Categories and logical syntax

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Title: Categories and logical syntax
Issue Date: 2014-09-04

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Academisch Proefschrift

ter verkrijging van de graad van doctor
aan de Universiteit Leiden
op gezag van de Rector Magnificus prof. mr. C.J.J.M Stolker
volgens besluit van het College voor Promoties
te verdedigen op donderdag 4 september 2014
klokke 10.00 uur

door

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geboren te Lørenskog, Noorwegen
in 1982

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This research was supported by the Promoties in de Geesteswetenschappen programme of the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO).

Printed and bound by GVO Drukkers en Vormgevers.

Versteht man unter Kategorie die logische Form im weitesten Sinne, so muß der Begriff der Kategorie zum obersten Begriff der Logik werden.

Emil Lask

Introduction

Gilbert Ryle opens his paper *Categories* (1938) by remarking that "doctrines of categories and theories of types are explorations in the same field." He goes on to show how in particular Aristotle's doctrine of categories can be seen as a special case of the theory of types. That particular proposal will not be defended here, but a principle underlying Ryle's argument serves to organize what follows: categories, or types, and logical syntax are closely affiliated notions. It will be argued that Aristotle's as well as Kant's categories are close associates of what I shall call syllogistic syntax, namely a logical syntax that takes the form 'S is P' as basic; while type theory is a close associate of function–argument syntax, namely a logical syntax that takes the form $f(a)$, function applied to argument, as basic.

The dissertation is divided into two chapters, the first dealing with syllogistic syntax and the second with function–argument syntax. The traditional doctrine of categories and type theory each invite different sets of reflection. A discussion in one chapter is therefore in general not mirrored by a discussion in the other chapter, and neither does a discussion in one chapter in general depend on one in the other; but certain themes from the first chapter recur in the second; and it is hoped that the first chapter adds depth and perspective to the second. Apart from the relation of categories to logical syntax, recurring themes are the generality of the categories and the nature of the items categorized. Throughout, our emphasis is on logical aspects of categories. For an overview of the content of each chapter the reader is referred to the table of contents and to the synopses at the beginning of each chapter.

The most important contribution of this dissertation, to my mind, is the map it presents of a large range of conceptions of category and type, indeed of most such conceptions that are of relevance to logic. The map drawn in chapter 1 is one in which Aristotle's categories, the parts of speech, and Kant's categories all find their place. It requires less ecumenical effort to synthesize the various notion of type that have been salient in the logico-philosophical debate since the time of Frege and Russell; but I am not aware of any such effort of the same extent as that documented in chapter 2 below. A dissertation must, however, do more than just report on existing speculations pertaining to a certain topic. In chapter 2 we enter into several current debates, in particular debates concerning category mistakes, Frege's problem of the concept horse, sortal concepts, and generality. We give an introduction to Martin-Löf's constructive type theory and argue for its relevance to these debates. In chapter 2 we moreover deal with various exegetical issues, for instance the relation between the pairs function/argument and function/object in

Frege's works; the role of the notion of range of significance in Russell's type theories; and the proper understanding of Husserl's pure grammar in relation to categorial grammar. The chapter ends with a brief discussion of the notion of material categories, with which we leave the domain of logic and enter that of phenomenology. A thorough treatment of that topic therefore lies outside the scope of this dissertation. Chapter 1 is more historical in character, but I have tried to avoid painstaking exegesis; not that there is anything wrong with painstaking exegesis, but it would be out of place in a work with a scope as broad as the current one. Besides the synthesis already mentioned, the main contributions of chapter 1 are found in its sections 1 and 2, in particular in their examination, firstly, of the items categorized by, and secondly, of the generality of Aristotle's categories.

A definition of the notion of category will not be provided in the current work. Instead we shall study different notions of category already present in the philosophical literature. In that regard the work is an historical one. The bits of history that we do, however, are motivated by systematical concerns. We rarely consider historical causes and historical consequences; and we generally ignore historical contexts. Our main aim is not to understand the history of the notions of category and type, but to understand these notions themselves. In that regard the work has systematical aspirations. But whether the pudding offered here contains systematical fruits can only be proved by eating it.

Contents

Acknowledgements

I am very grateful to my supervisor, Göran Sundholm, for all the guidance, inspiration, and support he has given me since I first met him at his house in April 2007. How he has informed my thinking and my approach to philosophy cannot easily be expressed in words, but I think it shows itself on almost every page of what follows.

With Stefan Roski and Robert Goené I have had countless discussions about categories, types, and related topics. Many thanks are due to them as well as to the students of a seminar on categories that I gave at Leiden in the autumn of 2012.

The influence of Per Martin-Löf on the present work will be evident to anyone who is acquainted with his lectures on the philosophical aspects of constructive type theory. I am grateful for the opportunity I had to discuss constructive type theory with him at Leiden in February 2014.

Without the critical remarks from Frans de Haas on an early draft of my PhD fellowship application I believe that the application would never have been successful; for those remarks I remain thankful. The Philosophical Institute at Leiden University has offered a relaxed atmosphere during the last four years; special thanks are due to its administration for assisting me in all matters extra-philosophical.

A precondition for this dissertation's having been written is the PhD fellowship very kindly granted to me by the Dutch research council, NWO, through its programme *Promoties in de Geesteswetenschappen.* As well as expressing my gratitude to that institution I wish to express my hope that it will continue to finance independent PhD projects in the humanities.

Finally, I wish to thank my parents for their support in various matters pertaining to the writing and public defence of this dissertation.

Bibliographical abbreviations

Works by Aristotle

Unless otherwise noted below, the translations used are the Oxford translations of Barnes (1984). The Greek texts consulted are the relevant editions of the Oxford Classical Texts.

Cat	*Categories*	
Int	*De Interpretatione*	
APr	*Prior Analytics*	(Striker, 2009)
APo	*Posterior Analytics*	
Top	*Topics*	(Smith, 1997)
SE	*Sophistical Refutations*	
Phys	*Physics*	
DA	*De Anima*	
Met	*Metaphysics*	
Rhet	*Rhetoric*	
Poet	*Poetics*	

Commentaries on Aristotle

These are abbreviated *in APr*, *in Cat*, and *in Int*, for *In Aristotelis Categorias commentarium*, etc. The Greek texts consulted are the relevant editions of *Commentaria in Aristotelem Graeca* (*CAG*). Page references (also found in the translations) are to these volumes. The following translations are used.

Alexander of Aphrodisias	*in APr*	Barnes et al. (1991)
Ammonius	*in Cat*	Cohen and Matthews (1991)
Ammonius	*in Int*	Blank (1996)
Dexippus	*in Cat*	Dillon (1990)
Porphyry	*in Cat*	Strange (1992)
Porphyry	*Isagoge* (*Isag*)	Barnes (2003)
Simplicius	*in Cat*	Chase (2003)

Other ancient works

DL	Diogenes Laertius, *Lives of Eminent Philosophers*
GG	*Grammatici graeci*
GL	*Grammatici latini*

References are to book and pager number. Text and translation of *DL* is found in Hicks (1931). Translations of Plato are taken from Cooper (1997).

Works by Frege

Bs	*Begriffsschrift*	Frege (1879)
GLA	*Die Grundlagen der Arithmetik*	Frege (1884)
FB	*Funktion und Begriff*	Frege (1891)
BG	*Begriff und Gegenstand*	Frege (1892a)
SB	*Sinn und Bedeutung*	Frege (1892b)
Gg	*Grundgesetze* I and II	Frege (1893, 1903a)
UGG	*Über die Grundlagen der Geometrie* I and II	Frege (1903b, 1906)
WF	*Was ist eine Funktion?*	Frege (1904)
Ged	*Der Gedanke*	Frege (1918)
Vern	*Die Verneinung*	Frege (1919)
NS	*Nachgelassene Schriften*	Frege (1983)
WB	*Wissenschaftlicher Briefwechsel*	Frege (1976)

Works by Kant

KrV	*Kritik der reinen Vernunft*	(Kant, 1787)
Prolegomena	*Prolegomena zu einer jeden künftigen Metaphysik*	(Kant, 1783)
Jäsche Logik		(Kant, 1800)

Page references are to the *Akademie* editions.

Works by Russell

PoM	*Principles of Mathematics*	Russell (1903)
PM	*Principia Mathematica*, volume 1	Russell and Whitehead (1910)

Works by Husserl

References to the *Logische Untersuchungen* (Husserl, 1901) are of the form *LU* + investigation number + paragraph number. References to the *Ideen zu einer reinen Phänomenologie und phänomenologische Philosophie* are of the form Ideen + book number + paragraph number. (Only the first book, (Husserl, 1913), was published during Husserl's lifetime. The other books, (Husserl, 1952a) and (Husserl, 1952b), are based on *Nachlass* material; cf. the editorial overview in (Husserl, 1952a).)

CHAPTER 1

Categories and Syllogistic Syntax

Synopsis

In this chapter we study various traditional doctrines of categories: Aristotle's doctrine, the grammatical doctrine of the parts of speech, and Kant's doctrine. The main point of attention is Aristotle's doctrine of categories; the discussions of the parts of speech and of Kant's categories are both set against that background.

In the first section we consider the question of what the items are that fall under Aristotle's categories. We argue that these are terms in the sense of syllogistics. The starting point of the second section is the traditional interpretation of Aristotle's categories as highest genera. This motivates a study of the so-called predicables, of which genus is one. We argue that Aristotle's categories are not highest genera, but rather classes of terms. The nature of items categorized is taken up again in section 3. Terms are linguistic in nature. We report on an ancient thesis that the items categorized are linguistic and argue against a recent contention that the category scheme in chapter 4 of the *Categories* differ from that in chapter I.9 of the *Topics*. The doctrine of parts of speech is introduced in the fourth section. We look at elements of its early history and its relation to Aristotle's categories. In section 5 we suggest a view of the relation of Kant's categories to those of Aristotle in terms of the distinction between categorematic and syncategorematic terms. There we also consider various ways of "deriving" the categories from certain other notions.

1. Items categorized

In order to understand what sorts of items fall under Aristotle's categories it is natural to look to chapter 4 of the *Categories*, where the categories are first introduced (the list can be found in Appendix 1 at the end of this chapter (p. 66)). They are said there to collect "things said without combination" (*ta kata mēdemian symplokēn legōmena*).[1] Section 1.1 investigates this and related notions in Aristotle's work; it is proposed that items categorized are terms in the technical sense of syllogistics. It is sometimes held that items categorized are predicates; this view is scrutinized in section 1.2 and shown when properly understood to be subsumed by the view that items categorized are terms.

1.1. Things said without combination. The examples of things said without combination that Aristotle lists in chapter 4 of the *Categories* include 'man', 'white', 'four-foot', 'double', 'in the Lyceum'; as examples of things said *with* combination he lists in chapter 2 of the same work 'man runs' and 'man wins'. But apart from these examples Aristotle gives only a negative characterization of the notion: being said *with* combination is a prerequisite for being true or false. That truth and falsity presupposes combination is a claim one finds not only in the *Categories* ($2^{\mathrm{a}}10$, $13^{\mathrm{b}}10$), but at several places in Aristotle's works,[2] and it may be regarded as one of the main theses of Plato's *Sophist*. It is in fact reasonable to assume that Aristotle with his notion of thing said without combination alludes to this work of his teacher.[3] Plato there (261d–263d) notes that a *logos* comes to be when certain spoken sounds (*phōnai*) "fit together."[4] A list of verbs such as 'walks runs sleeps' remains a list, as does a list of nouns such as 'lion stag horse', for these words do not fit together. But when a noun and a verb are combined,[5] as in 'Socrates walks', the words fit, and the result is a *logos*. Things said without combination would therefore be the elements combined in a *logos*.

[1]We follow Ackrill in using 'thing said' as the translation of *legomenon*, a participle of *legō*; thus the 'thing' is supplied by the English.

[2]*Int* $16^{\mathrm{a}}11$, *DA* III.6 $430^{\mathrm{b}}2$, *Met* E.4 $1027^{\mathrm{b}}19$. Whereas the *Categories* speaks only of *symplokē*, these cited places uses the pair of terms *synthesis* (combination) and *diaresis* (separation). It is clear from the cited *DA* passage that Aristotle sees a close parallel between combination and separation, so for ease of exposition I shall omit separation from the discussion. For the claim that a term by itself is not yet a proposition, see also *Top* $101^{\mathrm{b}}26$–28.

[3]As was perhaps first noted by Trendelenburg (1846, pp. 11–12).

[4]Plato was probably reacting to "the problems of predication" raised by Parmenides and Antisthenes: how is false, and how is non-tautological predication possible? (cf. Nuchelmans, 1973, pp. 9–12) The problem of falsity (and non-being) is a main theme throughout the greater part of the *Sophist*. Antisthenes's problem of non-tautological predication is raised at 244c: "Surely it is absurd for someone to agree that there are two names when he maintains that there's only one thing" (cf. 251b–c).

[5]Plato uses both the noun *symplokē* (262c6) and a participle of the corresponding verb *symplekō* (262d4) to describe this combination of noun and verb.

Let us not worry now about the fact that Plato calls these elements noun and verb; as we shall see in the next paragraph Aristotle introduces a different terminology, and as we shall see in section 4.1, he reserves 'noun' and 'verb' for making more purely grammatical distinctions.

The notion of *logos* that Plato describes in the *Sophist* is taken over by Aristotle with his notion of *logos apophantikos*, translated by Ackrill as 'statement-making sentence'. This is a sentence in which there is truth or falsity (*Int* $17^{\mathrm{a}}2$); hence it is a thing said with combination. Moreover, it says something of something (*ti kata tinos*, $17^{\mathrm{a}}25$). Thus, firstly, a statement-making sentence is a combination, and secondly, it says something of something. It is reasonable therefore to suggest that what is combined in a statement-making sentence are the two "somethings" of the formula 'something of something', namely the something said of something else and the something else of which that something is said. In the context of Aristotle's syllogistics these are both called terms (*APr* $24^{\mathrm{b}}16$):

> I call a term that into which a proposition is resolved, that is, what is predicated and what it is predicated of, with the addition of to be or not to be.

What is here called a proposition (*protasis*) is "a sentence (*logos*) affirming or denying something of something" (*APr* I.1 $24^{\mathrm{a}}16$), and thus coincides with the notion of a statement-making sentence as defined in the *De Interpretatione*. Instead of "somethings" this definition speaks of 'what is predicated' and 'what it is predicated of', and baptizes these things 'terms'. Since a statement-making sentence was taken to be a combination of the two "somethings" in the formula 'something of something', it can therefore also be said to be a combination of terms: a statement-making sentence is a combination of terms.

The phrase 'with the addition of to be or not to be' in the quoted passage presumably refers to the copula. Thus, there are the terms S and P and the addition of to be or not to be in the shape of the copula. The proposition is therefore assumed to have the form 'S is P', the basic form of the syntax of syllogistics. Not all statement-making sentences are of this form, however: 'man runs', for instance, is not. If a combination of terms has the form 'S is P', it may therefore be asked whether all statement-making sentences are combinations of terms. Aristotle can be viewed as dealing with this question in chapter 10 of the *De Interpretatione*, where he argues that an important class of statement-making sentences is reducible to syllogistic form. Aristotle distinguishes the three forms of simple statement-making sentence illustrated by the following examples.

(1) man is (not)
(2) man is (not) recovering
(3) man recovers/does not recover.

Here (2) is of syllogistic form, and Aristotle holds that (3) is reducible to (2): replacing the finite verb 'recovers' in (3) by a copula and a participle we get 'man is recovering'.[6] Aristotle says little about the first form, and the reason is perhaps that it is difficult to see that it involves any combination at all.[7] In fact, when Brentano in the 19th century suggested a revision of logical syntax, taking the form (1), 'S exists (not)', as basic, he did so partly on the grounds of a conviction that not all judgements involve a combination.[8] However that may be, Aristotle disregards this form in his logic, and his reduction of (3) to (2) allows him to concentrate on the form 'S is P', and that is a combination of terms. But if a thing said with combination is a combination of terms, then a thing said without combination must be a term; hence we propose to identify things said without combination with terms. The proposal is therefore also that items categorized by Aristotle's categories are terms.

In the *Analytics* Aristotle gives many examples of terms, including 'there being a single science' (*to mian einai epistēmēn*, *APr* I.36) and 'there is knowledge of the good, that it is good' (*tou agathou estin epistēmē hoti agathon*, *APr* I.38). Hence one sees that terms need not be of the simple form exhibited by 'man' and 'white', but may have a varying degree of complexity. In *APr* I.35 Aristotle in fact distinguishes between simple and complex terms, calling the former *onomata* and the latter *logoi* ($48^{a}29$–30). But if a term is a thing said without combination, then we need to understand the complexity of a complex term otherwise than as the sort of combination yielding a thing said with combination. To that end let us first consider the word *logos*, employed by Aristotle for complex terms.[9]

According to the *De Interpretatione* a *logos* is a significant spoken sound that has parts significant in separation, while an *onoma* is a significant spoken sound that does not have parts significant in separation. Ackrill translates *logos* in this context as 'sentence'. Hence, as a sentence is presumably a thing said with combination, one is led to think that having parts significant in separation is just the same as being said with combination. A more thorough examination shows that such an identification

[6]The same reduction is made at *Met* Δ.7 $1017^{a}27$ff..
[7]Alexander of Aphrodisias (*in APr* 15,15–15,23) argues that this form as well is reducible to the first form, namely as 'man is being', hence with 'being' as the predicate term. This view is not compatible with our interpretation, since it would force us to assign a category to 'being'.
[8]Brentano first presents his revision of syllogistic syntax in the seventh chapter of *Psychologie vom empirischen Standpunkt* (Brentano, 1874, pp. 271–289).
[9]For the following cf. the similar considerations of Moravcsik (1968, pp. 126–135).

cannot be upheld. The translation of *logos* as 'sentence' is not compatible with Aristotle's terminology in *APr* I.35, where it means complex term; but that may be just another instance of the famous polysemy of this Greek word. There are, however, reasons to think that this is not so. In *Poetics* 20, a chapter that repeats almost verbatim the definition of *logos* given in the *De Interpretatione*, Aristotle on the one hand says that there are *logoi* without verbs, hence sub-sentential units, and on the other hand he calls the *Illiad*, a super-sentential unit, a *logos* (cf. *APo* II.10 93^b35). Thus, the word *logos* seems to signify complex sayings quite generally.[10] Nothing in Aristotle's definition of *logos* at *Int* 4 and *Poet* 20 excludes such an interpretation, and even in the *De Interpretatione* (16^a21) one finds an example of a sub-sentential unit, namely 'white field', being called a *logos* (cf. *Int* 17^a16). That a *logos* need not be a thing said with combination is in fact confirmed by the example 'rational mortal animal'. This phrase, or whatever one takes to be the definition of man, is a *logos* according to *Poet* 1457^a25; but it is synonymous to 'man', since a defined term is synonymous to its definition (*Cat* 1^a6); but 'man' is a thing said without combination (*Cat* 1^a19); hence, on the reasonable assumption that two synonymous phrases are either both said with or both said without combination it follows that 'rational mortal animal' must also be a thing said without combination. We cannot therefore identify the two notions of having parts significant in separation and being said with combination, for there are items having parts significant in separation that are not said with combination.

This is not to say that there are no *logoi* that are things said with combination: statement-making sentences, *logoi apophantikoi*, are precisely that. We thus have a genus of *logos*, defined as a sign having parts significant in separation, with the two species under it of complex terms on the one hand and things said with combination on the other. These are species of *logos*, and so are complex sayings, but they differ in the nature of their complexity: on the one hand we have the complexity pertinent to terms and on the other the complexity pertinent to sentences. A similar distinction has been drawn in more recent times by Jespersen (1924, esp. pp. 96–144) between what he calls *nexus* and *junction*. A junction is a complex term, typically generated by adding one or more qualifiers to a simple term (ibid. pp. 108–114); so it has parts significant in separation in Aristotle's sense. A nexus is a sentence or a sentence-like combination (ibid. esp. pp. 117–122), corresponding to Aristotle's thing said with combination. That a nexus need only be *sentence-like* means that the notion comprehends apart from sentences—be it as main or subordinate clauses—also accusatives with infinitive

[10] Barnes (2007, p. 180) translates *logos* in this context as 'saying', while the German translation of Weidemann (2002) uses *Wortgefüge*.

and the combination of a verbal or adjectival noun with a genitive, as in 'the doctor's arrival'. Like Aristotle, Jespersen relies on our intuitive grasp of the difference between these two manners of combination, and the only general characterization he offers are elucidations of a junction as "lifeless, stiff, rigid" and as "a single idea," and of a nexus as "having life in it," being "pliable, as it were, animate and articulated" and "always containing two ideas which must necessarily remain separate" (ibid. pp. 115–116). We have maybe reached a point here where crisp descriptions must give in, and elucidations by means of metaphors or otherwise take their place.

In any event we must distinguish complex terms from sentences. Complex terms have parts significant in separation, but are not said with combination; sentences are things said with combination. A simple term must then be a term that has no parts significant in separation. As is seen from a discussion in Porphyry's commentary on the *Categories* (3rd century AD), this definition is problematic. Porphyry maintains that verbs in the first-person as well as certain idiomatic third-person forms are, even when said by themselves, things said *with* combination.[11] Although the subject is not then expressed, it is implicit in the verbs themselves (*in Cat* 87,38ff.).[12] Among these idiomatic third-person forms Porphyry counted 'it rains', in Greek expressed by the one word *huei*.[13] According to Aristotle's definition of the verb (*Int* $16^{\text{b}}6$), it is a sign having no parts significant in separation. A verb is therefore a simple term according to the suggested definition. If Porphyry is right, however, some verbs are also things said with combination. Verbs of the kind Porphyry points to would therefore be counterexamples to the identification of simple terms with words having no parts significant in separation.

Until now we have assumed, as Aristotle also did, that a simple term is a simple expression and a complex term a complex expression. We have in effect twice found that simplicity of expression cannot be used as an indicator of saying without combination: there are complex terms, such as 'mortal rational animal', that say things

[11]Cf. Wittgenstein's remark (TLP 4.032) that the simple sign *ambulo* is composite.

[12]This view was shared by Ammonius in his discussion of the same problem at *in Int* 28,11–28,28.

[13]According to Porphyry *huei* has 'Zeus' as implicit subject. This appears to have been the common construal of the sentence among the Greeks (cf. Miklosich, 1883, p. 7). Brentano held, supported by the work of Miklosich (ibid.), to the contrary that such sentences are truly subjectless (cf. Brentano, 1925, pp. 183–187). Subjectless sentences, as they were sometimes called, were in fact much discussed among 19th century logicians (in lecture notes from 1917 Husserl (1996, p. 172) speaks of "die endlose Literatur" on this topic). Herbart, who was among the first to deal with subjectless judgements in the context of syllogistic syntax did so by construing their subject to have no content (and therefore maximum extension); see Herbart (1837, § 63). See the entry on *Subjektlose Sätze* in *Eisler*'s *Wörterbuch der philosophischen Begriffen* for an overview of what various other authors said on this matter.

without combination as well as simple terms, such as *huei*, that say things with combination. But the notion we are after is perhaps not simplicity of expression, but rather simplicity of meaning. Another commentator, Ammonius (5th century AD), held that a thing said without combination is what has both a simple meaning and a simple expression (*in Cat* 32,26–33,4):

> For (1) we can use a simple expression (*phōnē*) with a compound meaning (*sēmainomenon*), as when I say *trechō* ('I run'), for I refer to myself and to my action; (2) the expression may be compound but its meaning simple, as in 'mortal rational animal', for the expression is compound, but its meaning is man; (3) both may be compound, as when I say 'Socrates runs'; or (4) both may be simple, as the categories themselves.

Ammonius here assumes that we have a means of recognizing a compound meaning in a simple expression and a simple meaning in a compound expression. But what are such means? We seem to have a good grasp of the distinction between what is sentential and what is not, and on that basis we may settle such cases as *trechō* and 'Socrates runs'. Moreover, if we know of an expression that it has simple meaning, then we can infer that its definition will have simple meaning, as in 'mortal rational animal'. These seem to be the means of recognition of compound meaning implicit in Ammonius's examples. It is clear that they leave many cases unsettled. 'White man' is a compound expression, but does it have a compound meaning? Defining 'Whan' to mean 'white man', does 'whan' have a simple meaning?[14] One chapter in the *Organon* that could be taken to deal with such questions is *Int* 11, but it is not clear what it says on the matter; as Barnes (2007, p. 130) notes, it "obscures rather than illuminates." Indeed, no general test for the simplicity or otherwise of the meaning of an expression is forthcoming, it seems to me.

