s}^2}$. Top to bottom: time plot, phase plot, power spectrum, Poincaré section. The Poincaré sections are obtained by evaluating $(y(t), \dot{y}(t))$ at integer multiples of $T_d = \frac{2\pi}{\omega_d}$.

The equation of motion for a Duffing oscillator is equal to the one for a harmonic oscillator (eq. (2.24)) plus a cubic term:

$$m\ddot{z}(t) + \gamma\dot{z}(t) + k_z z(t) + \delta z^3(t) = F_{ext}(t). \quad (2.42)$$

The Duffing parameter δ determines the amount of non-linearity of the oscillator.

The Duffing equation (2.42) is notoriously hard to solve. In some limiting cases, however, it gets a little easier. In our case, for instance, the spring constant vanishes at the extrema of the force curve $k = \frac{dF}{dz} = 0$. At low levitation currents, gravity will pull the Zeppelin down towards the extremum below $z = 0$.

So suppose that gravity has pulled the Zeppelin all the way down to this extremum. Also, suppose that we drive the Zeppelin with $F_{ext} = F_0 \cos(\omega_d t)$ at $\omega_d = 1 \frac{\text{rad}}{\text{s}}$ and that $\frac{\delta}{m} = 1 \frac{\text{N}}{\text{m}^3}$. This particular form of the Duffing oscillator is also known as the ‘Ueda oscillator’[30]. Figure 2.10 shows some solutions to this equation.

There are many interesting features about these solutions, but here we only want to point out the form of the time signals and the emergence of sub-harmonics in the power spectrum. In our experiments, we have found that the Lead Zeppelin sometimes produces time-signals that greatly resemble the ones of figure 2.10. We will show some of those in section 4.3. Depending on the shape of the Zeppelin, the amplitude of its motion and the equilibrium position on the force curve, there will be a high degree of chaos (c), or less chaos, with an almost ‘square’ time trace (a). The fact that sub-harmonics show up, is interesting to keep in mind in future attempts to link peaks in the power spectrum to Zeppelin resonances.

2.5 Lockin Measurement

A typical measurement that we perform involves exciting the Lead Zeppelin with the excitation coil connected to a function generator. The function generator supplies an AC voltage at a frequency f of amplitude V_{fg} , which we can convert to an excitation force:

$$F_{ext}(t) = \kappa I_{exc}(t) = \kappa \frac{V_{fg}}{R_{fg}} \cos(2\pi f t), \quad (2.43)$$

where R_{fg} is the sum of the function generator’s impedance (50 Ω) and the resistance in the wiring to the excitation coil, which is usually $\sim 10 \Omega$, making it a total of $R_{fg} = 60 \Omega$. Via the pickup coil and the SQUID, the Lead Zeppelin’s response to this force is measured with a lockin amplifier.

When the Lead Zeppelin obeys the equation of motion (2.24), and armed with $\Phi_{SQ}(z)$ from section 2.4.1, we can model our SQUID lockin measurements. The signal from the levitating Lead Zeppelin causes a voltage across the SQUID, but we must be careful, because so does the signal coming from the excitation coil. Without this crosstalk, a measurement with a lockin amplifier yields the amplitude and phase of the motion of the Zeppelin for a given excitation frequency.

We will derive here what we expect to measure including crosstalk. Also, because the Lead Zeppelin has relatively low frequency and high Q , measurements tend to be rather time consuming. We shall discuss our lockin measurements quite detailed, because we often measure more quickly than what is necessary to allow the Zeppelin to equilibrate properly.

The typical measurement is to do a frequency sweep. We excite the Zeppelin at a time t_A with a certain frequency f_A such that the amplitude of oscillation is x_A . After having done this for a time Δt we move on to the next frequency f_B . The new steady state amplitude of oscillation x_B is found through (2.26), but it will take some time before it is reached. To avoid having to use the full transient solution, we instead simply state that the change to the new amplitude should be exponential and will depend on the Q -factor:

$$x_{A \rightarrow B}(t) = x_A + (x_B - x_A) \left(1 - e^{-t/\tau_{zep}}\right), \quad (2.44)$$

where $\tau_{zep} = \frac{Q}{\pi f_0}$.

Now, in reality, a frequency sweep is often done faster than what is dictated by τ_{zep} . The amplitude of oscillation will then not yet have reached x_B . To incorporate this into our model, we feed the motion of the Zeppelin through the steady state solution by setting up a frequency vector $\omega_i = 2\pi f_i$ with the frequencies at which we measure and by then calculating

$$x_i(t') = x_{i-1}(\Delta t) + (x_{\text{ampl}}(\omega_i) - x_{i-1}(\Delta t))(1 - e^{-t'/\tau_{zep}}), \quad (2.45)$$

where we let $t' = 0 \rightarrow \Delta t$ for each i , and where we take $x_{\text{ampl}}(\omega_i)$ from equation (2.26). Depending on the actual measurement being performed, we initialize $x_1(t' = 0) = 0$ or $x_1(t' = 0) = x_{\text{ampl}}(\omega_1)$ (although this detail usually doesn't really matter).

For each i , we describe the Zeppelin's motion by

$$x_{zep,i}(t') = x_i(t') \cos(2\pi f_i t' + \theta_i). \quad (2.46)$$

We assume that the phase changes instantaneously to its new value:

$$\theta_i = \theta(\omega_i), \quad (2.47)$$

with $\theta(\omega)$ as in equation (2.27). This will cause steps between consecutive $x_{zep,i}$, because the cosines don't match up. The real, physical Lead Zeppelin will not show this behaviour, because there cannot be a discontinuity in its position. However, in the current model, we will keep this as is; steps are a high frequency effect, and since the model involves low-pass filtering anyway, this doesn't make much of a difference to our result. Again, we are simply avoiding having to use the full solution to the equation of motion.

The flux in the pickup coil in time per frequency is

$$\Phi_i(t') = \Phi(x_{zep,i}(t') + z_{eq}). \quad (2.48)$$

We also insert the term z_{eq} to incorporate the fact that the Lead Zeppelin has an equilibrium position away from $z = 0$.

The flux $\Phi_i(t')$ can be converted to a voltage in the SQUID. Namely, we know how the flux changes from pickup coil to SQUID by equation (2.22), and we have a calibration of the voltage per flux in the SQUID, V_Φ . We can measure the current per flux for our SQUID I_Φ and multiply this by the feedback resistance R_{fb} and the gain G of our room-temperature pre-amplifier. Typical values are $I_\Phi = 2.83 \frac{\mu A}{\Phi_0}$, $R_{fb} = 100 \text{ k}\Omega$ and $G = 1$ so

$$V_\Phi = I_\Phi R_{fb} G = 0.283 \frac{V}{\Phi_0}. \quad (2.49)$$

Then the voltage in the SQUID due to the motion of the Lead Zeppelin is

$$V_{SQ,zep,i}(t') = \frac{\Phi_i(t')}{220} V_\Phi. \quad (2.50)$$

In our setup, there is also a lot of crosstalk: the flux in the pickup coil is not only given by the Zeppelin, but by the excitation coil as well.

$$V_{SQ,ct,i}(t') = V_{ct} \cos(2\pi f_i t' + \theta_{ct,i}). \quad (2.51)$$

The parameter V_{ct} depends, among other things, on the mutual inductance of the excitation coil to the pickup coil. We will keep it as a fit parameter, obtainable from the SQUID signal off Zeppelin resonance.

In total, we measure

$$V_{SQ,i}(t') = V_{SQ,zep,i}(t') + V_{SQ,ct,i}(t') - V_{\text{offset}}. \quad (2.52)$$

We account for the fact that the SQUID is insensitive to flux offsets prior to being flux-locked by subtracting a constant voltage V_{offset} . This $V_{SQ,i}(t')$ is fed to the lockin.

The lockin amplifier takes the SQUID voltage, and outputs its amplitude R and its phase Θ . It does so by multiplying the input signal with a reference signal, and feeding this through a low-pass filter.

$$\begin{aligned} \hat{X}_i(t') &= V_{SQ,i}(t') \times V_{ref} \sin(2\pi f_{ref} t' + \theta_{ref}), \\ \hat{Y}_i(t') &= V_{SQ,i}(t') \times V_{ref} \cos(2\pi f_{ref} t' + \theta_{ref}). \end{aligned} \quad (2.53)$$

If the input signal were a simple sine, then $\hat{X}$ and $\hat{Y}$ contain (co)sines at the sum and difference frequencies $f_{sig} \pm f_{ref}$. In particular, for $f_{ref} = f_{sig}$ there is a component at $2f_{sig}$, which is filtered away by the low-pass filter, and a DC component that scales with the (co)sine of the phase difference between signal and reference.

$$\begin{aligned} \hat{X}_i(t') &= \frac{V_{sig} V_{ref}}{2} \cos(\theta_{sig} - \theta_{ref}) + \dots \\ \hat{Y}_i(t') &= \frac{V_{sig} V_{ref}}{2} \sin(\theta_{sig} - \theta_{ref}) + \dots \end{aligned} \quad (2.54)$$

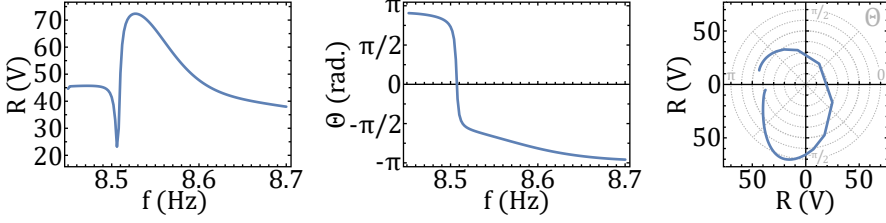


Figure 2.11: A simulation of a lockin measurement of the driven Lead Zeppelin. Left to right: the amplitude of the signal R , the phase of the signal Θ and a phase plot $R \cos \Theta$ vs. $R \sin \Theta$. The strength of the excitation has been taken as $F_0 = 1$ nN, which was chosen based on $I_{lev} = 0.3$ A, such that $\kappa \approx 10^{-8} \frac{\text{N}}{\text{A}}$ (eq. (2.37)), and on $I_{exc} = 0.1$ A. We also used the following values, which were chosen with $I_{lev} = 0.3$ A in mind: $k = 2 \times 10^{-3} \frac{\text{N}}{\text{m}}$, $m = 7 \times 10^{-7}$ kg, $\gamma = 3 \times 10^{-8} \frac{\text{N}}{\text{m/s}}$ and $z_{eq} = -700$ μm , such that $f_0 = 8.507$ Hz, $Q = 1247$ and $\tau_{zep} = 47$ s. Also $\tau_{LI} = 0.3$ s, $\Delta t = 2.0$ s, $\Delta f = 2$ mHz, $V_{SQ,ct} = 40$ V, $\theta_{ct} = \frac{\pi}{2}$, $\theta_{ref} = 0$.

In our case, these functions won't look as simple as this; the above is just an illustration of the working principles of a lockin amplifier.

Filtering is done by an RC -filter. For some input signal across a resistor and a capacitor in series, the voltage across the capacitor is the first order low-pass filtered version of the input signal. In the time domain, this can be expressed as a convolution with an exponential, which in the case of $\hat{X}$ and $\hat{Y}$ looks like:

$$\begin{aligned} X_i(t') &= \frac{1}{\tau_{LI}} \int_0^{t'} \hat{X}_i(s) e^{(s-t')/\tau_{LI}} ds + X_{0,i} e^{-t'/\tau_{LI}}, \\ Y_i(t') &= \frac{1}{\tau_{LI}} \int_0^{t'} \hat{Y}_i(s) e^{(s-t')/\tau_{LI}} ds + Y_{0,i} e^{-t'/\tau_{LI}}. \end{aligned} \quad (2.55)$$

The lockin time constant $\tau_{LI} = RC$ can be chosen by the user on the lockin front panel. The initial voltages $X_{0,i} = X_{i-1}(\Delta t)$ and $Y_{0,i} = Y_{i-1}(\Delta t)$ are exponentially repressed, but when Δt is similar to τ_{LI} , we can't forget about them.

Finally, the lockin output $R_i(t')$ measures the amplitude of the input signal and $\Theta_i(t')$ measures $\theta_{sig} - \theta_{ref}$:

$$\begin{aligned} R_i(t') &= \frac{2}{V_{ref}} \sqrt{X_i^2(t') + Y_i^2(t')}, \\ \Theta_i(t') &= \arctan \left(\frac{Y_i(t')}{X_i(t')} \right). \end{aligned} \quad (2.56)$$

We read out at $t' = \Delta t$: $R_i(\Delta t)$, $\Theta_i(\Delta t)$, at each frequency f_i for each i . This model captures the ringing up in time of the Zeppelin during the lockin measurement of R and Θ , as well as our impatience in waiting for the ring-up to be completed.

In figure 2.11 we show the results for R , Θ and the corresponding polar plot across the frequency sweep, of a simulation of a lockin measurement of the driven Zeppelin motion for some typical values. While for a high Q resonator without crosstalk the resonance is represented in a polar plot as a circle beginning and ending at the origin, its north pole coinciding with the origin, the presence of crosstalk moves the circle. For certain values of the crosstalk $V_{SQ,ct}$ with phase θ_{ct} , the circle encircles the origin causing R to have a dip before its peak and Θ to drop by 2π across the Zeppelin resonance. After the resonance, the signal relaxes to its off-resonant value as dictated by τ_{zep} .

2.6 Damping Mechanisms

The motion of the Lead Zeppelin will be damped by several factors. Here, we discuss two obvious causes of damping: the damping due to interaction with surrounding Helium gas and the damping due to eddy currents in normal metals in the vicinity of the Lead Zeppelin.

2.6.1 Background Gas

To have good thermalization, we leave the Helium background pressure in the vacuum container at around $P = 10^{-2}$ mbar. We cannot go much lower in pressure because the experiment in its current form depends on the cooling provided by the background gas: e.g. the Niobium shortcut wires of the PCS (chapter 3) heat up when injecting a current into the levitation coils, and need to be kept cold. The Helium will set a limit on the Q factor of the levitating Lead Zeppelin.

When the Lead Zeppelin is a sphere of radius R_{zep} , the damping due to collisions with the surrounding gaseous Helium is given by[31]:

$$\gamma = \frac{4\pi R_{zep}^2 P}{\bar{v}_{Th}} = 8.2 \times 10^{-9} \frac{\text{N}}{\text{m/s}}, \quad (2.57)$$

for $P = 1 \text{ Pa}$ ($= 10^{-2}$ mbar) and $R_{zep} = 325 \text{ }\mu\text{m}$. Also, $\bar{v}_{Th}$ denotes the thermal root mean square velocity of the gas molecules, given by $\bar{v}_{Th} = \sqrt{\frac{3k_B T}{m_{\text{He}}}} = 162 \frac{\text{m}}{\text{s}}$, when $T = 4.2 \text{ K}$ and $m_{\text{He}} = 4 \text{ amu} = 6.64 \times 10^{-27} \text{ kg}$.

This contribution to the damping could be lowered by using a smaller Zeppelin, and by decreasing the Helium background pressure, which requires better thermalization of the experiment.

2.6.2 Eddy Currents

The clamp that holds the levitation coils is made from brass, because it is stiff and easily machinable. However, being a normal, resistive metal also at low temperatures, the movement of the Lead Zeppelin will cause dissipation in it,

because the moving Zeppelin constitutes a time-varying flux: $P_{\text{diss}} = \frac{1}{R} \left(\frac{d\Phi}{dt} \right)^2$. When we call $v = \frac{dz}{dt}$, we can write the dissipation as $P_{\text{diss}} = \frac{1}{R} \left(\frac{d\Phi}{dz} \right)^2 v^2$. This can be compared to $P_{\text{diss}} = \mathbf{F} \cdot \mathbf{v} = \gamma v^2$, which follows from the damping force $\mathbf{F} = \gamma \mathbf{v}$. Therefore the damping coefficient is

$$\gamma = \frac{1}{R} \left(\frac{d\Phi}{dz} \right)^2. \quad (2.58)$$

In section 2.4.1 we calculated $\frac{d\Phi}{dz}$ for the pickup coil with which we read out the motion of the Lead Zeppelin. We recalculate it with R_p the radius of the part of the brass clamp of interest and z_p its distance from the Zeppelin.

The resistance R depends on the geometry of the clamp and the material from which it is made. The parts of the brass clamp that keep the Macor levitation coil holders in place, are in the current design (fig. 2.4) hollow cylinders of dimensions $r_{\text{in}} = 3.5$ mm, $r_{\text{out}} = 7$ mm, $h = 4$ mm. Let us assume that here eddy currents flow in the φ -direction. The resistance can then be calculated from $R = \rho \frac{l}{A}$, with ρ the specific resistance: dividing the cylinder up in infinitesimal volume elements with dimensions dr , ds and $d\varphi$, we write $R = \int \rho \frac{rd\varphi}{drds}$. Since we assume the current to flow in the φ -direction, resistivity is summed over $d\varphi$, but conductivity $\frac{1}{R}$ is summed over dr and ds . Therefore, the total resistance of our hollow cylinder is

$$R = \frac{2\pi\rho}{h \ln \frac{r_{\text{out}}}{r_{\text{in}}}}, \quad (2.59)$$

and the damping due to eddies in the surrounding normal metal is

$$\gamma = \frac{1}{2\pi\rho} \int_{r_{\text{in}}}^{r_{\text{out}}} dr \int_{s_{\text{min}}}^{s_{\text{max}}} ds \left[\frac{1}{r} \left(\frac{d\Phi}{dz}(R_p = r, z_p = s) \right)^2 \right]. \quad (2.60)$$

We calculate the flux derivative in a loop of radius $R_p = 5.25$ mm at location $z_p = 9.5$ mm to be

$$\frac{d\Phi}{dz} = -1.18 \times 10^8 \frac{\Phi_0}{\text{m}}, \quad (2.61)$$

and assume the flux derivative to have this value in the whole cylinder. The specific resistance of Copper at 4 K is [32] $\rho_{\text{Cu}}(4 \text{ K}) = 2 \times 10^{-11} \Omega \text{ m}$. This assumes ultra-pure, annealed Copper. Our clamps are in fact made of an (unannealed) Copper-alloy: brass. We estimate brass's specific resistivity at 4 K to be roughly 100 times higher than that of pure Copper, on the basis of its room temperature specific resistivity being about 5 times higher and its residual resistance ratio being much lower:

$$\rho_{\text{brass}}(4 \text{ K}) = 2 \times 10^{-9} \Omega \text{ m}. \quad (2.62)$$

We add the two contributions from the upper and lower cylindrical part of the clamp together, and get

$$\gamma = 2.6 \times 10^{-8} \frac{\text{N}}{\text{m/s}}. \quad (2.63)$$

If we were to make the coil clamps out a non-metallic material, this contribution to the damping will have gone away.

2.7 The Sensitivity of the Lead Zeppelin

In this section we will explore the sensitivity of the Lead Zeppelin and check the feasibility of a gravitational experiment with small source masses. First we will cover some of the basics of measurements on the Lead Zeppelin and their Fourier analysis. We will look at the Power Spectral Density (PSD) and root mean square (RMS) amplitude of signals, such that in the following subsections we can describe the thermal motion and force noise experienced by the Lead Zeppelin, as well as the influence of external vibrations, and compare it to the gravitational forces generated by a test mass oscillating with a given amplitude some distance away.

To do a (gravitational) measurement of driven Zeppelin motion, there are several important criteria:

$$\begin{aligned} S_F &> S_{F,Th}, \\ S_x &> S_{x,lim}, \\ S_x &> H^2(\omega) \times S_{x,E}. \end{aligned} \quad (2.64)$$

The force on the Zeppelin must be larger than the thermal force noise $S_{F,Th}$ acting on it. Also, the Zeppelin motion must be larger than the position detection limit $S_{x,lim}$ of our measurement. Finally, and sadly very importantly, the external vibrations $S_{x,E}$ reaching the Zeppelin via transfer function $H(\omega)$ must be sufficiently small.

2.7.1 Basics of Fourier Transform Measurements

Our definition of the continuous Fourier transform is

$$\mathcal{F}\{x(t)\}(\omega) = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt, \quad (2.65)$$

with $\omega = 2\pi f$. When we measure for a time T_m and sample the time signal in N chunks, we use the discrete Fourier transform:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i\omega_k t_n}, \quad (2.66)$$

with $X_k = X(\omega_k)$, $x_n = x(t_n)$ and $\omega_k = 2\pi f_k$, $f_k = k\Delta f = k\frac{1}{T_m}$, $t_n = n\Delta t = n\frac{T_m}{N}$. The discrete Fourier transform is (approximately) equal to the continuous Fourier transform on the interval $(0, T_m)$, up to a factor Δt , through a Riemann summation

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i\omega_k t_n} \approx \frac{1}{\Delta t} \int_0^{T_m} x(t) e^{-i\omega_k t} dt = \frac{1}{\Delta t} X(\omega_k). \quad (2.67)$$

Let us assume that we drive the Lead Zeppelin at exactly a frequency ω_d with an infinitely narrow linewidth, i.e. a δ -peak in frequency space:

$$\mathcal{F}\{F_0 \cos(\omega_d t)\}(\omega) = \pi F_0 \left(\delta(\omega - \omega_d) + \delta(\omega + \omega_d) \right). \quad (2.68)$$

Now, if the system behaves linearly (which we assume), the Zeppelin's response (eq. (2.26)) will also be a δ -peak in frequency space.

We will not be too concerned with phase information and negative frequency components, since the PSD that we will be calculating throws away phase information and is symmetric around $\omega = 0$.