We are primarily seeking instruction on the sort of item that is categorized by Aristotle's categories. Perhaps it is possible to make progress on that matter without first explicating the nature of the complexity of a complex term and the simplicity of a simple term. It seems reasonable to say that if a term t is of category C, then adding a suitable qualifier (e.g. an adjectival, adverbial, or prepositional phrase) to t yields a new term belonging to the same category C, even though the qualifier taken by itself may be of another category C'. For instance, a white man is a man and therefore a substance, even though white is a quality. Ackrill (1963, pp. 73–74) holds that 'white

[14] This question is in effect raised by Ackrill (1963, p. 73); Barnes (2007, pp. 132–133) discusses it and related questions.

man' does not fall into any category, for it "introduces two items from two categories." But it seems like an unnecessary restriction on the scope of Aristotle's category theory if qualified terms could not be said to belong to a category. Man, let us assume, is defined as rational animal. Hence, if man belongs to the category of substance, then rational animal does as well. But in 'rational animal' we have two items from two categories. That a qualified term belongs to a category is thus presupposed by the practice of definition. More specifically, the doctrine of definition by proximate genus and differentiae seems to presuppose that a qualified term belongs to the same category as its head. The proximate genus, namely, must belong to the same category as the defined term, although the differentiae need not do so.[15] Could there be other rules apart from qualification for generating categorized terms? Conjunctions such as 'Plato and Socrates' or 'walking or conversing' could perhaps be called complex substances and complex actions, but to a conjunction such as 'cuts while being cut' it is not possible to assign a category since it conjoins an action and an affection.[16]

The foregoing suggests that if we were to supply syllogistics with category specifications, then we should have to assume a regimented language of some form. The problems surrounding the notion of simple term suggest that these would have to be laid down by fiat, and each assigned a category. From the simple terms new terms would be generated by formation rules formulated such that each term could be assigned a category in some sense consistent with the category of the terms from which the new term was generated. These formation rules would include a rule of qualification that would take, say, a substance term and a quality term and yield a substance term; more generally, we should need to stipulate which kind of term may qualify others, and what the category of the qualified term should be. Besides there could be rules such as intra-categorial conjunction. Assuming, as we do here, that items categorized are terms, such formation rules would also determine the extension of the concept of a term: each term belongs to a category; what does not belong to a category is not a term. It would presumably follow that Aristotle's example of a

[15] Morrison (1993) argues convincingly that P's being a differentia of S does not determine its category (e.g. it need not be of the same category as S). An important assumption of Morrison's is a distinction between categories of predication and ontological categories. I shall argue in section 3.2 below that this distinction is not Aristotelian; to my mind, the true Aristotelian pendant of Morrison's "categories of predication" are the predicables (substance as a category of predication corresponds to the predicable of genus, while quality as a category of predication corresponds to the predicable of differentia (on which Aristotle was not quite clear, see section 2.1.2 below)). I am grateful to Donald Morrison for sending me a copy of his paper.

[16] Conjunction of terms was regarded by Porphyry as one of two ways of saying with combination (*in Cat* 71,5). Simplicius *in Cat* 71,18 holds that a verb in the middle voice will signify both an action and an affection, and so may not be categorized.

complex term ‘there being a single science’ is in fact not a term. It is not a term since it cannot be assigned a category. That appears not to raise a problem, however, since the sentence Aristotle wishes to formalize, ‘Of contraries there is a single science’, is simply not formalizable in syllogistic syntax.

1.2. Subject and predicate. Plato noted that any proposition (*logos*) is about something and says something of that thing (*Sophist* 262e–263d). In effect he thereby drew the now traditional distinction between the subject and the predicate of a proposition. As a definition of this pair of notions there are certainly shortcomings in saying merely that the subject is (that which signifies) what the proposition is about while the predicate is what is said of that thing; for as Ryle (1933) noted, ‘about’ has many meanings.[17] ‘Peter’ is the subject of ‘Peter loves Mary’; but instead of saying that this sentence is about Peter we may perhaps equally well say that it is about loving Mary; that is, it is about loving Mary and says that this is a characteristic of Peter.[18] To this one could object, rightfully I think, that the original sentence was not ‘loving Mary is a characteristic of Peter’, but rather ‘Peter loves Mary’, and it is only the first of these that allows the suggested analysis. So the sentence ‘Peter loves Mary’ is not about loving Mary; but could we not say that it is about Mary, predicating Peter’s loving her of her?[19] As a foolproof definition of the notions of subject and predicate is not needed for our purposes, that is a question I shall not attempt to answer here.[20] That we grasp the distinction seems enough for what follows; and I take it as an unproblematic assumption that at least speakers of English do so. We have practiced that grasp as schoolchildren analyzing sentences, and again when applying the rule $\mathbf{S} \rightarrow \mathbf{N} + \mathbf{VP}$ in exercises of generative syntax.

The tradition is not always clear whether subject and predicate are linguistic or other sorts of items. Aristotle, for instance, vacillates at this point, taking subject and predicate now to belong to the level of *logos* (*APr* $24^{\mathrm{b}}16$ff.), now to the level of

[17]The definition of what a proposition is “absolutely about” defended by Goodman (1961) is of little use in defining the notion subject, since it entails (cf. ibid. pp. 9–10) both that ‘Socrates is identical to himself’ is not about Socrates and that ‘Cows are animals’ is about the class of non-cows.

[18]This point was made by Ramsey (1925b, p. 404).

[19]Geach (1962), who defined a subject of a sentence S as “an expression for something that S is about” (p. 23), insisted that ‘Mary’ in fact is a subject of ‘Peter loves Mary’.

[20]Apart from Geach, Strawson has written extensively on the distinction from the point of view of philosophy (cf. esp. his 1959, Part II; 1971; 1974). He regards the case where the subject is singular as basic and the general case as derived from this (1974, esp. pp. 35–36, 125–132), though he never offers anything like a definition of the general case. For a treatment from the point of view of linguistics, see Lyons (1968, ch. 8), who notes, for instance, that case may not be a foolproof indicator of what is the subject of a proposition. In some languages, namely, the case of what is the “goal” of a transitive verb is also the case of the subject of an intransitive verb; hence the subject (or “actor”) of a transitive verb will here not be in the same case as the subject of an intransitive verb (ibid., pp. 340–342).

pragma (*Int* 17ᵃ40).[21] According to the Stoics a predicate is a "deficient *lekton*," hence not an expression, but rather something like Fregean sense (*DL* VII.63). Ammonius (*in Int* 7,30), on the other hand, speaks of subject and predicate as vocal sounds (*phōnai*). In the more recent treatments of Geach (1962) and Strawson (1974) subject and predicate are understood to be expressions, and that is the practice we shall follow here. Thus we take 'Peter' and not Peter to be the subject of 'Peter loves her'. Peter himself may rather be said to be the topic of the proposition. Likewise 'loves her' is the predicate of 'Peter loves her', while loving Mary may be called the comment of the proposition, relating to the topic as the predicate relates to the subject.[22]

One might ask whether characterizing subject and predicate is what Plato in fact does at the place in question; might the characteristics he states not just be yet other characteristics of noun and verb? Indeed, in his examples illustrating the distinction it is always a noun that serves the former role and a verb that serves the latter. In that respect the examples are deceptive, for they all consist of only two words. Consider instead the following sentence.

> A young Norwegian mathematician who came from a poor family and died at the age of 26 proved that there is no general solution by radicals to quintic equations.

Here we should say that the first part 'A young...the age of 26' is the subject, while the rest is the predicate. But then one sees that there is no limit to the complexity of the subject and the predicate of a proposition, in particular, that they do not have to be single nouns and verbs respectively. One sees, moreover, that there may be noun phrases embedded in the predicate and verb phrases embedded in the subject. The fundamental contrast between subject and predicate on the one hand and noun and verb on the other lies, however, not in matters of complexity. The fundamental contrast lies in the fact that subject and predicate, unlike noun and verb, are relative notions. It is only in the context of a proposition that it makes sense to speak of subject and predicate. By contrast, a word is a noun or a verb, and a phrase a

[21]For more examples, see Barnes (2007, pp. 114–123).

[22]The pair of terms 'topic' and 'comment' seems to stem from Hockett (cf. Lyons 1968, p. 335 and *OED* on 'topic') but 'topic' is found with what must be the same meaning in Jespersen (1924, p. 146). A similar pair of notions is that of psychological subject and predicate, introduced by Von der Gabelentz (1869, p. 378): "ich nenne das, woran, worüber ich den Angeredeten denken lassen will, das psychologische Subject, das, was er darüber denken soll, das psychologische Prädicat."

noun phrase or a verb phrase, independently of its occurring in any given proposition.[23] Thus one speaks of the grammatical or logical roles or functions of subject and predicate in contrast to the parts of speech or word classes of noun and verb.

Our interest here is in propositions of syllogistic syntax, 'S is P'. As instances of this form we should, however, count not only 'Peter loves Mary' (analyzed as 'Peter is loving Mary') or 'man is mortal', but also 'some men are bald' and 'some men are not philosophers'. In general, a proposition of syllogistic syntax is determined not by its terms alone, but by its terms together with what is traditionally called its quantity and quality (and in modal syllogistics, its modality as well).[24] These are aspects of the proposition indicating whether the predicate is said of all or of some S's, and whether it is in fact said or rather denied of these. There is therefore a question whether, in 'some men are bald', one should count 'some men' or only 'men' by itself as the subject. A grammarian would presumably choose the former alternative (and would again realize the problem of defining the subject in terms of what the sentence is about: which men is 'some men is bald' about?). I am inclined towards choosing the latter, that is, towards saying that the term S by itself is the subject. For if the quantifier is counted as part of the subject term, then by symmetry we should also have to count negation as part of the predicate in a negative proposition; that is, we should have to countenance negative terms. It is unclear, however, what for instance a non-man is: whether it is an angel, or any substance whatsoever which is not a man, or something else. Fortunately, whether we say that 'some men' or rather that 'men' is the subject makes no difference to what follows.

A related question is whether P by itself or rather the phrase 'is P' should be viewed as the predicate of a syllogistic proposition. According to the stipulations of Geach (1962, p. 22ff.) it is 'is P' which is the predicate; likewise, the grammarian Jespersen (1924, p. 150) calls P by itself the *predicative* of the proposition and 'is P' its predicate. The view that P alone is the predicate can be found in Alexander of Aphrodisias (*in APr* 15,2ff.) and Ammonius (*in Int* 7,30ff.), in the *Port-Royal* Logic (Part II ch. 3) and in the *Jäsche Logik* (§ 24); let us quote Mill (1843, Bk. I ch. 1 § 2), however, who is conveniently explicit about the matter:

> Every proposition consists of three parts: the Subject, the Predicate, and the Copula. The predicate is the name denoting that which is affirmed or denied. The subject is the name denoting the

[23]Cf. Chomsky (1965, pp. 68–70), who notes that in one and the same sentence the same word may serve as the subject of one verb phrase and as the object of the other; thus in 'John was persuaded by Bill to leave' 'John' is, according to Chomsky, the object of 'persuade' but the subject of 'leave'.

[24]For this terminology, see section 4.3 below.

person or thing which something is affirmed or denied of. The copula is the sign denoting that there is an affirmation or denial, and thereby enabling the hearer or reader to distinguish a proposition from any other kind of discourse. Thus, in the proposition, The earth is round, the Predicate is the word *round*...

Mill thus takes the predicate not to include the copula. It seems to be mainly a matter of convention whether we decide for the one or for the other terminology, but it is important for our discussion here to have recognized both options.

In Aristotle's Greek *katēgoria* sometimes means 'predicate'. He says at *Cat* 3^a36 that "from a primary substance there is no predicate (*katēgoria*)," and at *Int* 21^a29 he speaks of predicates (*katēgoriai*) "containing no contrariety" (where 'dead' is taken to contain a contrariety to 'man', since a dead man is not a man). Hence, in light of Aristotle's introducing the categories in *Topics* I.9 (and at *APo* 83^b15) as *genē tōn katēgoriōn*, genera of predicates,[25] we should ask whether items categorized are not terms but rather predicates. A positive answer to this question seems presupposed in the traditional designation of the categories in Latin as *praedicamenta*, that which is predicated.[26] Which answer one ought to give depends on how one understands the notion of predicate in syllogistic syntax.

If such a predicate is taken to be of the form 'is P', then it is clear already on grammatical grounds that items categorized are not predicates. For Aristotle gives 'man' as an example of the category of substance, where no 'is' is to be found. To this it may be objected that in a Greek predication the 'is' is optional, thus one can say, for instance, *ho Sokratēs anthōpos*; hence, whereas 'is man' is not an Aristotelian substance, what is a substance is 'man' *when said of something*, that is, when predicated. Thus a predicate is a term *qua* predicated, whether or not an 'is' be attached to it.[27] At *Cat* 3^a36 Aristotle said that "from a primary substance there is no

[25]Frede (1981, pp. 32–35) argues that *katēgoria* in the *Topics* generally should be rendered 'predication', thus taken to signify a full proposition. But at *Top* I.9 Aristotle says that whatever is an accident, genus, property, or definition belongs to the *genē tōn katēgoriōn*, and whatever is one of these is not a full proposition, but a predicate as is clear from *Top* I.4 101^b26–27: "none of these [i.e., whatever is an accident, genus, etc.] said by itself is a proposition or problem [i.e., a predication]." For further criticism of Frede's argument, see Ebert (1985, p. 130, fn. 29).

[26]Martianus Capella, *The Marriage of Philology and Mercury* IV.362, 383 (early 5th century AD), speaks of the categories as *praedicationes*, but Boethius translates (early 6th century AD) *katēgoria* in the relevant sense as *praedicamentum*, hence at *Cat* 10^b19,21 and at Porphyry *Isag* 4.15; 4.21; 6.7 (see e.g. the relevant volumes of *Aristoteles Latinus*). In fact, Augustine refers to Aristotle's categories as *praedicamenta* at *Confessions* IV.16.29 (around 400 AD); on the Latin translation he made use of cf. Minio-Paluello (1945, pp. 65–68).

[27]This appears to be the position of Apelt (1891), who says, for instance, that the categories are "Arten der Begriffe, inwiefern und wie sie im Urteil als Prädikate auftreten" (p. 128).

predicate." This I interpret to mean that a primary substance by itself cannot serve as a predicate. That is to say, a primary substance, such as 'Socrates' or 'Bucephalus', cannot by itself be predicated of anything. But primary substances are substances, and therefore items categorized. Hence, 'Socrates' is an item categorized that is not a predicate in the relevant sense. Items categorized can therefore not be identified with predicates as predicated, since primary substances are not such things.[28]

If, however, a predicate is understood simply as the P of 'S is P', then we may well say that items categorized are predicates. For in Aristotle's logical syntax the terms S and P are syntactically similar; that is to say, whatever is an S of one proposition can be the P of another, and vice versa.[29] That principle is presupposed by Aristotle's proof method of conversion, which is fundamental to syllogistics' being more than just a list of valid moods, namely also a system for reducing imperfect moods to perfect ones (*APr* I.4–6). By the principle of syntactic similarity any term may be a P, and so the class of terms coincides with what may be a P, hence so does the class of items categorized. The syntactic similarity of terms means that the category of a term has no influence on the syntactic properties of a term: terms will in general differ in category, but they are always syntactically similar. As we shall see in the next chapter (esp. section 1.3), this is a point at which syllogistic syntax parts ways with Fregean function–argument syntax. In the latter the category of a term—function of a certain order and kind, or object—determines the kinds of syntactic relations into which it may enter—a first-level unary function, for instance, can never be substituted for an object.

If items categorized are terms, then primary substances are singular terms. Following what we just said, a singular term is syntactically similar with all other terms, although it signifies an individual. It is clear from *APr* I.33, where Aristotle considers terms such as 'Aristomenes' and 'Mikkalos', that he countenances singular terms in syllogistics. The question then arises how to accommodate this with *Cat* $3^{\mathrm{a}}36$, the statement that there is no predicate from a primary substance. Indeed, independently of the identification of primary substances with singular terms the question arises of how to interpret a proposition whose P is a singular term. One possible answer is to say that the copula in this case must be understood as the 'is' of identity. A better answer to my mind is to say that the singular term P must be understood as a general term satisfied by P alone, namely as an "individual concept." This suggestion also

[28] Considerations along these lines are the reasons offered by Ryle (1938, pp. 190–191) and De Rijk (2002, 368–374) for preferring to say that items categorized are terms and not predicates.

[29] Geach (1972, p. 47) saw in the acceptance of the syntactic similarity of subject and predicate a change from the logical syntax presupposed in the *De Interpretatione*. He deemed this change "a disaster, comparable only to the Fall of Adam."

supplies an answer to the question of what the quantifier is in a proposition whose subject is singular: it may be particular as well as universal.[30]

2. The generality of the categories

As understood in the philosophical tradition categories are concepts of a very general kind. The commonest way of explaining the generality of Aristotle's categories is to identify them with highest genera. A famous statement of this identification is found in the so-called *Introduction*, or *Isagoge*, of Porphyry (*Isag* 6.7–6.13):

> Let it be supposed, as in the *Categories*, that the first genera are ten—ten first origins, as it were... The highest genera [*ta genikōtata*], then, are ten.

Statements to the same effect are found, among the ancient commentators, in Alexander of Aphrodisias (*in APr* 291,17ff.), Ammonius (*in Cat* 13,15ff.), and Simplicius (*in Cat* 17,19ff.); among modern commentators in Bonitz (1853, pp. 591–623), Trendelenburg (1846, e.g. p. 20), Brentano (1862, p. 100), Ross (1949, p. 25), and Ackrill (1963, pp. 79); the identification is moreover implicit in such recent works on the *Categories* as Wedin (1997) and Studtmann (2008b). This section offers a critical examination of the identification.

2.1. The predicables. In traditional logic a genus is one of the four or five so-called predicables, and it is as such that we must understand highest genera when identified with categories. The doctrine of predicables originates in Aristotle's *Topics*, but has perhaps more often been associated with Porphyry's *Introduction*.

2.1.1. *Aristotle's Topics.* Aristotle's *Topics* is structured around the notions of definition, *idion*, genus, and accident (cf. *Top* I.6 $102^{b}35$–$103^{a}1$): roughly, *Top* II–III deal with accident, *Top* IV with genus, *Top* V with *idion* and *Top* VI-VII with definition. Aristotle does not employ a technical term for these notions collectively, but they have come to be called predicables.[31] The predicables characterize the relation of the predicate to the subject in a true categorical proposition. When we make explicit, what Aristotle does not, the reference to such a true categorical proposition '*S*

[30] For this suggestion, see Barnes (2007, pp. 154–167).

[31] This word in the appropriate sense apparently originated with Abelard (cf. Baumgartner and Kolmer, 1989, p. 1179). Kant uses the name *Prädikabilien* in an altogether different sense, namely for a priori concepts derived from the categories (KrV A82/B108); he might knowingly have gone against the tradition, as his own "Transcendental Topics" was based on a different set of notions, the so-called concepts of reflection (cf. A268–269/B324–435). Geach (1962, p. 25) defines a predicable as "an expression that gives us a proposition about something if we attach it to another expression that stands for what we are forming the proposition about," noting (ibid. p. 24) that "the older use of the noun 'predicable' is too little current in recent philosophical literature to stop me from staking out my own claim to the term."

is P', the definitions read as follows (cf. *Top* I.5). The predicate P is the definition of the subject S if it is a *logos* signifying the essence (*to ti ēn einai*) of S. It is an *idion*, or *proprium*, or (unique) property, of S if it is not a definition of it, yet nevertheless counterpredicates with S, that is, is such that 'P is S' is true. Thus, (neglecting plucked hens) 'featherless biped' is an *idion* of man, since it does not reveal the essence of man, yet is nevertheless such that the converse 'featherless biped is man' is true. The predicate P is a genus of S if "it is predicated in the what it is (*en tōi ti esti katēgoroumenon*) of many items differing in species" ($102^{\mathrm{a}}31$). A predicate is predicated of S in this manner, Aristotle explains further, if the proposition in question is what would appropriately be given in answer to the question of what something is, as it is appropriate if the subject is man to say that it is an animal. Aristotle gives two definitions of what it is for P to be an accident of S. Firstly, if P is neither the definition, nor an *idion*, nor a genus of S, then P is an accident of S. Secondly, and less trivially, if P is such that it does, but need not, belong to S, then P is an accident of S. According to both criteria, being-seated is an accident of Socrates, assuming that he is sitting.

At *Top* I.8 Aristotle presents what he takes to be a deductive proof (*pistis dia sullogismou*) that his list of predicables constitutes a complete classification of the ways in which a predicate may truly be said of a subject—in other words, that in any true categorical proposition the predicate P is either the definition, an *idion*, a genus, or an accident of the subject S. The proof is by division and runs as follows. We assume that 'S is P' is true. Now, the converse 'P is S' is either true or false. If 'P is S' is true, then P is either the definition or an *idion* of S, depending on whether or not P reveals the essence of S. If 'P is S' is false, then P is either a genus or an accident of S, depending on whether or not P is said in the definition of S (*en tōi horisōi legomenōn*). *QED*. Two remarks on this proof are worth making. Firstly, Aristotle holds that a definition "is composed of genus and differentiae" (e.g. *Top* I.8 $103^{\mathrm{b}}14$). Hence, if 'P is S' is false and P is said in the definition of S, it would seem to follow that S is *either* a genus *or* a differentia of S. Whence it would seem that Aristotle has forgotten to include differentiae on his list, indeed that his own proof of completeness presupposes differentia to be a predicable. As we shall see in more detail shortly, Aristotle subordinates the notion of differentia to that of genus, and that allows him to infer that P in this case must be a genus of S. Secondly, it is plain that Aristotle in the proof employs the first of his two definitions of accident, namely as the residue of the other three predicables. Hence the result established is rather "unexciting," as Smith (1997, p. 73) remarks: if you classify some P's as A, others as

B, others as C, and then say that every P which is neither an A nor a B nor a C is a D, then it needs no proof that any P is one of A, B, C, or D.

The relational character of the predicables is worth emphasizing: no term is a genus just by itself, but only a genus of another term; no term is an accident just by itself, but only an accident of another term; and likewise for *idion* and definition. This contrasts with the categories, for a category is not relational in this sense: which category a term belongs to is not relative to its occurrence in a true categorical proposition. For this reason the name 'figures of predication', translating Aristotle's *ta schēmata tēs kategorias*, seems to me an unhappy description of the categories. Figures of predication would rather seem to describe the predicables, for they classify ways in which the predicate is predicated of the subject, while the categories classify predicates in isolation. Aristotle employs the name 'figures of predication' for instance at *Met* Δ.7 ($1017^{\mathrm{a}}22$ff.), in one of his distinctions of the different senses of 'being': "the senses of being are just as many as the figures of predication." Recalling our reading (p. 4 above) of "with the addition of to be or not to be" at *APr* I.1 $24^{\mathrm{b}}18$ to refer to the copula, it seems natural to interpret this passage from *Metaphysics* Δ as identifying the categories with the different senses of the copula, thus to hold that the being in question here is that expressed by the copula.[32] But, again, it seems more reasonable to say that the copula is said in as many ways as there are, not categories, but predicables, for the predicables classify precisely different ways in which S *is* P. Thus, we could say when P belongs to S as its definition that S is nothing but P; when P belongs to S as a genus that S is essentially or generically P; when P belongs to S as an *idion* that S is properly P; and when P belongs to S as an accident that S is accidentally P. But we would not say that S is substantially P or qualitatively P or quantitatively P or relatively P, etc., but simply that P is a substance or a quality or a quantity or a relative, etc.

A slightly different conception of the relation between categories and figures of predication is defended by Brentano (1862, esp. pp. 108–122). According to him there is no actual identity between these things, but only a one-one correspondence; in particular, each category corresponds to a way of predicating a predicate of a primary substance. If S is a substance, then 'S is P' and 'S is P'' differ in figure of predication if and only if P and P' differ in category. If S is a singular non-substance, namely an "individual accident" such as the particular whiteness of this table,[33] then there is a predication 'S is P' if and only if S and P belong to the same category; but they all share the same figure of predication, namely that found in 'Socrates is

[32] That view is argued for by Apelt (1891).

[33] On individual accidents, see ch. 2 section 1.2.

a man', viz. the figure corresponding to the category of substance. The figure of 'S is P' where S is general is the same as the figure of 'S' is P' where S' is singular. The figures of predication are therefore just as many as the kinds of predicate. There are objections one could raise against this argument, but I shall not do so here. Instead I want to question, once again, the notion of figure of predication, for also on Brentano's reading is it difficult to make sense of the idea that these correspond to the categories. Consider for instance the predication 'Socrates is six feet tall'. Here a quantity is predicated of Socrates. Brentano's view must be that this is a quantitative predication, that the predicate is predicated in the figure of quantity. But then one is forced to say that in this predication 'six feel tall' is predicated *quantitatively* of Socrates, and that seems to me to be either nonsensical or pleonastic. In any event it fails to clarify the notion that categories are, or correspond to, figures of predication.

2.1.2. *Porphyry's Introduction.* In Thomas Blount's *Glossographia* of 1656 one reads that "In Logick there are five Predicables, otherwise called Prophyries five terms." Porphyry had in fact distinguished five rather than four predicables, also known in the tradition as the *quinque voces*: genus, species, differentia, *idion*, accident. He defines genus as "what is predicated in answer to, What is it?, of several items which differ in species" (*Isag* 2.15), which repeats almost verbatim the definition Aristotle gave at *Top* I.5. At *Top* I.4 Aristotle had expressly treated the notion of differentia as a specimen of genus: "the differentia, since it is genus-like should be placed together with the genus" (*Top* 101$^{\mathrm{b}}$18). At *Top* IV.6 (128$^{\mathrm{a}}$20–29), however, Aristotle in effect denies that differentiae are to be identified with genera, and he gives three criteria for distinguishing the two. Firstly, "the genus is said of more items than the differentiae";[34] secondly, "in presenting the what it is it is more fitting to say the genus than the differentia"; thirdly, "the differentia always signifies a quality of the genus, but not so the genus of the differentia."[35] It is indeed doubtful whether Aristotle's definition of genus as "what is said in the what it is of several items differing in species" covers the notion of differentia. Thus, in his completeness proof at *Top* I.8 Aristotle operates with a notion of genus simply as what is said in the definition of the subject; the proof would presumably not go through had it relied on the the official definition of genus from *Top* I.5. Given this ambivalence, it is not surprising that Porphyry adds

[34]Here it is plain that Aristotle has in mind the *divisive*, and not the *constitutive*, differentiae of the genus in question; these notions are, as far as I know, not explicated in Aristotle's work, but Porphyry explains them in his chapter on differentia (*Isag* 9.25–10.21).

[35]This third characteristic is also found at *Top* IV.6 144$^{\mathrm{a}}$18–22 and in the first definition of quality at *Met* Δ.14 (1020$^{\mathrm{a}}$33ff.). It does not imply that all differentiae fall into the *category* of quality: see Morrison (1993) and Barnes (2003, pp. 350–356). On how the Neoplatonic commentators dealt with the categorial status of differentae cf. De Haas 1997, pp. 180–250.

differentia to the list of predicables, and that a latter-day traditional logician such as Joseph (1916, p. 74) has followed him in doing so. Porphyry gives several accounts of the notion. According to one (*Isag* 11.8ff.) "a differentia is what is predicated as a qualification (*en tōi poion ti esti*) of several items which differ in species," where one can recognize Aristotle's second criterion above, that "the differentia always signifies a quality of the genus." Aristotle's two other criteria for distinguishing differentiae from genera can likewise be found in Porphyry's text (cf. *Isag* 11,11 and 14,14).

Even though species (*eidos*) does not occur in Aristotle's list of predicables in *Top* I.5, the notion is presupposed in his definition of genus as what is predicated essentially of several items differing in species. It is therefore natural to ask why Aristotle did not include species on his list of predicables. To say that S is a species of P is usually to say that P is a genus of S; hence the name 'species' indicates in this case not how a predicate relates to the subject, but how a subject relates to the predicate. This is presumably what Ross (1949, p. 33) alludes to when he says that "this is Aristotle's classification of predicables which Porphyry later muddled hopelessly by reckoning species as a fifth predicable." If there is any place for the notion of species in a set of predicables, it would therefore have to be so as to cover singular essential predications; indeed, that seems to be presupposed by Porphyry's characterization that "a species is what is predicated in answer to 'What is it?' of several items differing in number" (*Isag* 4,12). It is, however, clear that none of Aristotle's predicables classify such predications: not accident or *idion*, for these are not predicated essentially; not genus, for it is predicated of "several items differing in species" (*Top* 102^{a}31), so in particular not of individuals as such; and not definition, for a definition converts with its subject (*Top* I.8 103^{b}9), but a singular essential judgement does not convert—Socrates is a man, for instance, but man is not Socrates. Since Aristotle thought he had covered all possible predications in his table of predicables, it is natural to conjecture that Aristotle in the *Topics* did not countenance singular essential judgements (he may have countenanced accidental or proper singular judgements);[36] this must therefore be the reason why species is not on his list of predicables.

Porphyry's treatment of *idion* and accident has no relevance for the following, so we omit discussion of them.

2.2. The ordering of genera. Given two terms g and s, let us write $s < g$ to mean that g is a genus of s. Our aim in this section is to investigate this

[36]According to Smith (1997, p. xxix) there are no singular judgements at all in the *Topics*.

relation.[37] We shall see that it is a strict ordering among general terms with the property that above any g there is at most one greatest element (i.e., a highest genera). If $s < g$, then both s and g are general terms, for a genus is a general term said of another general term. We may therefore take the field of the relation $<$ to consist of general terms. Rohr (1979, p. 383) remarks that Aristotle's discussion of the so-called Third Man argument, reported by Alexander of Aphrodisias (*in Met* 84,27–85,3), suggests that he would deny reflexivity of the $<$-relation; a lesson of the Third Man is precisely that self-predication of a genus leads to an infinite regress. Another reason why Aristotle should deny the reflexivity of $<$ is that it would make definition impossible. A definition, according to Aristotle, states the genus and differentae of its definiendum.[38] Hence if g was its own genus, it would feature in its own definition, whence the definition would be circular, hence not really a definition. That the $<$-relation is asymmetrical seems to be what Aristotle expresses at *Top* IV.1 121$^{\mathrm{a}}$12: "it is clear that the species partake of the genera, but not the genera of the species." But asymmetry can also, just as irreflexivity, be argued for by appeal to the notion of definition: if g is a genus of s, then g will feature in the definition of s; if s in turn would be a genus of g, then it would feature in the definition of g, hence by unravelling the definition of s (namely by replacing in it g by its own definition)[39] we should find that s features in its own definition, and so again the definition would be circular. Rohr (1979) argues at length, and convincingly to my mind, that the relation $<$ is transitive; indeed, Aristotle seems to say as much at *Cat* 1$^{\mathrm{b}}$10ff. Hence we conclude that the relation $<$ is a strict ordering.

Aristotle says at *Top* IV.2 121$^{\mathrm{b}}$29: "for it seems that whenever one species falls under two genera, the one is embraced by the other."[40] In order-theoretic language this means that whenever $s < g_1$ and $s < g_2$, then either $g_1 < g_2$ or $g_2 < g_1$ (that these cases mutually exclude one another follows from the fact that $<$ is asymmetrical). Let us say that an ordering which satisfies this condition is tree-like; this name is motivated by the fact that in such an ordering there is only one way upward along $<$, just as in a tree there is only one way down towards the stem. Aristotle therefore says in the quoted passage that it seems that the ordering $<$ is tree-like; but he is noncommittal about the matter, "for some think that prudence is both virtue and

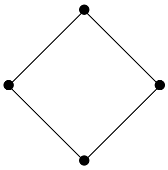
[37]We build on the investigations of Rohr (1979) and Berg (1983).

[38]Cf. the already quoted *Top* I.8 103$^{\mathrm{b}}$15: "definition is composed of a genus and differentiae." See further *Top* 139$^{\mathrm{a}}$28–29; 141$^{\mathrm{b}}$25–27; 153$^{\mathrm{b}}$14.

[39]This notion of unravelling a definition is Aristotelian, cf. *Top* II.2 110$^{\mathrm{a}}$5: "to replace the words of a definition by (their) definitions, and not to stop until one has reached what is familiar." A technical discussion of this notion is given by Curry (1963, pp. 101–110).

[40]The same principle is mentioned at *Top* VI.2 140$^{\mathrm{a}}$1, where it seems to be taken as universally vaild.

knowledge and that neither of its genera is embraced by the other" ($121^{\mathrm{b}}31$). Prudence should therefore be a case of an s for which there were two terms g_1 (virtue) and g_2 (knowledge) such that $s < g_1$ and $s < g_2$, but such that neither $g_1 < g_2$ nor $g_2 < g_1$ holds. Aristotle claims, however, that in this case there will be a yet higher genus by which both g_1 and g_2 are embraced, for instance, both knowledge and virtue are states ($121^{\mathrm{b}}33$–38). Order-theoretically this means that whenever $s < g_1$ and $s < g_2$, then there is a g_3 such that $g_1 < g_3$ and $g_2 < g_3$; let us say that an ordering satisfying this condition is diamond-like. A diamond-like ordering need not be tree-like, as witnessed by



But Aristotle believes that any part of the $<$-ordering is either tree-like or diamond-like ($122^{\mathrm{a}}1$): "for if the genera are subordinate neither the one to the other nor both to the same thing, then what is given is not a genus." Thus, for given genera g_1 and g_2 of s, it holds either that $g_1 \lesseqgtr g_2$ or else that there is a g_3 such that $g_1 < g_3$ and $g_2 < g_3$. It is readily seen that in an ordering satisfying this property there is at most one highest genus above any term s. Hence, while Aristotle may not admit that the ordering of genera in general is tree-like (though he could insist that some parts of it is), he is committed to the view that above any species there is at most one highest genus. Thus, if it should turn out that the categories coincide with highest genera, what we shall call the principle of the mutual exclusion of the categories would follow: it is not the case that the same term falls into two categories, for that would mean that there were two highest genera above that term[41]

2.2.1. *Trees in Plato and Porphyry.* Before considering the relation between this ordering of genera on the one hand and the categories on the other, it is worth briefly remarking on a well-known historical antecedent and an equally well-known historical succedent to it. Plato's method of division (*diairhesis*) is certainly in the background of Aristotle's account of genera, species, differentiae, and their ordering.[42] Plato's idea seems to have been that by "cutting along natural joints" one should reach the true definition of a given term (*Phaedrus* 266a, cf. *Statesman* 262b-263a). When

[41]This point was made already by Brentano (1862, p. 128) in commenting on this passage ($122^{\mathrm{a}}1$).
[42]On Platonic division and its role in Plato's philosophy, see e.g. Philip (1966) and Ackrill (1971). On the related method of collection (*sunagōgē*), see Menn (1998).

Plato carries out a division to reach the definition of, for instance, the sophist or the statesman,[43] what gets divided and what a particular division results in are variously called genus and species (these terms seem to be used synonymously by Plato). And a genus is divided by means of differentiae; for instance, at *Sophist* 219e hunting is divided into the hunting of living things and the hunting of lifeless things. Hence the structure that results from a Platonic division is a tree-like ordering $\prec$ such that if $s \prec g$ then g is a genus of s, and such that s may be obtained from g by the addition of one or more differentiae; this is indeed the picture we have just seen in Aristotle. That division should always be dichotomous appears to be dictated by the description of the method at *Phaedrus* 266a,[44] but Plato elsewhere admits that we may not be able to divide a kind into only two subkinds, in which case "we must always cut into the nearest number as far as we can" (*Statesman* 287c). At the place in question Plato in fact makes a sevenfold division of arts that contribute to the caring of citizens; elsewhere he divides spoken sound (*phōnē*) into vowel, stop, and continuant (*Philebus* 18b–d). In such cases of polytomous division the question remains of course whether a sequence of dichotomous divisions is possible that would end in the polytomous one; that indeed happens in the case of spoken sound: whereas both Plato and Aristotle (in *Poetics* 20) divides spoken sound directly into three, the *Tekhne grammatikē* first divides it into vowel and consonant, and thereafter divides consonant further into stop and continuant.[45] To the best of my knowledge Aristotle says nothing in the *Topics* that commits him one or the other way regarding dichotomy. In *Cat* 8 he speaks about four genera of quality (one of them is in fact called a species), suggesting a division of the genus of quality into four; but it is not obvious that Aristotle thinks of quality as a genus in the technical sense (more on this below).

What is known as The Tree of Porphyry[46] most likely derives from a creative reading of the following passage in Porphyry's *Introduction* (4,21–4,25):

> Substance is itself a genus. Under it is body, and under body animate body, under which is animal; under animal is rational

[43]For an overview of the divisions in the *Sophist* and the *Statesman*, see Gill (2010).

[44]Boole (1854, pp. 50–51), having shown that the equation $x(1-x)=0$, which for him is the expression of the law of non-contradiction, is derivable from the second degree equation $x^2 = x$ remarks:

> it is a consequence of the fact that the fundamental equation of thought is of the second degree that we perform the operation of analysis and classification by division into pairs of opposites, or, as it is technically said, by *dichotomy*.

[45]This observation is due to Menn (1998, p. 295, fn. 5).

[46]The terms *arbor Porphyriana*, *arbor Porphyrii* are not recorded before the Middle Ages; an early occurrence is in Peter of Spain's *Summulae Logicales*, Tractatus II cap 11 (Dinneen, 1990, p. 19).

animal, under which is man; and under man are Socrates and Plato and particular men.

In mediaeval logic textbooks, but not in Porphyry's *Introduction* itself, one finds a drawing as in Figure 1, a tree in the literal sense whose trunk is made up by genera, and whose leaves are differentiae.[47] This, however, is not an ordering of genera of the kind we have seen in Aristotle; it is indeed a tree-like ordering, but it contains a chain such as the following

$$\text{substance} > \text{corporeal} > \text{body} > \text{animate} > \ldots$$

That is, it contains a chain that places differentiae under genera, while in Aristotle's ordering it is only species that get placed under genera.

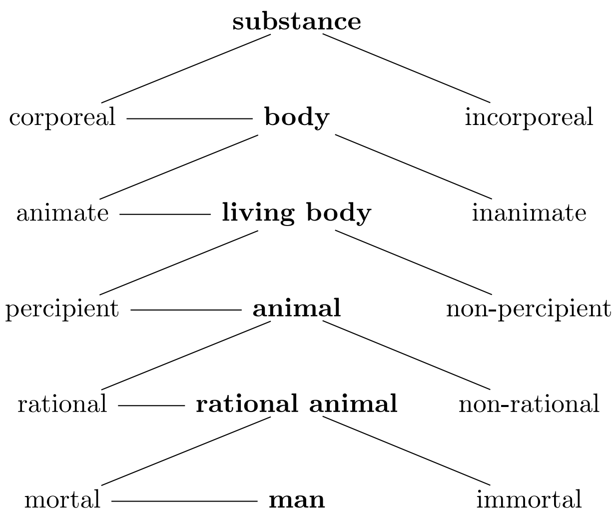


FIGURE 1. Porphyry's tree

2.3. Categories and the ordering of genera. In *APo* I.19 Aristotle asks whether, given a term g, it is possible (1) to form an infinitely ascending sequence $g < g' < g'' < g''' < \ldots$ or (2) to form an infinitely descending sequence $\ldots < g''' < g'' < g' < g$. He moreover asks (3) whether, given two terms $g < g'$ it is possible to form an infinite sequence $g < h < h' < h'' < \ldots < g'$ or $g < \ldots < h'' < h' < h < g$. He deals with question (3) in *APo* I.20, assuming negative answers to questions (1) and (2); and indeed assuming that there are no infinitely ascending or descending

[47]This drawing derives from the one given in Barnes (2003, p. 110); for a more embellished tree, see for instance Kretzmann (1966, p. 54).

sequences it is plain that we cannot find infinitely many $h, h', h'', h''', \ldots$ such that either $g < h < h' < h'' < \ldots < g'$ or $g < \ldots < h'' < h' < h < g$, since in both of these cases we should produce an infinitely ascending or descending sequence. Question (1) is dealt with in *APo* I.22 ($82^{\mathrm{b}}37$–$83^{\mathrm{a}}1$). Aristotle there offers the following argument, the virtues of which need not be assessed here. Its first premise is the claim that knowledge of a term g presupposes knowledge of all $g' > g$. Knowledge of g is knowledge of the definition of g; that definition contains another genus $g' > g$, which itself must be known if the definition is to be known; but knowing g' means knowing its definition, appealing to a yet higher genus. Whence all $g' > g$ are involved in the knowledge of g. The second premise says that an item of knowledge is finite: "you cannot survey infinitely many items in thought" ($83^{\mathrm{b}}6$). The third premise says that knowledge of terms is indeed possible. From this we are invited to draw the conclusion that any $<$-sequence above g must be finite: all terms above g are genera found when unravelling the definition of g, and these must all be known if g is to be known; since knowledge of g is possible and an item of knowledge is finite, there can be only finitely many terms above g. In particular, above any g, if it is not already a highest genus, there is a highest genus. We saw in section 2.2 that above any g there is at most one highest genus. Hence we may conclude that if g is not already a highest genus, there is a unique highest genus $g' > g$.

Aristotle takes up question (2) in *APr* I.27. The argument amounts essentially to the claim that individuals are not predicated of anything else. One may ask why singular predication should be relevant to the order of genera, which, as we have seen, has only general terms in its field. Anyhow, even if we did extend the field of $<$ to individual terms, it would not follow from Aristotle's claim that we can have no infinitely descending sequence in $<$. Man is said of Callias, and Callias is an individual; but that does not exclude the possibility of there being a species below man which in its turn is predicated of Callias—male could be such a species.

So this last argument is not valid;[48] but our interest here is in highest genera. As we have now seen there is, according to Aristotle, a diamond-like ordering (some sections of which may be tree-like) of genera with a unique maximal point above any genus; that is, if g is not itself already a highest genus, then there is a unique highest genus above it. It is remarkable that Aristotle, in contrast to modern and ancient commentators, nowhere identifies these highest genera with the categories. There are even suggestions in Aristotle's text that highest genera have a relatively low degree of generality (*Cat* $14^{\mathrm{a}}24$): "good and bad are not in a genus (*en genei*) but are themselves

[48] It is then interesting to note that the *Jäsche Logik* § 11 denies that there are lowest species (and also that there is a next highest genus.

actually genera of certain things (*genē tinōn ontōn*)." Thus, good and bad appear to be regarded as highest genera, not "being in" any other genus.[49] At several places Aristotle calls the categories 'genera', either simply (*Cat* $11^{a}37$, *DA* $402^{a}23$),[50] or in the combinations 'genera of predicates' (*APo* $83^{b}13$–17, *Top* $103^{b}22$–23, $152^{a}38$, *SE* $178^{a}5$) or 'genera of beings' (*APo* 88^{b}, *DA* $412^{a}6$). 'Genus' thus employed does, however, not have the technical sense given to it in *Top* I.5, pertaining to the relation of the predicate to the subject in a true categorical proposition, but rather a non-technical sense, meaning 'class' or 'type' or something along those lines. For instance, when Aristotle at *Top* $103^{b}22$–23 calls the categories 'genera of predicates' it is obvious that 'genus' is used in a non-technical sense, for the genera in this sense are said to contain the genera in the technical sense.[51]

The view that a category is a class of terms (or predicates) seems to be presupposed in the following instructive passage on the relation of the categories to the ordering of genera (*Top* $120^{b}26$–$121^{a}9$).

> Moreover, see whether the genus and the species are not found in the same division, but the one is a substance while the other is a quality, or the one is a relative while the other is a quality, as snow and swan are each a substance, while white is not a substance but a quality, so that white is not a genus either of snow or of swan. . . . To speak generally, the genus ought to fall under the same division as the species; for if the species is a substance, so too should be the genus, and if the species is a quality, so too the genus should be a quality; for instance, if white is a quality, so too should colour be; likewise in other cases.

Categories are here spoken of as "divisions" (*diairheseis*) within which genera and species fall,[52] and a division is to my mind more like a class than like a highest genus. In this passage Aristotle presents a *topos* by means of which one can attack a dialectical proposition claiming that g is the genus of s: see whether s and g fall within the same division—if they do not, then g cannot be a genus of s. Let us write $g : C$ to mean that g falls under the category C. The general principle Aristotle appeals to in justifying this *topos* says that if $s : C$ and $s < g$, then $g : C$. We may

[49]Cf. *Phys* V.4 $227^{b}11$ "where it happens that the genus is at the same time a species," suggesting that this is not the rule.

[50]The Oxford translation has *summa genera* at *DA* $402^{a}23$, but there is nothing in the Greek corresponding to *summa*.

[51]Kapp (1920, pp. 226–228) makes the same point referring to *Top* $107^{a}3$–30; $152^{a}38$–$152^{b}2$.

[52]Cf. *Cat* $10^{a}19$, where the category of quality is spoken of as a division (*he peri to poion diairhesis*); and *SE* $166^{b}14$, which refers to the categories as "the divisions previously made."

call this the principle of upward categorial closure. The corresponding principle of downward categorial closure says that if $g : C$ and $s < g$, then $s : C$. Contrary to what one would expect, Aristotle denies the latter principle (*Top* $124^{\mathrm{b}}15$–24):

> if the species is a relative, so too is the genus, as is the case with double and multiple; for each is a relative. If, on the other hand, the genus is a relative, there is no necessity that the species should be so as well; for knowledge is a relative, but not so grammar.

Here Aristotle first restates the principle of upward categorial closure (for the category of relatives); but he goes on to deny the principle of downward categorial closure. Knowledge apparently provides a counterexample, for while knowledge itself is a relative, its species grammar is not a relative, but a state, and therefore a quality. The categorization of knowledge must have presented a problem for Aristotle. Since knowledge is knowledge of an object, it is a relative according to him (e.g. *Cat* $6^{\mathrm{b}}5$, $11^{\mathrm{b}}26$); but the various species of knowledge, such as grammar,[53] are said to be states, and therefore qualities (*Cat* $8^{\mathrm{b}}28$, $9^{\mathrm{a}}6$–7). Indeed, grammar is not a relative since grammar is not grammar of something, rather grammar is knowledge of something, so it is only in virtue of the genus that it is a relative; but what we possess when we possess grammar is a quality (*Cat* $11^{\mathrm{a}}20$–38).

The most troubling aspect of Aristotle's denial of downward closure is not the lack of symmetry it entails when compared to the acceptance of upward closure. The most troubling aspect is rather the fact that acceptance of upward categorial closure together with the denial of downward closure entails the denial of the principle of mutual exclusion—the principle that the same term cannot fall under two categories. Namely, the species of knowledge are qualities, so by upward closure knowledge is itself a quality; but knowledge is also a relative; hence knowledge is both a quality and a relative, contrary to the principle of mutual exclusion. Aristotle draws the consequence at *Cat* $11^{\mathrm{a}}38$–39:

> Moreover, if the same thing really is a quality and a relative there is nothing absurd in its being counted in both the genera.

Aristotle never states the principle of mutual exclusion explicitly, so he does not contradict himself outright when he here rejects it; but it is implicit in his introducing the categories at *Cat* 4 by saying "of things said without combination each signifies

[53]Grammar (*grammatikē*) is here understood quite literally as the knowledge of letters (*grammata*); cf. *Top* VI.5 $142^{\mathrm{b}}31$: "for instance, if he defines grammar as the knowledge of writing from dictation; for he ought also to say that it is knowledge of reading." Similarly at Plato *Sophist* 253a–b the expert in grammar is said to be one who knows which letters blend and which do not, so presumably someone who knows how to read and write.

either substance *or* quantity *or* quality *or*..." as well as in his calling the categories *divisions* at *Top* 120^{b}26: one and the same term cannot belong to two divisions, so if a category is a division, neither can one and the same term belong to two categories. Porphyry argues that denial of mutual exclusion is quite unproblematic (*in Cat* 140,27–141,5):[54]

> Socrates, for instance, can be shown to be subject to a number of affections: insofar as he is a man, he is a substance; insofar as he is three cubits tall, let us suppose, he is a quantity; insofar as he is a father or a son, he belongs to the relatives; insofar as he is temperate, he is qualified; and in this way he is brought under the different categories in virtue of various differentiae. If, then, Socrates, who is a single thing, is found to fall under different categories when he is considered in different respects, what is absurd about a state being in one respect a relative, and in another a quality?

But this is not a way out. We can of course predicate terms of all categories of Socrates, but it does not follow that he, Socrates the substance, therefore is a categorial chameleon. Each term has its category independently of what is said of it and what it is said of; the fact that one term t falling under a category C can be truly predicated of a term t' falling under a different category C' does not entail that t and t' nevertheless both belong to the same category. Indeed, if a substance were also all the things that can be truly said of it, it would belong to all categories, whence one could ask why categorial distinctions had been made in the first place.[55]

Frede (1987b, p. 13) speculates that *Cat* 11^{a}38–39, the passage recently quoted where mutual exclusion is denied, might be spurious. But this seems unlikely, since, as we have noted, Aristotle there simply draws the consequence of views explicitly stated elsewhere in the *Categories* and in the *Topics*. Concentrating on Aristotle's treatment of knowledge seems to me more promising. One could ask whether the proposed counterexample of knowledge—the only one Aristotle offers—instead of showing that the principle of mutual exclusion is false rather reveals confusion in Aristotle's conception of the notion of a relative? Aristotle's criteria for calling a term a relative are indeed rather unclear. Knowledge is a relative since knowledge is always knowledge of something (6^{b}5); and a state is a relative because a state is always the state

[54]Cf. the similar considerations of De Rijk (2002, p. 133–134).

[55]I would raise similar objections to one of the main arguments of Morrison (1992) against what he aptly calls the taxonomical interpretation of the categories. Morrison holds (pp. 26–28) that when Aristotle says of white that it is not properly called large, but only accidentally (*Cat* 5^{a}38–5^{b}2), then he assumes a notion of "accidental categorials"; so white belongs to the category of quantity accidentally. But even if large is predicated accidentally of white, it does not follow that 'white' itself is a quantity.

of someone or something (ibid.)—but the sense in which an instance of knowledge is the knowledge of the Pythagorean Theorem is altogether different from the sense in which a given state of virtue is the state of Socrates. Large is a relative because something is large only relative to a certain comparison class ($5^{\mathrm{b}}16$–21); and lying, standing, and sitting are called relatives ($6^{\mathrm{b}}11$–12) presumably because a particular lying, standing, or sitting is always someone's lying, standing, or sitting. Aristotle insists that a wing is a relative, since "a wing is a wing of a winged" ($6^{\mathrm{b}}38$–$7^{\mathrm{a}}5$), while he is willing to revise his definition of relatives in order to avoid having to say that an arm—which one would think was in the same category as a wing—is a relative ($8^{\mathrm{a}}18$–$8^{\mathrm{b}}21$). And why, according to *Cat* 4, is cutting a doing (*poiein*) rather than a relative, for a cutting is always the cutting of something; and why is being-cut an affection (*paschein*) rather than a relative, for a being-cut is always the being-cut by someone; and indeed why is four-foot, or any other quantity, a quantity rather than a relative, for a four-foot is always someone's or something's measure? It is difficult to see how such questions can be settled on the basis of Aristotle's definition of relatives (cf. Ackrill, 1963, p. 99); but instead of improving this rather imprecise definition, Aristotle chose to reject the principle of mutual exclusion.