The δ -peak will be smeared out over a wider range of frequencies, by the fact that we measure for a finite time T_m . For a rectangular window $w_R(t) = 1$ ($0 \leq t \leq T_m$ and zero otherwise), i.e. we do not modify the signal apart from breaking it in T_m -long chunks, the Zeppelin signal is convolved with a sinc function in frequency space. Namely, up to a complex phase factor, the Fourier transform of $w_R(t)$ is

$$W_R(\omega) = T_m \operatorname{sinc} \left(\frac{\omega T_m}{2} \right), \quad (2.69)$$

and $\mathcal{F}\{x(t)w_R(t)\}(\omega) = (X * W_R)(\omega)$.

The sinc function is defined as $\operatorname{sinc} x = \frac{\sin x}{x}$. The height of the main peak of the window is $W_R(0) = T_m$. We define the width δf of the sinc as the Full Width at Half Maximum (FWHM) of the main peak of the square of the sinc. It requires solving $x^2 = 2 \sin^2 x$ for x , and results in $\delta f = \frac{0.88}{T_m}$. The longer we measure, the higher and narrower the measured peak becomes.

The convolution of a function with a δ -function, yields the function itself. Therefore, at a δ -peak drive of the Zeppelin, we will measure a sinc function with finite height and linewidth whose peak coincides with the δ -peak.

Another widely used window is the Hann window. The Fourier transform of the Hann window $W_H(\omega)$, from the time-window $w_H(t) = \sin^2 \left(\frac{\pi t}{T_m} \right)$ ($0 \leq t \leq T_m$ and zero otherwise), is

$$W_H(\omega) = \left(1 + \frac{\omega}{2} \frac{1}{\omega - \frac{2\pi}{T_m}} + \frac{\omega}{2} \frac{1}{\omega + \frac{2\pi}{T_m}} \right) \frac{T_m}{2} \operatorname{sinc} \left(\frac{\omega T_m}{2} \right). \quad (2.70)$$

At $\omega = 0$, it is only half as high as the rectangular window. However, it has reduced spectral leakage as compared to the rectangular window, meaning that closely adjacent frequency components are better distinguishable.

The PSD is a handy tool in the frequency analysis of a signal, as it gives a handle on how strongly a certain frequency is present in the signal. In discrete time, it is calculated from the Fourier transform X_n as

$$S_n = \frac{1}{\text{NPBW}} \frac{1}{\Delta f} \frac{1}{N^2} |X_n|^2. \quad (2.71)$$

The factor NPBW is the Noise Power Bandwidth of the Window, a correction factor for the used window; e.g. for a rectangular window it is equal to 1, for a Hann window it equals 1.5. When we use a window other than the rectangular window, we should in the above replace X_n with its convolved counterpart.

When x_n is measured in meters, the units of the PSD are $\frac{\text{m}^2}{\text{Hz}}$. Often we will be talking of $S_n^{1/2}$, rather sloppily, as the power spectral density too, even though it really is an amplitude spectral density.

We can calculate what the value is of the peak in the PSD due to driving the Lead Zeppelin with an infinitesimally sharp excitation at a frequency ω_d . The Zeppelin will acquire an amplitude $x_{\text{ampl}}(\omega_d)$, according to equation (2.26), and move as a perfect cosine at ω_d (we ignore the phase). In frequency space, this becomes a δ -function, which is smeared out (depending on the chosen window, let us choose a simple rectangular window here) to a sinc function. Going to discrete time and frequency, we find that

$$X_n = \frac{1}{2} N x_{\text{ampl}}(\omega_d) \text{sinc} \frac{(\omega_n - \omega_d) T_m}{2}. \quad (2.72)$$

It should be noted that in going from the continuous to the discrete Fourier transform, strictly speaking the Dirac δ -function becomes a Kronecker δ , which, in our definitions, means that we have had a factor of 2π to correct for.

We can now calculate the PSD:

$$S_n = \frac{T_m}{4} x_{\text{ampl}}^2(\omega_d) \text{sinc}^2 \frac{(\omega_n - \omega_d) T_m}{2}, \quad (2.73)$$

or

$$S_n^{1/2} = \frac{1}{2} \sqrt{T_m} x_{\text{ampl}}(\omega_d) \left| \text{sinc} \frac{(\omega_n - \omega_d) T_m}{2} \right|. \quad (2.74)$$

If we were to choose T_m , N and ω_d such that there is a frequency $\omega_n = \omega_d$, at this frequency the value will be

$$S_{n @ \omega_d}^{1/2} = \frac{1}{2} \sqrt{T_m} x_{\text{ampl}}(\omega_d). \quad (2.75)$$

When we choose the Hann window instead of the rectangular window, the peak value is

$$S_{n@{\omega_d}, \text{Hann}}^{1/2} = \frac{1}{2\sqrt{6}} \sqrt{T_m} x_{\text{ampl}}(\omega_d). \quad (2.76)$$

It is not always the case that T_m , N and ω_d are precisely such that $\omega_n = \omega_d$ exists. For an ω_n slightly off ω_d , the measured peak value will be lower than it would otherwise be, resulting in a wrong estimation of $x_{\text{ampl}}(\omega_d)$. In this case, we need to do a little more work.

We begin by invoking Parseval's Theorem, which states that

$$\sum_{n=0}^{N-1} |x_n|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |X_n|^2. \quad (2.77)$$

When we take the sum over x_n to be a Riemann sum and convert it to an integral as in equation (2.67), and when we convert X_n to S_n with equation (2.71), we find that

$$\int_0^{T_m} |x(t)|^2 dt = \frac{1}{\text{NPBW}} \sum_{n=0}^{N-1} S_n. \quad (2.78)$$

The integral can be calculated easily, as we have $x(t) = x_{\text{ampl}}(\omega_d) \cos(\omega_d t)$ and therefore

$$\int_0^{T_m} |x(t)|^2 dt = x_{\text{ampl}}^2(\omega_d) \left(\frac{T_m}{2} + \frac{\sin(\omega_d T_m)}{4\omega_d} \right). \quad (2.79)$$

When $T_m \gg \frac{1}{2\omega_d}$, which is normally true in our measurements, the rightmost term can be ignored, and we are left with

$$\frac{1}{2} T_m x_{\text{ampl}}^2(\omega_d) = \frac{1}{\text{NPBW}} \sum_{n=0}^{N-1} S_n. \quad (2.80)$$

When we constrain the sum to PSD values in the very vicinity of the peak, which carry the most energy of the original signal and therefore contribute the most, throwing out the copy of the peak at $-\omega_d$ as we go, we write

$$x_{\text{ampl}}^2(\omega_d) = 4 \frac{1}{T_m} \frac{1}{\text{NPBW}} \sum_{n \sim \omega_d} S_n. \quad (2.81)$$

This result coincides with the earlier result where $\omega_n = \omega_d$ exists: in this case only the peak value of the sinc survives, whilst all the other frequency bins coincide with the zero-points of the sinc. This new result, however, can also deal with the more general case.

We want to remark here that the summing of PSD components in a peak allows us to be somewhat impatient, in the sense that we can get away with measurement times shorter than what is dictated by the quality factor of a resonance. If the Zeppelin motion is a (sharp) Lorentzian, whose width is given by $\delta f = \frac{f_0}{Q}$, then only for measurement times longer than $T_m = \frac{Q}{f_0}$ will there be more than one PSD data point in the peak. For typical values $Q = 3000$ and $f_0 = 10$ Hz, this means $T_m > 300$ s. Luckily, Parseval's Theorem means that the energy in the peak is conserved in the PSD components; it is just spread to adjacent frequency bins. Summing them yields a good estimate for the amount of energy in the actual Zeppelin motion. On the other hand, we cannot measure for too short either, because then the peak gets spread out so much that it may not be visible over the detection noise.

Finally, we discuss what happens when the Lead Zeppelin is driven by white noise, rather than an infinitely sharp drive at ω_d as in the above. When driven by white noise, it is hard to assign an amplitude x_{ampl} to the motion $x(t)$ of the Zeppelin. We instead use the Root Mean Square (RMS) amplitude x_{RMS} , which is defined on the time interval T_m as

$$x_{\text{RMS}} = \sqrt{\langle x^2(t) \rangle}, \quad \langle x^2(t) \rangle = \frac{1}{T_m} \int_0^{T_m} x^2(t) dt. \quad (2.82)$$

In the previous simple case that $x(t) = x_{\text{ampl}} \cos(\omega t)$ and we average over a single cycle, we find $x_{\text{RMS}} = \frac{x_{\text{ampl}}}{\sqrt{2}}$.

When we invoke the continuous version of Parseval's Theorem

$$\int_0^{T_m} |x(t)|^2 dt = \frac{1}{2\pi} \int_0^\infty |X(\omega)|^2 d\omega, \quad (2.83)$$

as well as the continuous version of the PSD

$$S(\omega) = \frac{1}{T_m} |X(\omega)|^2, \quad (2.84)$$

we may write

$$x_{\text{RMS}}^2 = \frac{1}{2\pi} \int_0^\infty S(\omega) d\omega. \quad (2.85)$$

When the Zeppelin is excited by a white noise source $\eta_x(\omega)$, it acquires a position noise $S_{x,\eta}$ through the equation of motion (2.26), with $k = m\omega_0^2$ and $S_F^{1/2} = kS_x^{1/2}$, of

$$S_{x,\eta}^{1/2}(\omega) = \frac{\omega_0^2}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2}} \eta_x(\omega). \quad (2.86)$$

The integral of a Lorentzian evaluates to

$$\int_0^\infty \frac{d\omega}{(\omega^2 - \omega_0^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2} = \frac{\pi}{2} \frac{Q}{\omega_0^3}, \quad (2.87)$$

such that the RMS amplitude of the Zeppelin motion becomes

$$x_{\text{RMS}} = \frac{1}{2} \sqrt{\omega_0 Q} \eta_x(\omega). \quad (2.88)$$

(As an example, when the Zeppelin is driven thermally, $\eta_x = \frac{1}{k} \sqrt{4k_B T \gamma}$ (section 2.7.3), we find that $k x_{\text{RMS}}^2 = k_B T$.)

2.7.2 Detection Noise

The Zeppelin signal $S_n^{1/2}$ should be higher than the detection noise. Our SQUIDs have a noise floor of typically $S_{\Phi, SQ}^{1/2} = 100 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$. In equation (2.23) we calculated that the SQUID flux derivative to Zeppelin motion is $\frac{d\Phi_{SQ}}{dz} = 3.0 \times 10^5 \frac{\Phi_0}{\text{m}}$. This leads to a position detection limit of

$$S_{x, \text{lim}}^{1/2} = \frac{S_{\Phi, SQ}^{1/2}}{\left(\frac{d\Phi_{SQ}}{dz}\right)} = 3.3 \times 10^{-10} \frac{\text{m}}{\sqrt{\text{Hz}}}. \quad (2.89)$$

Note that this number can be improved upon in two ways: using a less noisy SQUID and placing the pickup coil at a better position. It is not uncommon for SQUIDs to have a noise floor of $\sim 1 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$, even at a temperature of 4 K, which means two orders of magnitude improvement of $S_{x, \text{lim}}^{1/2}$. Moving the pickup coil can also be beneficial (figure 2.7), possibly gaining us about two orders of magnitude if we're optimistic.

Currently, at $T_m = 100$ s, the Lead Zeppelin needs to have an oscillation amplitude of at least (eq. (2.74))

$$x_{\text{ampl}}(\omega_d) > \frac{2}{\sqrt{T_m}} S_{x, \text{lim}}^{1/2} = 6.7 \times 10^{-11} \text{ m} \quad (2.90)$$

to be discernible above the SQUID noise. As we will see in the coming sections, the thermal motion of the Zeppelin and the motion of the Zeppelin because of external vibrations are both well above this value.

2.7.3 Thermal Motion

From the equipartition theorem, we know that the force noise on a damped harmonic oscillator due to thermal fluctuations is[3]

$$S_{F, Th} = 4k_B T \gamma, \quad (2.91)$$

with k_B Boltzmann's constant, T the temperature and γ the damping factor from the equation of motion (2.24). The levitating Lead Zeppelin, when driven only by this thermal noise, will have a thermal displacement noise of (eq. (2.26))

$$S_{x,Th}(\omega) = \frac{4k_B T \gamma / m^2}{(\omega^2 - \omega_0^2)^2 + \left(\frac{\omega \omega_0}{Q}\right)^2}. \quad (2.92)$$

The value at resonance $S_{x,Th}(\omega_0) = \frac{4k_B T Q}{m \omega_0^3}$, so

$$S_{x,Th}^{1/2}(\omega_0) = 1.2 \times 10^{-9} \frac{\text{m}}{\sqrt{\text{Hz}}}. \quad (2.93)$$

at $T = 4.2$ K, $m = 2$ mg, $\frac{\omega_0}{2\pi} = 10$ Hz and $Q = 3000$. This is well above the detection noise, equation (2.89) — we should already be able to measure the thermal motion of the Lead Zeppelin. However, we will see in section 2.8 that currently the external vibrations give larger motion than the thermal motion.

2.7.4 Gravitational Experiments

In this section we calculate what the requirements are to use a Lead Zeppelin to measure the gravitational force between it and a source mass, at frequencies between 10 Hz and 100 Hz. The smallest source mass used to date to measure gravitational forces is about 90 grams [33].

Suppose we have a block of mass $M = 1$ g at a distance $r = 3$ cm from the Lead Zeppelin, and we shake it around with an amplitude of $a = 1.5$ mm. The gravitational force between M at rest and the Zeppelin, of mass $m = 1$ mg, is

$$F_g = G \frac{mM}{r^2} = 74 \text{ aN}. \quad (2.94)$$

At the two end points of oscillation of the mass

$$F_g(r \pm a) = \frac{GmM}{(r \pm a)^2} \approx \frac{GmM}{r^2} \left(1 \mp 2\frac{a}{r}\right). \quad (2.95)$$

So the amplitude of the gravitational driving force on the Zeppelin is given by

$$F_0 = \frac{1}{2} \Delta F_g = 2a \frac{GmM}{r^3} = 7.4 \text{ aN}. \quad (2.96)$$

This gravitational driving force should be obtainable with a simple electromagnetic drive motor, as described in e.g. [34]. Of course, there is the subtlety that the drive motor and the Lead Zeppelin must be vibrationally decoupled from each other, such that the motor doesn't deliver its drive through the experimental setup rather than purely gravitationally — which may prove to be difficult at $r = 3$ cm in a cryostat. Alternatively, one can think of a block of mass $M = 1$

kg at a distance $r = 1$ m and amplitude $a = 5$ cm, which yields the same gravitational driving force. This corresponds roughly to a (heavy) cylinder piston in a car engine [35], which can be made to oscillate between 600 rpm (10 Hz) and 6000 rpm (100 Hz) easily.

If the mass M is made to oscillate at a resonance of the Zeppelin, the Zeppelin will acquire an amplitude (equation (2.26)) of

$$x_{\text{ampl}}(\omega_0) = \frac{F_0}{m} \frac{Q}{\omega_0^2} = 2a \frac{GM}{r^3} \frac{Q}{\omega_0^2} = 5.6 \text{ pm}, \quad (2.97)$$

for typical numbers $\frac{\omega_0}{2\pi} = 10$ Hz and $Q = 3000$. From equation (2.74), this leads to a PSD component with the value

$$S_{x,g}^{1/2}(\omega_0) = \frac{aGM}{r^3} \frac{Q}{\omega_0^2} \sqrt{T_m} = 2.8 \times 10^{-11} \frac{\text{m}}{\sqrt{\text{Hz}}}, \quad (2.98)$$

when $T_m = 100$ s and the window is rectangular. In our current experimental setup, this is below the noise floor by an order of magnitude, eq. (2.89). However, there are several factors that can pull this number towards measurability. As stated before, lowering the SQUID noise can gain us as much as two orders of magnitude, and placing the pickup coil in a more optimal position will also be beneficial. Then, the gravitational force between the oscillating mass M and the Lead Zeppelin will be visible over the detection noise. We can also play with the other parameters: e.g. when Q were $10\times$ bigger or when we were more patient $T_m = 10^4$ s ($= 2\frac{3}{4}$ hours), then too the Zeppelin begins to respond measurably to the gravitational force of M .

A big caveat comes from the criterion on the external vibrations. As we will see in the next section, the vibrations of the environment are, at present, much too large for the gravitational measurement to be feasible.

2.8 External Vibrations

In order to shield the experiment from vibrations, we hoist the Helium Dewar, in which the experiment is mounted, off the floor with a crane attached to the ceiling of our laboratory using bungee cords (figures 2.12, 2.16). The movement of the Dewar $x_d(t)$ caused by movement of the crane $x_c(t)$ is governed by the equation

$$m_d \ddot{x}_d(t) + \gamma_d \dot{x}_d(t) + k_d(x_d(t) - x_c(t)) = 0. \quad (2.99)$$

Here, m_d is the mass of the Helium Dewar, k_d is the spring constant of the bungee cords and γ_d the damping of the system. We assume that the major source of damping that the crane experiences does not come from the bungee cords. Going to frequency space, we find the transfer function

$$H_{cd}(\omega) = \left| \frac{X_d(\omega)}{X_c(\omega)} \right| = \left| \frac{\omega_d^2}{\omega_d^2 - \omega^2 + i \frac{\omega \omega_d}{Q_d}} \right|. \quad (2.100)$$

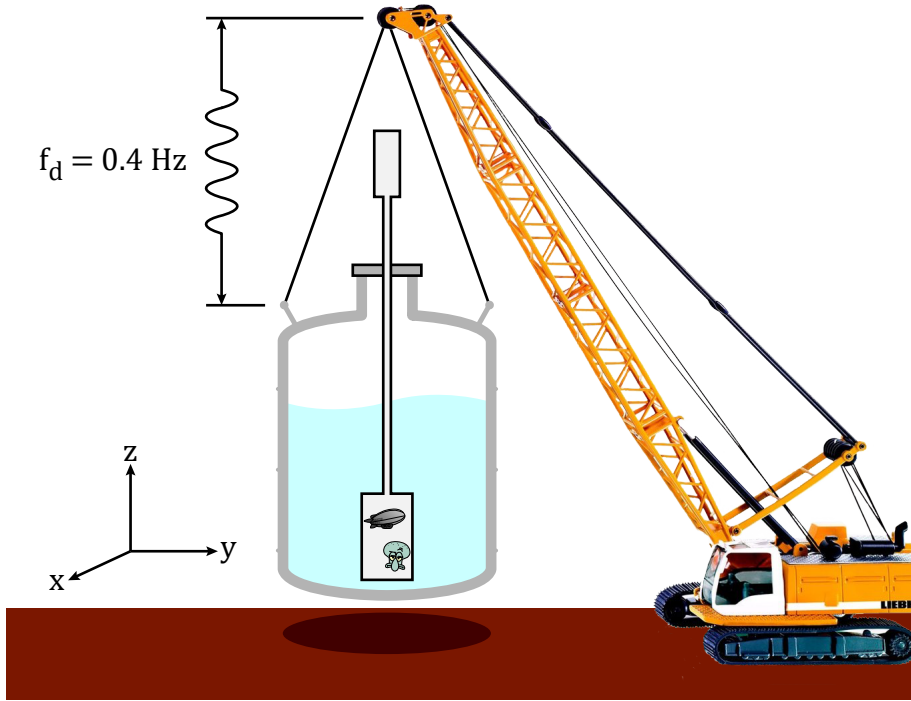


Figure 2.12: To keep external vibrations from reaching the experiment, we hoist the Helium Dewar off the floor using a crane and bungee cord. This acts as a 2nd order filter on movement of the crane beyond the resonance of the hanging Dewar $f_d \approx 0.4 \text{ Hz}$.

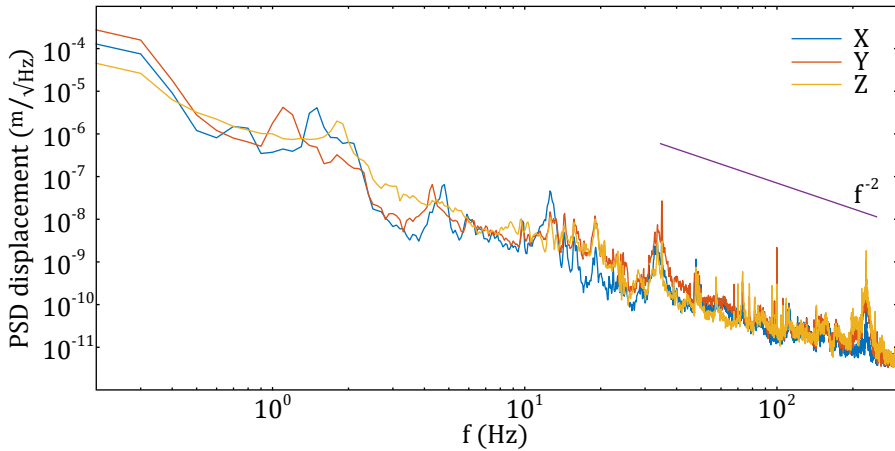


Figure 2.13: The movement of the hanging Helium Dewar, measured with geophones. The frequency resolution is $\Delta f = 0.1 \text{ Hz}$. Each PSD is an average of 10 PSDs.

We used $\gamma_d = \frac{m_d \omega_d}{Q_d}$ and $\omega_d = \sqrt{\frac{k_d}{m_d}}$.

We note that $H_{cd}(\omega \ll \omega_d) = 1$. At low frequencies, the Dewar follows the crane motion exactly. Also, $H_{cd}(\omega \gg \omega_d) = \left(\frac{\omega_d}{\omega}\right)^2$. Above the resonance of the Dewar, the Dewar-bungee cord (mass-spring) system serves as a second order filter on movement of the crane. The lower ω_d , therefore, the better vibrations from the crane are isolated from the Dewar.