Let us nevertheless suppose with Aristotle that downward closure (and with it, mutual exclusion) fails. Rohr (1979, pp. 384–385) observes that in this case categories cannot be highest genera: the transitivity of the $<$-ordering together with the assumption that the categories themselves are genera entail the principle of downward categorial closure—if $g : C$ and C is itself a genus, then $g < C$, hence by transitivity it follows that $s : C$ for all $s < g$, which is just the principle of downward categorial closure. (More precisely, the category of relatives cannot be a highest genus, but this conclusion should generalize to all the categories.) To my mind Aristotle should have revised his conception of relatives and refined his analysis of knowledge so that it would not provide a counterexample to downward closure. That principle, as well as mutual exclusion, would therefore remain intact. This revised Aristotle would thus not accept one of the premisses of Rohr's argument. But the revised Aristotle would still have good reasons to reject the identification of categories with highest genera. This identification presupposes that categories are themselves terms, namely the maximal elements of the $<$-ordering. A category C is then predicated of terms falling under it in precisely the same way that for instance 'animal' is predicated of 'man' or 'horse'. A category can itself, without further ado, be the predicate of an 'S is P' proposition. But as we shall see in more detail in the next chapter (section 4), such categorial predications are of an altogether different kind from ordinary predications.

3. The skopos of the Categories

In the opening of the *De Interpretatione* Aristotle introduces a version of what is sometimes called "the semantic triangle"[56] and distinguishes thereby between the word as an acoustic or graphic entity, the thought (*noēma*) corresponding to the word,[57] and the thing signified (*pragma*). In our discussion in section 1 above we neglected the fact that in the *Categories* Aristotle appears to confuse these various levels, and leaves one with the question of which compartment items categorized belong to.[58] The aim in what follows will not be to neutralize all passages that may go against our interpretation of items categorized as terms, hence as linguistic in some sense, but rather, in section 3.1, to report on an ancient discussion of this question, whose conclusion agrees quite well with our interpretation; and in section 3.2 to criticize a reading more recently put forward by several commentators according to which in the *Categories* the items categorized are ontological in nature, while in the *Topics* they are linguistic in nature.

3.1. The commentators. The list of categories is introduced in chapter 4 of the *Categories* as what is *signified* by "things said without combination," which could be taken to mean that items categorized are things signified.[59] But later in the same chapter Aristotle declares that none of the "items mentioned is said just by itself in any affirmation, but from a combination of these with each other an affirmation is produced" ($2^{a}5$–7), where the "items mentioned" must refer to the categories, which suggests that the items categorized are things said, since only things said combine to form affirmations. In chapter 2 of the *Categories* Aristotle sets out to divide *ta onta*, what there is; at the opening of his discussion of substance in chapter 5 (*Cat* $2^{a}14$–16) he then classes primary substances into one of these divisions, suggesting that the category of substance is made up of things; but a few columns later he speaks of primary substances *signifying* "a certain this" (*tode ti sēmainein*, $3^{b}10$–12), thus apparently assuming that they are things said.

Aristotle's vacillation at this point appears to have prompted already the Peripatetics who in the 1st century BC were the first to write commentaries on the

[56]In fact it is a quadrangle, cf. Kretzmann (1974). On its relation to the triangle of Ogden and Richards, see Lieb (1981). On the semantic triangle of the Stoics, see section 4.2 below.

[57]Aristotle initially calls this "affections of the soul" (*ta pathemata tēs psuchēs*), but at $16^{a}10$ they are identified with thoughts (*noēmata*). The acoustic entity is said to be a a symbol of these affections at *Int* $16^{a}2$, but to follow the thought (*to en tēi dianoiai*) at *Int* $23^{a}33$.

[58]A well-known complaint; see e.g. Kneale and Kneale (1962, pp. 25–27, 196–197).

[59]Though, as suggested by Malink (2007, p. 277) on the basis of numerous relevant passages, we may have to do here with a more or less technical sense of *sēmainei*, signify, used simply to indicate membership in a category.

Categories to ask the question, into which compartment the items categorized fall.[60] The Neoplatonic commentators, starting with Porphyry in the 3rd century AD, paid particular attention to this question and called it the question of the *skopos*, or aim, of the *Categories*. The reason why it was thus called is not clear to me,[61] but the idea seems to have been that the answer to this question would provide a principle to be appealed to in dealing with various *aporiai*. We find for instance the commentator Simplicius (6th century AD) referring to the *skopos* again and again in his commentary.[62]

The question of the *skopos* was as a rule taken up in the preamble of a commentary.[63] Three positions held by nameless precursors were first distinguished:

> Let us now examine the questions raised a little way back; the first concerns the aim (*skopos*). Notice that commentators have differed on this, some saying that the Philosopher is discussing words (*phōnai*), some, things (*pragmata*), and some, concepts (*noēmata*).[64]

Those arguing[65] that the work is about words had pointed to Aristotle's key term *legomena*, 'things said', or 'what is said'. Those arguing that the work is about things had pointed to Aristotle's phrase 'of things there are' (*tōn ontōn*, $1^{a}20$), and to the judgement that it is not for the philosopher to study mere words. Those arguing that the work concerns itself with concepts may have insisted that things said are in fact conceptual in nature, on a par with the Stoic notion of *lekton*. Instead, the Neoplatonic commentators held that the *skopos* of the work is a synthesis of all these three views:

> The Philosopher's aim here, therefore, is to treat words that mean things through mediating concepts.[66]

To distinguish this reading from the one holding that items categorized are words (and perhaps for other reasons besides) Porphyry introduced the notions of the primary and secondary imposition (*prōtē, deutera thesis*), presented by him and the

[60]Cf. Gottschalk (1990, esp. p. 70) and Sharples (2008, esp. pp. 279–281). From the discussions of Porphyry (*in Cat* 59,16) and Simplicius (*in Cat* 11,22ff.; 13,16) it appears that the question had been discussed by Boethus of Sidon, a student of Andronicus of Rhodes (1st century BC).

[61]Ammonius explains the terminology thus (*in Cat* 7,18-20): "for just as an archer, for example, has a mark toward which he shoots and which he wants to hit, so also a writer has some end in view, which he is eager to attain."

[62]See e.g. *in Cat* 16,15; 21,7; 24,22; 40,18; 69,1ff.; 73,30.

[63]These preambles had quite a regimented form; see Praechter (1909, pp. 523–531) and Hadot (1987, esp. pp. 99–106, 120–121).

[64]Ammonius *in Cat* 8,20–9,3.

[65]For the following arguments, cf. Ammonius *in Cat* 9,3–9,11 and Simplicius *in Cat* 9,4–10,2.

[66]Ammonius *in Cat* 9,17.

later commentators as an ontogeny of language,[67] but in fact being a valuable semantical distinction, closely related to the mediaeval distinction of first and second intentions.[68] In the primary imposition man gave names to the things around him. In the secondary imposition he "reflected upon the expressions from another point of view" (*in Cat* 57,30) and invented words such as 'noun' and 'verb'; thus he gave names to words themselves.[69] Items categorized are words instituted by the first imposition; indeed the (names of the) categories themselves are words of this kind, whereas names relevant to an investigation of words *qua* words are instituted by the second imposition.

According to the Neoplatonic commentators, therefore, items categorized are words insofar as they signify things. This view is in line with the interpretation pursued here, according to which items categorized are terms, which of course are linguistic in some sense. There is also the dual view, that items categorized are things insofar as they are signified by words. That seems to be the view of Ackrill (1963). At several places in his commentary Ackrill insists that items categorized are things, but he admits that (p. 78):

> Though the items in categories are not expressions but 'things', the identification and classification of these things could, of course, be achieved only by attention to what we say.

In a categorization, or "identification and classification" of things, one needs to attend to what we say; so the view presupposed by Ackrill must be that items categorized are things insofar as they are signified by words, since it is only through this signification that we can effect the categorization. Hence, both Ackrill and the Neoplatonic commentators recognize the central place that the meaning relation plays in categorization, but they emphasize different nodes in it. De Rijk (2002, p. 134), who has forcefully argued that categories are "classes of names,"[70] summarizes his view as follows:

[67]For a reconstruction of Porphyry's doctrine of imposition and the concomitant ontogeny of language, see Ebbesen (1990, pp. 146–162); for the doctrine of imposition in Simplicius and other later commentators, see Hoffmann (1987, pp. 78–90). Use of the word *thesis* in the sense of name giving can be found already in Plato's *Cratylus* 390d (*he tou onomatos thesis*); on *thesis* in ancient thought about language, in particular in relation to *physis*, see Fehling (1965, pp. 218–229). The exact phrase *prōte thesis* is used in the *Tekhnē grammatikē* § 12 to characterize primitive, in contrast to derivative, nouns.

[68]See Knudsen (1982) for an account of the doctrine of first and second intention also discussing the relation to the doctrine of imposition (ibid. 484–485); Ockham draws both distinctions at *Summa Logicae* I.11–12 (Loux, 1974, pp. 72–75).

[69]And presumably also to other things, as has been emphasized by Lloyd (1990, p. 36–43): Dexippus *in Cat* 15.24ff. classifies 'whole' and 'part' as words of the second imposition; and it seems that 'genus' and 'species' were also thus classified.

[70]See especially De Rijk (1980) and De Rijk (2002, pp. 358–471).

> what is classified is not *things* by themselves, not *names* by themselves, but *things according to their mode of being* expressed by a categorial designation,

That is, it seems, items categorized are things as signified by words. I take it, then, that the view of Ackrill and others following him, on the one hand, and the view of the Neoplatonic commentators and De Rijk, on the other, are basically in agreement with each other, in that they both defend a semantic conception of the categories: items categorized are meaning entities, be they words as signifying things or things as signified by words.

Is there any alternative to this semantic conception of the categories? Porphyry contrasts the categories with what he calls the genera of being, maintaining that the latter are in some unspecified sense prior to the former, indeed that the categories simply reproduce the catalogue of being (*in Cat* 58,12ff.):[71]

> Since beings are comprehended by ten generic differentiae, the words that indicate them have also come to be ten in genus and are themselves also so classified. Thus the predicates (*katēgoriai*) are said to be ten in genus, just as beings themselves are ten in genus.
> [...] Words are like messengers that report to us about things, and they get their generic differentiae from the things about which they report.

In Porphyry's picture there are thus ten genera of being, and to each such genus γ there corresponds a unique category C_γ such that, (1) every category is equal to some C_γ, and (2) a term t falls under C_γ if and only if the signification of t (the thing about which t reports) falls under the genus γ. The genera of being would thus seem to present an alternative to semantically conceived categories: pure being divided into genera altogether independently of language, and indeed providing the blueprint for the categorization of language. It is not uncommon today to conceive of category schemes independently of language;[72] but it is doubtful whether one can conceive of Aristotle's categories that way: according to Aristotle's doctrine, 'man' is a substance, while 'slave' is a relative, hence if the genera of being would mirror the categories—which they do in Porphyry's picture—then a man and a slave would

[71]Cf. Simplicius *in Cat* 11,1–11,22, reporting Porphyry's view.
[72]E.g. Chisholm (1996).

belong to different genera, and that is an idea I for one find it difficult to make sense of.[73]

That the items categorized are words fitted well into the view of the commentators on the order of Aristotle's works and their place in the Neoplatonic curriculum. Already Aristotle's compiler Andronicus of Rhodes had classified the *Categories* as a logical work, and had apparently held that logic should be studied before all other subjects.[74] This view was shared by the Neoplatonic commentators, who regarded logic as a tool for studying other subjects,[75] which should therefore be studied before ethics, physics, mathematics, and theology (metaphysics).[76] Among logical works the *Categories* is the first to be studied. The argument for this, presented by Ammonius (*in Cat* 4,28–5,30) and Simplicius (*in Cat* 14,21–15,25), may be familiar.[77] The aim of logic is to teach the method of demonstration;[78] but a demonstration is a syllogism productive of knowledge,[79] hence one must know what a syllogism is before one can know what a demonstration is. A syllogism consists of propositions,[80] and a proposition is made up from terms; hence, before teaching what a syllogism is logic should teach what propositions are, and before teaching what a proposition is logic should teach what terms are. This order of things was recognized in Aristotle's writings: the *Categories* deals with terms, the *De Interpretatione* with propositions, the *Prior*

[73]Cf. the remark on the category of relatives of De Rijk (2002, p. 135). According to Apelt (1891, p. 107), a relative is "lediglich eine Geburt unseres Verstandes, ohne ein entsprechendes Ding in der Wirklichkeit."

[74]Gottschalk (Cf. 1990, p. 66).

[75]There was an ancient discussion whether logic is a separate part of philosophy, as the Stoics held (*DL* VII.39–40), or an instrument (*organon*) for philosophy, as the Peripatetics as well as the Neoplatonist commentators held (e.g. Alexander of Aphrodisias *in APr* 1,1–6,13); see e.g. Lee (1984, pp. 44–54) or Kneale and Kneale (1962, p. 139). Ammonius *in Cat* 4,28 and Simplicius *in Cat* 4,23 describe Aristotle's logical works as *organika*, but the name *Organon* apparently has no ancient authority (Gottschalk, 1990, p. 66, fn. 58).

[76]On whether ethics should be taught before logic, see Ammonius *in Cat* 5,30–6,5 and Simplicius *in Cat* 5,17–6,5.

[77]For a critical reading of Simplicius's argument, see Morrison (2005). On the later mediaeval treatment of the *Categories* as a treatise of logic and of the question of the subject matter of the *Categories* more generally, see Pini (2002, esp. pp. 19–44, 138–170).

[78]"…so that one may be able to distinguish the true from the false and the good from the bad" (Ammonius *in Cat* 4,29–5,4, Simplicius *in Cat* 14,21–25). The moral benefit of logic is not often emphasized today, but was not lost on Hawes in his *Pastime of Pleasure* (ch. 5, verse 2): "You shall, quod she [sc. Logyke], my scyence wel lerne,/ in tyme and space, to your gret utilite;/ So that in lokynge you shal than decerne/ A frende from fo, and good from iniquyte:/ Ryght from wronge ye shall know in certainte./ My scyence is all the yll to eschewe,/ and for to knowe the false from the trewe."

[79]*syllogismos epistemonikos* is Aristotle's definition of demonstration (*apodeixis*) at *APo* I.2 71^{b}18; adopted by Ammonius *in Cat* 5,9; Simplicius *in Cat* 14,33 gives the less informative definition 'demonstrative syllogism' (*syllogismos apodeiktikos*).

[80]Ammonius *in Cat* 5,11, Simplicius 14,29: a syllogism is a certain aggregate (*syllogē*) of propositions (*logoi*).

Analytics with syllogism, and the *Posterior Analytics* with demonstration.[81] Hence, since the teaching of philosophy must begin with logic, and the teaching of logic must begin with the doctrine of terms, the *Categories* is the first work the student of philosophy must study: it is "the prologue to the whole of philosophy," as Simplicius remarks in the opening of his commentary (*in Cat* 1,3–1,7).

3.2. A recent contention. Apart from chapter 4 in the *Categories*, the only other place in Aristotle's works where one finds a list of ten categories is in chapter I.9 of the *Topics*. Traditionally these two lists are taken to coincide.[82] Malcom (1981), Frede (1981), Ebert (1985), and Malink (2007) have argued that the two lists do not coincide, and—as far as the *skopos* is concerned—that the category scheme of *Cat* 4 is ontological, a division of things, while the category scheme of *Top* I.9 is linguistic or logical, a division of terms or predicates. For the purposes of this section let us, following Malink, call the categories as presented at *Cat* 4 C-categories, and those presented at *Top* I.9 T-categories; and let us call the reading of *Top* I.9 offered by these interpreters the novel reading.

The main (and perhaps only) reason offered by the authors cited for preferring the novel reading is that the first C-category is called substance (*ousia*), while the first T-category is called essence (*ti esti*). The term *ti esti* recurs several times in the rest of *Top* I.9, where in general it cannot be taken to mean substance. Hence we have the choice either of charging Aristotle with equivocation or else to say that T-categories differ from C-categories.[83] Choosing the latter, and basing itself on what Aristotle says here and elsewhere in the *Topics*, the novel reading says the following about the relation between C- and T-categories. In *Top* I.9 Aristotle assumes as already understood the C-categories, since these are listed at $103^{b}27$–29 and referred to in the following.[84] The T-categories are then explained in terms of the C-categories. To the T-category of essence belong all substances as well as all genera of non-substance C-categories; examples of the latter are colour and magnitude: colour is the genus of white, while magnitude is the genus of two-cubit. To the other T-categories belong the non-generic terms of the corresponding C-category; for instance, to the T-category

[81]Thomas Aquinas in his commentary on the *De Interpretatione* (cf. *Intro* nn. 1–2) relates these works further to the three operations of the mind (*operationes intellectus, rationis*), viz. the operations traditionally known as simple apprehension, judgement, and reasoning. That correspondence is, as far as I know, not found in the Neoplatonic commentators.

[82]So for instance by Alexander of Aphrodisias *in Top* 65,14 and Trendelenburg (1846, p. 34).

[83]For this formulation, see esp. Frede (1981, pp. 36–37).

[84]As admitted by Malcom (1981, p. 665), Ebert (1985, p. 132) and Malink (2007, p. 273). Frede (1981, p. 38) admits that the categories listed at $103^{b}27$–29 are not T-categories, though he seems to want to deny any involvement of C-categories in the *Topics* (e.g. ibid. p. 31).

of quality belongs white, but also coloured (in contrast to colour).[85] Hence we get two altogether different category schemes. Not only does the T-category of essence not coincide with the C-category of substance, but no T-category coincides with the corresponding C-category. Having thus argued for the distinction between T- and C-categories, the novel reading is free to go on to claim that while C-categories are ontological, T-categories are linguistic or logical.

We have already seen that Aristotle is simply not clear on the matter whether C-categories are linguistic or ontological; hence, even if the argument offered for preferring the novel reading was successful, it would not follow that C-categories are ontological. There are, however, good reasons to have reservations about the novel reading. Firstly, it seems to trade one ambiguity for another. For since both T- and C-categories are mentioned and discussed in *Top* I.9, and since for instance 'quality' means different things whether one has in mind T-categories or C-categories, it follows that the names of the non-substance categories are used ambiguously in this chapter: 'quality' sometimes mean the T-category of quality and sometimes the C-category of quality. The disambiguation of *ti esti* in this chapter is therefore bought at the cost of introducing ambiguity of the names of all the other T-categories. With this exchange of ambiguities the novel reading seems to me to loose its force, since it was proposed precisely in order to avoid equivocation. Secondly, according to the novel reading, 'white' is a T-quality and 'cubit' a T-quantity; but this seems to be contradicted by what Aristotle says at $103^{\mathrm{b}}29$ ff. He there says that in predicating white of a white colour or cubit of a cubit magnitude one "says the essence"; this technical phrase is taken by the novel readers to imply membership in the T-category of essence; but that contradicts their classification of 'white' and 'cubit' as a T-quality and T-quantity respectively. Indeed, if such terms as 'white' and 'cubit' do not count as non-essences, then it is difficult to see which terms do. Thus, Malink (2007, p. 289) admits that 'white' is ambiguous: it may signify the T-category of essence or the T-category of quality. Again the disambiguation of *ti esti* therefore comes at the cost of introducing other ambiguities. Thirdly, at several places of *Topics* VI Aristotle uses the word 'substance' (*ousia*) where he evidently means essence (*to ti ēn einai*, *to ti esti*). In a general discussion of definition at *Top* $139^{\mathrm{a}}29$–31 he says "of the elements of a definition the genus is what primarily signifies the substance (*ousia*) of what is defined," where what is meant, presumably, is that the genus, in contrast to the differentiae, is what primarily indicates the essence (*ti esti*) of the

[85]See especially the list given by Malink (2007, p. 291) and the definition he offers at pp. 280–281. See also Malcom (1981, pp. 666–668) and Ebert (1985, p. 125, 137–138). Frede's article is mainly destructive, and offers few constructive remarks on what T-categories should look like.

definiendum. Likewise, at *Top* 143$^{\text{a}}$18 Aristotle discusses the definition of justice, saying "the substance (*ousia*) of a thing involves the genus," where it is implied that justice has a substance, but where it is meant that justice has an essence (*ti esti*).[86] If we regiment Aristotle's language in the *Topics* as the novel reading proposes—taking *ousia* always to mean the C-category of substance, and *ti esti* always to mean the T-category of essence—then we cannot make sense of these passages.

4. The parts of speech

Items categorized are terms, and these are linguistic in nature; but the classification of linguistic items effected by the categories differs of course from the classification into parts of speech. Section 4.1 gives a brief historical introduction to the topic, while section 4.2 examines, on the basis of historical examples, the relation between parts of speech and categories. Section 4.3 surveys various characterizations of syncategorems; as we shall see in section 5 the notion of syncategorem provides a way of understanding the relation between Aristotle's and Kant's categories. The reader may prefer to ignore the many footnotes with which the text in section 4.1 are equipped; most of them are of a philological character.

4.1. Introduction. The little manual known as the *Technē grammatikē*, traditionally attributed to Dionysius Thrax (2nd century BC), is the canonical reference for the parts of speech of Greek grammar.[87] The Greek phrase in the *Technē* translated by 'part of speech', *meros tou logou* (*GG* I.1 23,1), literally means part of the sentence, since *logos* is there defined as "a combination of words in prose conveying a meaning that is complete in itself" (*GG* I.1 22,5), and that is a description of the sentence.[88] We shall, however, follow the tradition (in English going back at least to the early 16th century, cf. *OED*) of calling these parts 'parts of speech', employing as well the more recent term 'word classes' (probably deriving from German *Wortklasse*, recorded 1817 in Grimm). The parts of speech, then, or word classes recognized in

[86]See also *Top* 150$^{\text{b}}$24.

[87]On the work of Dionysius Thrax, see Pfeiffer (1968, pp. 266–267), and see the whole of Pfeiffer's book for the historical context. Already in ancient times doubts were raised as to the authenticity of the *Technē* (cf. e.g. Lallot, 1989, pp. 20–21). On account of the work of Di Benedetto (1958, 1959) it appears to be generally assumed among historians of linguistics today that the *Technē* as we know it is spurious, although opinions vary as to the precise genealogy of the text and its proper place in the history of grammar (cf. e.g. Taylor 1987, Law and Sluiter 1995, Robins 1995). A summary of Di Benedetto's arguments may be found in Pinborg (1975, pp. 103–106).

[88]This description may be of Stoic origin (cf. *DL* VII.63), as may be the phrase *meros tou logou* (*DL* VII.57, cf. *Stoicorum veterum fragmenta* 2.131, translated by Frede 1978, p. 327, a fragment apparently from Chrysippus employing *ta tou logou moria* in the relevant sense). Aristotle *Poet* 1456$^{\text{b}}$20 uses *merē tēs lexeōs*, to which belong not only the parts of speech, but also letters and syllables as well as the sentence itself (*logos*).

the *Technē* are: noun, verb, participle, article, pronoun, preposition, adverb, and conjunction.[89] Adjectives are missing from this list, but they are included under the class of nouns as one of its 24 "species." Articles were omitted when the list was adapted by Latin grammarians to their language,[90] there being no article in Latin, but the interjection was added as an eighth *pars orationis*. Noun, verb, participle, pronoun, preposition, adverb, conjunction, and interjection are therefore the eight parts of speech recognized in the very influential Latin grammars of Donatus (4th century AD) and Priscian (6th century AD); so influential in fact were these works that, for instance, *The Royal English Grammar* of Greenwood (1737) follows them in omitting the article as a separate part of speech, considering it instead an adjective, and therefore a noun (cf. ibid. pp. 27, 41).[91]

The *Technē* employs in its descriptions of the various word classes three kinds of characteristics. One kind of characteristic is morphological, pertaining to the acoustic or graphical shape of the words in the respective class; another kind of characteristic is semantic, pertaining to their signification; a third kind of characteristic is syntactic or functional, pertaining to the role a word of the class plays in grammatical constructions. The classes of noun, verb, participle, article, and pronoun are all characterized in part by their so-called accidents (*parepomena*),[92] which are typically manifest in the morphology of the language, namely in the patterns of inflection characteristic of the given part of speech. In particular, the accidents of the noun include gender,[93] number, and case; the accidents of the verb include tense, number, person, mood, and voice; and the accidents of the pronoun include person, gender, number, and case. But nouns are in addition said to signify "a body or a thing" (*sōma ē pragma sēmainon*,

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[89]According to Quintilian, *Institutio Oratoria* I.4,20, Aristarchus, the teacher of Dionysius Thrax (on the life and work of Aristarchus, cf. Pfeiffer, 1968, pp. 210–233), recognized eight parts of speech, where it is clear from the context that these are the eight parts distinguished in the *Technē*. The reliability of Quintilian's testimony has been defended (convincingly to my mind) by Ax (1991) and Matthaios (1999) against doubts raised by, e.g., Pinborg (1975, p. 107) and Frede (1977, p. 341).