The bungee cord stretches approximately $u = 1.5$ meters before the Helium Dewar comes off the floor, from which the resonance frequency follows:

$$f_d = \frac{1}{2\pi} \sqrt{\frac{g}{u}} \approx 0.4 \text{ Hz}, \quad (2.101)$$

with $g = 9.8 \frac{\text{m}}{\text{s}^2}$ the gravitational acceleration.

We measured the motion of the hanging Helium Dewar $S_{x,\text{Dewar}}$ in the three spatial directions using three geophones, see figure 2.13.

The experimental setup is mounted on a dipstick and stuck in the liquid Helium, so the movement of the Dewar will affect the movement of the Lead Zeppelin. Let us assume that the motion of the Dewar equals the motion of the experiment $S_{x,E} = S_{x,\text{Dewar}}$. Then, the displacement noise of the Zeppelin due to the motion of the Dewar is

$$S_{x,\text{zep}}^{1/2}(\omega) = S_{x,E}^{1/2}(\omega) H(\omega). \quad (2.102)$$

Here, $H(\omega)$ is the same as $H_{cd}(\omega)$ from equation (2.100), but now using the Zeppelin's resonance frequency ω_0 and quality factor Q , instead of the Dewar parameters.

In some limiting cases of interest, we rewrite $H(\omega)$ to

$$H(\omega) = \begin{cases} 1, & \omega \ll \omega_0, \\ Q, & \omega = \omega_0, \\ \left(\frac{\omega_0}{\omega}\right)^2, & \omega \gg \omega_0. \end{cases} \quad (2.103)$$

For frequencies much smaller than a Zeppelin resonance ω_0 , the Zeppelin will follow the motion of the Dewar exactly. However, since we are assuming that the Dewar and the experiment move together rigidly, there is no motion between the Zeppelin and the excitation coil and the levitation coils. Therefore, this causes no flux change in the pickup coil, and this is no problem. For frequencies much bigger than ω_0 the motion of the Dewar is damped to second order.

The big problem lies with the motion of the Dewar at ω_0 . Here, $H(\omega_0) = Q$: the Dewar motion is enhanced by Q . In order for this to be no problem, the motion of the Dewar has to be at least Q times lower than the Zeppelin motion of interest.

When we want to measure the thermal motion as described in section 2.7.3, which is $S_{x,Th}^{1/2}(\omega_0) = 1.2 \times 10^{-9} \frac{\text{m}}{\sqrt{\text{Hz}}}$ at $\frac{\omega_0}{2\pi} = 10$ Hz and $Q = 3000$ (equation (2.93)), the external vibrations must stay below

$$S_{x,E}^{1/2}(\omega_0) < \frac{1}{Q} S_{x,Th}^{1/2}(\omega_0) = 3.9 \times 10^{-13} \frac{\text{m}}{\sqrt{\text{Hz}}}. \quad (2.104)$$

When it is the gravitational interaction as described in section 2.7.4 that we want to measure, where we found that $S_{x,g}^{1/2}(\omega_0) = 2.8 \times 10^{-11} \frac{\text{m}}{\sqrt{\text{Hz}}}$ (equation (2.98)), the criterion gets even more stringent:

$$S_{x,E}^{1/2}(\omega_0) < \frac{1}{Q} S_{x,g}^{1/2}(\omega_0) = 9.4 \times 10^{-15} \frac{\text{m}}{\sqrt{\text{Hz}}}. \quad (2.105)$$

At present, we can see in figure 2.13 that the Dewar motion at 10 Hz is about $10^{-9} - 10^{-8} \frac{\text{m}}{\sqrt{\text{Hz}}}$.

The seismically isolated islands in the newly constructed measurement hall of Leiden University, which will be opened shortly, have been measured to have a displacement noise of $\sim 2 \times 10^{-10} \frac{\text{m}}{\sqrt{\text{Hz}}}$ at 10 Hz[36]. This would make our vibration sensitive measurements a lot closer to being doable, but it is not quite enough yet. What is more, the vibration isolation system of Advanced LIGO, most likely the best in the world, reaches an impressive displacement noise level of under $10^{-12} \frac{\text{m}}{\sqrt{\text{Hz}}}$ at 10 Hz[37]. It is clear that we need to revert also to other measures than just vibration isolation if we want the Lead Zeppelin to be a useful force sensor.

In the case of the thermal noise, we can go to a higher frequency: the thermal noise peak drops as $\omega_0^{-3/2}$, but simultaneously, the Dewar motion falls off as ω^{-2} , and life gets easier as $\sqrt{\omega_0}$. Similarly, lowering the Zeppelin's mass means increasing the thermal noise, but the Dewar motion is unaffected; it then gets easier as $\sqrt{m}$. A higher Q increases the thermal noise peak as $\sqrt{Q}$, but the Zeppelin motion due to the Dewar motion goes as Q ; it gets harder as $\sqrt{Q}$.

In the case of the gravitational measurement, tinkering with ω_0 , m or Q has no net effect. Here, the easiest solution is to use longer measurement times T_m , which scales as $\sqrt{T_m}$. But, in order for these not to become unrealistically long, we can additionally go for a larger source mass M (linear scaling), at a smaller distance r (scaling r^{-3}), with a larger amplitude a (linear scaling).

A final remark on the vibrations reaching the Lead Zeppelin is in order. In the above we have been assuming that the experiment moves rigidly with the Dewar. We can get away with this only to a certain extent. The levitation coils are tightly held in place by the brass clamp, which is in turn tightly screwed to the Niobium tube in which it is put, which is in turn tightly screwed to the dipstick (figure 2.16). The resonances of all of these stages are likely to be a lot higher in frequency than the Zeppelin resonances that we measure, so we can assume that the stages' transfer functions roughly equal unity (eq. (2.100)). The motion of the dipstick, then, is fed one-to-one to the levitation coils.

The dipstick itself, however, is essentially a long rod with a mass (the experiment) attached to its end, clamped on one end to the Dewar. It is possible for the boiling off of Helium, a process known as boiling noise[38], to set the dipstick in motion. At the moment, we do not know how serious this contribution to the displacement noise is, but we keep in mind that this process exists.

2.9 Measures Against Electromagnetic Noise

A SQUID is a notoriously sensitive magnetometer, with a sensitivity of typically $\sim 1 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$. To give an idea of the smallness of this: $1 \mu\Phi_0$ is equal to the amount of flux through the bottom of a coffee cup caused by a refrigerator magnet 10 km away. It is clear that we must take special care to shield external (magnetic) noise.

A major cause of noise is the current source that we use to power the coils in the experiment. Any noise in the current creates flux noise in the levitation coils, which is picked up by the pickup coil and fed to the SQUID. In the ideal case that the pickup coil is located exactly in the middle between the levitation coils, both levitation coils are wound exactly identically, and there is no crosstalk to the leads between pickup coil and SQUID, this noise is not picked up. Real life, however, is of course not so ideal.

Say, for instance, that one of the levitation coils has one winding more than the other and that the only noise is the current noise specified by the manufacturer of the current source[39] $I_n = 0.1 \text{ mA}_{\text{RMS}}$. For typical numbers of the experiment $R_{\text{winding}} = 2.5 \text{ mm}$, $R_p = 375 \mu\text{m}$, and taking the distance between the extra winding and the pickup coil as the largest possible in our setup (i.e. the least-bad-case scenario) $l = 3.75 \text{ mm}$, we find that the mutual inductance $M = 10^{-11} \text{ H}$ (by equation (2.14)) and the noise in the SQUID (by equation (2.22)):

$$\Phi_{SQ,n} = 50 \text{ m}\Phi_{0,\text{RMS}}. \quad (2.106)$$

Clearly, this is bad — and this is the best case scenario!

Indeed, in our case we quickly found that the amount of noise coupling into the SQUID is many times worse. It is in fact so bad, that the SQUID modulation is completely drowned in the noise, which renders the SQUID unusable. We took several measures to overcome this problem, which can be seen in figure 2.15.

By putting the levitation coils in the Persistent Mode, a lot of the noise troubles are dealt with. This is explained in more detail in chapter 3.

We took care to make the experiment sit inside a closed cage of Faraday. This mitigates electric noise, like radios and cell phones. Inevitably, the wiring to the coils and the wiring to the SQUID punch a hole in the cage of Faraday, which we kept as small as possible. With an Aluminium clamp, the dipstick is securely connected to the Helium Dewar with as low an electrical impedance as possible, making the dewar part of the cage of Faraday.

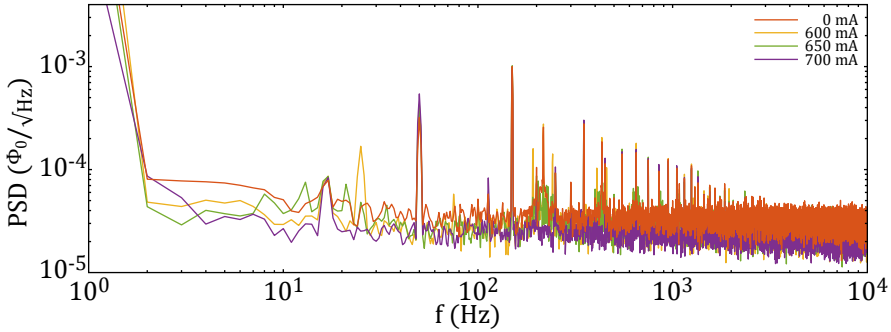


Figure 2.14: The PSDs of the SQUID signals with the levitation coils in the persistent mode, after installing the electromagnetic shielding, for some different levitation currents. Even though the noise floor is quite high for a SQUID at $\sim 30 - 40 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$, it does not change with changing I_{lev} (nor does it change when the current source is completely detached).

The experiment and the SQUID are each put in a Niobium tube. The wires from the pickup coil to the SQUID are led between these Nb tubes through a thin Lead capillary. Additionally, the vacuum pot of the dipstick is covered in a layer of Niobium foil. All of these measures together mitigate magnetic noise.

The SQUID wires going from the SQUID in the vacuum pot to a connector on the warm side of the dipstick are comprised of 4 twisted pairs jointly shielded by a CuNi-braiding. The SQUID read-out box¹ is placed directly on the connector.

The wires going from the coils to the outside world, are comprised of twisted pairs jointly shielded by a CuNi-braiding from the vacuum pot to a connector on the warm end of the dipstick. From there on, they are shielded per twisted pair, and jointly shielded as well. The coil wiring ends in a specially designed breakout box. In this breakout box, each twisted pair is run through a line filter² and each separate wire is run through a π -filter³ as well. The current source is connected via shielded twisted pairs, which are wound around ferrite rings.

For the wires designated to the excitation coil, an extra filter measure is taken: in addition to the line filter and π -filters, a transformer is added that electrically separates the function generator from the excitation coil. This is just an extra safety feature, made possible by the fact that the excitation coil will carry AC signals. The transformer⁴ has a flat frequency response between 20 Hz and 20 kHz. The function generator is connected to the breakout box with a BNC cable wound around a ferrite ring.

Figure 2.14 shows the SQUID noise at a few different levitation currents. The noise level is the same for all currents. Completely detaching the current source from the experiment, also does not change the noise level.

Figure 2.16 shows the actual implementation of the experiment.

¹Magnicon GmbH, XXF-1-6/3 SQUID Electronics.

²Epcos, TDK Europe GmbH, B84111AB60 filter, 6A, chassis.

³Tusonix, Inc., 4209-053LF filter, pi-section, 5500PF, 350V, 10A.

⁴JensenTM Transformers, Inc., JT-MB-CPC microphone transformer.

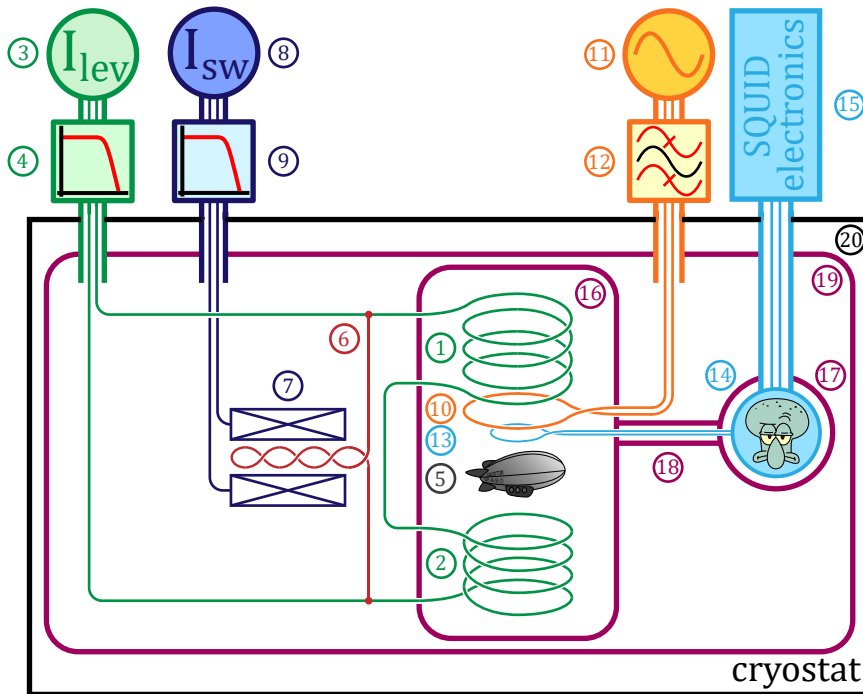


Figure 2.15: A more complete schematic of the experimental setup. We see the **upper** (1) and **lower** (2) **levitation coils**, connected to a **current source** (3). Added is **filtering** (4), which consists of a line filter and per wire a π -filter. Between the levitation coils we find the **Lead Zeppelin** (5). The levitation coils are shorted with a **Niobium wire** (6), which is twisted around itself and stuck in the **switch coil** (7) of the Persistent Current Switch (see chapter 3). By running a current I_{sw} through the switch coil with a **current source** (8), the Niobium shortcut wire is brought to its resistive state and a current can be injected into the levitation coils. Even though the Niobium shortcut wire is twisted around itself, thereby minimizing the mutual inductance between it and the switch coil, we still add **filtering** (9) to make sure there is as little noise as possible coupling into the levitation coils. This filtering consists of a line filter and per wire a π -filter. The **excitation coil** (10) is connected to a **function generator** (11) which is **filtered** (12) by a line filter, per wire a π -filter and additionally a transformer. The Zeppelin motion is picked up by a **pickup coil** (13) connected to a **SQUID** (14), which is read out with **SQUID electronics** (15). The levitation coils, excitation coil, pickup coil and Lead Zeppelin are placed inside **superconducting Niobium shielding** (16). The SQUID is also secured inside **Niobium shielding** (17). The wires from the pickup coil to the SQUID are led between the two shieldings through a **superconducting Lead capillary** (18). Around the whole cold part of the setup, we place an extra **Niobium shield** (19). We make sure that the Dewar in which we do our experiments (20) is securely attached to the shieldings of the various wires that leave the experiment to the outside world.

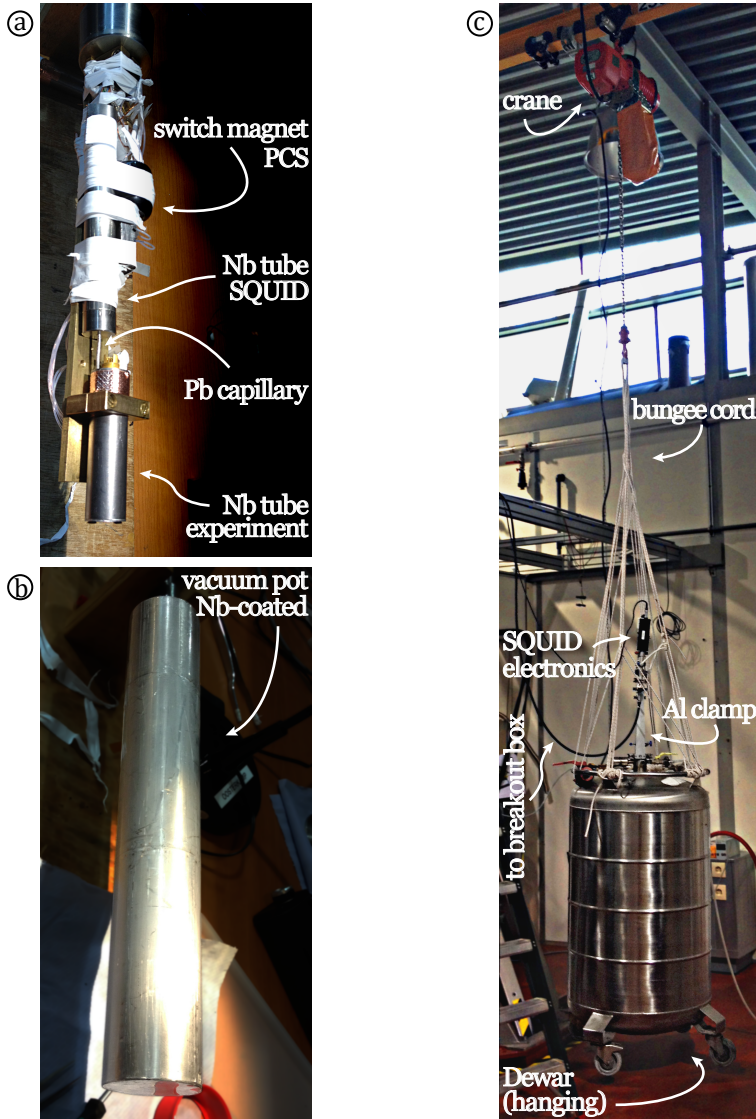


Figure 2.16: (a) The experiment as in figure 2.4 is placed in a Niobium tube and mounted on a dipstick. Via a Lead capillary, the wires of the pickup coil go to another Niobium tube in which the SQUID is placed. Also visible is the switch coil of the PCS. (b) We place a vacuum pot around the experiment, which is coated with Niobium foil. This way, the non-shielded parts of wiring are still shielded. (c) The dipstick is stuck in a Helium Dewar, which is hoisted off the floor with a crane and bungee cord. The SQUID electronics are placed directly on the dipstick. The dipstick and Dewar are connected to each other with an Aluminium clamp.

The following chapter has been published as: B. van Waarde, O. Benningshof and T. Oosterkamp, “A Magnetic Persistent Current Switch at milliKelvin Temperatures”, *Cryogenics*, vol. 78, p. 74, 2016.

In appendix B we explore whether we should worry that the levitation coils in the persistent mode, obeying flux conservation, cancel the excitation coil’s drive of the Zeppelin, and the flux in the pickup coil coming from the Zeppelin.

Chapter 3

A Magnetic Persistent Current Switch at milliKelvin Temperatures

We report the development of a magnetically driven Persistent Current Switch operated in a dilution refrigerator. We show that it can be safely used to charge a 60 mH coil with 0.5 A at 11 mK, which heats up the dilution refrigerator to 60.5 mK. Measurements at 4 K on a 440 μ H coil reveal a residual resistance of $R \leq 3.3$ p Ω .

3.1 Introduction

There are many applications in which a stable, low noise magnetic field is desired, such as in MRI, NMR or qubit studies. An elegant way of establishing such a magnetic field is by the use of a Persistent Current Switch (PCS). In a PCS a superconducting coil is shunted by a superconducting shortcut such that together they form a closed resistanceless circuit. The coil can be charged by briefly switching the shortcut to the resistive state. The circuit then becomes an RL -circuit, and a power source can be connected to inject a current into the coil. Back in the superconducting state, the flowing current is in principle stable and low-noise, and, hence, so too is the magnetic field induced by the current in the coil.

The switching of the shortcut can be accomplished in a number of ways: one can heat the shortcut to above its critical temperature, as is most often done for large magnets, or one can create a magnetic field higher than the shortcut's critical field[41–45], or even mechanically interrupt it[46, 47].

We intend to do SQUID-based experiments on the mixing chamber stage of a dilution refrigerator cryostat, which means that aside from being sensitive to magnetic noise, we are also concerned about heat input. It is our wish to place a PCS close to our experiment on the mixing chamber stage such that the wiring between PCS and experiment can be kept short — shorter wires are less

susceptible to noise and also mechanically less vulnerable — while keeping the heat input manageable. The magnetic PCS presented in this paper provides us with a solution that suits our needs.

3.2 Design and Fabrication

We set out to design a PCS that:

1. Can be operated in a dilution refrigerator, i.e. an environment with very little cooling power, on the order of 1 μW .
2. Introduces only a low (ideally zero) resistance such that it may be used with small coils of 0.1 mH – 0.1 H and still yield a long lifetime τ .
3. Can be charged with a user-adjustable current in the range of 0.01 A – 1 A.
4. Has reasonable dimensions, preferably smaller than 5 cm.

To our knowledge, a PCS that combines all of these features has not yet been constructed.

In a magnetic PCS we distinguish two coils. The experiment coil L_{exp} , which is to be charged with a current I_{exp} and used to perform the experiment of choice, and the switch coil L_{sw} with current I_{sw} , which is used to bring a superconducting shortcut to the normal state. Figure 3.1 gives a schematic overview.

We fabricate experiment coils from 100 μm diameter NbTi wire with a 13 μm Formvar insulating layer. We measure the inductances of our experiment coils by integrating the voltage response upon charging them:

$$\int V(t)dt = L_{exp}\Delta I_{exp}. \quad (3.1)$$

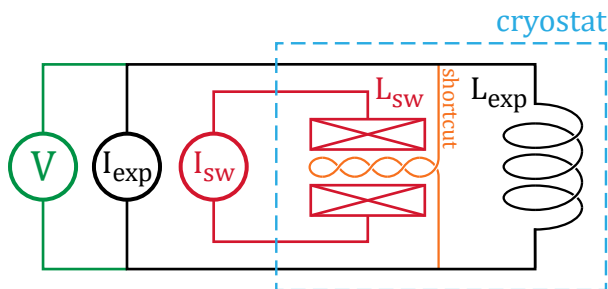


Figure 3.1: In black the basic electrical circuit for making a magnetic field with a coil L_{exp} and a current source. Added are a **superconducting shortcut** that is brought to its resistive state by a **switch coil** L_{sw} . The response is measured by a **voltmeter**. The **blue dashed box** contains the parts that are at low temperature inside a cryostat, either thermally attached to the mixing chamber stage of a dilution refrigerator, or in a vacuum dipstick inserted in a liquid Helium Dewar.

The experiment coils discussed in this paper have inductances $L_{exp} = 60$ mH and $L_{exp} = 440$ μ H. For convenience, we charge the experiment coil from empty $I_{exp} = 0$ A to $I_{exp} = I_0$ in a single step and we power up the switch coil as fast as our setup allows for. The energy dissipated in the shortcut while charging then totals $\frac{1}{2}L_{exp}I_0^2$, as we will see later.

Dissipation can also come from other sources. There is Ohmic dissipation in the current lines towards the coils, which we find to be negligible if care is taken to use superconducting wiring from 4 K to the mixing chamber stage of the dilution refrigerator and if care is taken to thermalize the wiring well. Further, the rapid change in magnetic field in both the experiment coil and the switch coil may cause dissipation due to eddy currents that are generated in normal metals in their vicinity.

The superconducting shortcut is made of an insulated Niobium wire, 50 μ m in diameter. We twist the wire around itself, such that the mutual inductance to the switch coil is minimized, thereby minimizing the noise input through this channel. We have chosen to make the shortcut out of Niobium because of its high critical current density and critical temperature and its relatively low critical field. This means that at the currents we intend to use, quasiparticle dynamics are of no concern, and that tests in liquid Helium are possible.

All superconducting connections are made by spot welding[48] the wires to Niobium sheets of 100 μ m thickness. In order to get a good superconducting connection we strip the wires of their Formvar insulation with a knife and clean them with IPA. We clean the Niobium sheets by sanding them lightly with sand paper and then wiping them with IPA. We spot weld the NbTi wires to the Nb sheets with 20 – 25 Watt-seconds and the thinner Nb shortcut wire with 8 – 10 Watt-seconds. The superconducting closed circuit comprised of the experiment coil and the Nb shortcut wire, contains two spot welded joints (Figure 3.1). To realize a residual resistance as low as possible, we use about 10 spot welds per wire, and furthermore we let the wires cross one another on the Nb sheet, such that they are spot welded to the Nb sheet, and also to each other. We have also tried to laser weld the connections, but this resulted in higher contact resistances as well as less mechanical stability.

The switch coil should be able to deliver at least $B_{c2} = 400$ mT, the upper critical field of Niobium at 0 K[49]. We construct it from 100 μ m diameter Copper clad single core NbTi wire, 62 μ m diameter NbTi core, with a 13 μ m insulating Formvar layer. The Copper cladding is necessary to thermalize the switch coil. The spindle on which the switch coil is wound is made from PEI and allows for a coil with an inner diameter of 9 mm, an outer diameter of 22 mm and a length of 18 mm. In the center of the spindle we leave a hole of 3 mm diameter through which the shortcut is put. For sturdiness, we give the spindle 4 mm thick walls on either side of the coil and apply a layer of Stycast 2850FT to the whole after winding. We were able to put $N = 8672$ windings on the spindle. The inductance of the switch coil is about $L_{sw} = 0.5$ H.

The NbTi wire leads of the switch coil are twisted and led from the mixing chamber stage of the cryostat, where the switch coil is mounted, uninterrupted to the 4 K stage, thermalized at the intermediary stages on Copper bobbins. From 4 K to room temperature, the wires are from Copper. We should avoid sending too high currents, because these could cause dissipation in the non-superconducting parts of the wiring. We aimed for a switching current I_{sw} on the order of 1 A, at which the switch coil makes a field of

$$B_{sw} = \frac{\mu_0 N I_{sw}}{l} = 600 \text{ mT} > B_{c2}. \quad (3.2)$$

3.3 Results and Discussion

We placed the PCS with an experiment coil of $L_{exp} = 60 \text{ mH}$ on the mixing chamber stage of a dilution refrigerator which reached a minimum temperature of 10.5 mK. After some tweaking, we found that we could charge and discharge the experiment coil without dangerously warming up the cryostat by using a switch current of $I_{sw} = 2 \text{ A}$ through the switch coil L_{sw} during 2 seconds. Figure 3.2 shows a typical voltage response for the case that we charge L_{exp} from $I_{exp} = 0$ A to 0.5 A.

If we left the switch activated for longer than $\sim 10 \text{ s}$, we saw a sudden, vigorous increase in temperature. The cause of this is, we suspect, the non-superconducting connection of the switch coil's leads at the 4 K stage of the cryostat. To avoid having to recondense the ^3He into the dilution refrigerator every time we switch, we therefore limit ourselves to quick switches. Alternatively, one can choose to charge the experiment coil at a higher temperature when there is more cooling power available and cool further down afterwards. As we will see, the superconducting contacts have such a low resistance that there is ample time before the stored current is appreciably diminished, making this a viable possi-

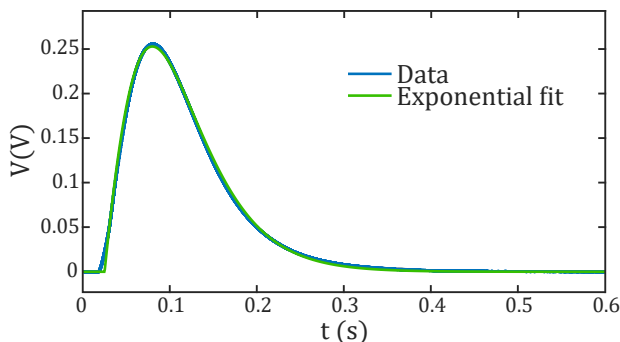


Figure 3.2: The voltage response of an $L_{exp} = 60 \text{ mH}$ coil at $T = 11 \text{ mK}$ when charged with $I_{exp} = 0.5 \text{ A}$ using $I_{sw} = 2 \text{ A}$. The shape of $V(t)$ is determined by the time-dependent normal state resistance $R_{pcs}(t)$ of the shortcut; assuming it be exponential, eq. (3.3), we find $R_{ns} = 1.4 \text{ } \Omega$ and $\tau_{ns} = 73 \text{ ms}$.

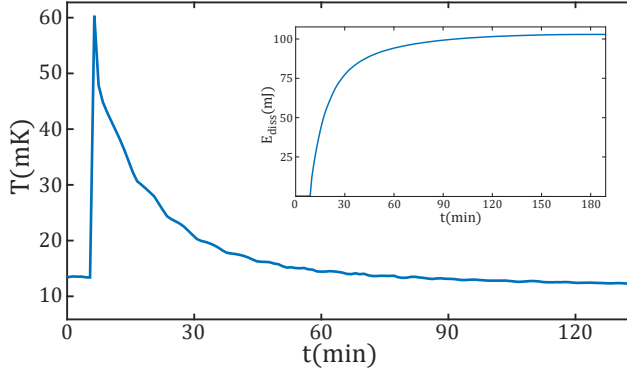


Figure 3.3: The temperature of the mixing chamber stage versus time, $\Delta t = 1$ minute. When charging the experiment coil of $L_{exp} = 60$ mH to $I_{exp} = 0.5$ A, the temperature increases to 60.5 mK. The inset shows the dissipated energy E_{diss} in time, which levels off to 103 mJ, and which we estimate by converting temperature to cooling power and integrating over time.

bility. However, we wanted to verify that it is possible to charge a coil even at the lowest temperature attainable in our cryostat without introducing a worryingly high heat input.

The shape of $V(t)$ can be understood by assuming that the normal state resistance of the superconducting shortcut arrives at its final value R_{ns} exponentially with a time constant τ_{ns} upon switching[41]

$$R_{pcs}(t) = R_{ns}(1 - e^{-t/\tau_{ns}}). \quad (3.3)$$

The values of R_{ns} and τ_{ns} depend on a multitude of factors including the switching field B_{sw} ($\propto I_{sw}$), temperature and cooling power.

The general solution for the PCS's voltage response with time-varying $R_{pcs}(t)$ when charged $I_{exp} = 0$ A $\rightarrow I_0$ is given by

$$V(t) = I_0 R_{pcs}(t) e^{-\int_0^t \frac{R_{pcs}(t')}{L_{exp}} dt'}. \quad (3.4)$$

Fitting this to the measurement, using the exponential $R_{pcs}(t)$ of eq. (3.3), we find that $R_{ns} = 1.4 \Omega$ and $\tau_{ns} = 73$ ms.

The energy that is dissipated in the shortcut due to (dis)charging of the experiment coil is

$$E_{diss}(t) = \int_0^t P_{diss}(t') dt' = \int_0^t \frac{V^2(t')}{R_{pcs}(t')} dt'. \quad (3.5)$$

For large t , it is easy to show that $E_{diss}(t)$ converges to $\frac{1}{2} L_{exp} I_0^2$: the energy that needs to be dissipated in the shortcut is exactly the energy that is stored in the experiment coil. The heating effect from this contribution to the dissipation can be made smaller by charging the experiment coil in a number of steps rather

than in a single step as is done now. However, in our experiment it is not the dominating factor — presumably because eddy currents near the switch coil dominate, which we shall explain shortly.

During (dis)charging of the coils, we monitor the temperature of the mixing chamber stage onto which the PCS is mounted. Figure 3.3 shows the temperature of the mixing chamber stage as a function of time during the switch of figure 3.2. At the moment of switching, the temperature quickly increases to 60.5 mK and gradually decreases again afterwards. Our calibration of the cooling power versus temperature $P_{cool}(T)$ allows us to convert the temperature to power dissipation, and, consequently, integration yields the energy extracted by the dilution refrigerator from the mixing chamber stage during the temperature peak. We estimate $E_{diss} \approx 100$ mJ, see the inset in figure 3.3.

Note that this is much more than the energy dissipated in the shortcut, $\frac{1}{2}L_{exp}I_0^2 = 7.5$ mJ when $L_{exp} = 60$ mH and $I_0 = 0.5$ A. (Also, E_{diss} is much less than the energy in the switch coil $\frac{1}{2}L_{sw}I_{sw}^2 = 1$ J, for $L_{sw} = 0.5$ H and $I_{sw} = 2$ A, which we expect to be dissipated (mostly) in the internal resistance of the current source.)

We attribute the measured E_{diss} to eddy currents: for the sake of thermalization our coils are securely fastened to the mixing chamber stage of the dilution refrigerator, which is a gold-coated copper disk of 1 cm thickness. Being a normal metal, induced eddy currents are dissipated here and cause a heating that scales with $(dB/dt)^2$ [50]. A straightforward way of reducing this heat input would be to ramp the switch current I_{sw} up and down more gradually and to divide the charging of the experiment coil into several small steps. The eddy currents could be further reduced by keeping the switch and experiment coils away from normal metals or encasing them in a superconducting shield and use normal metal only to provide cooling to the coils.

We measured the quality of the spot welded superconducting joints by evaluating the residual resistance R in a long-lasting measurement. Because of the exceptionally low resistance of the joints, there is no measurable decrease in I_{exp} even after a few days. We therefore placed a PCS with an experiment coil of $L_{exp} = 440$ μ H in a vacuum dipstick inserted in a liquid Helium Dewar, charged it from $I_{exp} = 0$ A to 350 mA, and left it untouched for a little under 17 days before discharging. Figure 3.4 shows the two voltage responses. Fitting to equation (3.4) gives $R_{ns} = 375$ m Ω and $\tau_{ns} = 1.1$ ms.

The integration of $V(t)$, eq. (3.1), when discharging $I_{exp} = I_W \rightarrow 0$ A yields the current still flowing in the experiment coil I_W after waiting a time t_W . It is related to the injected current I_0 as

$$\frac{I_W}{I_0} = e^{-t_W/\tau}, \quad \tau = \frac{L_{exp}}{R}. \quad (3.6)$$

The amount of current still flowing thus allows us to measure the residual resistance in the superconducting circuit R . We measure $I_W = 346$ mA, which translates to $\tau \geq 4.2$ years and $R \leq 3.3$ p Ω .

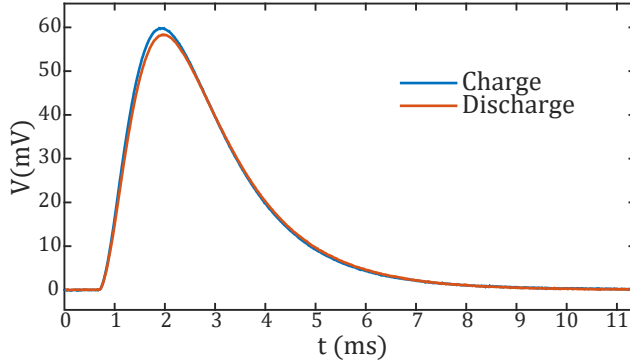


Figure 3.4: The voltage responses at 4 K when charging an experiment coil of $L_{exp} = 440 \mu\text{H}$ to $I_{exp} = 350 \text{ mA}$, waiting for a time $t_W = 16.8$ days and then fully discharging. For the discharge voltage response we plot $-V(t)$ to emphasize how little current has been lost. We measure the current still flowing in the experiment coil to be $I_W = 346 \text{ mA}$, therefore $\tau \geq 4.2$ years and $R \leq 3.3 \text{ p}\Omega$.

3.4 Summary

We have shown that it is possible to put a 60 mH coil in the persistent mode carrying a current of 0.5 A in a dilution refrigerator at 11 mK using a magnetic Persistent Current Switch. We find that spot welding all wires to each other via Nb sheets ensures a resistance of less than $3.3 \text{ p}\Omega$.

3.5 Acknowledgements

The authors wish to thank J.J.T. Wagenaar and M. de Wit for helpful discussions and G. Koning and F. Schenkel for technical support. This research is part of the Single Phonon Nanomechanics project of the Dutch Foundation for Fundamental Research on Matter (FOM).

Chapter 4

Flying the Lead Zeppelin

In this chapter we present the experimental results that we obtained. To begin with, we will show some of the first results that were acquired with an older version of the experimental setup, which helped us develop many of the improvements of the experiment described in chapter 2.

We then continue to show the results obtained with the experimental setup of chapter 2. We will find out how the resonance frequencies of the Lead Zeppelin depend on the levitation current, and what the quality factors are of the resonances. Subsequently, we discuss the severity of external vibrations in our measurements. We end by analysing which measured resonances are the Zeppelin's natural frequencies, and which are caused by non-linear mixing amongst themselves and between them and the external vibrations.

All the measurements presented in this chapter have been performed with circular levitation coils. As we noticed while performing the experiments, we might want to change to elliptical levitation coils: combined with an aspherical Zeppelin, namely, this will push the rotational resonances to higher frequencies, and make the x and y translational resonance more distinct. We will come back to this in the discussion, chapter 5.

Several Lead Zeppelins are used in the results that are about to be shown. To keep track of which particle was used in which experiment, we will refer to them mostly by their estimated mass, but also by other distinctive qualities; see table 4.1. See figure 2.2 for a photo of the particles.

Location in figure	Mass (mg)	Size (μm)	Description
Bottom right inset	1.4	670×570	<i>Chunky</i>
Top right corner, main panel	1.6	650	<i>Almost spherical</i>
Bottom left corner, main panel	2	870×660	<i>Aspherical</i>
Top right inset	0.8	520×500	<i>Almost spherical</i>

Table 4.1: The Lead Zeppelins used in our experiments, and some properties by which we will refer to them. Based on the photos of figure 2.2.

4.1 Liftoff

Early results were obtained with a chunky Zeppelin (scraped off of a Lead block, but not torched) of size $\sim 670 \times 570 \mu\text{m}$, $m \approx 1.4 \text{ mg}$, see the bottom right inset in figure 2.2. At the time of these measurements, the PCS was not yet installed, and the pickup coil was an old transformer chip of 44 closely packed, flat spiral, Niobium windings (figure 2.3).

We did frequency sweeps and measured the SQUID signal with a lockin amplifier as described in section 2.5. The result of this can be seen in figure 4.1, where we show for three different levitation currents $I_{lev} = 0.65, 0.70, 0.75 \text{ A}$ the measured lockin amplitude R , phase Θ and a phase plot (R, Θ) . We notice that as I_{lev} is increased, the resonance frequency also increases; this is indeed

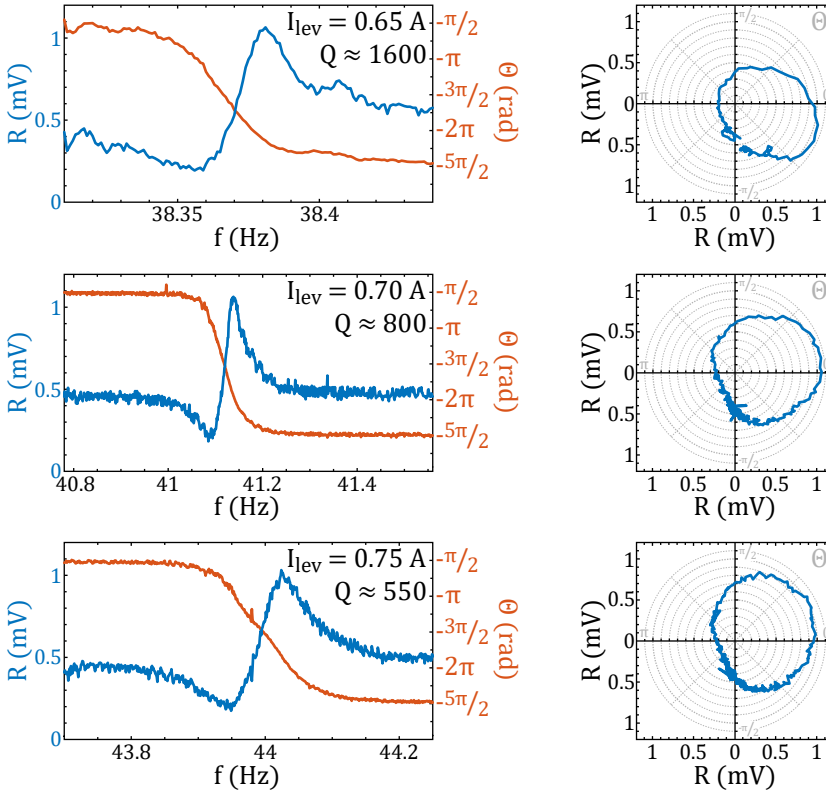


Figure 4.1: Lockin measurements of a chunky 1.4 mg Zeppelin at $I_{lev} = 0.65, 0.70, 0.75 \text{ A}$, with the settings $V_{exc} = 100 \text{ mV}$, $\Delta f = 1 \text{ mHz}$, $\Delta t = 1 \text{ s}$, $\tau_{LI} = 0.3 \text{ s}$. We see similar behaviour to the simulation of figure 2.11. The lockin amplitude output R first goes down, then up and settles to its off-resonant value again (if we were patient enough), while the phase Θ drops by 2π . The phase plots are displaced circles. The Q factors indicated in the figures were obtained with ringdown measurements.

I_{lev} (A)	f_{res} (Hz)	Q	γ ($10^{-7} \frac{N}{m/s}$)
0.65	38.37	1600	2.1
0.70	41.12	800	4.5
0.75	44.00	550	7.0

Table 4.2: The resonance frequencies and quality factors of the Zeppelin for three levitation currents, as seen in figure 4.1, acquired by ringdown measurements. The damping factors are calculated as $\gamma = \frac{m\omega_{res}}{Q}$, with $m = 1.4$ mg.

the Lead Zeppelin that we are measuring.

Due to a combination of signal attenuating factors, the Zeppelin motion was higher than the noise floor only when driven by the excitation coil.

The graphs are phenomenologically the same to what we simulated in section 2.5. R has a dip before its peak, Θ drops by 2π across the resonance and the phase plot is a moved circle.

We measure the Q s of the resonances with ringdown measurements: we ring up the Zeppelin with the function generator and measure the decay in time of the signal at resonance with the lockin. Fitting an exponential to this yields the decay time τ_{zep} and subsequently the quality factor $Q = \pi f_{res} \tau_{zep}$. We added the measured quality factors to figure 4.1.

As we increase I_{lev} , the quality factors go down. This could be due to the fact that we are close to the critical magnetic field of Lead at these levitation currents; as we increase the levitation current, a larger part of the Zeppelin becomes a normal conductor. Because of the expulsion of magnetic fields by the superconducting pickup coil, which distorts the levitating magnetic field, we do not know the exact shape of the levitating magnetic field, and therefore which fraction of the Zeppelin sits in too high a field. However, at $I_{lev} = 0.8$ A there is no Zeppelin response at all anymore, indicating that we must have been indeed very close to the critical field.

When we take a look at what the SQUID measures when we send a reference (off-resonant) signal through the excitation coil and calculate what we should have measured, we find a big discrepancy: the measured signal is a (conservative) factor of $\sim 10^7$ too low! Possibly a wire from the pickup coil to the SQUID has (partly) snapped, or some of the superconducting wiring is not superconducting, or a wirebond is oxidized. In any case, rather remarkably, this is not all bad. Without this extreme attenuation of signal, the noise coming from the current source, which is not reduced by a PCS (because we didn't have one at the time) would be far too much for the SQUID to handle and we would not have been able to measure anything at all (section 2.9).