[90]Apparently this was done already in the 1st century BC by Remmius Palaemon (cf. Quintilian *Institutio Oratoria* I.4,20).

[91]On the various systems of parts of speech adopted by tradtional English grammars, see Michael (1970, pp. 201–280); on the treatment of articles in particular, see ibid. pp. 350–360, and especially 354–356.

[92]The Stoics apparently spoke instead of *symbebēkota* (cf. Barwick, 1922, p. 107 ff.), which is the word *accident* typically translates in a philosophical setting (cf. *OED* on *accident*). The term 'accident' in this grammatical sense can still be found in a grammar such as Sweet (1900). In some authors, such as Jespersen (1924, p. 53), accidents are called 'syntactic categories'.

[93]The gender distinction of nouns (and under this name) appears to have been recognized already by Protagoras (cf. the witness of Aristotle *SE* $173^{b}19$ff., *Rhet* $1407^{b}7$).

GG I.1 24,3);[94] the verb to "express activity or passivity" (*energeian ē pathos paristasa*, *GG* I.1 46,5);[95] and the pronoun to be "indicative of definite persons" (*prosōpōn hōrismenōn dēlōtikē*, *GG* I.1 63,2). Noun, verb, and pronoun are therefore characterized not only morphologically by their accidents, but also by means of semantic criteria. The article, apart from being described by its accidents of gender, number, and case is in addition said to be "placed before or after[96] the inflection of the noun" (*GG* I.1 61,2), thus syntactically characterized. Likewise the adverb is characterized functionally by its relation to the verb, namely as being "said of the verb" (*GG* I.1 72,4), and morphologically as being uninflected. Finally, the preposition and the conjunction are purely syntactically characterized: "the preposition is a word placed before all parts of speech, in compounds as well as in grammatical constructions" (*GG* I.1 70,2) and "the conjunction is a word conjoining thoughts in order and revealing gaps in the expression" (*GG* I.1 86,3).[97]

A similar mixture of criteria is found in the remarks of Plato and Aristotle on what in the *Technē* are called the parts of speech.[98] In the *Sophist* Plato describes the noun and the verb by semantic as well as by syntactic or functional criteria. Thus, by appeal to semantics the verb is said to be "the sort of indication that is applied to an action" and the noun to be "the kind of spoken sign that is applied to a thing that performs the actions" (262a); but Plato moreover says that any *logos* is about something and says something about that thing (262e–263d), where it is clear from the context that the noun serves to pick out what the *logos* is about and the verb to say something about that thing. Thus the noun and the verb are syntactically characterized, corollated, in effect, with the roles of subject and predicate of a sentence, as already noted in section 1.2 above.

According to Aristotle's definitions of the noun and the verb in the *De Interpretatione* 2–3 their genus is "significant spoken sound no part of which is significant

[94]The 'or' here is not merely expletive: stone is given as an example of a *sōma* and education as an example of a *pragma*; perhaps on the basis of these examples Kemp (1987, p. 176) translates the former by 'something corporeal' and the latter by 'something non-corporeal'. Lallot (1989) translates the former by '*corps*' and the latter by '*action*'.

[95]According to a scholium (*GG* I.3 161,7) Apollonius Dyscolus said in his work on the verb (now lost) that Dionysius Thrax defined the verb as a word signifying a predicate (*rhēma esti lexis katēgorema sēmainousa*); as we shall see below the Stoics defined the verb by the very same formulation.

[96]The article placed after the noun is the relative pronoun.

[97]The second part of this definition "revealing the gaps in the expression" (*to tēs hermēneias kechēnos dēlousa*) has bothered editors and commentators; see Barnes (2007, pp. 183–184) and especially Lallot (1989, pp. 227–236) for more discussion. When Kemp (1987, p. 185) translates "*fills up* gaps in the expression" he must be relying on an alternative reading which substitutes *plērousa* for *delousa*; for reasons not accept this reading see Barnes (ibid. p. 184) and Lallot (ibid. p. 228).

[98]For a concise overview of the doctrine of parts of speech in Plato, Aristotle, and the Stoics, see Robins (1966).

in separation," familiar from our discussion in section 1.1 above, while the verb is distinguished from the noun by "additionally signifying time" (16^b6). The genus of being a significant spoken sound having no part significant in separation has both a semantic and a morphological component, for it refers both to signification and to the notion of a part of a word. The differentia of additionally signifying time may at first seem to provide a morphological criterion, met by the verb in its showing variation in tense. Aristotle holds, however, that a verb in the past or the future tense is in fact not a verb (16^b16–17), hence variation in tense cannot be a characteristic of verbs; moreover, Aristotle explains that 'recovers' (*hugiainei*) is a verb, for unlike the word 'recovery' (*hugieia*) "it additionally signifies something's holding now" (16^b9). Additionally signifying time thus seems to furnish a semantic criterion. To these partly morphological and partly semantic descriptions Aristotle adds functional descriptions. In effect, like Plato, he identifies the verb with the predicate of the sentence (16^b6, 16^b10) and the noun with the subject (19^b5). Thus, at *Int* 20^b1 'white' is called a verb, but 'white' does not additionally signify time, so it is a verb only because it functions as a predicate. In *Poetics* 20 two parts of speech in addition to noun and verb are identified, called *syndesmos* and *arthron*. It is unclear from the text which words are to be counted as *arthra*,[99] but *syndesmoi* are most likely conjunctions (cf. *Rhet* 1407^a21–31; *Int* 17^a9,16). Whichever word classes they be, they are defined by means of functional criteria: their genus is non-significant sound (which thus excludes semantic criteria in their differentiae); but where the *syndesmos* "produces out of several significant sounds one significant sound" (1457^a4–6), the *arthron* "reveals the beginning, end or middle of the *logos*" (1457^a6–7).

The Stoics are generally held to have played an important role in the development of grammar.[100] They distinguished at least five parts of speech: in addition to verb, article, and conjunction they divided the class of nouns into two separate parts,

[99]Lucas (1968, p. 202): "It is impossible to say what kinds of non-significant word A. here intends." Likewise, Van Bennekom (1975, p. 406): "The definitions themselves [the text gives two definitions] hardly give a clue as to what sort of words may be meant." Adding to the difficulties are the facts 1) that according to the witness of Dionysius of Halicarnassus *De compositione verborum* 2 Aristotle distinguished only three parts of speech, namely noun, verb, and *syndesmos*; and 2) that *arthron* is nowhere else in the Aristotelean corpus (apart from the spurious *Rhetoric to Alexander* 1435^b13–16) employed as a term of grammar (cf. Pinborg 1975, pp. 72–75 or Schramm 2005, esp. pp. 187–193 for a fuller overview of the difficulties and of solutions proposed in the literature). Van Bennekom and Schramm (ibid.) argue that the class of *arthra* comprises articles and prepositions; articles (including relative pronouns) are indeed what the *Technē* terms as *arthra*, whereas the only certain example of an *arthron* in the text of the *Poetics* is the preposition *peri*. I am grateful to Michael Schramm for sending me an offprint of his paper.
[100]So, e.g., Pinborg (1975, pp. 77–103).

common nouns (*prosēgoria*) and proper nouns (*onoma*/*kyrion onoma*).[101] Judging from the testimony of Diogenes Laertius (*DL* VII.58) purely semantic criteria were employed in defining the common noun, the proper noun, and the verb. A common noun was said to signify a common quality (*koinē poiotēs*), a proper noun a peculiar quality (*idia poiotēs*), and a verb was said to signify what the Stoics called a predicate (*katēgorēma*), which is explained as an incomplete *lekton* (*DL* VII.64). According to the same testimony, the article and the conjunction were defined by reference to their function as well as their morphology. An article is "a declinable element of speech distinguishing gender and number," while a conjunction is "an indeclinable part of speech conjoining the parts of speech."[102] It has, however, been argued by Pinborg (1975, p. 99–100) and others[103] that the Stoics gave purely semantic definitions also of the article and the conjunction. The Stoic Posidonius is reported by Apollonius Dyscolus (2nd century AD) to have written a work "on conjunctions" where he opposes those (including Aristotle) who held that the conjunction does not signify anything (cf. *GG* II.1 214,4ff.). The same Apollonius characterized the article (or at least the demonstrative and indefinite pronouns, considered articles by the Stoics)[104] as signifying existence (*ousia*) without quality (*GG* II.1 9,9–10), and Pinborg argues that this definition is in fact of Stoic origin.

4.2. Categories and the parts of speech. The question seems to have occurred already to the ancient commentators on Aristotle what the relation is between the parts of speech and the categories. Some recent commentators hold that the relation was a tight one in Stoic doctrine (4.2.1). Some have also seen a relation between Aristotle's categories and the parts of speech, though we shall follow the ancient commentators in emphasizing the contrast between the two (4.2.2)

4.2.1. *In the Stoics.* The presumably Stoic definition of the article as signifying existence without quality, together with the definitions of the two kinds of noun, have led scholars to see a connection between the Stoic parts of speech and what is known

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[101] According to the scholium referred to in fn. 95 above (*GG* I.3 160,26) Dionysius Thrax treated common noun and proper noun as separate parts of speech. Matthaios (1999, pp. 214–244) argues that Aristarchus classed them as one part of speech, as indeed reported by Quintillian (cf. fn. 89 above).

[102] The Stoics are said to have conceived of prepositions as "preposed conjunctions" (so e.g. Priscian, *GL* II 54,20–22; for more passages, see Schmidt 1979, p. 136–137, and for a discussion Barnes 2007, pp.190–192). Cf. the view of Jespersen (1924, p. 89): "The so-called conjunction is really, therefore, a sentence preposition."

[103] E.g. Lloyd (1971, pp. 67–69) and Frede (1978, pp. 330–332).

[104] Apollonius (*GG* II.1 5,13–9,6) also criticizes the Stoics for their treating pronouns in general as belonging to the same part of speech as articles.

as their categories. The Stoics are said to have assumed four categories:[105] substrate (*hypokeimenon*), qualified (*poion*), somehow disposed (*pōs echon*), and somehow disposed in relation to something (*pros ti pōs echon*). Since the two kinds of noun are both said to signify quality, and the article presumably substrate, the following correspondence between parts of speech and the categories has been suggested.[106]

Part of speech	article	common and proper noun	intransitive verb	transitive verb
Category	substrate	qualified	somehow disposed	somehow relatively disposed

There is, however, an obvious problem with this table, for at *DL* VII.58 nouns are said to signify a quality (*poiotēs*) and not a qualified (*poion*), while a verb is said to signify what the Stoics called a predicate, i.e. an incomplete *lekton* (*DL* VII.64), and not something's disposition. An elegant solution is offered by Christensen (1962, pp. 43–52): one must take account of all vertices in the Stoic "semantic triangle"—the sign, the sense (let us adopt that word here for what resides at the level of the *lekton*), and the reference. This "triangle" is sketched by Sextus Empiricus in *Adversos Mathematicos* 8.11–12 (text 33B in Long and Sedley 1987):[107]

> three things are connected with one another, the sense (*sēmainomenon*), the signifier (*sēmainon*), and the reference (*tygchanon*). Of these the signifier is a vocal sound, for instance 'Dion', the sense is the very thing (*pragma*) revealed by it, and which we apprehend as it subsists in our thought, and which foreigners do not understand even though they hear the utterance; and the reference is the external substrate, for instance Dion himself.

The parts of speech obviously belong at the level of the signifier. And for the Stoics, items categorized reside at the level of reference. The level of reference is not free of language, according to Christensen: the Stoic categories are "reference classes" (p. 51), that is, "classes of objects in so far as these are denotata of meanings of the basic types" (p. 48). The Stoic conception of the categories is, accordingly, a semantic

[105] For this, see especially Menn (1999), but also Christensen (1962, pp. 48–52), Rist (1969, pp. 152–172), and Lloyd (1971). The status of the whole doctrine is uncertain; Barnes (2005, p. 26) concludes a general discussion of the sources that "Bref, la théorie stoïcienne des catégories est un mythe."

[106] The first to have done so appears to be Schmidt (1839, p. 37), who, however, placed conjunction instead of transitive verb as corresponding to relative disposition. For the table below, see Lloyd (1971, p. 69), and Pinborg (1975, p. 101).

[107] Its relation to Aristotle's semantic triangle is discussed by, for instance, Christensen (1962, pp. 44–47) and Barnes (1993).

conception (cf. section 3.1 above), where the categories cannot be thought apart from the meaning relation.

The solution to the problem is then to point out that what *DL* VII.58 describes the various parts of speech as signifying resides not at the level of reference, but at the level of sense. In particular, quality is now taken to reside at the level sense, together with the predicate. Common nouns as well as proper nouns have as their sense a quality (*poiotēs*), and as their reference a qualified (*poion*).[108] Corresponding to the distinction between intransitive and transitive verbs the Stoics distinguished between unary and binary predicates (the latter were called direct predicates, *ortha katēgorēmata*, *DL* VII.64). An intransitive verb has as its sense a unary predicate, and as its reference a disposition, while a transitive verb has as its sense a binary predicate, and as its reference a relative disposition. It remains to account for the article. Here we have Apollonius's characterization, reported above, that the article reveals (*dēlein*) only existence (*ousia*), and is as such contrasted with the noun, which reveals (*epangellesthai*) a quality; hence, assuming that this was also the Stoic characterization, the mark of existence is taken to be the sense of the article, while its reference is substrate. The following table results (cf. Christensen, 1962, p. 50).[109]

Part of speech	article	common and proper noun	intransitive verb	transitive verb
Sense	existence	quality	unary predicate	binary predicate
Category (reference)	substrate	qualified	somehow disposed	somehow relatively disposed

4.2.2. *In Aristotle.* In light of this apparent correspondence between categories and parts of speech in Stoic doctrine it is natural to ask how Aristotle's categories relate to the parts of speech. In traditional grammar it was common to appeal to Aristotelian categories in the definitions of noun and verb. Priscian, for instance, defined the verb by appeal to the categories of action and affection, and the noun by appeal to the categories of substance and quality:

[108] The Stoic notion of *ptōsis* appears to have satisfied the following relation: a verb stands to a noun as a predicate stands to a *ptōsis* (cf. esp. Plutarch, *Questiones Platonicae* 1009c). The question therefore arises how this notion of *ptōsis* relates to that of quality. Pinborg (1975, p. 81) insists that they are in fact one and the same, "with quality implying the physical reality behind language, *ptōsis* the logical structure as seen in itself." Frede (1994, p. 20) appears to make the same identification.

[109] Menn (1999, pp. 226–227) argues that it is participles which have as their denotation the somehow disposed and the somehow relatively disposed. According to Priscian (*GL* II 54,9ff.) the Stoics counted participles among the verbs (cf. Schmidt, 1979, p. 135, for more examples), so Menn's view may be reconcilable with that presented above.

The characteristic of the verb is to signify action or affection or both. . .[110]

The characteristic of the noun is to signify substance and quality.[111]

A similar definition of the verb was given by Donatus in his *Ars Minor*,[112] and there is reference to action and affection also in the definition of the verb in the *Technē* (*GG* I.1 46,5), although there *energeia* and not the Aristotelian *poiein* or any of its derivatives is used as the name for the category of action. However, I know of no attempt in traditional grammar at defining all the various parts of speech solely in terms of Aristotle's categories. Indeed, as we have just seen, more often than not both morphological and syntactic characteristics played a role in that definition besides semantical ones. The only attempt I am aware of at defining all parts of speech purely in terms of Aristotle's categories was carried out by the Danish linguist Viggo Brøndal in his work *Ordklasserne* (1928).[113] Brøndal there defined a system of 15 word classes, many of them with subclasses, from the Aristotelian categories of substance, relation, quantity, and quality. The system purported to be universal in the sense that all word classes of all languages are among these 15. It lies outside the scope of this dissertation to go further into Brøndal's work.

Robert Kilwardby (13th century) denied that there could be any correspondence between the parts of speech and Aristotle's categories:

> The parts of speech are not distinguished after the distinctions of things, but after the distinctions of modes of signifying [. . .] Things of all categories can be signified by the noun, e.g., quantity and quality and the rest. For that reason there are not ten parts of speech as there are ten categories of things.[114]

[110] *GL* II 55,8–9: "Proprium est verbi actionem sive passionem sive utrumque cum modis et formis et temporibus sine casu significare."

[111] *GL* II 55,6: "Proprium est nominis substiantiam et qualitatem significare."

[112] *GL* IV 359,4–5: "uerbum quid est? pars orationis cum tempore et persona sine casu aut agere aliquid aut pati aut neutrum significans"

[113] See also his *Morfologi og Syntax* (1932), where the Aristotelian categories of substance, relation, quantity, and quality are used in defining various functional categories, such as subject, predicate, and object. The class of prepositions is studied in Brøndal (1940).

[114] Kilwardby, *in Priscianus minorem*, (cited by Pinborg 1967, p. 48): "Non distinguuntur partes orationis secundum distinctionem rerum, sed secundum distinctionem modorum significandi. Possunt autem omnes res eodem modo significari, scilicet permodum habitus; ideo res omnium predicamentorum possunt per nomen significari, ut quantitas et qualitas et sic de aliis. Et hac de ratione non sunt decem partes orationis, sicut sunt x predicamenta rerum."

A similar sentiment is expressed in Kilwardby's commentary on the *Categories* (cited by Ebbesen 2005, p. 259): "Est igitur, ut dicit Boethius, scientia Praedicamentorum de X vocibus X prima rerum genera significantibus. Non enim est de vocibus penes diversas figurationes vocum, quae sunt inflectio casuum aut temporum, sed de vocibus in quantum sunt significativae."

According to Kilwardby, the noun provides a counterexample, for it may signify elements of any category. In fact, the modistae, of which Kilwardby may be counted a member, held that, in general any thing may be the signification of all the various parts of speech (cf. Pinborg, 1967, p. 81). One and the same *dictio*, which is a sound (*vox*) furnished with reference, can be formed into any part of speech so long as the reference is compatible with the relevant form, the relevant "mode of signifying" (*modus significandi*),[115] that determines the part of speech. The words *dolor*, *doleo*, *dolens*, *dolenter*, and *heu!*, for instance, all signify pain, but differ in their mode of signifying it, namely as a noun, a verb, a participle, an adverb, and an interjection respectively (cf. Pinborg, 1982, pp. 257). So according to the modistae the correspondence between the parts of speech and the categories fails in both directions: neither is the category of the thing signified determined by the part of speech of the signifier, nor is the part of speech of the signifier determined by the category of the thing signified.

Aristotle's own language in *Categories* 4 indicates that he had not conceived of any correspondence between parts of speech and the categories: the examples of each of the four categories of action, affection, position, and having are all equally verbs. One could, however, argue that these are different kinds of verb, and in the extension of this set up a correspondence between the categories and a finer grouping of the parts of speech. That was indeed done by Trendelenburg (1846, pp. 23–24), who suggested the correspondence of substance to the noun, quantity and quality each to a kind of adjective, when and where to adverbs of place and time respectively, relative, at least its prototypical cases, to the comparative form of the adjective, and the four other categories to four kinds of verbs—action to verbs in the active voice, affection to verbs in the passive voice, position to "at least a part of the intransitives," and having to verbs in the perfect tense.[116] Trendelenburg was certainly well aware that no such correspondence is indicated in Aristotle's works, and indeed that Aristotle had not made the necessary grammatical distinctions; but he nevertheless thought that grammatical reflection was instrumental to Aristotle's conceiving of the categories (cf. section 5.2 below).

[115]See Pinborg (1967, pp. 30–46) for the genealogy of this notion.

[116]Benveniste (1966, pp. 66–70), without citing Trendelenburg, argues for the same correspondence except that he lets position correspond to verbs in the middle voice. For some discussion of Benveniste and Trendelenburg, see Kahn (1978, pp. 233–237). More recently Baumer (1993) has suggested the correspondence of substance to noun (and other "nominal forms"), quality to adjective, relative to oblique cases of the noun, quantity to grammatical number, when to tense, doing to the active voice, affection to the passive voice, and where to preposition (cf. ibid. page 428). It is not clear to me how one should understand this, since a noun always has number and may well be in an oblique case (does it then signify substance, quantity, and relative?) and a verb has tense as well as voice (so it signifies a when as well as an activity or a passivity?).

It is in any event clear that the division of linguistic items into categories differs in character from its division into parts of speech; let us try to specify some of the differentiae. According to Kilwardby, the parts of speech do not follow "the distinctions of things." We saw (p. 32) that according to Porphyry the division of language into *categories* does indeed follow the distinctions of things, namely into genera of beings; but Porphyry introduced the notions of primary and secondary imposition precisely so as to be able to differentiate the division into categories from the division into parts of speech (*in Cat* 58,30–59,14; cf. 57,29–58,6). Ammonius, in his commentary on the *De Interpretatione*, raised the question (9,28–10,1) "why, when he has treated of simple vocal sounds [=things said without combination][117] at book length in the *Categories*, he here again undertakes to speak about name and verb, each of which is obviously a simple vocal sound." His response (10,4–10,12) nicely spells out Porphyry's point:[118]

> For when we consider that simple vocal sounds are significative of the things to which they have been assigned, this is all we call them—simple vocal sounds—since we do not here distinguish names from verbs; but when we have seen some lack of correspondence among these and find that some of them are combined with articles while others are not, or also that some additionally signify a certain time, while others do not, then we distinguish them from one another and we call those which are combined with articles and do not additionally signify time 'nouns'; and those which cannot be combined with articles but are said according to a certain time we call 'verbs'.

Given our identification of the notion of term with that of thing said without combination, and so with Ammonius's "simple vocal sounds," and generalizing Ammonius's response to all parts of speech, Porphyry's point is the following. Terms are divided into categories by considering them primarily as signifying things (and, according to Porphyry, by letting them inherit the generic differences of the things they signify). Words are divided into parts of speech by considering them not only according to their signification, but also according to their grammatical properties; the latter requires a reflection on the words themselves (secondary imposition) and not only on their signification (primary imposition). Hence the criterion of classification in the two cases concern different aspects of the linguistic items classified. Moreover, the two divisions divide different items. Items categorized are terms; but not all words

[117]For this identification, cf. e.g. Ammonius *in Cat* 11,19.

[118]See the similar passage of Ammonius *in Cat* 11,7–11,17, where he employs the terminology of primary and secondary imposition.

of a given language are terms. In fact, if we adhere strictly to the 'S is P'–form, then only noun, participle, and pronoun are grammatically suited as terms (a verb we may perhaps think of as a "proto-term," namely proto to its various participial forms). On the other hand, any word belongs to a part of speech. And, as we argued above, terms may be of arbitrary complexity; but the members of the parts of speech are single words.

4.3. Syncategorems. In his commentary on the *De Interpretatione* Ammonius divided the parts of speech into three classes: noun, verb, pronoun, and participle are "significant of certain natures or simply of persons or activities or some combination of these" (11,9); the main function of the adverb is "to make clear some relation of the predicate to the subject" (11,15);[119] while article, preposition, and conjunction are "absolutely without significance by themselves" (12,14). What occasioned this distinction in Ammonius was the question, just quoted, why Aristotle in the *De Interpretatione* had discussed only noun and verb among the eight parts of speech. About 400 years earlier Plutarch (1st century AD), in his tenth Platonic question, had asked the same question with regard to Plato and as a response drawn, albeit somewhat less perspicuously, distinctions along similar lines.[120] It is in the context of a discussion of such distinctions, moreover, that we first meet the word 'syncategoremata'. Priscian reports that "according to the dialecticians" there are only two parts of speech, namely noun and verb, while all other words are called syncategorems (*GL* II 54,5–7). Since this division was suggested by dialecticians, it is presumably meant to separate the words that can function as terms from those that cannot; hence 'noun' and 'verb' must here be understood, not in the manner of the grammarian, but rather so as to include both pronoun and participle.[121] Ammonius may have been among the dialecticians Priscian had in mind, for at one place in his commentary Ammonius says that only noun and verb are parts of speech (*logos*), while adverb, conjunction, article, and preposition are merely parts of diction (*lexis*) (*in Int* 12,16–13,6). Here a part of diction is any articulate sound, hence unlike a part of speech it need not be significant.[122]

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[119]On Ammonius on adverbs and his related treatment of modality, see Barnes (1991).

[120]Plutarch's *Platonic Questions* are found in Book XIII of his *Moralia*. Luhtala (2005, pp. 129–137) collects parts of this as well as several of the other texts discussed in this paragraph.

[121] Hence the following statement by Apuleius (2nd century AD), grouping pronouns and participles together with adverbs and conjunctions, is odd: "Indeed, adverbs, pronouns, participles, conjunctions and other such things which grammarians list are no more parts of speech than ornamented curved sterns are parts of ships and hair of men; or at least they are fit to be classed in the general structure of speech like nails, pitch, and glue" (Londrey and Johansen, 1987, p. 85).

[122]According to Aristotle *Poet* 1456^{b}20, where the roots of Ammonius' terminology presumably lie, the parts of *lexis* are letter (*stoicheion*), syllable, *arthron*, *syndesmos*, noun, verb, case, and *logos* itself.