In an attempt to make the signal bigger, we tried to get the Lead Zeppelin closer to the pickup coil by increasing the lower levitation current and simultaneously decreasing the upper one; that is, instead of $I_{up} = I_{down} = I_{lev}$, we did $(I_{up}, I_{down}) \rightarrow (I_{up} - \Delta I, I_{down} + \Delta I)$, with ΔI typically some multiple of 10 mA. This has the effect of moving the equilibrium position more towards the pickup coil. What we found is that, as the Zeppelin is brought closer to the pickup coil,

the signal strength remains about the same, but the resonance frequency goes down. We believe that this is due to the Meissner effect of the pickup coil: being effectively a superconducting slab with a small hole in the middle, it distorts the levitating magnetic field. As the Zeppelin gets closer to the pickup coil, the magnetic field gets weaker, thus giving rise to a lower resonance frequency.

To reduce this effect, we wound the new coil visible on the right in figure 2.3. Also, by making this new coil from sturdy Niobium wire, and by going straight to the input coil of the SQUID, we prevent the 10^7 mismatch from occurring.

4.2 Levitation Current versus Zeppelin Frequency

We installed a Persistent Current Switch in the experiment, and placed the new pickup coil. At first, we put each levitation coil in its own, separate persistent mode. We wanted to keep the possibility open to charge each coil with a different current, thereby having control over the equilibrium position of the Lead Zeppelin. However, we were unable to find any Zeppelin resonances. We suspect that this is due to the effect of flux reduction of the Zeppelin flux as explained in appendix B, despite the fact that this should only make for a factor of $\sim 5 - 10$ difference in signal. We then put the levitation coils in series, in anti-Helmholtz configuration (that is, with opposite polarity), controlled by one PCS.

We fabricated the Lead Zeppelin which is roughly spherical with diameter 650 μm , $m = 1.6$ mg, of figure 2.2's top right corner. We then swept the levitation current in the range $I_{lev} = 200 \rightarrow 450$ mA in steps of 10 mA. At each levitation current, we measured the SQUID signal 10 consecutive times for 10 seconds, calculated the Power Spectral Density (PSD) of each 10 s time trace, and took the (quadratic) average of the resulting 10 PSDs. The result is shown in figure 4.2.

We look whether among the many peaks in the PSD we find moving peaks, as they are tell-tale signs of a Zeppelin resonance. There is indeed movement of some peaks, two of which have been indicated with coloured lines in the figure. What is immediately clear is that the peaks are not moving linearly with I_{lev} , which is what we predicted in section 2.4.2. However, we remember figure 2.8: at the lower levitation currents, we are in the non-linear part of the force curve. It is then not at all strange that we see a deviation from linearity also in the change of resonance frequencies.

Furthermore, we notice that peaks start to emerge from $I_{lev} = 250$ mA upwards. Below this levitation current, the gravitational force is still stronger than the levitation force.

Above $I_{lev} = 410$ mA, the signal begins to disappear. We attribute this to the fact that at this levitation current, the levitating magnetic field strength starts to become similar to the critical magnetic field of Lead at 4.2 K. We calculated in equation (2.41) that

$$B_{c,\text{Pb}}(4.2 \text{ K}) = 13.9 \text{ mT.} \quad (4.1)$$

The levitating magnetic field at the underside of the Zeppelin, $R_{zep} = \frac{1}{2} \times 650 \mu\text{m}$, for $I_{lev} = 410 \text{ mA}$ is

$$B_{lev} = G_z(410 \text{ mA}) \times (-R_{zep} + z_{eq}) = 11.5 \text{ mT}, \quad (4.2)$$

for the typical numbers $G_z(1 \text{ A}) = -34 \frac{\text{T}}{\text{m}}$ (eq. (2.11)), and $z_{eq} = -0.5 \text{ mm}$ (figure 2.9).

Of the many peaks in the PSD of the Zeppelin motion, some are not coming from the Zeppelin. As we are sensitive to any motion between the coils and the Zeppelin, a vibration of e.g. the dipstick might be picked up. Also, notoriously, 50 Hz is visible. Importantly, none of these non-Zeppelin resonances will move to a different frequency when we change the levitation current. Of course, making matters more complicated, the non-Zeppelin motions probably mix with the Zeppelin motion, leading to peaks at the sum and difference frequencies. As the Zeppelin resonances shift, so do the sum and difference frequencies. We go into more detail regarding this in section 4.6.

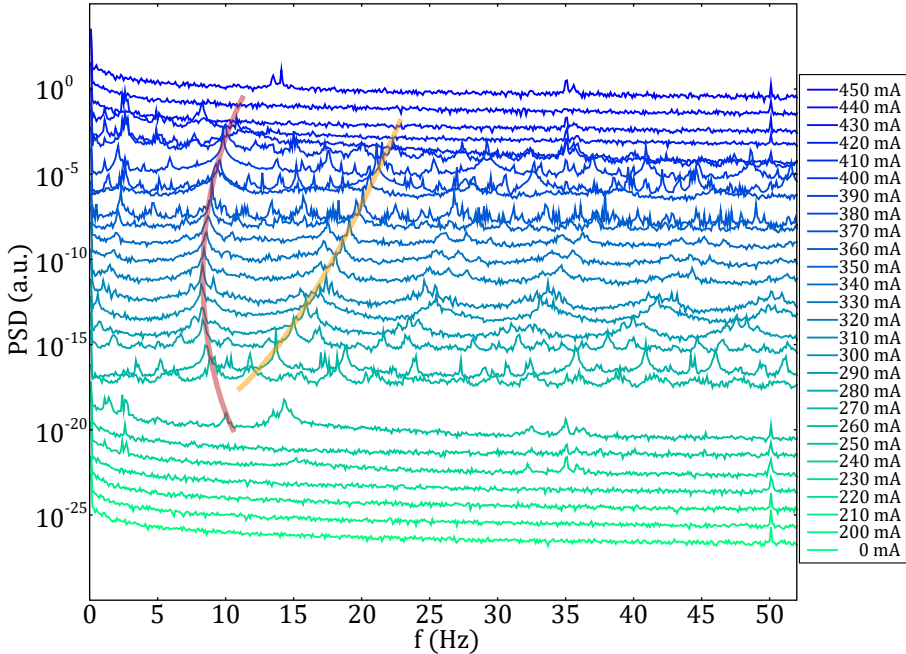


Figure 4.2: The PSDs ($\Delta f = 0.1 \text{ Hz}$) of the Zeppelin signal for different levitation currents. In this measurement, the Zeppelin is roughly spherical of $650 \mu\text{m}$ diameter and mass $m = 1.6 \text{ mg}$. The PSDs have been pulled apart to make the movement of resonances better visible. The coloured lines are meant as a guide to the eye, indicating the shift of resonances as the levitation current is changed.

4.3 Quality Factors

When we give the Lead Zeppelin a kick, its resonances will gain higher amplitudes: we observe an increase of peak heights in the PSD of the signal. The decay in time back to their rest-values is governed by the quality factor of the resonance.

We administer kicks to the levitating Zeppelin in two ways. Inevitably, whenever we change the levitation current, the levitating force on the Zeppelin suddenly changes, which changes the equilibrium position, which thus constitutes a kick. More crudely, we can actually give a (small) kick to the hanging Helium Dewar in which the experiment is put, which has the effect of shaking the levitation coils around the Zeppelin. Such a kick is delivered by gently tapping the dipstick with e.g. a screwdriver handle or a rubber hammer.

We should remark here that we cannot measure the Zeppelin motion whilst the levitation coils are being charged; if they are not in the persistent mode, the input of noise is too big for the SQUID to handle. It is for this reason that we revert to physically kicking the Dewar to get the Zeppelin in motion, such that we need not tamper with the levitation currents every time.

When the Zeppelin hovers in the non-linear part of the levitation force curve (figure 2.8), or when we kick it hard enough to drive it into the non-linear regime, we can expect that the peaks in the PSD will be coming from the 6 modes of the Lead Zeppelin (3 translational and 3 rotational) and mixing terms between these modes. Also, some will come from the experimental surroundings' resonances, and mixing terms can appear between those and the Lead Zeppelin. We study this in more detail in section 4.6.

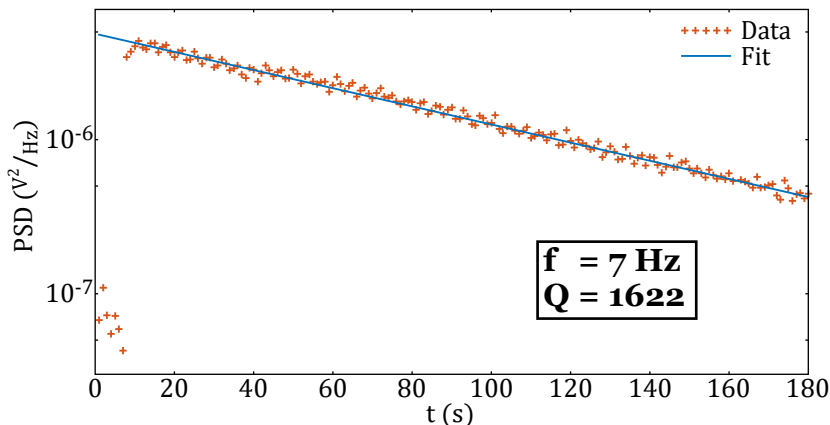


Figure 4.3: The peak height of the PSD component at 7 Hz in time when we apply a kick to the Dewar, with the 2 mg Zeppelin levitating and $I_{lev} = 350$ mA. We fit an exponential decay function $Ae^{-t/\tau}$, and calculate the quality factor $Q = \pi f_{res}\tau = 1622$. It takes a few minutes for the Zeppelin to relax to its equilibrium again.

2 mg Zeppelin				0.8 mg Zeppelin			
f (Hz)	τ (s)	Q	γ ($10^{-8} \frac{\text{N}}{\text{m/s}}$)	f (Hz)	τ (s)	Q	γ ($10^{-8} \frac{\text{N}}{\text{m/s}}$)
$I_{lev} = 300$ mA				$I_{lev} = 340$ mA			
8	89	2217	4.5	8.0	112	2806	1.4
17	43	2295	9.2	8.44	109	2901	1.5
23	45	3198	9.0	8.6	115	3112	1.4
26	68	5481	5.9	9.0	105	2972	1.5
31	41	3924	9.8	11.9	36	1338	4.5
40	67	8308	6.0	16.88	58	3089	2.7
48	38	5619	10.6	17.06	51	2710	3.2
$I_{lev} = 350$ mA				18.2	113	6479	1.4
7	74	1622	5.4	19.0	113	6728	1.4
8	83	2068	4.8	20.4	52	3349	3.1
18	81	4560	5.0	25.4	37	2958	4.3
19	78	4665	5.1	26.7	47	3964	3.4
33	53	5464	7.6	29.0	51	4629	3.1
$I_{lev} = 400$ mA				35.1	37	4120	4.3
1.9	170	1013	2.4	36.4	61	6917	2.6
3.7	66	765	6.1	$I_{lev} = 350$ mA			
5.6	54	952	7.4	17.0	65	3494	2.5
8.7	190	5191	2.1	18.0	56	3180	2.9
16.1	69	3504	5.8	18.7	81	4731	2.0
17.0	61	3246	6.6	19.6	41	2536	3.9
18.0	145	8177	2.8				
18.9	80	4753	5.0				
19.8	54	3368	7.4				
20.2	78	4946	5.1				
20.7	173	11270	2.3				
22.6	96	6788	4.2				
24.8	47	3700	8.4				
25.2	44	3463	9.1				
27.0	78	6611	5.1				
28.9	150	13628	2.7				
29.4	90	8322	4.4				
30.8	80	7640	5.1				
37.6	79	9334	5.1				

Table 4.3: The exponential decay times τ , quality factors Q and damping coefficients γ for several resonances of the Lead Zeppelin, for two different Zeppelins (left the aspherical 2 mg Zeppelin; right the almost spherical 0.8 mg Zeppelin) and some levitation currents. The τ s are obtained by giving the Helium Dewar a small kick, and fitting an exponential decay function to the ringdown of the emergent peaks in the PSD (see figure 4.3). Subsequently, we calculate $Q = \pi f_{res} \tau$ and $\gamma = \frac{2m}{\tau}$. These results are gathered from several data sets with differing frequency resolutions.

We fabricated the aspherical Lead Zeppelin that can be seen in the bottom left corner of figure 2.2. Based on its dimensions, we estimate it to have a mass $m = 2$ mg. We gave it a small kick, at $I_{lev} = 350$ mA, and recorded the PSD of the SQUID signal in time. Figure 4.3 shows the height of the peak at $f_{res} = 7$ Hz from this Zeppelin at this levitation current decreasing in time. We have fitted an exponential to it, which yields the ring-down time τ and consequently the quality factor $Q = \pi f_{res} \tau$. In this case, we find a Q of about 1600. Table 4.3 shows more quality factors obtained in this way.

The damping coefficient $\gamma = \frac{2m}{\tau}$ is a frequency independent comparator between different Zeppelins and levitation currents. It also sets the thermal force noise on the Zeppelin motion through $S_{F,Th} = 4k_B T \gamma$, which ultimately is the limiting force noise in a gravitational experiment (section 2.7.4), although this is not yet our concern as long as the external vibrations are limiting us. We added the damping coefficients to table 4.3. We see that for the resonances of the 2 mg Zeppelin, the damping is mostly around $\sim 5 - 10 \times 10^{-8} \frac{\text{N}}{\text{m/s}}$, except for 5 of them at $I_{lev} = 400$ mA who have $\gamma = 2.1 - 2.8 \times 10^{-8} \frac{\text{N}}{\text{m/s}}$. As we will detail in section 4.6, the resonances with higher damping can be considered non-linear mixing terms of these modes, and will decay more quickly.

We repeated the ‘kick-and-ringdown’ measurements with a slightly smaller Zeppelin, of almost spherical dimensions 520×500 μm . We estimate it to have a mass $m = 0.8$ mg (table 4.1). We added the obtained quality factors and damping coefficients to table 4.3.

If the dominating cause of damping comes from eddy currents in the surrounding non-superconducting metal, one would expect to see an increase in damping for increasing levitation current. However, for both particles this is not what we observe. The damping is lower for the smaller particle, which is an indication that at present, it is the surrounding Helium gas that provides the leading cause of damping. Indeed, when we use equation (2.57) in reverse, we can find out what the pressure must have been at these γ s, assuming it is the ambient Helium that’s causing the damping. For the 2 mg particle, we fill in $R_{zep} = 380$ μm (based on figure 2.2) and $\gamma = 3 \times 10^{-8} \frac{\text{N}}{\text{m/s}}$ to find that $P = 2.7 \times 10^{-2}$ mbar. The smaller 0.8 mg particle, with $R_{zep} = 250$ μm and $\gamma = 2 \times 10^{-8} \frac{\text{N}}{\text{m/s}}$, has $P = 4.1 \times 10^{-2}$ mbar. These values lie nicely in the pressure range that we did the experiments in, which was kept between $10^{-2} - 10^{-1}$ mbar for thermalization of the experiment.

4.4 External Vibrations

In our experimental setup, we find that vibrations from external sources couple to the Lead Zeppelin noticeably. Having suspended the Helium Dewar from the ceiling as discussed in section 2.8 helps a great deal, in that for instance the opening and closing of doors in the laboratory doesn’t affect the experiment,

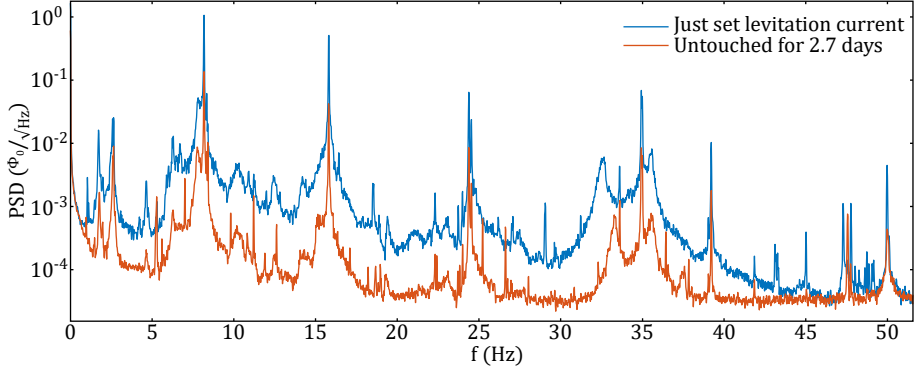


Figure 4.4: In blue the PSD of the SQUID signal after just having set the levitation current to $I_{lev} = 350$ mA. The Zeppelin signal is then very big, also off resonance. In orange, the PSD of the SQUID signal after having left the experiment untouched for 2.7 days. The signal is diminished, but it has by no means sunk into the SQUID noise. The PSDs are the average of 20 consecutive 50 s time traces ($\Delta f = 0.02$ Hz). The Zeppelin used here is the 2 mg aspherical particle.

nor does the act of sitting down in a chair; but the motion of the laboratory ceiling still couples measurably to the Dewar through the bungee cords and, subsequently, measurably to the Lead Zeppelin.

Figure 4.4 shows the difference in the PSDs of the Zeppelin signal between that when it has just gotten a jolt due to a change in levitation current, and that after having left it untouched for a long time. The signal shortly after the jolt shows higher peaks on Zeppelin resonances, but also the off-resonant signal is increased. This could be due to the non-linear nature of the levitating force, see figure 2.10. We also see that after waiting a long time, the signal has decayed somewhat, but has not nearly died out yet. In fact, we know from e.g. figure 4.3 that, depending on Q , already after a few minutes will the peaks have settled down to their rest values.

When the Zeppelin were driven solely thermally, its resonances would reach peak heights typically of around $S_{x,Th}^{1/2} \sim 1.18 \times 10^{-9} \frac{\text{m}}{\sqrt{\text{Hz}}}$ at 10 Hz (eq. (2.93)), or $S_{\Phi,Th}^{1/2} = 3.5 \times 10^{-4} \frac{\Phi_0}{\sqrt{\text{Hz}}}$. A glance at figure 4.4 reveals that currently the Zeppelin moves quite a bit more than that — about 3 orders of magnitude.

Having taken special care to shield electromagnetic noise from coupling into the levitation coils, we do not believe that this could be driving the Lead Zeppelin to these heights (section 2.9). Instead, it is much more likely that there are external vibrations, shaking about the levitation coils around the levitated Zeppelin, that cause this excitation.

That being said, there are a number of potential candidates. Evidently, as discussed, the Helium Dewar suspended from the laboratory ceiling has a certain motion, which will couple into the experiment. Likewise, the dipstick on which

the experiment is mounted, will be moving; it moves according to the movement of the Dewar, but also the boil-off of Helium will cause it to fluctuate.

From figure 4.4, we can estimate how much Zeppelin movement there remains after leaving it untouched for a long time. Equation (2.81) relates the RMS movement x_{RMS} to the PSD peak at a frequency f_p

$$x_{\text{RMS}} = \sqrt{\frac{2}{T_m \text{ NPBW}} \sum_{n \sim f_p} S_n}, \quad (4.3)$$

with in this case $T_m = 50$ s and $\text{NPBW} = 1.5$ (we used a Hann window). We let the sum extend over 5 frequency bins, of which the middle one coincides with the highest measured PSD value. Using $\frac{d\Phi_{SQ}}{dz} = 3.0 \times 10^5 \frac{\Phi_0}{\text{m}}$, and pretending that all peaks are z -resonances, we can convert the measured S_Φ to S_x . We calculate the Zeppelin movement for a bunch of prominent peaks. Also, we calculate the effective temperature of the motion with equation (2.88), namely

$$T_{\text{eff}} = \frac{m\omega_0^2}{k_B} x_{\text{RMS}}^2, \quad (4.4)$$

where $m = 2$ mg. The results are listed in table 4.4. At all peaks the motion of the Zeppelin is very large and its effective temperature is very high, compared to a thermal excitation, where for $T = 4.2$ K at 10 Hz we have $x_{\text{RMS}} = 86$ pm.

Note that we have been assuming that all the motions are in the z -direction: we did not calculate $\frac{d\Phi}{dr}$ in section 2.4.1, although we do know that it has to be smaller than $\frac{d\Phi}{dz}$ on account of the levitating magnetic field being smaller when moving in the r -direction as opposed to in the z -direction (see figure 2.6). If a motion is not in the z -direction, then, its calculated value of S_x is larger, and, subsequently, so are the values for x_{RMS} and T_{eff} .

f (Hz)	x_{RMS} (nm)	T_{eff} (10^6 K)
2.64	22	0.020
8.18	339	44
15.8	107	16
24.38	23	1.9
24.54	6.0	0.12
34.94	21	3.0
35.0	16	1.7
39.22	4.2	0.16
47.56	2.3	0.068

Table 4.4: The movement of the 2 mg Zeppelin, taken from the ‘Untouched for 2.7 days’ data of figure 4.4. We assume all motions to be along the z -axis to get x_{RMS} . These numbers are much higher than a thermal excitation $x_{\text{RMS}} = 86$ pm at 10 Hz and at 4.2 K, which is also reflected by the effective temperatures of the motions.

4.5 Duffing Behaviour

Figure 4.5 shows two time traces of the Zeppelin signal. The top one shows the ‘long-term’ behaviour of the spherical 1.6 mg Zeppelin at $I_{lev} = 400$ mA, a good while after setting the levitation current. Notice the ‘square’ shape of the signal, which shows the non-linearity of the Lead Zeppelin. The signal goes on like this forever. It does not decay, nor does it change appreciably otherwise, for days on end: this motion is driven, and most likely by the Dewar.

The bottom time trace is measured right after having set the levitation current to $I_{lev} = 300$ mA with the aspherical 2 mg Zeppelin. We attenuated the signal $10\times$ here, to keep our measurement devices happy. We see that the movement of the Zeppelin is a lot bigger here, and there is more chaos. More chaos, as well as mixing between modes, is expected because we are at a lower levitation current, with a less symmetrical Zeppelin and at a larger excitation. When left to decay, this signal becomes similar to the top figure.

These types of time signals are what one would also observe when dealing with a (driven) Duffing oscillator (section 2.4.4). The top time trace resembles a weakly driven Duffing oscillator, and the bottom time trace a strongly driven Duffing oscillator.

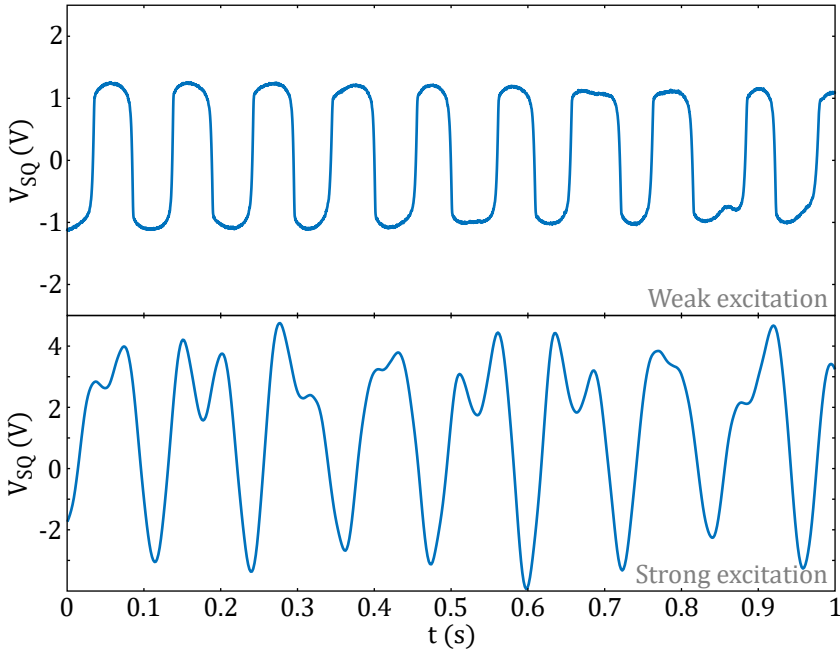


Figure 4.5: Two time traces of the measured SQUID signal. Top: the spherical 1.6 mg Zeppelin, at $I_{lev} = 400$ mA, some time after having set the levitation current. Bottom: the aspherical 2 mg Zeppelin, at $I_{lev} = 300$ mA, shortly after having set the current, attenuated $10\times$. This is very similar to what one would expect for the non-linear Duffing oscillator at weak (top) and strong (bottom) excitations, see section 2.4.4.

4.6 Naming Resonances

In this section we want to see if we can assign peaks in the PSD to resonances of the Lead Zeppelin. We will be looking for 6 resonances, 3 translational and 3 rotational. Also, there will be mixing between them, because of the non-linearity of the levitating force and because the Zeppelin movement is quite big — especially when we kick it.

When the Zeppelin is a perfect sphere, there are only translational resonances: a spherical particle has no restoring force in its rotational degrees of freedom due to the symmetry of the setup, and when it spins, this does not lead to flux changes in the pickup coil. With an aspherical Zeppelin, there is a restoring force on rotations. The rotations about the x and y axes are readily measurable, and the rotation about the z axis becomes measurable when $r \neq 0$ (a 2nd order effect). Of

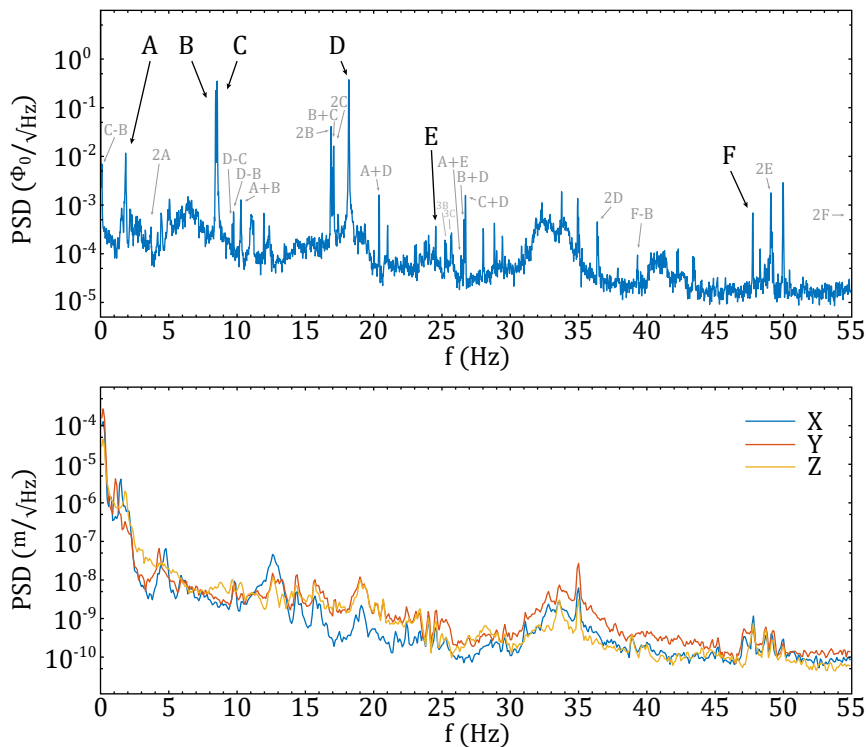


Figure 4.6: Top: the Zeppelin signal of the 0.8 mg spherical Zeppelin at $I_{lev} = 340$ mA after a kick, $\Delta f = 0.02$ Hz. The labelling A–F attempts to understand which peaks belong to the Zeppelin; see table 4.5. Bottom: the geophone measurement of the Dewar motion plotted in the same frequency range for comparison. In both measurements, there is a prominent peak at 35 Hz. Clearly, this Dewar resonance seeps through to the Zeppelin. The bulge around this peak in the geophone measurement, also shows up in the Zeppelin measurement.

#	A	B	C	D	E	F
f (Hz)	1.84	8.44	8.54	18.18	24.54	47.76

$\pm$	A	B	C	D	E	F
A	3.7	10.28		20.02	26.4	
B		16.88	16.98	26.62		56.2
C		0.1	17.08	26.72		
D		9.74	9.66	36.38		
E					49.08	
F		39.32				

Table 4.5: Assigning the peaks of figure 4.6 to resonances and their mixing. On the diagonal and above, we fill in the location of a peak that corresponds to the sum of column and row (e.g. 2A, A+B, etc...), below the diagonal the difference (B−A, etc...). A blank entry means there is no corresponding peak in the PSD. All numbers are in Hertz. It is likely that B, C and D are the x , y and z translational resonances, respectively.

course, due to imperfections of the setup and Zeppelin, there will in reality always be a little stiffness to all rotations; e.g. when the levitation coils are not wound perfectly circularly. We expect that for the Zeppelins that we use the restoring force to rotations is quite weak, resulting in low rotational spring constants and therefore low resonance frequencies. (If the rotational spring constants are *really* weak, the Zeppelin could even ‘skid through’ its (local) potential minimum and start spinning unhindered; in that case it is hard to assign a frequency to the motion.)

By using elliptical levitation coils, rather than circular ones, in combination with a Zeppelin which is deliberately aspherical (like the 2 mg particle), we ought to be able to bring the rotational resonances to higher frequencies. See also chapter 5.

Furthermore, even though we calculate that the translational resonance frequencies in x and y are equal when using circular coils, regardless of the shape of the Zeppelin, we should keep in mind that this is the case only to 1st order in the Zeppelin dimensions and its movement. With our non-spherical Zeppelins, we can expect to see a decoupling of the x - and y -resonances. Moreover, when the levitation coils are not perfectly circular, there will be a decoupling of x and y even if the Zeppelin were a perfect sphere.

As mentioned earlier, we cannot ignore the motion of the experimental surroundings. The motion of the Dewar and dipstick will seep through to the Zeppelin, and their resonances (should) mix with the Zeppelin’s. Figure 4.6 shows the Zeppelin signal for the almost spherical 0.8 mg Zeppelin shortly after a kick (by setting the levitation currents) together with a geophone measurement of the motion of the Dewar for comparison.

If the Zeppelin is a Duffing oscillator, we can expect a whole lot more than just sum and difference frequencies (figure 2.10). Even without mixing, there

could be peaks popping up everywhere (e.g. at sub-harmonics), depending on the amount of non-linearity. Here, we will constrain ourselves to look only for the ‘easy’ mixing terms, in order to be able to say something about the Zeppelin’s resonances.

In table 4.5 we have assigned the labels A through F to 6 peaks from figure 4.6. The criteria to this labelling are the narrowness of the peaks (the Zeppelin has a high Q , the Dewar most likely does not), their height, and the presence of peaks at the location of mixing terms ($f_A \pm f_B$, etc...).

We find that many peaks have a sister at double their frequency, a manifestation of the non-linear nature of the Lead Zeppelin. We include these double frequencies in the table.

A could be a rotational resonance, being the peak lowest in frequency. B, C and D are likely to be the x , y and z translational resonances, respectively, because B and C are very close to one another, and because all simple mixing terms between them show up. Though not listed in table 4.5, B and C also have a triple frequency. E and F are added to the list because they explain some mixing terms. If B, C and D are translational resonances, then A, E and F are either rotational modes of the Zeppelin, or resonances from the experimental setup. A, low in frequency, could well be a rotation. For E and F, quite high in frequency, we find this harder to believe: they may be the rotations around the x and y axes, but they could also simply be Dewar or dipstick resonances. All three peaks A, E and F have quite a low height, which makes sense if it are rotations: a rotation would cause less flux change in the pickup coil than a translation, especially in the case of the near-spherical $520 \times 500 \mu\text{m}$ Zeppelin.

Using the same analysis, we can turn to the 2 mg Lead Zeppelin for which the location of peaks and their quality was presented in table 4.3. If at $I_{lev} = 400 \text{ mA}$ we single out the 5 peaks with the longest ringdown times, we find that many of the other peaks can be considered mixing terms, because they are located at sum and difference frequencies. Not only are the frequencies related, the ringdown times must also be related. Namely: a resonance at f_{AB} which is due to the mixing of two resonances at f_A and f_B will have an amplitude A_{AB} which is proportional to the amplitudes A_A and A_B of its parents:

$$A_{AB} \propto A_A A_B. \quad (4.5)$$

This implies that when both parent resonances decay in time exponentially, i.e. $A_{A,B} \propto e^{-t/\tau_{A,B}}$, the daughter resonance amplitude does so too with a shorter time constant

$$A_{AB} \propto e^{-t/\tau_{AB}} \propto e^{-t/\tau_A} e^{-t/\tau_B}, \quad (4.6)$$

leading to

$$\frac{1}{\tau_{AB}} = \frac{1}{\tau_A} + \frac{1}{\tau_B}. \quad (4.7)$$

f (Hz)	Designation	Measured τ (s)
1.9	A	170
8.7	B	190
18.0	C	145
20.7	D	173
28.9	E	150

f (Hz)	Designation	Measured τ (s)	Calculated τ (s)
16.1	C–A	69	78
20.2	E–B	78	84
22.6	D–A	96	86
27.0	E–A	78	80
29.4	B+D	90	91
30.8	A+E	80	80
37.6	B+E	79	84

Table 4.6: We assign the labels A–E to the 5 PSD peaks with the longest ringdown times τ of the 2 mg Zeppelin at $I_{lev} = 400$ mA from table 4.3. By declaring A–E to be fundamental Zeppelin modes, many of the other peaks can be understood as mixing terms on account of both their frequency, as well as their shorter ringdown times.

In table 4.6 we compare the measured ringdown times of mixing terms to what we calculate with the above formula. We see that the measured and calculated τ s indeed agree with each other.

Chapter 5

Discussion and Outlook

We have shown that we can make a superconducting particle levitate by means of a suitable magnetic field, and read out its motion. There are, naturally, improvements that can be made which will turn the Lead Zeppelin into a force sensor that can compete with the state-of-the-art, e.g. starting from being able to detect the thermal motion, towards a gravitational force measurement, and ultimately to a quantum measurement. We want to discuss some of the improvements here, that we thought of while doing the experiments — a few of which have even already been realized, awaiting testing.

Vibration Isolation

There are some ideas on how we can improve our vibration isolation. As mentioned in section 2.8, the new measurement hall of Leiden University has vibration isolated islands with a displacement noise of $\sim 2 \times 10^{-10} \frac{\text{m}}{\sqrt{\text{Hz}}}$ at 10 Hz[36]. Compared to a typical laboratory movement of $\sim 10^{-8} \frac{\text{m}}{\sqrt{\text{Hz}}}$, this is an improvement of 2 orders of magnitude. If we were to put a crane on such an island, and hang our Helium Dewar on it with bungee cords, the Dewar motion should drop by 2 orders of magnitude as compared to now, figure 2.13.

This is still neglecting the boiling noise coming from the evaporating Helium. This contribution to the displacement noise of the Lead Zeppelin is difficult to calculate, as it depends on many factors like the exact geometry of the Helium Dewar and dipstick, as well as the boiling rate (which depends on the geometry, the amount of liquid Helium left in the Dewar, the heat input, ...). To know how bad this is, really one should measure it; we have not done this, so we are not sure whether it is something to worry about. Then again, when we repeat our measurements on a vibration isolated island, and the Zeppelin motion isn't diminished, it was probably the boiling noise that caused the major source of trouble.

Another idea then is to damp external motion directly at the experiment.

Rather than screwing the experimental chamber tightly to the dipstick, as in figure 2.16, we could instead suspend it from soft springs, see figure 5.1, which could be placed on a dipstick, but also in a cryostat. When we succeed in getting the resonance frequency to be (much) lower than a Zeppelin resonance ω_0 , we can diminish the dipstick motion reaching the Zeppelin; as explained in section 2.8, the transfer function that takes external motion to Zeppelin motion then is $H(\omega) = \left(\frac{\omega_0}{\omega}\right)^2$. In fact, when we assume that the boiling noise from evaporating Helium is negligible in at least the z -direction, which seems to make sense, this frame acts as a second vibration isolation stage on motion from the laboratory ceiling, which is then filtered to 4th order.

The frame from figure 5.1 has a resonance frequency of a few Hz. The Zeppelins of this thesis have resonances of around 10 Hz, and these can be pushed upwards as we explain below. Unfortunately, the thermalization of the experiment in this frame, without spoiling its ability to damp external vibrations, is difficult and no successful experiment has been performed yet.

Sensitivity

When the externally driven Zeppelin motion gets smaller, at some point we will want to lower the SQUID noise and increase the coupling to the pickup coil.

We have tried already to get the Zeppelin closer to the pickup coil by introducing an asymmetry in the levitation currents, i.e. $I_{up} \neq I_{down}$. This way, one can manoeuvre the Zeppelin to a position where $\frac{d\Phi}{dz}$ is larger. However, this led to unintuitive results in our earliest experiments without a PCS (section 4.1), and hasn't (yet) resulted in a successful measurement after instalment of a PCS at each levitation coil (section 4.2).

Similarly, placing the pickup coil at a more optimal position can increase the signal a great deal. As is clear from e.g. figure 2.7, a lot can be won from this simple modification.

One can also use a SQUID with a lower noise level. Our SQUIDs remained at best at about $35 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$, and more typically around $100 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$. This number is far higher than what is possible with a SQUID: the commercial Quantum Design SQUIDs that we use should be able to reach $5 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$ at 4 K down to below 1 Hz[26], and SQUIDs with still better figures do in fact exist.

Higher Resonance Frequencies

A higher resonance frequency makes many envisioned experiments easier. At the moment most importantly, the Dewar motion becomes less of a nuisance.

Ideally, we simply crank up the levitation currents; not only does this increase the resonance frequencies, but also will the Zeppelin hover closer to the $z = 0$ point, which means less non-linear (Duffing) behaviour as well as a higher



Figure 5.1: By suspending the experiment (housed in the Nb tube) in a frame with some soft springs, external vibrations are mitigated.

coupling to the pickup coil. Of course, we cannot just increase the levitation currents unlimitedly, because we cross the critical magnetic field quickly (section 4.2). However, there are easy adjustments that we can make that do enable this.

For instance, we can use a smaller Zeppelin. Indeed, let us calculate how high a levitation current we can allow for when we use a smaller Zeppelin of, say, $R_{zep} = 5 \text{ } \mu\text{m}$. The critical field at 4.2 K for Lead, equation (2.41), is $B_{c,\text{Pb}}(4.2 \text{ K}) = 13.9 \text{ mT}$. When we assume the Zeppelin to float at $z = 0$, which for smaller particles and higher levitation currents becomes true rapidly (figure 2.9), we find that in our setup the maximum levitation current is, using equation (4.2),

$$I_{lev,c} = 81 \text{ A.} \quad (5.1)$$

From this levitation current onwards, the critical magnetic field is exceeded at the edge of the Zeppelin. At this levitation current, the resonance frequencies are (equation (2.36))

$$f_{x,y} = 1.4 \text{ kHz,} \quad f_z = 2.2 \text{ kHz.} \quad (5.2)$$

Of course, for smaller and smaller particles, levitation may be more difficult to achieve, due to the Van der Waals force that will make the particle stick to the

surface of the Macor holder in which we put it. At this point, one would have to implement some sort of launching device that can fling the Zeppelin towards the zero point of magnetic field; for instance a short voltage burst on a piezo element on which one places the Zeppelin. Also, one must be careful not to cross the critical current density of the superconducting wiring, nor that of the spot welded wire joints.

For even smaller particles, on the order of the glass nanospheres that are used in optical levitation experiments (radii 20 – 500 nm, e.g. [5, 19]), say $R_{zep} = 100$ nm, we would have $I_{lev,c} = 4$ kA and $f_{x,y} = 67$ kHz, $f_z = 110$ kHz. Here, it will be even more challenging to get the Zeppelin off the surface and into the zero point of magnetic field.

The resonance frequencies can also be increased when we make a Zeppelin out of another superconducting material than Lead. As was hinted to in section 2.3, one could go for a Niobium Zeppelin or an Aluminium Zeppelin. Because of the smaller densities of these materials compared to Lead ($\rho_{Pb} = 11.34 \times 10^3 \frac{\text{kg}}{\text{m}^3}$ versus $\rho_{Nb} = 8.57 \times 10^3 \frac{\text{kg}}{\text{m}^3}$ and $\rho_{Al} = 2.70 \times 10^3 \frac{\text{kg}}{\text{m}^3}$), they will have higher resonance frequencies and they will levitate closer to $z = 0$, already at the same levitation currents.

The downsides of Aluminium are its low critical temperature $T_{c,Al} = 1.18$ K and critical field $B_{c,Al}(T = 0 \text{ K}) = 10.5$ mT[50, p. 224]. This means that measurements in liquid Helium are out of the question and that the increase in resonance frequency as a consequence of having a smaller density is partially lost on a lower maximum levitation current.

A Niobium Zeppelin is not all that less dense than Lead, and spherical Zeppelins won't be made in-house owing to the high melting temperature of Niobium (although such particles can be bought), but it has the big advantages of higher critical temperature $T_{c,Nb} = 9.25$ K and critical fields $B_{c1,Nb}(T = 0 \text{ K}) = 174$ mT, $B_{c2,Nb}(T = 0 \text{ K}) = 400$ mT[49]. Even if one wants to stay below $B_{c1,Nb}$, e.g. to avoid vortex dynamics, then still much higher levitation currents can be used than with Lead. As an example: at 4.2 K we find that $B_{c,Pb} = 13.9$ mT and $B_{c1,Nb} = 138$ mT; an order of magnitude difference.

An idea we had to get the rotational modes to higher frequencies, is to wind elliptical levitation coils, rather than circular ones, combined with a Zeppelin that is (very) aspherical. This will help to clamp rotational modes more firmly. The rotational frequencies about the x - and y -axis are probably increased most effectively by the amount that the Zeppelin deviates from being spherical. The rotation about the z -axis we can help a little further by introducing the rotational asymmetry in the levitation coils. At the same time, the translations in x and y will be decoupled, which will help us identify which peak belongs to which resonance. We have fabricated such coils, but no successful experiments were performed yet.

Lower Damping, Thermalization

We found that in our experiments the quality factor of the resonances seems to be limited by the ambient Helium gas, which was typically at a pressure between $P = 10^{-2} - 10^{-1}$ mbar, and resulted in damping factors on the order of $\gamma \sim 10^{-8} \frac{\text{N}}{\text{m/s}}$, see table 4.3. The obvious thing to do to achieve lower damping is to go to lower pressures. However, in our setup, the thermalization of the various components depends on having enough background gas; e.g. the SQUID modulation becomes worse, and completely disappears below $\sim 8 \times 10^{-3}$ mbar, but also the PCS can no longer be operated.