Ammonius was, however, not of the opinion that articles, prepositions, and conjunctions are altogether void of significance. They are "absolutely without significance by themselves" (ibid. 12,14); but that is probably to say that they are significant only in conjunction with other expressions. Such a view was held already by Apollonius Dyscolus, who compared articles, prepositions, and conjunctions to consonants, and the other parts of speech to vowels (*GG* II.2 13,1–14,2): vowels can be pronounced by themselves, but consonants need for their pronunciation the company of one or more vowels, as do for instance /biː/ and /keɪ/. Such, in fact, is the main characteristic of syncategorems according to Priscian. He glosses this apparently Greek word[123] as "*consignificantia*" (*GL* II 54,7), which he in turn glosses as "signifying when conjoined with other items, but not in itself" (*GL* III 114,19–20). The Latin verb *consignificat* was Boethius's translation of *prossēmainei*, a word Aristotle had employed in the *De Interpretatione* for three not obviously related modes of signification:

- the verb's signifying time in addition to whatever else it signifies ($16^{\mathrm{b}}6$);
- the mode of signification of the copula 'is': "by itself it is nothing, but it *prossēmainei* some combination, which cannot be thought of without the components" ($16^{\mathrm{b}}24$);
- the mode of signification of the quantifiers 'every' and 'no', which *prossēmainousi* that the subject is "taken universally."

If any of these senses survive in Priscian's characterization of syncategorems as "signifying when conjoined with other items, but not in itself" it must be the second: the copula signifies nothing by itself but joined to two terms it comes to signify a combination.

That the meaning of a symbol is to be explained within a larger context is characteristic of what Russell and Whitehead in the *Principia Mathematica* (p. 66) called incomplete symbols; such symbols have only a "definition in use," or what is sometimes called a contextual definition. A syncategorem is, however, not an incomplete symbol in Russell's sense. If, as seems reasonable, we take definite descriptions as paradigms of such incomplete symbols, then their incompleteness is one which is not apparent in the surface grammar, but which is seen only after logical analysis. This is unlike syncategorems, which already on the surface is seen to require the company of other expressions in order to have any use at all. Definite descriptions could seem on the surface to be of use in naming entities, though according to Russell's analysis that is not the case—although the definite description appears as a subject term in a

[123]The components *syn-* and *katēgorēma* are Greek, but *synkatēgorēma* has apparently not been found in any Greek sources (cf. Meier-Oeser, 1998, p. 787). The form of the word listed in the *OED* is 'syncategorem', which will therefore be used here.

proposition, there is no subject in the defining expression which the definite description abbreviates; rather, this is split up into an existential quantifier, a uniqueness stipulation, and a predication. Syncategorems, by contrast, do not, not even on the surface , seem to be of use in naming entities, or of any other use that does not involve other expressions.

This could be taken to suggest that syncategorems have another sort of incompleteness appealed to in modern logic, namely the incompleteness that according to Frege is characteristic of function, predicate, and relation symbols (see section 1 of chapter 2 below). Their incompleteness, or unsaturation, is evident on the surface, hence in that regard they are closer to syncategorems than are Russellian incomplete symbols. One should, however, not identify syncategorems with functions in Frege's sense. As we shall in section 1.3 of chapter 2, function symbols, just as all other symbols of Frege's logic, must be regarded as categorems of that logic, for they are all assigned a type. The general point is that the notion of syncategorem can only be understood in the context of traditional logic and grammar, where a distinction is made between form and matter, and syncategorems are taken to be form elements (more on this a few paragraphs below); in function–argument syntax, by contrast, no distinction is made between form and matter. Notions of incompleteness found in modern logic and grammar can therefore not be used in elucidating the incompleteness of syncategorems. Thus, if following Barnes (2007, pp. 246–250) one seeks to clarify the incompleteness of a syncategorem a by saying that its meaning can only be explained in a context Xa, then this must be understood against the background of traditional logic and grammar; for in modern logic and grammar that clarification will apply to Fregean function symbols and Russellian incomplete symbols, neither of which are syncategorematic.

Owing most likely to the problems that syncategorems caused in the logical analysis of various propositions there developed in the late 12th and early 13th century a separate literature entirely devoted to their study.[124] In this literature the notion of consignification is never described along Priscianic lines. In what appears to be one of the earliest instances of this literature consignification is rather glossed as "signifying nothing that is complete and limited in itself (*in se ipsis terminatum vel finitum*),

[124]See Braakhuis (1979) for a study of this literature. The historical background in the logical analysis of problematic propositions is treated extensively there (pp. 27–90), and is emphasized in the overview article of Kretzmann (1982). One can find treatises on syncategorems published as late as the early 16th century (cf. Meier-Oeser, 1998, p. 788).

but signifying the dispositions and circumstances of things (*dispositiones et circumstantias rerum*)" (Braakhuis, 1979, p. 117).[125] It is not clear to me in which sense this characterizes a notion of consignification, but be that as it may; there is namely another, more syntactic, characterization of syncategorems that appears to have been dominant in this literature. It is found, if not expressly in the description of what in general characterizes a syncategorem, so at least implicitly in the treatment of the individual syncategorems (cf. ibid. p. 385). William of Sherwood, for instance, in his *Syncategoremata* treatise describes syncategorems as "determinations of principal parts [=categorems] insofar as they are subjects or predicates" (Kretzmann, 1968, p. 15). Syncategorems are thus not characterized in terms of any notion of consignification, but rather by their role as specifying how the predicate is to be predicated of the subject or the subject subjected to the predicate. Thus, the quantifier 'every' specifies how the subject is to be taken as subject of the predicate, namely that it is "universally subjected to the predicate" (ibid. p. 17); and the modal adverb 'necessarily' specifies how the predicate is to be taken as predicate of the subject, namely that its composition with the subject is necessary (ibid. p. 101). William moreover distinguishes between categorematic and syncategorematic uses of the same word, and at least in some cases (such as 'all' and the numerical quantifiers) this distinction comes down to whether the word is to be considered as part of the subject or predicate, or whether it modifies them along the lines indicated for 'all' and 'necessarily'. But it was not part of William's characterization that those words are syncategorems which cannot function by themselves as subject or predicate. This characterization, which is implicit in the earlier tradition from Plutarch to Priscian, seems to appear again only in the 14th century; it is found expressly in Albert of Saxony's *Logic* I.3:

> a categorem is said to be what, taken significatively, can be a subject or a predicate... a syncategorem is said to be what, taken significatively, cannot be the subject or the predicate[126]

Buridan (14th century) in his *Treatise on Consequences* I.7 connects syncategorems with the notion of form of the proposition. Having explained the notions of formal and material consequence, Buridan remarks that he must add "what we take to be the form of a consequence or proposition and what matter." He continues

[125] Similar glosses are given in the logic of William of Sherwood (Kretzmann, 1966, p. 24) and in the *Syncategoremata* treatise of Henry of Ghent (Braakhuis, 1979, p. 351).

[126] Albert von Sachsen (2010, p. 23): "Terminus categorematicus dicitur, qui significative acceptus postest esse subiectum vel praedicatum vel pars subiecti vel pars praedicati propositionis categoricae... Sed terminus syncategorematicus dicitur, qui significative acceptus non posse esse subiectum vel praedicatum vel pars subiecti vel pars praedicati propositionis categoricae..."

> by the matter of a proposition or consequence we understand the purely categorematic terms, namely the subjects and predicates, as distinguished from the syncategorematic terms adjoined to them, by which they are connected or negated or distributed or determined to some particular manner of supposition; but we say that everything else pertains to form.[127]

Thus Buridan identifies the matter of a proposition with "the purely categorematic terms," and holds that the syncategorematic words "pertain" to form. From the ensuing discussion it appears that what he means by this pertaining is that the syncategorems contribute to determining the form of the proposition.[128] From the same discussion it is also clear that more factors contribute to determining this form, in particular, the order of the syncategorems and the possible relation of the categorems to each other (e.g. repetition of a term). Similar notions of the form and matter of a proposition are found in Alexander of Aphrodisias's commentary on the *Prior Analytics* (2nd–3rd century AD).[129] At the opening of *APr* I.2 Aristotle had said ($25^{\mathrm{a}}1$–5):

> every proposition states either that something belongs or that it belongs of necessity or that it may belong, and of these some are affirmative, others negative,..., and again of the affirmative and negative propositions some are universal, others particular, and others indeterminate...

Alexander calls the property of being affirmative or negative the proposition's quality and its property of being universal, particular, or indeterminate its quantity (*in APr* 11,29–34);[130] the property of stating that something belongs, or belongs of necessity, or that it may belong, Alexander calls the mode of the proposition (*in APr* 26,25–28,30). Commenting on the quoted passage Alexander notes that "there will be in all three times six propositions differing from one another in form" (*in APr* 27,20–21). Thus he holds that the proposition's quantity, quality, and mode all contribute to the form of a proposition, in the sense that a difference in one of these yields a difference in form. Alexander moreover speaks quite freely of terms as the matter of a

[127] Buridan (1976, p. 30): "Per materiam propositionis aut consequentiae intelligimus terminos pure categorematicos, scilicet subiecta et praedicata, circumscriptis syncategorematicis sibi appositis, per quae ipsa coniunguntur aut negantur aut distribuuntur vel ad certum modum suppositionis trahuntur; sed ad formam pertinere dicimus totum residuum."

[128] For a helpful discussion, see Moody (1953, pp. 16–18).

[129] See Lee (1984, pp. 37–44) and Barnes (1990, esp. pp. 39–55).

[130] This terminology is also found in Apuleius (Londrey and Johansen, 1987, p. 82/83–84/85).

proposition. For instance, Aristotle is said to use "letters in his exposition in order to indicate to us that the conclusions do not depend on the matter" (*in APr* 53,29).[131]

A final characteristic of syncategorems that will be relevant for the following is their role as connectors. It was of course part of the definition of the conjunction that it conjoins other parts of speech, but, discussing the relation of categorems to syncategorems, Ammonius suggests that all syncategorems—not only conjunctions—conjoin other parts of speech; conjunctions, articles, prepositions, "and even adverbs" (*in Int* 13,1–3)

> are useful for combining and constructing the parts of speech with one another, just as a bond is useful for adding unity to things bound and glue to the things held together by it.

This view, and especially the comparison of the syncategorems with glue, is used or mentioned by several philosophers and grammarians in Antiquity.[132] But the characteristic of having as their main function the binding of other parts of speech was, as far as I know, never taken to define syncategorems, neither in Antiquity nor in the Middle Ages. It, or something very close to it, was, however, used by Locke in defining what he called particles. This group of words corresponds more or less to the syncategorems, for it includes the copula as well as prepositions and conjunctions. Locke introduced the notion thus (*Essay* III.vii.1):

> Besides Words, which are names of *Ideas* in the Mind, there are a great many others that are made use of, to signify the *connexion* that the Mind gives to *Ideas, or Propositions, one with another*.

So the distinction here is between words that signify ideas[133] and words that signify connections established by the mind of ideas or propositions. It is then worth mentioning that in the *Port-Royal* grammar of Arnauld and Lancelot (1676) a similar

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[131]From many, perhaps most, of the relevant passages it appears that for Alexander hylomorphism applies not at the level of the proposition, but at the level of the syllogism; so for instance at the often quoted *in APr* 6,26–28: "The figures are like a sort of common matrix (*tupos tis koinos*)—by fitting matter into them, it is possible to mould the same form in different sorts of matter." But we have seen that Alexander also talks of the form of a proposition; and from *in APr* 36,2–9 it is likewise clear that terms may be thought of as the matter of a proposition.

[132]Besides Ammonius *in Int* 12,25–13,6, see Dexippus *in Cat* 32,17–33,8 and the corresponding passage in Simplicius *in Cat* 64,18–65,2. See moreover the passage from Apuleius quoted in footnote 121 above. Plutarch used a different simile, saying that syncategorems contribute to speech "as salt does to a dish of food and water to a barley-cake" (1010C). For grammatical references, see *GG* I.3 515,19ff. and *GL* II 551,18.

[133]E.g. *Essay* III.ii.2: "*Words in their primary or immediate Signification, stand for nothing, but the* Ideas *in the Mind of him that uses them.*"

distinction led to quite another division of the parts of speech. There, words signifying "objects of thought" are distinguished from words signifying the "form and manner of thought" (p. 30). The former class is associated with the first operation of the mind in traditional logic, the act of conceiving, while the latter class is associated with the second operation, the act of judging (pp. 26–30). Since the verb is seen as primarily signifying affirmation (p. 101), it is placed in the latter class together with conjunction, which signifies the operation of the mind that joins or disjoins or negates propositions, considering them absolutely or conditionally (p. 151);[134] to this group interjections belong as well, for an interjection is a word signifying (naturally rather than by convention) "the movements of our soul"(p. 153). The preposition, on the other hand, is not placed together with the conjunction, for it signifies an objective relation which itself is an object of thought (p. 88),[135] and so belongs to the former group together with the noun, pronoun, and participle. In this group Arnauld and Lancelot also include the adverb, which apparently is merely an abbreviation of a prepositional phrase, as in the equation X-ly = with X-ness (p. 93); as well as the article, for it merely specifies the meaning of the noun (p. 52). In other words, for Arnauld and Lancelot the categorematic parts of speech, those that are "names of ideas in the mind," are noun, pronoun, participle, article, preposition, and adverb; while the syncategorematic parts of speech, those that "signify connexion," are verb, conjunction, and interjection.

5. Kantian themes

The discussion of syncategorems in the previous section may shed light on the relation of Aristotle's to Kant's categories. One can say that Kant's categories stand to syncategorems as Aristotle's categories stand to categorems. Like syncategorems, Kant's categories are associated with the form of a proposition and with the notion of connection; like categorems, Aristotle's categories are associated with the matter of a proposition and what is connected by the syncategorems. In a formula: Kant's categories synthesize what are categorized by Aristotle's categories.

5.1. Kant's table of categories. According to Kant, his table of judgement provides "the clue to the discovery" of the table of categories (the latter can be found in Appendix 2 on page 66 below). The elements of the table of judgement are first

[134] This is a slight distortion of the truth, since conjunctions at the cited place are said to signify "l'operation mesme de nostre esprit, qui joint, ou disjoint les choses, qui les nie, qui les considere absolument, ou avec condition"; so it is not propositions, but things, which are joined or disjoined, etc. by conjunctions.

[135] "les Cas & les Propositions avoient esté inventez pour le mesme usage, qui est de marquer les rapports que les choses ont les unes aux autres."

introduced as "functions of the understanding (in judging)" and "moments of thinking (in judging)" (A70/B95ff.), though Kant also speaks of these items as forms of judgement.[136] These functions or moments or forms, of which there are twelve in total, are placed in four groups of three under the headings of 'quantity', 'quality', 'relation', and 'modality'.[137] Under the heading of 'quality', for instance, one finds the forms of general, particular, and singular, and under the heading of 'relation' the forms of categorical, hypothetical, and disjunctive. As the study of Tonelli (1966) shows, all of Kant's twelve forms of judgement were recognized in logic books of the time, indeed most of them are part of Aristotelian syllogistics; but the precise combination assumed by Kant is apparently original, as is the idea of listing them in a table.[138]

The argument that the table of judgement can serve to uncover the table of categories is roughly as follows. The categories are primitive pure concepts; concepts are the business of the understanding; being pure, the categories must therefore somehow lie in the understanding in advance of all experience, and being primitive, they are not derived from other concepts; the categories are *Stammbegriffe* (A81/B107) or *Elementarbegriffe* (A64/B89, *Prolegomena*, § 39, p. 323). According to Kant, the exercise of the understanding is exhausted by its exercise in judgement,[139] hence we can discover the categories only by paying attention to the notion of judgement itself; not by paying attention to the possible terms of a judgement since these in general have an empirical origin, but to the forms of judgement, or to the "functions of the understanding" by means of which the terms of the judgement are unified.[140] In fact,

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[136]See, for instance, *Prolegomena* § 22 (p. 304). In another context (A266/B322) Kant reports that, according to logicians, the form of a judgement is the relation in it by means of the copula of the "given concepts."

[137]As we have seen above, the use of 'quantity' and 'quality' for forms of judgement can be traced back at least to the 2nd century AD, being present in both Alexander of Aphrodisias and Apuleis; Alexander also speaks of the *tropos* of a proposition as its stating that "something belongs or that it belongs of necessity or that it may belong," and 'mode' is just the translation of *tropos* in this sense. What Kant calls the relation of a judgement (which name is not found before Kant, cf. Tonelli 1966, p. 151) was called its "substance" by William of Sherwood (Kretzmann, 1966, pp. 27–29).

[138]Tonelli (1966, p. 140) reprints a table found in a logic of a certain Boehm, published in 1749, but this has a very different structure from Kant's table. As Kneale and Kneale (1962, p. 356) point out, Kant's tabulation is confusing. In syllogistics each proposition has a quantity, a quality, and a modality, but that is no longer so when one includes conditional and other complex judgements: what, for instance, is a negative hypothetical judgement?

[139]A69/B94: "Wir können aber alle Handlungen des Verstandes auf Urteile zurückführen, so daß der Verstand überhaupt als ein Vermögen zu Urteilen vorgestellt werden kann." These acts of the understanding include apart from judging itself what is traditionally known as simple apprehension, and reasoning (cf. footnote 81 above). For a detailed explication of Kant's argument as set out in the surrounding text, see Wolff (1995, esp. pp. 87–110).

[140]Cf. A78/B104: "Aber nicht die Vorstellungen [∼terms], sondern die reine Synthesis der Vorstellungen auf Begriffe zu bringen, lehrt die transz. Logik."

the categories are the concepts derived from the unity that these functions of the understanding bring to a manifold of intuition.[141]

Kant famously concludes (A79/B105) that

> In this manner there arise just as many pure concepts of the understanding, which relate *a priori* to objects of intuition, as there in the previous table were logical functions in all possible judgements.[142]

Not only do the categories and the forms of judgement have the same number, as Kant states here, but their tables have a similar construction: the categories are divided into the same four headings with three items under each standing in a one-one correlation with items of the table of judgements. Precisely how the forms of judgement and the categories relate is, however, a complicated question. On the one hand there is the statement of the complete coincidence of the categories with the "logical functions of thinking," that is, the forms of judgement (B159). On the other hand there is the statement that the categories require apart from these logical functions of thinking some aspect of sensibility (the so-called "schemata"), for without that we do not yet have concepts (A245). But to get into the details of all of this would only take us off track.[143]

Following the table of categories Kant glosses 'category' as "original pure concept of synthesis" (A80/B106); a few pages earlier he had described a category as a "pure synthesis generally represented" (A78/B104). This hints at the fundamental role played by the categories in Kant's critical epistemology: from the Transcendental Deduction (especially in the B-edition) and the System of Principles it emerges that the categories are what primarily bring about connection and unity among our representations, thereby making experience (*Erfahrung*) possible. Such a conception of the categories, as "conditions for the possibility of experience" (e.g. A94/B126), indeed as the "originator" (*Urheber*) of experience (B127), is of course not to be found in Aristotle, nor in any other doctrine of categories that we shall deal with in this dissertation, but is a peculiarity of Kant's doctrine.

[141] A79/B104–105: "Dieselbe Funktion, welche den verschiedenen Vorstellungen in einem Urteile Einheit gibt, die gibt auch der bloßen Synthesis verschiedener Vorstellungen in einer Anschauung Einheit, welche, allgemein ausgedruck, der reine Verstandesbegriff heißt." For a helpful discussion of this passage, see Allison (2004, pp. 152–156).

[142] "Auf solche Weise entspringen gerade so viel reine Verstandesbegriffe, welche a priori auf Gegenstände der Anschauung überhaupt gehen, als es in der vorigen Tafel logische Funktionen in allen möglichen Urteilen gab: denn der Verstand ist durch gedachte Funktionen völlig erschöpft, und sein Vermögen dadurch gänzlich ausgemessen."

[143] One relevant distinction worth mentioning, in effect made by Kant at A245, is that between pure and schematized category, for which see, for instance, Paton (1936, vol. 1, pp. 260–261).

Above we saw that syncategorems are described both as pertaining to form in contrast to the matter of a proposition, and as signifying a connection by the mind of ideas signified by categorems. The foregoing shows that these two characteristics also fit Kant's categories: they stand in an intimate relationship with the forms of judgement and are indeed themselves described as "forms of thought" (*Gedankenformen*, B150, B305); and they serve to connect and unify our representations. Both of these characteristics, being the form of thought and what unifies it, are alluded to when Kant calls the categories "the mere form of connection as it were" of experience (*Prolegomena* § 39, p. 323). A syncategorem was, however, also described as what cannot by itself be the term of a syllogistic proposition. Hence, according to our interpretation of what falls under Aristotle's categories, syncategorems are precisely those elements of a syllogistic proposition that are not categorized. But then we see that Kant and Aristotle must have been led by quite different motives. For if we consider a propositional schema of modal syllogistics, such as 'all A are possibly B', and ask which elements of this proposition are of relevance to the doctrine of categories, then we shall get directly opposite answers according as to whether we assume Aristotle's or Kant's doctrine. According to Aristotle's doctrine it is the terms A and B that are of relevance to category theory, for these are then the items categorized. According to Kant's doctrine, however, it is all the other elements that are of relevance, namely the universal quantity, the affirmative quality, the categorical relation, and the problematic modality, for these correspond to the categories. Thus the locus of the proposition from the point of view of Kant's doctrine is the complement of its locus from the point of view of Aristotle's doctrine.

This complementarity of the two doctrines may help to explain the difficulty in answering in the case of Kant the questions we posed in sections 1 and 2 regarding Aristotle's categories. What are the items categorized by Kant's categories? They are said to be concepts of objects *überhaupt* (B128, A242, A290/346), so one could perhaps say, quite straightforwardly, that it is objects which fall under the categories. But the main role of the categories is to bring synthetic unity to a manifold of intuition, and that happens when such a manifold is brought under one or more of the categories. Hence it seems that one could equally well say that it is manifolds of intuition that fall under the categories; but a manifold of intuition in the relevant sense is not yet an object, indeed it is an object only when subsumed under one or more categories (cf. A104–105, B137). Matters do not get more tractable when considering the individual categories. I would not know what to say to the question of which items fall under the category of plurality (*Vielheit*); or under this category rather than under the category of totality (*Allheit*); or under the category of negation? Under each of

Kant's categories of relation there would seem to fall not single items, but rather pairs or even greater pluralities of items. That is clear enough for the categories of cause and effect, and of community, but even the category of substance is not one under which single items fall, since Kant's category is in fact that of *substance and accident*, corresponding to the subject and predicate of a categorical judgement (cf. B128–129). The categories of modality are said to express the relation of a concept to the capacity for knowledge (*Erkenntnisvermögen*), and not help determining the object itself (A219/B266); so these would seem to be modifications of concepts rather than concepts themselves.

With no clear answer to the question of what the items categorized are according to Kant's doctrine, it is not easy either to characterize the generality of Kant's categories. They are conditions for the possibility of thought of objects—that may be taken to entail high generality, since the categories must then be involved somehow in *all* thought of objects; but it is not a description of the kind of generality that pertains to them as concepts. We should probably want to say that this is a formal kind of generality, for the categories are called the intellectual form of experience (A310/B367) and contrasted with its matter (cf. A86/B118). In section 6.2 of chapter 2 below we shall consider a notion of generality, or rather formality, that may seem pertinent to Kant's categories. But that, as we shall see, is a kind of generality to be explained by analogy with the relation of a constant term to a variable, an explanation which hardly is adequate in the case of Kant's categories: what would be a variable corresponding to the category of negation, or to the category of existence and non-existence?

On the basis of Kant's logical doctrine of concepts—in particular the doctrine of the "extension and intension" of concepts[144]—and some remarks at the end of the Amphiboly of Concepts of Reflection (A290–291/B346–348), Tolley (2012, pp. 433–440) has suggested that we understand the generality of the categories in terms of extension and intension. Being of high generality, the categories are concepts of minor intension but vast extension. In particular, the categories are conceived to have only the concept of an object *überhaupt* and a very few other concepts in their intension. Indeed, the suggestion is that the categories are reached by a number of divisions

[144]The doctrine is found in the Port-Royal *Logique* (I.vii): "J'appelle *comprehension* de l'idée, les attributs qu'elle enferme en soi, & qu'on ne lui peut ôter sans la détruire... J'appelle *étendue* de l'idée, les sujets à qui cette idée convenient." Kant employs *Inhalt* for the former and *Umfang* or *Sphäre* for the latter (cf. *Jäsche Logik* § 8). Hamilton in his *Lectures on Logic* introduced 'intension' instead of 'comprehension' (cf. Kneale and Kneale 1962, p. 318; *OED*), a word that after Carnap (1947) has taken on a different significance in logic. For the doctrine of the extension and intension of concepts in Kant, see De Jong (1995, pp. 622–627).

from the concept of an object *überhaupt*, where a division must consist in adding marks to the intension of a concept so as to obtain a more specific concept.[145] As there are $12 = 2 \cdot 2 \cdot 3$ categories, they should be reached after three such divisions, and one of these divisions would need to be trichotomous; that would presumably be the division of each of the four headings of quantity, quality, relation, and modality into the three categories under each. Kant calls the categories of quantity and quality 'mathematical' and those of relation and modality 'dynamical' (B110), and that could perhaps be taken to correspond to the first division (cf. Tolley ibid. fn. 47).