A better thermalization can be achieved by screwing components tightly onto the dipstick with metal clamps, rather than with a load of Teflon tape, figure 2.16. This would certainly make the SQUID be operable at lower pressures, as well as the components of the PCS and its various coils: we know that all of that works just fine in a dilution refrigerator at 10 mK in a cryogenic vacuum[40]. If need be, a (thin) strip of metallic material could provide cooling to components that still require a better thermalization; for instance, maybe the levitation coils need this once we remove the brass clamp that holds them together.

When thermalization is taken care of, namely, the next spoiler of γ is this brass clamp (section 2.6.2). To get around this, we will replace it with a clamp made from the plastic PEI. Please note that one has to be very careful not to add too much normal metal around the Lead Zeppelin for cooling purposes, as this will set a new stage for eddy currents to take place.

Using smaller particles also helps us here. With the eddy current damping eliminated, the contribution from gaseous Helium is brought down as R_{zep}^2 (eq. (2.57)). The proposed Zeppelin of size $R_{zep} = 100$ nm would result in a 7 – 8 orders of magnitude improvement of γ over the current Zeppelins, even without changing the pressure.

Future Prospects

Finally, we compare the Lead Zeppelin's force sensing abilities to other techniques around. As far as our measurement scheme is concerned, the detection noise is sufficiently low to detect the thermal motion. The (on-resonance) force detection limit of the Lead Zeppelin is

$$S_{F,lim}^{1/2}(\omega_0) = \frac{m\omega_0^2}{Q} S_{x,lim}^{1/2} = 8.7 \times 10^{-16} \frac{\text{N}}{\sqrt{\text{Hz}}}, \quad (5.3)$$

for $m = 2$ mg, $Q = 3000$, $\frac{\omega_0}{2\pi} = 10$ Hz and $S_{x,lim}^{1/2} = 3.3 \times 10^{-10} \frac{\text{m}}{\sqrt{\text{Hz}}}$ (eq. (2.89)), whereas the thermal force noise at 4.2 K is

$$S_{F,Th}^{1/2} = \sqrt{4k_B T \gamma} = 3.1 \times 10^{-15} \frac{\text{N}}{\sqrt{\text{Hz}}}, \quad (5.4)$$

for the same typical numbers and $\gamma = \frac{m\omega_0}{Q}$. However, as we know already from section 2.8, the force noise on the Lead Zeppelin brought in by the motion of the Helium Dewar is much larger, and as a matter of fact presently limits our force detection:

$$S_{F,zep}^{1/2} = m\omega_0^2 S_{x,Dewar}^{1/2} = 7.9 \times 10^{-12} \frac{\text{N}}{\sqrt{\text{Hz}}}, \quad (5.5)$$

for a typical Dewar motion at 10 Hz of $S_{x,Dewar}^{1/2} \sim 10^{-9} \frac{\text{m}}{\sqrt{\text{Hz}}}$, figure 2.13, and where we used $S_{x,zep}^{1/2} = H(\omega) S_{x,Dewar}^{1/2} = Q S_{x,Dewar}^{1/2}$ on resonance, eq. (2.103).

To cross from the vibrationally limited domain, eq. (5.5), into the thermally limited domain, eq. (5.4), using a smaller Lead Zeppelin offers a solution: the vibrational force noise scales in the Zeppelin size R_{zep} as $S_{F,zep}^{1/2} \propto m \propto R_{zep}^3$ and the thermal force noise as $S_{F,Th}^{1/2} \propto \sqrt{\gamma} \propto R_{zep}$, because when the background Helium provides the damping we have $\gamma \propto R_{zep}^2$. So when we reduce R_{zep} by a factor of 50 (i.e. from $\sim 250 \mu\text{m}$ down to the earlier proposed $5 \mu\text{m}$), leaving everything else as it is, the thermal force noise and the vibrational force noise are on a par at $S_{F,zep}^{1/2} = S_{F,Th}^{1/2} = 6 \times 10^{-17} \frac{\text{N}}{\sqrt{\text{Hz}}}$. The damping then is $\gamma = 4 \times 10^{-12} \frac{\text{N}}{\text{m/s}}$, and $Q = 3 \times 10^7$. A still smaller particle will be limited by thermal noise, rather than by vibrations.

In table 5.1 we compare the force noise on the Lead Zeppelin and its damping coefficient to those of the magnet on cantilever of Nichol et al.[2] and that of Vinante et al.[4], as well as to those of the optically levitated nanoparticle of Gieseler et al.[5]. We include in the table the scenario in which we make the Lead Zeppelin 50 times smaller, leaving the rest of the experiment as it has been throughout this thesis. We also add the ‘Thermal LZ’, explained below, in which we do not change the size of the Zeppelin, but assume that our vibration isolation is improved to the point where it no longer dominates the force noise.

The force noise only has to be so low as to allow for a measurement of the force of interest. In the table we therefore also include the mass m of the force sensors, their resonance frequency f_0 and the gravitational force F_g between them and an oscillating test mass M as in equation (2.96)

$$F_g = 2a \frac{GmM}{r^3}. \quad (5.6)$$

Here, r is the distance between M and m , and a is the amplitude of the oscillation of M . To compare between the different sensors, we fill in the same values as we did in section 2.7.4, $M = 1 \text{ g}$, $r = 3 \text{ cm}$ and $a = 1.5 \text{ mm}$ (or equivalently $M = 1 \text{ kg}$, $r = 1 \text{ m}$ and $a = 5 \text{ cm}$).

Considering that $m \propto R_m^3$ and $M \propto R_M^3$, where $R_{m,M}$ are the sizes of m and M , and further that the distance between them is $r \geq R_m + R_M + a$, the gravitational force scales as $F_g \propto \frac{aR_m^3R_M^3}{(R_m+R_M+a)^3}$, or $F_g \propto R_m^3$. In other words, when we are limited by vibrations, as is the case for the Lead Zeppelin, a

System	$S_F^{1/2} \left(\frac{\text{N}}{\sqrt{\text{Hz}}} \right)$	$\gamma \left(\frac{\text{N}}{\text{m/s}} \right)$	$m \text{ (kg)}$	$f_0 \text{ (Hz)}$	$F_g \text{ (N)}$
LZ	8×10^{-12}	10^{-8}	10^{-6}	10	10^{-17}
Small LZ [†]	6×10^{-17}	10^{-12}	10^{-11}	10	10^{-22}
Thermal LZ [†]	3×10^{-15}	10^{-8}	10^{-6}	10	10^{-17}
MoC 8 K[2]	2×10^{-18}	10^{-15}	10^{-17}	10^{5-6}	10^{-28}
MoC 28 mK [†] [4]	8×10^{-19}	10^{-13}	10^{-13}	10^{3-4}	10^{-24}
OLN [†] [5]	2×10^{-20}	10^{-20}	10^{-18}	10^{4-5}	10^{-29}

Table 5.1: The force noise and damping coefficient on the Lead Zeppelin (LZ) compared to state-of-the-art Magnet on Cantilever (MoC) and Optically Levitated Nanoparticle (OLN) force detection. For the ‘Small LZ’ we reduce the size of the Zeppelin, but we leave the rest of the experiment unaltered. The ‘Thermal LZ’ shows the case that we install the proposed vibration isolation measures. We add the mass and resonance frequency to the table, and calculate the gravitational interaction with a test mass (see main text). A dagger (†) indicates that the force noise comes from thermal fluctuations.

gravitational measurement does not become any harder or easier when reducing R_m . We are less susceptible to vibrations, which is good, but at the same time we will have a smaller gravitational interaction. When limited by thermal noise, reducing R_m actually makes the gravitational measurement more difficult.

The resonance frequency of the Zeppelin is much lower than the other force sensors, meaning that the gravitational force will be easier to generate. Further, we see that although a gravitational measurement in the current Lead Zeppelin experiment would require very long measurement times T_m , the same is true for the other techniques.

Carrying out the improvements discussed in this chapter, a measurement of the gravitational interaction with a small source mass within reasonable measurement times, should be possible. For instance, when we move our experiment to the new measurement hall of Leiden University, we immediately gain 2 orders of magnitude on $S_F^{1/2}$, due to the lower vibrations present there. When we also install the vibration isolation frame of figure 5.1 (for which we need to get its thermalization in order), the force noise will readily be dominated by thermal fluctuations. See the ‘Thermal LZ’ in table 5.1. At this point, a gravitational measurement will be possible within a very reasonable measurement time of a few hours, and could be brought down further by e.g. a modest increase of the resonance frequency (less vibrations, f_0^{-4} scaling) combined with going to a lower pressure (lower damping, less thermal noise).

Appendix

A Force Exerted by Excitation Coil

In this appendix we show that the excitation coil in our experimental setup can be used to excite the Lead Zeppelin in all 3 spatial directions — not just the z -direction. To do so, we expand the levitating force of section 2.4.2, equation (2.28), as follows:

$$\begin{aligned}\mathbf{F}_{tot} = & (k_x \varepsilon_x + \alpha_x \varepsilon_x I_{exc}) \hat{\mathbf{x}} + (k_y \varepsilon_y + \alpha_y \varepsilon_y I_{exc}) \hat{\mathbf{y}} \\ & + (k_z \varepsilon_z + \alpha_z \varepsilon_z I_{exc} + \kappa I_{exc}) \hat{\mathbf{z}}.\end{aligned}\tag{A.1}$$

We already calculated the spring constants $k_{x,y,z}$ and κ . The coefficients $\alpha_{x,y,z}$ are given, in full glory, by:

$$\begin{aligned}\alpha_x = \alpha_y = & \frac{1}{2} K_I (R_{lev}^2 (2R_{exc}^2 (R_{exc}^2 + 3R_{exc} R_{lev} + R_{lev}^2) \\ & + (4R_{exc}^2 + 6R_{exc} R_{lev} - 3R_{lev}^2) z_{exc}^2 + 2z_{exc}^4) \\ & + (R_{exc}^2 (-3R_{exc}^2 + 6R_{exc} R_{lev} + 4R_{lev}^2) \\ & - 6(R_{exc}^2 - R_{exc} R_{lev} + R_{lev}^2) z_{exc}^2 - 3z_{exc}^4) z_{lev}^2 \\ & + (2R_{exc}^2 - 3z_{exc}^2) z_{lev}^4), \\ \alpha_z = & -K_I (R_{lev}^2 (2R_{exc}^2 (R_{exc}^2 - 3R_{exc} R_{lev} + R_{lev}^2) \\ & + (4R_{exc}^2 - 6R_{exc} R_{lev} - 3R_{lev}^2) z_{exc}^2 + 2z_{exc}^4) \\ & - (R_{exc}^2 (3R_{exc}^2 + 6R_{exc} R_{lev} - 4R_{lev}^2) \\ & + 6(R_{exc}^2 + R_{exc} R_{lev} + R_{lev}^2) z_{exc}^2 + 3z_{exc}^4) z_{lev}^2 \\ & + (2R_{exc}^2 - 3z_{exc}^2) z_{lev}^4),\end{aligned}\tag{A.2}$$

with

$$K_I = \frac{3\mu_0}{2\pi} \frac{L_x L_y L_z R_{exc} R_{lev} z_{exc} z_{lev}}{(R_{exc}^2 + z_{exc}^2)^{7/2} (R_{lev}^2 + z_{lev}^2)^{7/2}} I_{lev}.\tag{A.3}$$

When we fill in the experimental parameters as found in table 2.1 and further the values $I_{lev} = 1.0 \times N_{lev}$ A, $L_x = 300$ μm , $L_y = 400$ μm , $L_z = 500$ μm , then

$$\alpha_x = \alpha_y = 9.75 \times 10^{-6} \frac{\text{N}}{\text{m} \cdot \text{A}}, \quad \alpha_z = 7.57 \times 10^{-6} \frac{\text{N}}{\text{m} \cdot \text{A}}.\tag{A.4}$$

We see that the $\alpha_{x,y,z}$ can indeed be safely ignored: they are multiplied by $\varepsilon_{x,y,z}$, which are at best about a micron, as well as the excitation current I_{exc} to get their force on the Zeppelin. However, we also note that because the coefficients $\alpha_{x,y,z}$ are non-zero, for positions of the Zeppelin away from the origin, we can in principle use the excitation coil to excite the motion of the Lead Zeppelin in all directions.

B Flux Reduction due to PCS

In this appendix we discuss some of the effects of the PCS on the Lead Zeppelin experiment. When the levitation coils are put in the persistent mode, there are some issues that need our reconsideration. Firstly, since the levitation coils are no longer connected to a current source of infinite impedance, but rather have zero resistance now, we can wonder whether the impedance of the excitation coil as met by the function generator has changed significantly. The excitation coil is, after all, close to either levitation coil: if the mutual inductance is large enough, this could mean that we can send down less power through the excitation coil than without the PCS. We will see, though, that at the low frequencies that we use, this makes not much of a difference.

Secondly, any flux introduced to the levitation coils could be very efficiently counteracted, as the levitation coils are keeping their flux constant whilst in the persistent mode. This effect will reduce the flux coming from the excitation coil, leading to a less efficient drive of the Zeppelin, and it might also affect the flux produced by the moving Zeppelin, meaning that there is less signal to be measured. We will see that when the levitation coils are each put separately in the persistent mode, this effect is quite severe. When we place the levitation coils in series with opposing polarity (anti-Helmholtz), and put them in the persistent mode together, this effect is small.

B.1 Effect on Impedance

We hook up a function generator to the excitation coil and call the combination of the function generator's $50\ \Omega$ impedance and the wires to the excitation coil R_{fg} . The wires typically have a resistance of $10\ \Omega$, making it a total of $R_{fg} = 60\ \Omega$. When only one of the levitation coils is present, having a mutual inductance M to the excitation coil, and accounting for the possibility it is in the normal state with resistance R_{ns} , the total impedance Z that the function generator sees follows from Kirchhoff's Laws and can be written as

$$Z = R_{fg} + i\omega L_{exc} + k^2 L_{exc} \frac{i\omega - \omega_{ns}}{1 + \left(\frac{\omega_{ns}}{\omega}\right)^2}. \quad (\text{B.1})$$

Here, $L_{exc} = 0.2\ \mu\text{H}$ is the inductance of the excitation coil, $\omega_{ns} = \frac{R_{ns}}{L_{lev}}$, with $L_{lev} = 220\ \mu\text{H}$ the inductance of the levitation coil, and k comes from $M = k\sqrt{L_{exc}L_{lev}}$, the mutual inductance between the levitation coil and the excitation coil.

The effect that the levitation coil has on the impedance, is very small. When $R_{ns} = 0\ \Omega$, the full effect is governed by k^2 ; however, it starts to be noticeable only at a frequency $\omega = \frac{R_{fg}}{L_{exc}} = 2\pi \times 48\ \text{MHz}$. For our purposes, this is completely negligible.

When $R_{ns} = 1\ \Omega$, $\omega_{ns} = 2\pi \times 723\ \text{Hz}$. This is also outside our region of interest.

B.2 Effect on Efficiency of Excitation Coil

The question remains what the net flux is that the excitation coil produces when the levitation coils are in the persistent mode. In the persistent mode, any coil obeys flux conservation: if externally some flux is offered, it will produce a countercurrent such that the total flux through the coil remains the same.

Let's begin with the on-axis magnetic field $B(z)$ of a circular current loop of radius R in the (x, y) -plane centered at the origin::

$$B(z) = \frac{\mu_0 I}{2R} \times \frac{R^3}{(z^2 + R^2)^{3/2}}. \quad (\text{B.2})$$

In the above form, one can recognize the $B(0)$ -term which is multiplied by a dimensionless distance correction factor. Let us assume that the flux decreases in the same way:

$$\Phi(z) = \Phi(0) \times \frac{R^3}{(z^2 + R^2)^{3/2}} \equiv \Phi \times P_{\text{dis}}(z, R). \quad (\text{B.3})$$

The flux in another circular loop of radius L , coaxial to the first, some distance s away, can be calculated with the above formula combined with an area correction factor. Namely, when $R > L$ we say, feeling comfortable in assuming that the field produced by the first, bigger loop is roughly constant across the area of the second, smaller loop:

$$\Phi_L = B_R(s) A_L = \Phi_R(s) \frac{A_L}{A_R}. \quad (\text{B.4})$$

The flux has been further diminished by the ratio of the areas of the two loops.

In the case that $L > R$, something similar is going on. Consider that $\Phi = MI$: the flux through the larger loop due to a current I in the smaller loop, is exactly the same as the flux in the smaller loop due to the same current I in the larger loop. In other words, the flux that is 'lost' on the way is the same in both cases.

The flux is therefore diminished by the ratio of the area of the smallest loop to the largest loop:

$$P_{\text{area}}(R, L) \equiv \begin{cases} \left(\frac{L}{R}\right)^2, & R > L, \\ \left(\frac{R}{L}\right)^2, & L > R. \end{cases} \quad (\text{B.5})$$

And, combining:

$$\Phi_L = \Phi_R P_{\text{dis}}(s, R) P_{\text{area}}(R, L). \quad (\text{B.6})$$

In the case of the excitation coil and the levitation coils, we simply need to make the right combinations of P_{dis} and P_{area} . We place the center of the setup

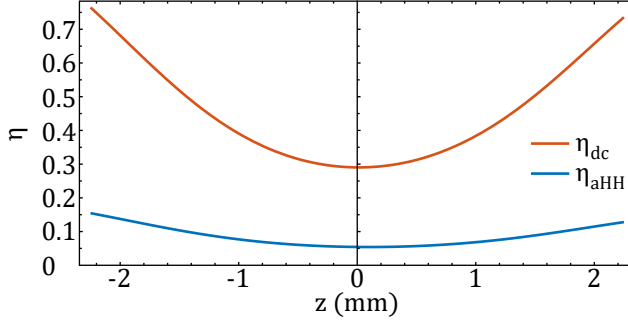


Figure B.1: The loss factors η as a function of z , $z = 0$ being the center of the experimental setup as in figure 2.4, for the two cases that the levitation coils are decoupled from each other and each connected to their own PCS (η_{dc}), and when they are coupled in an anti-Helmholtz configuration to a single PCS (η_{aHH}). The excitation coil is placed at $z_{exc} = 0.2$ mm, and we further used the other experimental parameters of table 2.1. These loss factors tell us how much more difficult it is to drive the Lead Zeppelin due to the levitation coils being in the persistent mode. The coupled loss factor η_{aHH} goes to zero for all z when the excitation coil is placed at $z_{exc} = 0$.

at $z = 0$, such that the flux Φ of an excitation coil of radius R_{exc} at position z_{exc} is given as a function of z as

$$\Phi_{exc}(z) = \Phi P_{dis}(z - z_{exc}, R_{exc}). \quad (B.7)$$

When the levitation coils are decoupled and both in their own separate persistent mode, the flux that reaches either of them is perfectly counteracted. For the upper levitation coil, the counteracting flux as a function of z is:

$$\Phi_{up} = -\Phi P_{dis}(z_{up} - z_{exc}, R_{exc}) P_{area}(R_{exc}, R_{up}) P_{dis}(z - z_{up}, R_{up}). \quad (B.8)$$

The same goes for $\Phi_{down}(z)$, with (up $\leftrightarrow$ down).

The effective excitation flux is the sum of the three contributions:

$$\Phi_{eff,dc}(z) = \Phi_{exc}(z) + \Phi_{up}(z) + \Phi_{down}(z). \quad (B.9)$$

The loss factor $\eta_{dc}(z)$ describes the amount of flux ‘lost’ compared to what was going to be lost anyway already

$$\eta_{dc}(z) = 1 - \frac{\Phi_{eff,dc}(z)}{\Phi_{exc}(z)}. \quad (B.10)$$

We will ignore higher order reduction effects: e.g. the counteraction of the flux of the upper levitation coil by the lower levitation coil is small and is not taken into account.

When the levitation coils are coupled and in anti-Helmholtz configuration, the flux that reaches the upper levitation coil minus that which reaches the

lower levitation coil is what is counteracted. We place absolute signs around the difference of the two fluxes, which is the easiest way to get all the signs right.

$$\Phi_{up,aHH}(z) = -\Phi \left| P_{dis}(z_{up} - z_{exc})P_{area}(R_{exc}, R_{up}) - P_{dis}(z_{down} - z_{exc})P_{area}(R_{exc}, R_{down}) \right| P_{dis}(z - z_{up}, R_{up}) \quad (B.11)$$

Again, for $\Phi_{down}(z)$ we do (up $\leftrightarrow$ down). The effective excitation flux is this time

$$\Phi_{eff,aHH}(z) = \Phi_{exc}(z) + \Phi_{up,aHH}(z) + \Phi_{down,aHH}(z), \quad (B.12)$$

and the loss factor

$$\eta_{aHH}(z) = 1 - \frac{\Phi_{eff,aHH}(z)}{\Phi_{exc}(z)}. \quad (B.13)$$

The two loss factors are shown in figure B.1. At a typical Zeppelin equilibrium position $z_{eq} = -700 \mu\text{m}$, we find the values

$$\eta_{dc}(z_{eq}) = 34 \%, \quad \eta_{aHH}(z_{eq}) = 6.6 \%. \quad (B.14)$$

Decoupling the levitation coils means that we can drive the Lead Zeppelin significantly less efficiently. Coupling them means that there is also a reduction, but it is a lot smaller.

B.3 Effect on Detection by Pickup Coil

Now, the loss factors of the flux picked up by the pickup coil due to the Lead Zeppelin will follow the same basic rules. The changing flux expulsion of the Lead Zeppelin as it moves through the levitating magnetic field, will be counteracted by the levitation coils.

When the levitation coils are coupled to a single PCS, and when the Lead Zeppelin has its equilibrium position at $z_{eq} = 0$, due to symmetry there is no flux reduction. As soon as the equilibrium position is away from the origin, a reduction will kick in. Decoupling the coils with two separate PCSs will mean that there is always a reduction, no matter where the Zeppelin has its equilibrium. We therefore choose to couple the levitation coils to minimize a loss of signal.

References

- [1] C. L. Degen, M. Poggio, H. J. Mamin, C. T. Rettner, and D. Rugar, “Nanoscale Magnetic Resonance Imaging,” *PNAS*, vol. 106, no. 5, p. 1313, 2009.
- [2] J. M. Nichol, E. R. Hemesath, L. J. Lauhon, and R. Budakian, “Nanomechanical Detection of Nuclear Magnetic Resonance Using a Silicon Nanowire Oscillator,” *Physical Review B*, vol. 85, no. 5, p. 054414, 2012.
- [3] H. B. Callen and T. A. Welton, “Irreversibility and Generalized Noise,” *Physical Review*, vol. 83, no. 1, p. 34, 1951.
- [4] A. Vinante, A. Kirste, A. den Haan, O. Usenko, G. Wijts, E. Jeffrey, P. Sonin, D. Bouwmeester, and T. H. Oosterkamp, “High Sensitivity SQUID-Detection and Feedback-Cooling of an Ultrasoft Microcantilever,” *Applied Physics Letters*, vol. 101, p. 123101, 2012.
- [5] J. Gieseler, L. Novotny, and R. Quidant, “Thermal Nonlinearities in a Nanomechanical Oscillator,” *Nature Physics*, vol. 9, p. 806, 2013.
- [6] E. Schrödinger, “Die Gegenwärtige Situation in der Quantenmechanik,” *Naturwissenschaften*, vol. 23, no. 48-50, pp. 807–812, 823–828, 844–849, 1935.
- [7] A. O. Caldeira and A. J. Leggett, “Influence of Dissipation on Quantum Tunneling in Macroscopic Systems,” *Physical Review Letters*, vol. 46, no. 4, p. 211, 1981.
- [8] L. Diósi, “Models for Universal Reduction of Macroscopic Quantum Fluctuations,” *Physical Review A*, vol. 40, no. 3, p. 1165, 1989.
- [9] P. C. E. Stamp, “Environmental Decoherence versus Intrinsic Decoherence,” *Philosophical Transactions of the Royal Society A*, vol. 370, p. 4429, 2012.
- [10] A. Einstein. (a) *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*, “Zur Allgemeinen Relativitätstheorie,” vol. XLIV, p. 778 and vol. XLVI, p. 799, 1915, “Erklärung der Perihelbewegung des Merkur aus der Allgemeinen Relativitätstheorie,” vol. XLVII, p. 831, 1915, “Die

- Feldgleichungen der Gravitation,” vol. XLVIII, p. 844, 1915. (b) *Annalen der Physik*, “Die Grundlage der Allgemeinen Relativitätstheorie,” vol. 49, p. 769, 1916.
- [11] T. H. Oosterkamp and J. Zaanen, “A Clock Containing a Massive Object in a Superposition of States; What Makes Penrosian Wavefunction Collapse Tick?,” *arXiv:1401.0176*, 2014.
 - [12] L. Rademaker, T. van der Reep, N. Van den Broeck, B. van Waarde, M. de Voogd, and T. Oosterkamp, “The Instability of a Quantum Superposition of Time Dilations,” *arXiv:1410.2303*, 2014.
 - [13] J. van Wezel and T. H. Oosterkamp, “A Nanoscale Experiment Measuring Gravity’s Role in Breaking the Unitarity of Quantum Dynamics,” *Proceedings of the Royal Society A*, vol. 468, no. 2137, p. 35, 2012.
 - [14] R. Penrose, “On Gravity’s Role in Quantum State Reduction,” *General Relativity and Gravitation*, vol. 28, no. 5, p. 581, 1995.
 - [15] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, “Towards Quantum Superpositions of a Mirror,” *Physical Review Letters*, vol. 91, no. 13, p. 130401, 2003.
 - [16] D. E. Chang, C. A. Regal, S. B. Papp, D. J. Wilson, J. Ye, O. Painter, H. J. Kimble, and P. Zoller, “Cavity Opto-Mechanics Using an Optically Levitated Nanosphere,” *PNAS*, vol. 107, no. 3, p. 1005, 2010.
 - [17] O. Romero-Isart, A. C. Pflanzer, F. Blaser, R. Kaltenbaek, N. Kiesel, M. Aspelmeyer, and J. I. Cirac, “Large Quantum Superpositions and Interference of Massive Nanometer-Sized Objects,” *Physical Review Letters*, vol. 107, p. 020405, 2011.
 - [18] J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, “Sideband Cooling of Micromechanical Motion to the Quantum Ground State,” *Nature*, vol. 475, p. 359, 2011.
 - [19] T. S. Monteiro, J. Millen, G. A. T. Pender, F. Marquardt, D. Chang, and P. F. Barker, “Dynamics of Levitated Nanospheres: Towards the Strong Coupling Regime,” *arXiv:1207.1567*, 2012.
 - [20] O. Romero-Isart, L. Clemente, C. Navau, A. Sanchez, and J. I. Cirac, “Quantum Magnetomechanics with Levitating Superconducting Microspheres,” *Physical Review Letters*, vol. 109, p. 147205, 2012.
 - [21] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, “Cavity Optomechanics,” *Reviews of Modern Physics*, vol. 86, p. 1391, 2014.

- [22] R. Ghobadi, S. Kumar, B. Pepper, D. Bouwmeester, A. I. Lvovsky, and C. Simon, "Optomechanical Micro-Macro Entanglement," *Physical Review Letters*, vol. 112, no. 8, p. 080503, 2014.
- [23] V. Singh, S. J. Bosman, B. H. Schneider, Y. M. B. A. Castellanos-Gomez, and G. A. Steele, "Optomechanical Coupling Between a Multilayer Graphene Mechanical Resonator and a Superconducting Microwave Cavity," *Nature Nanotechnology*, vol. 9, p. 820, 2014.
- [24] W. Meissner and R. Ochsenfeld, "Ein neuer Effekt bei Eintritt der Supraleitfähigkeit," *Naturwissenschaften*, vol. 21, no. 44, p. 787, 1933.
- [25] J. Clarke and A. I. Braginsky, *The SQUID Handbook*, vol. 1. WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim, 2004.
- [26] Quantum Design, Inc., USA, *Model 50 DC SQUID Sensor*. www.qdusa.com.
- [27] E. Diez-Jimenez, J.-L. Perez-Diaz, and J. C. Garcia-Prada, "Local Model for Magnet-Superconductor Mechanical Interaction: Experimental Verification," *Journal of Applied Physics*, vol. 109, no. 6, p. 063901, 2011.
- [28] Q.-G. Lin, "Theoretical Development of the Image Method for a General Magnetic Source in the Presence of a Superconducting Sphere or a Long Superconducting Cylinder," *Physical Review B*, vol. 74, p. 024510, 2006.
- [29] J. Eisenstein, "Superconducting Elements," *Review of Modern Physics*, vol. 26, no. 3, p. 277, 1954.
- [30] T. Kalmár-Nagy and B. Balachandran, "Forced Harmonic Vibration of a Duffing Oscillator with Linear Viscous Damping," in *The Duffing Equation: Nonlinear Oscillators and Their Behaviour*, ch. 5, p. 165, John Wiley & Sons, Ltd., 1st ed., 2011. Reproduced with permission under license number 3705961494022.
- [31] O. Romero-Isart, M. L. Juan, R. Quidant, and J. I. Cirac, "Toward Quantum Superposition of Living Organisms," *New Journal of Physics*, vol. 12, p. 033015, 2010.
- [32] R. A. Matula, "Electrical Resistivity of Copper, Gold, Palladium and Silver," *Journal of Physical and Chemical Reference Data*, vol. 8, no. 4, p. 1147, 1979.
- [33] R. C. Ritter, C. E. Goldblum, W.-T. Ni, G. T. Gillies, and C. C. Speake, "Experimental Test of Equivalence Principle with Polarized Masses," *Physical Review D*, vol. 42, no. 4, p. 977, 1990.
- [34] J. Schmöle, M. Dragosits, H. Hepach, and M. Aspelmeyer, "A Micromechanical Proof-of-Principle Experiment for Measuring the Gravitational Force of milligram Masses," *arXiv:1602.07539*, 2016.

- [35] E. Tingwall, “Everything You Ever Wanted to Know About Modern Pistons (and Probably Some Things You Didn’t).” <http://www.caranddriver.com/features/everything-you-ever-wanted-to-know-about-pistons-feature>. Consulted 24th May, 2016.
- [36] M. B. S. Hesselberth (Leiden University). Private communication.
- [37] F. Matichard, B. Lantz, K. Mason, R. Mittleman, B. Abbott, S. Abbott, E. Allwine, S. Barnum, J. Birch, S. Biscans, D. Clark, D. Coyne, D. DeBra, R. DeRosa, S. Foley, P. Fritschel, J. A. Giaime, C. Gray, G. Grabeel, J. Hanson, M. Hillard, J. Kissel, C. Kucharczyk, A. L. Roux, V. Lhuillier, M. Macinnis, B. O’Reilly, D. Ottaway, H. Paris, M. Puma, H. Radkins, C. Ramet, M. Robinson, L. Ruet, P. Sareen, D. Shoemaker, A. Stein, J. Thomas, M. Vargas, and J. Warner, “Advanced LIGO Two-Stage Twelve-Axis Vibration Isolation and Positioning Platform. Part 2: Experimental Investigation and Tests Results,” *Precision Engineering*, vol. 40, p. 287, 2015.
- [38] O. V. Pakhamov and I. A. Khrebtov, “Acoustic Noise Spectrum of the Liquid Helium Boiling Process on Superconducting Bolometer Surface,” *AIP Conference Proceedings*, vol. 780, p. 147, 2005.
- [39] Agilent 6626A, *DC Power Supply Multiple Output Linear System*.
- [40] B. van Waarde, O. Benningshof, and T. Oosterkamp, “A Magnetic Persistent Current Switch at milliKelvin Temperatures,” *Cryogenics*, vol. 78, p. 74, 2016.
- [41] D. Ameen and P. Wiederhold, “Fast-Acting Superconducting Power Switches,” *The Review of Scientific Instruments*, vol. 35, no. 6, p. 733, 1964.
- [42] D. Hagedorn and P. Dullenkopf, “A Fast-Acting Superconducting Switch for Protection of Superconducting Coils in the Persistent Current Mode,” *Cryogenics*, vol. 14, no. 8, p. 429, 1974.
- [43] K. Noto, Y. Kono, M. Matsukawa, M. Itagaki, T. Ishida, and K. Chiba, “Development of a 50 A-Fast Response, Magnetically Controlled Persistent Current Switch,” *IEEE Transactions on Applied Superconductivity*, vol. 5, no. 2, p. 258, 1995.
- [44] K. Noto, H. Kimura, M. Matsukawa, S. Fujinuma, T. Segawa, T. Takahashi, N. Sadakata, T. Saito, K. Goto, O. Kohno, H. Honma, and C. Takahashi, “Development and Test Results of Magnetically Controlled Persistent Current Switches,” *Materials Transactions, JIM*, vol. 37, no. 4, p. 907, 1996.
- [45] K. Goto, N. Sadakata, T. Saitoh, and O. Kohno, “Development of a 1-kA, 25-ohm Magnetically Controlled Persistent Current Switch for SMES,” *IEEE Transactions on Applied Superconductivity*, vol. 9, no. 2, p. 173, 1999.

-
- [46] Y. Tsuda, A. Ohara, S. Nakamura, S. Washida, N. Oomyou, O. Tsukamoto, T. Numata, and M. Yamamoto, "Development of Superconducting Mechanical Persistent Current Switch," *IEEE Transactions on Applied Superconductivity*, vol. 10, no. 1, p. 804, 2000.
 - [47] M. Tomita, M. Murakami, S. Nariki, and K. Sawa, "Mechanical Persistent Current Switch Made of RE-Ba-Cu-O Bulk with Resin Impregnation," *Physica C*, vol. 378-381, p. 864, 2002.
 - [48] S. Phillip, J. V. Porto, and J. M. Parpia, "Two Methods of Fabricating Reliable Superconducting Joints with Multifilamentary Nb-Ti Superconducting Wire," *Journal of Low Temperature Physics*, vol. 101, no. 3-4, p. 581, 1995.
 - [49] D. Finnemore, T. Stromberg, and C. Swenson, "Superconducting Properties of High-Purity Niobium," *Physical Review*, vol. 149, no. 1, p. 231, 1966.
 - [50] F. Pobell, *Matter and Methods at Low Temperatures*. Springer, 3rd ed., 2007.

Samenvatting

De Lead Zeppelin is een klein brokje Lood dat zweeft door middel van magnetische levitatie. Lood is een supergeleider bij afdoende lage temperatuur — de experimenten beschreven in deze thesis vonden plaats bij 4.2 K, het kookpunt van Helium-4 — en kan daarom via het Meissnereffect zwevend gemaakt worden door een geschikt magneetveld aan te leggen. Wij doen dit door de Lead Zeppelin te omarmen met twee stroomdragende spoelen van tegengestelde polariteit, die zodoende een potentiaalput creëren waarin de Zeppelin gevangen zit. In dit proefschrift worden de ontwikkeling van het Lead Zeppelinexperiment en de daaropvolgend verrichte metingen beschreven.

Het idee voor de Lead Zeppelin is ontsproten aan de MRFM-experimenten die in onze groep gedaan worden. MRFM (Magnetic Resonance Force Microscopy) is een techniek die gebruik maakt van de minieme magnetische interactie tussen atomen en een uiterst gevoelige krachtsensor, die zich onder andere ten doel stelt complexe moleculen, zoals proteïnen en virussen, in beeld te brengen in hun natuurlijke habitat, in 3D en met atomaire resolutie. De krachtsensor in MRFM is een ferromagnetisch bolletje van enkele micrometers in diameter op het uiteinde van een hefboompje van 100 tot 200 micrometer lang ($1\text{ }\mu\text{m} = 10^{-6}\text{ m}$). Deze sensor is gevoelig voor krachten in de orde van zeptoNewtons ($1\text{ zN} = 10^{-21}\text{ N}$), wat, anders gezegd, overeenkomt met de zwaartekracht die u ondervindt van uw geëmigreerde collega in Australië. Deze gevoeligheid heeft het mogelijk gemaakt de magnetische kracht te meten tussen de sensor en ~ 500 protonen.

Een nog grotere gevoeligheid wordt in MRFM, in principe, belemmerd door zogenoemde klemverliezen van de hefboom; het is dan ook voor de hand liggend om de hefboom uit het experiment te willen verwijderen. Zodoende stappen we over op de magnetische levitatie van een supergeleider, die geen klemverliezen kan lijden door de afwezigheid van zo'n hefboom.

Het ontwikkelen van een in toenemender mate gevoelige krachtsensor dient nog een ander doel: het experimenteel onderzoeken van de quantum mechanica van zware objecten. Het is namelijk duidelijk dat quantum mechanica geen goede theorie is om grote objecten als planeten of stoelen of kopjes koffie mee te beschrijven, terwijl zij met ongekennde precisie het gedrag beschrijft van kleine objecten als quarks en elektronen. Ergens op de schaal van klein naar groot, gaat er iets mis.

De algemene relativiteitstheorie, die de zwaartekracht beschrijft, werkt juist heel goed voor grote objecten, maar is experimenteel ongetoetst voor objecten lichter dan zo'n 90 gram. De algemene relativiteitstheorie en quantum mechanica beschrijven elk hun eigen deel van de werkelijkheid om ons heen, maar zijn tegelijkertijd inherent onverenigbaar. Het is buitengewoon interessant om een experiment te doen juist in het schemerig gebied waar beide theorieën elkaar tegenkomen. Hoe gedraagt een object zich dat groot genoeg is om een merkbare zwaartekracht te vertonen, maar dat klein genoeg is om de wetten van de quantum mechanica te gehoorzamen?

We beginnen dit proefschrift met het beschrijven van de door ons ontworpen meetopstelling. We bespreken de basisbenodigdheden om een succesvolle meting te kunnen doen en de theorie die erachter schuil gaat. Ook bekijken we voor hoeveel problemen externe vibraties kunnen zorgen en we rekenen uit wat de vereisten zijn om van de Lead Zeppelin een krachtsensor te maken die kan meedingen met de state-of-the-art. Omdat wij onze metingen doen met behulp van een Superconducting QUantum Interference Device (SQUID) — een zeer gevoelige magnetometer — verdient het afschermen van het experiment van elektromagnetische verstoringen speciale aandacht.

In hoofdstuk 3 bespreken we in detail een onderdeel van de meetopstelling dat van cruciaal belang is gebleken: de Persistent Current Switch (PCS). De PCS vormt een schakelaar tussen het experiment en de ruizige buitenwereld, die ons in staat stelt de beweging van de Lead Zeppelin te meten zonder de door ons gebruikte SQUID-magnetometer te laten verdrinken in ruis.

In hoofdstuk 4 lanceren we de Lead Zeppelin: we tonen aan dat de Lead Zeppelin inderdaad zweeft en dat we dit kunnen meten. We zien ook dat de beweging wordt aangeslagen door externe vibraties. Bovendien bemerken we dat de Zeppelin een niet-lineaire component heeft in zijn bewegingsvergelijking, wat zich onder andere uit in het verband tussen de resonantiefrequenties van de Zeppelin en de levitatiestroom, alsmede in mengtermen tussen verschillende trillingsmodi van de Zeppelin. Uit de metingen blijkt verder dat de dempingscoëfficiënt van de Lead Zeppelin gedomineerd wordt door interactie met omringend Heliumgas.

We sluiten met hoofdstuk 5 af met een korte discussie en enkele tijdens de metingen bedachte verbeteringen. Het is bijvoorbeeld wenselijk om beter de externe vibraties te ontkoppelen van het experiment, waar we methoden voor aandragen, en om de damping op de Lead Zeppelin te verlagen teneinde zijn gebruik als sensor voor steeds kleinere krachten werkelijkheid te maken. Sommige van de voorgestelde verbeteringen zijn al gefabriceerd en hoeven nog slechts te worden getest; worden zij aan de praat gekregen, dan is een zwaartekrachtmeting tussen massa's lichter dan ooit tevoren binnen handbereik.

List of Publications

L. Rademaker, T. van der Reep, N. Van den Broeck, B. van Waarde, M. de Voogd and T. Oosterkamp, “The Instability of a Quantum Superposition of Time Dilations”, *submitted*, preprint available at *arXiv:1410.2303*.

B. van Waarde, O. Benningshof and T. Oosterkamp, “A Magnetic Persistent Current Switch at milliKelvin Temperatures”, *Cryogenics*, vol. 78, p. 74, 2016.

B. van Waarde, M. de Wit, G. Koning and T. Oosterkamp, “Flying a Lead Zeppelin, a Force Sensor without a Handle”, *in preparation*.

Curriculum Vitae

The author was born on November 16th, 1986, in Haarlem, the Netherlands. He went to Lyceum Sancta Maria in Haarlem for his gymnasium high school diploma, and graduated in 2005.

Following this, he started a Bachelor of Science in Applied Physics at Delft University of Technology. The Bachelor of Science degree was obtained in 2010, for which the author studied the mechanical properties of suspended graphene in the group of prof. dr. ir. H. S. J. van der Zant.

As there was no ‘hard cut’ (*harde knip*) between the Bachelor and Master programmes at the time, he could start with his Master of Science in 2009. During the master’s, he made a switch of universities, and a Master of Science degree in Theoretical Physics was obtained in 2011 at Leiden University, for which he studied string theory and the quantum mechanics of charged fermions in fancy curved spacetimes under supervision of prof. dr. K. E. Schalm.

In 2012, he started his PhD in Experimental Physics at Leiden University in the group of prof. dr. ir. T. H. Oosterkamp, where he manufactured and studied the magnetic levitation of a small superconducting particle — the Lead Zeppelin.

Acknowledgements

During the past four years of my PhD I have received the help and support from many people. Words can never adequately express my gratitude, nor can I thank everyone who bore with me all this time within the restraints of this short acknowledgement. Nevertheless, I want to devote some words of appreciation on at least some of these people here.

First, I want to thank my supervisor Tjerk for giving me the opportunity to do a PhD, and for his immense help and vast involvement in the many aspects of my experiments and this thesis. I feel lucky to have had you for my supervisor. I thank my colleagues Marc, Jelmer, Arthur, Martin, Tom, Lucia, Gesa and Olaf, in various subsets, for valuable discussions, the awesome road trip in the USA, help with setting up experiments and understanding data, proofreading this thesis, Whisky Friday, and overall their good companionship.

I thank Ellie, who took care of effectively all bureaucracy, and without whom the scientific output from our institute would certainly quickly grind to a halt. Also thanks for the coffee.

I thank the Department of Fine Mechanics for their creative solutions to my questions and their amazing craftsmanship; in particular Gert Koning, Fred Schenkel and Christiaan Pen. Likewise, the Electronics Department deserves my gratitude for help with electronics and computer related problems. I further thank Hans van Kuyk and Wilfred van der Geest, who have supplied me with much-needed liquid Helium throughout the course of my PhD.

I thank Marcel Rost, Wim Bosch and Kier Heeck for sharing their knowledge with me, and for their indispensable suggestions for improvements on the experimental design.

I want to thank my family, to whom I must have appeared to have completely faded from existence from time to time (sorry about that).

Finally, this PhD would have been impossible if not for my physics buddies and close friends Bart, Ruud, Robert-Jan, Chris, Daniël, Fran(çois), Victor and Bas, who helped me greatly during the course of my PhD, professionally as well as personally.