I am rather sceptical of this suggestion. Firstly, it owes us an an account of the subsumption of items under the categories. As long as it is unclear what it means for an item to fall under a category—and whether this means the same for all the categories—it is also unclear what it means to talk of the extension of a category. One could insist that the extension of a concept, according to Kant, consists of the concepts contained under it, while the complications discussed above concern what it means for an individual to fall under a category; these are two quite different things for Kant, as he did not accept individual concepts. If, however, an individual is subsumed under the concept 'man', then it is presumably also subsumed under all the concepts in the intension of 'man', and the suggestion was that in the intension of any concept there will be one or more categories. Hence, if one or more categories belong to the intension of any concept, then we should need an account of the subsumption of individuals under the categories. Secondly, a question Duns Scotus had asked concerning attempts to derive Aristotle's categories by division (a topic to be discussed in the next section) now arises with regards to Kant's categories: when the categories have several other concepts above them in the hierarchy of extension and intension, why is it that they are of such special interest; why are precisely the concepts reached after three divisions of such importance?

Another token of the complementarity of Aristotle's and Kant's notions of category lies in their relation to the principle of mutual exclusion discussed above, namely the principle that the same item does not fall under two categories. We saw that, although Aristotle denies this principle, it is nevertheless natural to assume it for his

[145]Both De Jong (1995, p. 624) and Tolley (2012, p. 434) connect the doctrine of extension and intension of concepts to the Tree of Porphyry. Indeed, according to the *Jäsche Logik* §§8–10, a concept A in the intension of another concept B, is called a genus of B. Since Porphyrian differentiae as well as genera belong to the intension of a concept, this conception of genus breaks with the Porphyrian doctrine, for it requires us to identify genera and differentiae. It is not clear to me that concepts ordered according to their extension and intension will in fact form a tree: directly above any concept apart from a highest there will be more than one concept; for instance directly above man there will be animal and rational; hence there is no unique way upwards in the ordering; but that is what characterizes a tree.

categories. In the case of Kant's categories, however, it is more natural to deny than to accept the principle. Already the table of categories suggests that an object is determined with respect to one category under each of the four headings of quantity, quality, relation, and modality. That is also the picture emerging from the System of Principles, where each trio of principles deals with a determination of the object not settled by any of the other trios. Paton (1936, vol. 1, pp. 226, 303), in fact, insists that the categories are universal concepts, that is, concepts under which all objects fall, hence conversely, that each object falls under all the categories.

5.2. Generating the categories. Kant famously objected that Aristotle in his conception of the categories followed no principle, but "amassed them as he stumbled upon them."[146] Without such a principle the list of categories remains a mere "rhapsody" and not a system on which one can build philosophical theory.[147] In Kant this demand for a principle of generation is coupled with a demand for a proof of completeness of the list of categories, a proof that the list contains all and only the categories. As we saw in section 2.1.1, Aristotle did offer a proof of completeness for his list or predicables, but we do not find anything similar in his writings for his list of categories.

5.2.1. *Completeness.* Kant, by contrast, held that his own derivation of the categories from the forms of judgement showed the former to be complete. The parallelism between the forms of judgement and the categories "provides a rule according to which the place of each concept of the understanding and the completeness of all of them together can be determined *a priori*" (A67/B92). Kant thus bases the assertion of the completeness of his table of categories on the assumption that his table of judgement is complete. The question then arises whether this assumption is correct. Since it is difficult to find in Kant's text any principle governing the construction of the table of judgement, it is natural to think that it was simply assembled by Kant from what he had found in logic textbooks of the time, with no guarantee that the outcome should be complete. This thought has been challenged in the classic work of Reich (1948) and in a more recent study of Wolff (1995). Reich argued that a principle for the construction of the table of judgement can be found in the "synthetic unity of apperception" and the definition of judgement following in its wake in the Transcendental Deduction of the B edition (B141). For Wolff the key to completeness is Kant's notion of function, defined in the section of the *Critique* where the table of judgement

[146]:A81/B107): "Es war ein eines scharfsinnigen Mannes würdiger Anschlag des Aristoteles, diese Grundbegriffe aufzusuchen. Da er aber kein Principium hatte, so raffte er sie auf, wie sie ihm aufstießen, und trieb deren zuerst zehn auf, die er Kategorien (Prädikamente) nannte."

[147]This point is emphasized in the *Prolegomena* § 39 (pp. 322–326).

is given, namely as "the unity of the action of ordering different representations under a common one" (A68/B93). Kant does, after all, say that the four headings in the table of judgement correspond to four "functions of the understanding" (A70/B95). A more detailed account of the arguments of Reich and Wolff lies outside the scope of this dissertation.

It is perhaps not so well known that Kant's criticism of Aristotle had ancient forerunners. Porphyry remarks in his commentary on the *Categories* that not everyone had accepted Aristotle's list as the list of categories or highest genera (*in Cat* 86,31):

> There are three sorts of objections: some object that his list contains too many items, some that it contains too few, and others that he has included some genera instead of others.

No attempt is found in Porphyry's text to refute these objections,[148] but Simplicius in his more extensive discussion of the problem of completeness suggests that one may derive the categories by a division (*diairesis*) in the sense of Plato (cf. section 2.2.1). Simplicius starts with the notion of beings (*ta onta*) and obtains the ten categories by successively adding differentiae (*in Cat* 67,26ff.). A similar idea is found in Olympiodorus (*in Cat* 54,4ff.) and Elias (*in Cat* 159,9ff.), although their division yields only the four categories of substance, quantity, quality, and relation, while the other six categories are obtained from these four by composition.[149]

Simplicius introduces his division in a dubitative tone, and it can indeed be questioned how Aristotelian it is, for it seems to render being, namely the top node of the division, a genus. Aristotle had, however, argued that being is not a genus (*Met* B.3 998^{b}21): the divisive differentiae of a genus do not fall under it as species (cf. *Top* 122^{b}20–23); hence if being were a genus its divisive differentiae, not falling under the genus of being, would not have being, which Aristotle assumes cannot be the case.[150] In the High Middle Ages the question of the completeness of Aristotle's list of categories became known as the question of *sufficientia praedicamentorum*, and was commented on by a number of authors.[151] Among these was Thomas Aquinas, who

148 The editor Busse suggests that there may be a lacuna at the place in the text (namely after the quoted passage) where such a refutation could have been found.

149 Brentano (1862, p. 179) cites a passage of Ammonius where one also finds this idea of obtaining the six latter categories by composition, but I have not been able to locate this passage in the *CAG* edition of Ammonius's commentary on the *Categories*.

150 At *Top* IV.1 *passim*; IV.6 127^{a}26–38 Aristotle considers both being (*to on*), unity (*to hen*), and object of belief (*to doxaston*, 121^{a}22) as candidate genera and species, but he does so in the context of examples, and these need not reflect Aristotelian doctrine (cf. *SE* 178^{a}19 where 'to see' is called a passivity, while according to *DA* II.5 seeing is precisely not a passivity (esp. 418^{a}2)).

151 For an overview, see e.g. Bos and van der Helm, A. C. (1998) and Pini (2003), as well as Pini (2002, pp. 185–189).

before responding to the question in his commentary on *Met* Δ.7 (*in Met* Δ lect. 9 nn. 889–894) repeats Aristotle's argument that being is not a genus. This may well have been an implicit criticism of Simplicius's suggested derivation;[152] his own division, in any event, is not one of being, but rather one of kinds of predication.[153]

What is perhaps the most sophisticated derivation of Aristotle's categories by division, hence the most sophisticated derivation in the tradition going back at least to Simplicius, is that offered by Brentano (1862, esp. 144–178). Brentano gives a division terminating in Aristotle's categories which not only can be reconciled with the doctrine that being is not a genus, but which also purports to be through and through Aristotelian, each branching being justified by reference to Aristotelian texts and doctrines (mainly taken from the *Metaphysics*).[154] An idea of considerable importance to Brentano's division is that of analogical unity. The unity of a notion may be of different sorts. In particular, a notion may have a weaker sort of unity than that possessed by a genus, namely what Brentano, following Aristotle *Met* Δ.6 $1016^{b}31$ff., calls analogical unity. According to Brentano, this is the sort of unity that the notion of healthiness has in its application to men as well as to their appearance, their diet, and their habits. A healthy appearance is indicative of a man's good health, while a healthy diet and healthy habits are productive and preservative of it. In each case 'healthy' means something else, so the word is homonymous, but all of its various senses are related to the idea of a man's good health, and this relation furnishes the notion of healthiness with unity, namely analogical unity, which it preserves through all of its applications. Aristotle famously argues that the same holds for 'being' (e.g. *Met* Γ.2): this term is homonymous across the categories, but all of its senses are in some way related to the notion of substance. Being is not a genus, but it has

[152] Simplicius's commentary on the *Categories* was translated into Latin by William of Moerbeke in 1266; it is reasonable to assume that Thomas read this work, since he apparently refers to it in the *Summa Theologica* (cf. McMahon, 1981, p. 86); he might have done so by 1268, when he begun writing the commentary on the *Metaphysics* (cf. Bos and van der Helm, A. C., 1998, p. 187). Radulphus Brito, writing around 1300, claims that his division agrees with that of Simplicius ("ista sufficientia concordat cum sufficienta Simplicii," McMahon, 1981, p. 91); this is not quite right, for the two divisions disagree already in their first branching: where Simplicius divides *ta onta* into "existences" (*hyparxeis*) and activities (*energeiai*), Radulphus divides *ens* into *ens per se substistens* and *ens in alio*; the latter corresponds rather to Simplicius's division of existences into those that have their being *per se* (*kath heautos echousi to einai*) and those that come to be in others (*en allois hyphestēkasin*).

[153] Cf. Wippel (1987) for more details.

[154] See Brentano (1862, p. 177) for an overview of passages justifying each branching. Brentano (ibid. pp. 147–148) even suggests that Aristotle himself would have known of the possibility of this division. If one accepts the reconstruction of the development of Aristotle's conception of the homonymy of 'being' offered by Owen (1960), or indeed simply that the *Categories* or the *Topics* were written before most of the *Metaphysics*, then the most one can say is that Aristotle saw the possibility of this division only after he had conceived of the categories.

analogical unity, and according to Brentano, so do all the various notions that feature in his division above the categories, such as the notions of accidence (*symbebēkota*) and passive state (*pathē*).

Brentano's division is therefore compatible with the doctrine that being is not a genus. The division is not one from a notion enjoying generic unity, but one from a notion enjoying analogical unity. We reach genera in the division only when we reach the categories. Brentano therefore has a response to the objection of Duns Scotus, already mentioned, that no derivation of the categories by division is possible, since any such division shows that the categories are not the most general terms, the elements higher up in the tree being more general:[155] according to Brentano these higher nodes are not themselves genera; it is only with the categories that the division yields genera. Brentano's division is, moreover, not affected by an objection raised by Bonitz (1853, p. 645): according to *APo* I.7 a demonstration presupposes an underlying genus, and in the case of a derivation of the categories that genus will have to be being; but being is not a genus; hence no derivation of the categories, be it by division or otherwise, can be Aristotelian. However, it is sufficient for Aristotle that the underlying domain has analogical unity: the case of ontology shows this, for its domain is being *qua* being, and that is a notion having only analogical unity (cf. Brentano, 1862, pp. 145–147).

A few remarks may be made here on the recent work of Studtmann (2008b), who purports to show by means of division that "Aristotle's categorial scheme is derivable from his hylomorphic ontology" (p. 15, repeated at p. 141).[156] As far as derivations

[155]This is one of Duns Scotus's arguments in the following passage from the *Questions on Metaphysics* V q. 5–6, quoted by Pini (2002, p. 188): "Notae: variae sunt viae divisivae ostendendi sufficientiam praedicamentorum, quae videntur dupliciter peccare. Primo, quia ostendunt oppositum propositi, scilicet quod divisio entis in haec decem non sit prima. Si enim prius fiat in ens per se et in ens non per se, et ultra unum membrum subdividatur vel ambo: aut quaelibet divisio erit tantum nominis aequivoci, in aequivocata, quod nihil est probare – quia nomina sunt ad placitum; aut aliquo istorum decem erit conceptus communior immediatior enti, et ita ens non immediate dividitur in decem. Exemplum patet: ponendo quod per divisiones multas subordinatas in genere substantiae tandem deveniritur ad decem species specialissimas, illae non primo dividerent substantiam. Secundo, quia omnes illae viae divisivae non probant. Oportet enim probare quod divisum sic dividitur, et praecise sic, et hoc ad propositum, scilicet quod dividentia constituant generalissima."

[156]Shields (2007, p. 168) writes that "an older tradition sought… to show how the theory of categories could in fact be derived from hylomorphism," and (p. 170) that "no genuine attempt has been made since the Middle Ages" at doing so, but he provides no references. The section in question, *Generating the categories* (pp. 159–172), seems, however, to rely heavily on the useful presentation of this topic in Studtmann (2008a), to which Shields does refer. Studtmann there cites a passage from Thomas' derivation (*in Met* Δ lect. 9 n. 892) where it is said that if a predicate is taken as being in a subject essentially and absolutely and as flowing from its matter (*ut consequens materiam*), then it is a predicate of quantity; and likewise for form and quality. This is part of Thomas' division, but only a part of it, namely that part yielding the categories of quantity and quality. There is no indication in Thomas' text that all of the categories can be derived

are concerned, however, what this work actually accomplishes is at most to show how the two categories of quality and quantity each may be divided into various species and subspecies by means of a certain understanding of the notions of form and matter developed by Studtmann in the first part of his book. That is, instead of deriving the categories of quantity and quality by division from some other notions, Studtmann derives by means of a certain understanding of hylomorphism various species falling under the categories of quantity and quality. But such a derivation of the various species of a category is not a derivation of the category in the relevant sense. We want to be shown a path taking us from certain notions—which in this case would be form and matter—to the categories, not a path taking us from the categories to various terms falling under this category. And even if Studtmann had given a derivation of the categories of quantity and quality from the notions of form and matter, one could still not talk of a derivation of the categories, for the whole point of such a derivation is to show that Aristotle's ten categories are *all* and only the categories. A derivation only of quantity and quality suggests that there are no other categories than these two, and so jeopardizes the whole project of showing the completeness of Aristotle's list.

5.2.2. *Derivation without completeness.* A derivation of the list of categories by means of division will, to the extent that it succeeds, also show the completeness of the list: being the result of a division from a universal concept or quasi-concept, they exhaust conceptual space. Kant's primary objection to Aristotle's list was, however, not that it came with no proof of completeness, but rather that it was not the outcome of an underlying principle. That proposing such a principle need not mean providing a proof of completeness is clear from two ways of accounting for how Aristotle may first have conceived his categories. One account is associated with Trendelenburg, the other with several interpreters from Ockham to Ackrill.

Trendelenburg (1846, pp. 23–34) was perhaps the first to attempt to defend Aristotle against Kant's criticism.[157] Although Trendelenburg does nothing to show the completeness of Aristotle's list, he argues that it is an outcome of an analysis of grammar.[158] The categories are thus taken to correspond to various grammatical distinctions. Apart from pointing to the relevant grammatical distinctions and the

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from the notions of form and matter, and as far as I know, there is no older tradition of attempting such a derivation. (There is a tradition, manifest e.g. in Porphyry *Isag* 11.12–11.17, and perhaps going back to Aristotle (*Met* $1045^{a}14$–$^{b}7$; cf. $1024^{b}8$), of likening genus to matter and differentia to form, but that is something else.)

[157] Brentano (1862, p. 144) associates the problem of the "deduction" of the categories with Simplicius; as far as I can see, he never mentions Kant.

[158] Trendelenburg (1846, p. 33): "... dass die logischen Kategorien zunächst einen grammatischen Ursprung haben und dass sich der grammatische Leitfaden durch ihre Anwendung durchzieht."

correspondence reported in section 4.2 above, Trendelenburg supports his account by adducing a number of passages from the *Sophistical Refutations* (*SE* $166^{b}10$ff.; ch. 22) that touch on the relation between the surface grammar of a word and its category. The point of several of these passages, however, is that the category of a term is not always indicated by surface grammar; hence, they cannot support the claim that the categories were conceived by reflection on surface grammar. Trendelenburg's more considered claim is therefore that only in the early stages of uncovering the categories did grammatical consideration play an important role, while "the content of the concept" led the way thereafter.[159] Trendelenburg offers two further lines of support for his interpretation.[160] Firstly (ibid. pp. 27–30), he argues that there is a close correspondence between the grammatical notion of *ptōsis* and that of a category.[161] Secondly (ibid. pp. 30–33), he refers to the role that grammatical case plays in Aristotle's definition of the category of relation: a relative term always requires a genitive or dative for its completion.[162] It is difficult to find in these considerations any support for the grammatical origin of Aristotle's doctrine of categories; but we may indeed take them to support the more moderate contention that reflection on grammar played an important role at the initial stage of its conception.

According to Ockham (*Summa Logicae* I.41) and Ackrill (1963, pp. 78–79) and others[163] the categories correspond to questions that may be asked about a given primary substance, typically a man. To the question of *where* a man is, only a term from the category of where is appropriate; to the question of *when* a man is, only a term from the category of when is appropriate; to the question of *what* a man is, only a term from the category of substance—sometimes called 'what it is' (*ti esti*) by Aristotle—is appropriate.[164] This thesis thus gains support from the fact that

[159] Cf. ibid. p. 25: "dass sich die Kategorien zunächst nach der Gestalt des Ausdrucks zurecht gefunden, sodann aber über diese hinaus den Inhalt des Begriffs verfolgen."

[160] For a critical discussion, see Bonitz (1853, pp. 626–640).

[161] Aristotle seems to have thought of all words derived from another (so-called "paronyms") as *ptōseis* of the parent word (*Cat* $1^{a}12$–15). Thus a noun in an oblique case (*Int* $16^{b}1$, *Poet* $1456^{b}19$–21), verbs not in the present tense (*Int* $16^{b}7$), as well as adverbs derived from adjectives, as 'justly' from 'just' (*Top* $106^{b}29$–$107^{a}2$, $114^{a}33$–36, $136^{b}15$–32), are all *ptōseis*.

[162] Cf. *Cat* $6^{a}37$: "We call relatives all such things as are said to be just what they are, of or than other things. . . " Here the "of or than other things" translates a genitive (*heterōn*), but Aristotle goes on to give examples where a dative complement is used (e.g. at $6^{b}9$).

[163] For instance Gomperz (1909, p. 29), Gillespie (1925), and Ryle (1938).

[164] Thus in Ockham *Summa Logicae* I.41 (translated by Loux, 1974, p. 130):

> the distinction among the categories is taken from the distinction among interrogatives appropriate to substance or an individual substance. The different questions which can be asked about a substance can be answered by different simple terms, and a simple term falls under a category accordingly as it can be used to answer this or that question about substance.

the category names 'where' and 'when' (or the Greek words they translate) may serve as interrogatives, and that one of Aristotle's names for substance, *ti esti*, in the appropriate context means, What is it? It gains further support from the fact that the Greek, unlike the English, names for the categories of quantity and quality, and likewise the Greek name for the category of relatives, may serve as interrogatives. The names of the four last categories are all verbs in the infinitive; but they correspond naturally to questions one may ask of a substance, or at least of a man: what is he doing; what is he undergoing; what is his position; what does he wear (what is his habit)?

The suggestion is thus that Aristotle had, perhaps while developing the method for dialectic presented in the *Topics*, found occasion to distinguish these various questions, and thence ordered the appropriate answers into "classes of predicates" (*genē tōn katēgoriōn*).[165] A proof of completeness for the list of categories cannot be derived from this account, since there are interrogatives in Greek, such as 'how' (*pōs*), that do not correspond to any categories;[166] and, as we just saw, there are categories whose names do not correspond to interrogatives. But the account may perhaps support the claim that Aristotle followed a principle in constructing the list, hence that he did not merely "amass the categories as he stumbled upon them."

The category of relatives, however, shows that simply considering the questions that may be asked of a substance cannot quite have been Aristotle's procedure. For the question *pros ti;*, which we may translate as 'relative to what?', asked of Socrates cannot be answered with a relative term such as 'father' or 'husband'.[167] Terms in the category of relatives are not in general felicitous answers to the question, Relative to what? On the contrary, the predication of a relative term of a subject is a predication that prompts the question, Relative to what? To ask, Relative to what?, of Socrates makes little sense, but if we say that he is a husband, we may ask, Relative to what is he that, who is his wife? Socrates is a father; relative to what is he that, who is his son? The possibility of asking this question is characteristic of relatives. To the predications 'Socrates is a man' and 'Socrates is white', for instance, it does not make sense to ask, Relative to what is he that? But it does make sense if we predicate a relative of Socrates; indeed, it is then often called for. Hence, while the category of relatives may be taken to correspond to the question derived from its name, *pros ti*,

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[165] For this reading, see Gillespie (1925, esp. pp. 81 ff.). For a detailed argument that the *Categories* is to be read as a manual of dialectic, see Menn (1995).

[166] Recall from section 4.2 above that two of the Stoic categories were called *pōs echon* and *pros ti pōs echon*, employing precisely this interrogative/adverb.

[167] The same holds for the question *tinos;*, or *cuius?*, suggested by Ockham (ibid.).

the nature of this correspondence is not as that between, for instance, the categories of where and when and the questions derived from their names. In the latter case the categories may be viewed as the classes of possible answers to the question associated with the category, while the category of relatives has to be viewed as the class of predicates to the predication of which it makes sense to ask the question, Relative to what? The conjecture of Kahn (1978, p. 243) is therefore plausible, that reflection on interrogatives was only a part of what led Aristotle to distinguish ten categories, the sort of linguistic considerations emphasized by Trendelenburg perhaps being another motivation, and logical or ontological "intuition" a third.

Appendix 1: Aristotle's categories

English	Greek	Latin
substance	*ousia*	*substantia*
quantity	*poson*	*quantitas*
quality	*poion*	*qualitas*
relative	*pros ti*	*relatio*
when	*pou*	*quando*
where	*pote*	*ubi*
position	*keisthai*	*situs*
having	*echein*	*habitus*
doing	*poiein*	*actio*
affection	*paschein*	*passio*

Appendix 2: Kant's table of categories

Quantität
Einheit
Vielheit
Allheit

Qualität
Realität
Negation
Limitation

Relation
Inhärenz und Subsistenz
Kausalität und Dependenz
Gemeinschaft

Modalität
Möglichkeit – Unmöglichkeit
Dasein – Nichtsein
Notwendigkeit – Zufälligkeit

CHAPTER 2

Categories and Function–Argument Syntax

Synopsis

In this chapter we study the theory of types in its various forms: the simple and the ramified type hierarchy, types of individuals defined as ranges of significance of propositional functions, and types in the sense of constructive type theory. In the first section we introduce the simple type hierarchy, both its logical variant going back to Frege, and the grammatical variant going back to Ajdukiewicz. Russell's ramified types are considered in section 2.1. Russell defined a type as a range of significance, but we shall see that ramified types are not ranges of significance. In both the simple and the ramified hierarchy there is just one domain of individuals. If a type is a range of significance, however, that domain must be split up into several domains. That idea is introduced in section 2.2. If a type is defined as a range of significance the notion of a type depends on the notion of significance. We scrutinize the relation between types and significance in the third section. The topic of the fourth section is what I shall call type predications, judgements in which an entity is assigned to a type. Several philosophers, including Frege, Wittgenstein, and Carnap have found such predications problematic. The notion of a sortal is also a notion of a type of individuals; it is discussed in section 5. In the last section Husserl's distinction between formal and material categories is presented and compared to similar distinctions in Frege and Carnap.

1. The simple type hierarchy

The simple type hierarchy is intrinsically tied to function–argument syntax. Both are introduced in section 1.1. A language directed variety of the simple type hierarchy is discussed in section 1.3. In section 1.2 we consider the relation between simple types and Aristotle's so-called ontological square.

1.1. Frege: function and argument, and function and object. Frege's revolution in logic was in large part a revolution in logical syntax. In place of the basic form 'S is P' Frege introduced the basic form '$F(a)$', function F applied to argument a. To be more precise, this is the form of the content of a Fregean *Begriffsschriftsatz* (*Gg* § 5), i.e. a of theorem of Frege's ideography.[1] The theorem itself is a judgement, indicated by $\vdash F(a)$, that a content of the form $F(a)$ is true. Frege's logical syntax thus draws a distinction between judgement and the content judged to be true, where it should be emphasized that this is a logical, and not a "merely psychological," distinction.[2] No such distinction was drawn in syllogistic syntax; there the copula was taken to serve in effect two functions, namely as a connector of the two terms and as a mark of affirmation. Indeed, we saw that according to a reasonable interpretation of Aristotle, an affirmation is for him just a combination of terms; and this combination is signified by the copula. Likewise, in the *Port-Royal* grammar of Arnauld and Lancelot (1676) the verb is taken primarily to signify affirmation (ibid. pp. 95, 101), but the "the verb itself" is also said "to serve no other function than to mark the connection which we make in our mind of the two terms of a proposition" (ibid. p. 96), there being therefore in fact only one true verb, namely 'is' (other forms of this verb, as well as all other verbs and their various forms being in principle dispensable but invented for the convenience they afford).[3] To make Frege's distinction in traditional logical doctrine one would therefore have to separate these two functions of the copula. A judgeable content is obtained by the combination of two terms S and P. The mere combination is, however, not yet a judgement—a judgement is

[1] For reasons to prefer 'ideography' as the translation of the German 'Begriffsschrift', cf. Barnes (2002, esp. pp. 75–76).

[2] On the importance of distinguishing judgement and the content judged to be true for systematical purposes, see e.g. Martin-Löf (1996, 1987) and Sundholm (2004); the most extensive study of the distinction in Frege is found in Stepanians (1998); a history of forms of judgement assumed in modern logic is given by Sundholm (2009). As emphasized in the works of Martin-Löf and Sundholm there is in addition to the act of judgement also the object of such an act; this object judged is different from the content judged to be true, and it belongs to the domain of logic and not to that of psychology; for a different account of the non-psychological nature of the judgement stroke, see Dummett (1973, pp. 311ff.).

[3] For yet another example, see the discussion of the copula in Joseph (1916, pp. 159–170), e.g. p. 163: "to think the copula *is* the synthesis of judgement," and p. 166: "to express a combination of which I think is real, I use the verb *to be*."

obtained by asserting that the judgeable content is true.[4] It is the Fregean judgeable content $F(a)$ that will be our point of interest here, and which, in spite of the slight inaccuracy this involves, we shall think of as a parallel to the syllogistic form 'S is P'.

The analysis of judgeable content into function and argument is offered in the *Begriffsschrift* (1879) as an alternative to the traditional analysis of propositions into subject and predicate, which we discussed in section 1.2 of the previous chapter. The notions of function and argument adopted in that work therefore share the relational character of the latter notions: as 'man' is the subject of 'man is mortal' but the predicate of 'Socrates is a man', so the same expression can now be the function and now the argument of a proposition. The characterization Frege gives of his pair of notions is, however, considerably different from the traditional characterization of subject and predicate. The subject of a sentence was the name of what the proposition is or says something about, while the predicate was what is said of the subject (or of what is signified by the subject). An argument in Frege's sense is, by contrast, an element in a sentence that is regarded as replaceable, while a function is the part not regarded as replaceable. Thus, given a sentence such as 'Caesar killed Cato' we may regard 'Caesar' as replaceable by 'Clodius', yielding 'Clodius killed Cato'; or as replaceable by 'Cato', yielding 'Cato killed Cato'; or as replaceable by any other name. On this analysis 'Caesar' is the argument to the function 'killed Cato', yielding the value 'Caesar killed Cato'. This notion of a value of a function is not quite in accordance with how the notion is usually understood in mathematics: the value of the factorial function ! applied to the argument 4 is not the expression '4!', but the number 24. Anscombe and Geach (1961, p. 143) therefore introduced the name 'linguistic function' for functions whose values are expressions in this way; linguistic functions will be important in section 1.3 below. Returning to our example sentence, 'Caesar killed Cato', we are in fact free also to regard 'killed Cato' as replaceable, for instance by 'conquered Gaul', yielding 'Caesar conquered Gaul', thereby regarding 'Caesar' as the function. In the sentence 'Caesar killed Cato' one and the same element 'Caesar' may therefore be regarded on one analysis as argument and on another as function; as Frege remarks, what is the function and what is the argument in a sentence is "purely a matter of point of view" (*allein Sache der Auffassung*, *Bs* § 9), that is, what is function and what is argument depends completely on the analysis we choose.

[4]That the two roles of the copula thus should be separated from each other was clearly perceived by Mill (1843, Bk. I ch. v § 1), though his insight did not usher in a revolution of logical syntax, perhaps because he regarded assertion as not belonging to logic (loc. cit.).

This distinguishes Frege's notion of function and argument from the notions of subject and predicate. For while there is only one analysis of a sentence into subject and predicate, there are in general several analyses into function and argument (or arguments).[5] In the sentence 'Caesar killed Cato', 'Caesar' is the subject while 'killed Cato' is the predicate; but as we have just seen, not only may we regard 'Caesar' as the argument to the function 'killed Cato', we may also regard 'Cato' as the argument to the function 'Caesar killed', or 'Caesar' and 'Cato' as the arguments to the binary function 'killed'. In fact, we may also regard 'Caesar' or 'Cato' or indeed 'Caesar... Cato' as functions of the arguments 'killed Cato', 'Caesar killed', and 'killed' respectively. Thus, at least for sentences of this simple form there is complete freedom: any part of our example sentence may be regarded as the function, hence also any part as argument. In sentences of more complex structure, this is probably not so: at the time of the *Begriffsschrift* Frege would probably not have been open to regard connectives as functions, a point I shall return to below. Natural languages offer means of rewriting sentences that preserve sense but change the subject: by employing, for instance, the passive instead of the active form of the verb our example sentence would read 'Cato was killed by Caesar', hence with 'Cato' as subject. But 'Cato' is now the subject of a new sentence. Frege's method, by contrast, allows 'Cato' to be the function according to one analysis and the argument according to another analysis of one and the same sentence. This flexibility of the notions of function and argument in contrast to the notions of subject and predicate is emphasized by Frege on several occasions and seems to have been a main reason for his replacing the latter pair with the former.[6]

In the preface (p. x) to the *Grundgesetze* (1893) Frege remarks that the distinction between function and object is drawn more sharply in the present work than what it was in the *Begriffsschrift*. Indeed, he now employs an absolute notion of function, one not depending on a given analysis of a sentence.[7] A function in the absolute sense

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[5]Geach (1962, §24) insists that one and the same sentence may admit of several subject-predicate analyses. Thus one analysis of 'Peter loves her' yields 'Peter' as subject and and another yields 'her' (cf. footnote 19 of the previous chapter). Here Geach is presumably following Frege rather than the tradition in linguistics (cf. the criticism of a suggestion like Geach's in Hockett, 1958, pp. 6–7).

[6]In addition to *Bs* §§ 3, 9 there is *UGG*1 p. 372, fn. 1 which makes precisely this point. At *BG* pp. 198–200 Frege may seem to say that we are free in regarding this or that as subject and predicate; but what he actually says is that the same thought can be expressed by different sentences (differing for instance in whether the verb is in the active or the passive voice), in which, therefore, what is subject and what is predicate varies.

[7]Given the identification of concepts with functions of a certain kind this absolute notion of function is presumably present already in the *Grundlagen der Arithmetik* (1884); this work famously urges its reader "never to loose sight of the distinction between concept and object" (p. xxii). Hints of the notion are found even in the following passage from *Bs* § 9:

is in general characterized by its being "unsaturated," "incomplete," or "in need of completion,"[8] and as such functions are contrasted with objects, which are "saturated," "wholes," or "not in need of completion."[9] Thus, expressions such as 'the king of' or '$+3 = 5$' are felt to be incomplete, unsaturated, to have a gap in them, while the 'the king of Norway' and '$2+3 = 5$' are felt to be complete, saturated; and this distinction at the level of expressions is taken by Frege to correspond to a distinction at the level of what is signified by these expressions, namely as one between unsaturated and saturated entities.[10] There are problems here of how one should understand the notion of signification, both for incomplete expressions and for sentences (such as '$2 + 3 = 5$'), but the primary task for one who wishes to understand the distinction between function and object is to understand what Frege alludes to by means of his metaphors. Frege made himself vulnerable to criticism by this use of metaphor; but he was clear that no foolproof mode of description was available for primitive notions such as those of function and object.[11] He therefore famously asked that his reader be willing to meet him halfway (that he have an *entgegenkommendes Verständnis*, *UGG*1 p. 372); in other words, he asked his readers to employ the metaphors as steppingstones to their own understanding of these primitive notions. It is perhaps only as it should be that not everyone has found the metaphors fit for this task.[12] That provides one motivation for our looking for a different characterization of functions.

Church (1956, p. 15) remarks that "it lies in the nature of any given function to be applicable to certain things and, when applied to one of them as argument, to yield a certain value." The essence of a function, Church says, is that it can be applied to certain things so as to yield a value. Indeed, whatever else a function might be, it is certainly something that can be applied to certain things, and that when applied to one of them as argument yields a value. It is perhaps this characteristic of being

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> Für uns haben die verschiedenen Weisen, wie derselbe begriffliche Inhalt als Function dieses oder jenes Arguments aufgefasst werden kann, keine Wichtigkeit, solange Function und Argument völlig bestimmt sind. Wenn aber das Argument *unbestimmt* wird [...] so gewinnt die Unterscheidung von Function und Argument eine *inhaltliche* Bedeutung.

[8]The descriptions *ungesättigt* and *ergänzungsbedurftig* are found e.g. at *Gg* § 1; *FB* p. 6 adds *unvollständig*. The description *ergänzungsbedurftig* is in fact found already in Frege's discussion of relations at *GLA* § 70.

[9]The description *gesättigt* is found e.g. at *Gg* § 2; *UGG*1 p. 371 adds "ein Ganzes, das keiner Ergänzung mehr bedarf," and *BG* p. 205 *abgeschlossen*.

[10]See *Gg* §§ 1–2, 26, and more explicitly *UGG*1 p. 371, *WF* p. 665 for a statement of this correspondence.

[11]See *BG* p. 193, *UGG*1 pp. 371–372, *WF* p. 665. At *UGG*2 p. 301 Frege introduces the name 'elucidation' (*Erläuterung*) for the mode of explanation appropriate for primitive notions; for further discussion of this notion of elucidation, see Klev (2011, § 6.2).

[12]Black (1954) is a classic example.

applicable to certain things as arguments and thereby yielding values that Frege aimed at with his metaphors of unsaturation and incompleteness. Frege in fact explains application in terms of completion: "the function is completed by the argument; that to which it is completed I call the value of the function for that argument."[13] But if application to an argument is completion, then applicability to an argument is "completability," which is just bad English for 'incompleteness': applicability to an argument is a form of incompleteness. The completeness characteristic of a Fregean object, on the other hand, consists in its not being thus applicable. Thus we have characterized Frege's notion of function and its difference from the notion of object by employing the notion of application, a notion with which anyone who has studied Calculus will be familiar.[14] This characterization should be less open to criticism of the kind leveled against Frege's metaphors of unsaturation and incompleteness.

What is a function according to the *Grundgesetze* is thus not a matter of our point of view, but lies in the nature of things. This is therefore a novel notion of function compared to that adopted in the *Begriffsschrift*.[15] One consequence of the novel characterization is that there can be functions of functions. In the *Begriffsschrift* there is no such thing: there is function and argument, and the question of whether the argument really is a function is ill posed, for a function is not what anything can be irrespective of an analysis of a judgeable content. According to the characterization in the *Grundgesetze*, however, a function is a function independently of any analysis of a given content, whence there may well be functions of functions.[16] Frege notes that functions of functions are common in mathematics, for a definite integral is just that (*FB* p. 27): in $\int_0^1 x^2\,dx$ the function $\int_0^1 \ldots\,dx$ is applied to the function x^2. Of more significance for his ideography, however, is Frege's characterization of the universal (and thereby also the existential) quantifier as the function of a function: in 'everything is equal to itself', the quantifier 'everything' is taken to be a function applied to the function 'is equal to itself'. If there is no such thing as the function of a function in the *Begriffsschrift* we are therefore led to ask how Frege there regarded the quantifier. Frege remarks already in *Bs* § 1 that variables by themselves serve to

[13]*Gg* § 1: "Durch das Argument wird die Function ergänzt; das, wozu sie ergänzt wird, nenne ich Werth der Function für das Argument."

[14]For other ways of formulating the distinction, see Strawson (1959, § 6.1.2), Geach (1962, § 27).

[15]When Frege remarks in the already cited passage at *Gg* p. x that "the essence of the function as distinct from the object is more sharply characterized than in the *Begriffsschrift*," he refers, to my mind, to the development from a relative to an absolute notion of function (and not a development from a linguistic to an objectual notion of function, as is sometimes suggested in the literature, perhaps first by Anscombe and Geach 1961, p. 151).

[16]Likewise, in the *Grundlagen* there are concepts of concepts (e.g. *GLA* § 53); but it is clear from *Gg* p. x that Frege did not regard the notion of a second-level function as present in the *Begriffsschrift*.

express generality:[17] the associative law, for instance, can be expressed by $k + (l + m) = (k + l) + m$. The mere use of letters to express generality, although it suffices for expressing simple laws of arithmetic, and although it suffices for expressing the generality of a negative content, as in $n + 1 \neq n$, does, however, not suffice for expressing the negation of a general content. A sign is needed that indicates the scope of the letters within the formula, a sign that shows to which part of the formula the letters are to confer generality. That is the role of the cavity according to Frege (*Bs* § 11): "it restricts the domain to which the generality indicated by the letter pertains." Thus, in the *Begriffsschrift* the quantifier is merely a scope indicator.[18] The quantifier does of course also serve as a scope indicator when it is regarded, as in the *Grundgesetze*, as a function—its scope is then simply its argument—but in the *Begriffsschrift* a scope indicator is all that the quantifier is.

Once one has admitted functions of objects as arguments to other functions, there is no reason not to continue, namely to consider, for instance, functions of functions of functions of objects, or binary functions whose first argument is a function of objects and whose second argument is an object. In the *Grundgesetze* Frege does precisely that. In line with his treatment of quantification over objects Frege treats quantification over functions of objects as a function of functions of functions of objects; and he notes that differentiation is a binary function whose first argument is a function of objects and whose second argument is an object. He remarks that such examples show "the great variety of functions," yet he does not carry these considerations of various kinds of function into infinity: in the *Grundgesetze* Frege never considers functions of arity higher than 2 or of level higher than 3 (a function of functions of functions of objects). The reason may have been that, lacking a method for giving a general description of the resulting infinite hierarchy, Frege restricted himself to describing the types of function that he would make use of in his development of arithmetic. It seems clear from a remark Frege makes at *Gg* § 25 concerning the possibility of limiting oneself to functions of level 3 that he would have no qualms with building the hierarchy higher had that been required.[19] And there is at least one place in Frege's

[17] Cf. the remarks on the function of Latin letters at *Bs* §§ 1, 11, *Gg* § 17, *UGG*2 pp. 307–308, 377ff. Frege's Latin letters correspond to what Peano (1901, p. 2), and following him Russell, called real variables; their apparent variables correspond to Frege's Fraktur letters.

[18] As is noted by Heck and May (2013, p. 833).

[19] *Gg* § 25: "Man könnte meinen, dass dies [sc. functions of level 3] noch längst nicht genügte; aber wir werden sehen, dass wir mit dieser auskommen, und dass auch sie nur in einzigen Sätze vorkommt. Es mag hier zunächst nur kurz bemerkt werden, dass diese Sparsamkeit dadurch möglich wird, dass die Functionen zweiter Stufe in gewisser Weise durch Functionen erster Stufe vertreten werden können, wobei die Functionen, die als Argumente jener erscheinen, durch ihre Werthverläufe vertreten werden." I find it difficult to follow Thiel (1965, p. 53), who refers to this passage and the similar *FB* p. 31 as grounds

works where he speaks of ternary functions (*NS* pp. 269–270);[20] he does so without any ceremony, suggesting that he would have had no reservations against employing n-ary functions for arbitrary n had that been required.

A hierarchy of functions, functions of functions, functions of functions of functions, etc., built over certain ground types we shall call a hierarchy of simple types. Such a hierarchy can be defined inductively; perhaps the first to do so was Carnap (1929, pp. 30–31).[21] Carnap's definition can be used to describe the infinite hierarchy latent in Frege's considerations. There is a ground type ι of objects in Frege's sense; and if $\alpha_1, \dots, \alpha_n$ are types, then $(\alpha_1, \dots, \alpha_n)$ is a type, namely the type of functions with n arguments of types $\alpha_1, \dots, \alpha_n$ respectively and values of the type ι of objects. The sign ι may be eliminated from this presentation, rendering $()$ as the type of objects; but keeping ι makes for easier readability. In this notation, (ι) is the type of functions of one argument of the type of objects; (ι, ι) is the type of functions of two arguments, both of the type of objects; $((\iota))$ is the type of functions of one argument of the type (ι); and $((\iota), \iota)$ is the type of functions of two arguments, the first of the type (ι) and the second of the type ι. In the simple type theory of Church (1940) there are two ground types, rather than just one as in Frege's hierarchy. That is, Church has a type ι of individuals and a type o of propositions. This distinction was not made by Frege, for he held that what ordinary grammar calls sentences are to be equated with names of truth-values (*SB* p. 34).[22] Church moreover restricts the hierarchy to unary functions. Since he allows functions to take values in types other than the ground types, this restriction does not imply any restriction of logical power. By the technique of "Currying," [23] namely, we have for any n-ary function f a unary function h satisfying the equation

$$f(a_1, \dots, a_n) = h(a_1) \dots (a_n)$$

for asserting that "Frege von einer unendlichen Stufenhierarchie der Funktionen ausdrücklich Abstand genommen hat."

[20]That this is a place in his *Nachlass* is of no consequence for the point to be made here.

[21]Carnap's hierarchy is one of classes and relations, but he remarks (1929, p. 20) that a similar hierarchy exists for functions.

[22]Dummett's Frege follows Church in distinguishing the two types. Dummett (1973, p. 50) says that it is only in Frege's "later doctrine" that sentences are regarded as singular terms; that may well be true, but it is only in the later doctrine as well that there is any type hierarchy.

[23]This technique in the case of $n = 1$ is employed by Frege in his treatment of so-called "Doppelwertverlauf" in *Gg* § 36; it is also employed by Dedekind (1888) in his defining addition, multiplication, and exponentiation as families of unary functions, e.g. $m + n := \varphi_m(n)$, where φ_m is defined by the recursion equations $m + 0 = m$ and $m + n' = (m + n)'$. Systematic employment of the technique stems from Schönfinkel (1924, § 2).

Thus, by applying h to a_1, applying $h(a_1)$ to a_2, applying $h(a_1)(a_2)$ to $a_3,\ldots$, and finally applying $h(a_1)\ldots(a_{n-1})$ to a_n, we reach the value $f(a_1,\ldots,a_n)$. The proof for the existence of the function h goes by successively reducing the arity of the original function according to the following scheme. Given a $k+1$-ary function g we define a k-ary function g' satisfying

$$g(a_1,\ldots,a_k,a_{k+1}) = g'(a_1,\ldots,a_k)(a_{k+1})$$

Thus, g' is a k-ary function whose values are of the type of functions from the type of a_{k+1} into the type of $g(a_1,\ldots,a_{k+1})$. By iterating this reduction we reach the unary function h. Employing a notation apparently due to Schütte,[24] Church's simple type hierarchy can be defined as follows. There are ground types ι and o; and if α and β are types, then $(\alpha)\beta$ is a type, the type of functions from α into β. The type $(\alpha_1,\ldots,\alpha_n)$ of the Fregean hierarchy then corresponds to to the type $(\alpha_1)\ldots(\alpha_n)\iota$ in Church's hierarchy. The notation $(\alpha)\beta$ will in this dissertation be the default notation for the type of functions from α to β.

It seems that we may say without any significant reservations that the hierarchy latent in Frege's description of the various kinds of function is the hierarchy of simple types.[25] Church thought that such an identification would be "based on a misunderstanding" (1976b, p. 409). He holds that "with Frege a function is not properly an (abstract) object at all, but is a sort of incompleted abstraction" (loc. cit.), and that presumably rules against such things forming a type. Frege assumes, however, that his functions may both be the arguments to other functions and lie within the range of quantifiers; it seems strange to deny that *such* things would form a type. So Church's argument should rather have to be that Frege's notion of function is incoherent: incomplete entities like Frege's functions cannot be the arguments to other functions nor lie within the range of quantifiers. But if incompleteness is understood in the way suggested above, as the characteristic of being applicable to certain things as arguments and yielding certain things as values, then Church's claim would in effect be that there can be no type of functions whatsoever, since any function must have this characteristic of being applicable etc. And that is not a plausible claim.[26] As a further argument Church maintains that Frege "vigorously rejects the characteristic feature of the simple theory of types" (p. 410) namely that the laws of logic

[24]Cf. e.g. Diller and Schütte (1971).

[25]As is done by Resnik (1965, p. 330) and Dummett (1973, pp. 44–45).

[26]Although Church may have accepted it, as he may not have accepted this characterization of functions: he says namely ibid. that "on the whole it is the *Werthverlauf* which corresponds to the notion of function as used mathematics"; but a *Werthverlauf* is a notion function for which application is not primitive, but needs to be introduced separately as Frege does at *Gg* § 34 (cf. section 4.2.5 below).

are stated first for the ground domain and thereafter restated successively for other domains. This judgement seems to me mistaken. Frege's basic laws for propositional logic apply only to the ground domain, and Frege states the rules of propositional logic for this domain. The rule relevant for higher types is the rule of universal instantiation, and Frege states separate such rules for each of the two domains he needs for his purposes,[27] obviously being aware that he should have to state new rules if he needed universal instantiation for yet other domains. Hence I do not share Church's misgivings and shall not hesitate to speak of the various kinds of function explicit or implicit in Frege's work as *types* of function.

An important tenet of type theory is that all symbols are assigned a type. Disregarding the special sign of assertion, which, as noted, does not occupy us here, this tenet is honoured by the ideography of the *Grundgesetze.* The universal quantifier over a type τ is of type $((\tau))$, a function of unary functions of elements of type τ. The truth-values of true and false are of the type ι of objects. Frege's treatment of sentences as names of truth-values allows the assignment of types to the connectives: implication is of type (ι, ι), a binary function of objects, while negation is of type (ι), a unary function of objects. This contrasts with Frege's treatment of the connectives in the *Begriffsschrift.* There can be no question of whether the propositional logical operators were there regarded as functions (*pace* Baker, 2005), for such a question presupposes an absolute notion of function, which Frege did not adopt in the *Begriffsschrift.* It is unlikely that Frege in the *Begriffsschrift* would have been open to regarding for instance negation as the function of an argument in a judgeable content of the form $\neg p$; for, given the flexibility afforded by the analysis into function and argument, Frege should then also have had to be open to regarding negation as the argument to a function. In light of the explanation of quantification in the *Begriffsschrift* that in turn would have allowed quantification into the position of negation; the latter makes good sense in the *Grundgesetze*, but does not seem in line with Frege's explanation of the logical connectives in the *Begriffsschrift*, which is not very different from that given already by the Stoics (*DL* VII.73).

Although the notion of function and argument in the *Grundgesetze* is no longer intrinsically tied to the analysis of judgeable contents, there is still the possibility of analyzing such a content into function and argument. Such an analysis will now have to be in accordance with how these notions are understood in the *Grundgesetze.* In particular, what is deemed the function in such an analysis needs to be a function in the absolute sense; and what is the argument needs to be the argument

[27] These are basic laws IIa–b and III (*Gg* § 47).

to the function in the absolute sense. An argument to a function F is according to the *Grundgesestze* just what fills the gap in F. Considering our example sentence from above, 'Caesar killed Cato', we see therefore that we cannot, according to the new doctrine, regard 'Caesar' as a function of the argument 'killed Cato', as we can according to the *Begriffsschrift*: Caesar is an object, and therefore not a function in the absolute sense. There is, however, a second-level function φ(Caesar) (cf. *Gg* § 22),[28] and we may regard the given content as the value of this function applied to the first-level function 'killed Cato'. So what might initially seem like a restriction on the analysis into function and argument in the *Grundgesetze* when compared to the *Begriffschrift* is overcome by noting that the change in point of view that in the latter allowed what was previously regarded as an argument to be regarded instead as a function corresponds in the former to a change in point of view of the composition of the proposition from its elements: these are not 'Caesar' and 'killed Cato' but 'φ(Caesar)' and 'killed Cato'. Thus Frege's notion of function and argument in the *Grundgesetze* affords the same degree of flexibility in analysis that he had emphasized in the *Begriffsschrift* as distinguishing this pair of notions from the pair of subject and predicate. Given the treatment of the quantifiers and the connectives as functions, it is moreover clearer now than earlier that the analysis into function and argument has much wider applicability than the analysis into subject and predicate: any judgeable content can be analyzed into function and argument, including content of forms not expressible in syllogistic syntax, such as hypothetical form, 'If it is day, then it is light', or multiply quantified form, 'For every ε there is a δ such that…'. Thus, elements that traditional logic would treat as syncategorems are treated as functions of one or more arguments of suitable type.

With each expression being assigned a type independently of its occurring in a given sentence, we may regard the hierarchy of types as a categorization of the language. In the previous chapter we interpreted Aristotle's categories as providing a categorization of the language of syllogistics. Our thesis here is that Frege's hierarchy likewise affords a categorization of languages of function–argument syntax, that is, of languages of the sort that is associated with "modern logic." More precisely, Frege's hierarchy affords a prototype for the categorization of such languages. If, for instance, one does not follow Frege in treating sentences as names, then Frege's hierarchy does not provide a categorization of the language; but a slight modification of it does, namely one that introduces another ground type. And one can think of several other modifications of the type hierarchy to match modifications in the syntax of the

[28]On the "type-raising" involved here, cf. p. 90 below.

language. The general point is that a language of function–argument syntax can be categorized by a hierarchy of functions, of which Frege's hierarchy is a special case. The categorization of the symbols of a function–argument language by a suitable hierarchy of types is, in a certain sense, more perfect than the categorization of the symbols of the language of syllogistics by Aristotle's categories. The latter is a categorization only of the terms of the propositions of syllogistic language: the syncategorematic words indicating the form of the proposition are not categorized. Frege's types, by contrast, categorize all the symbols of the language, including the quantifiers and the connectives.

1.2. Frege's types and Aristotle's ontological square. We thus propose to regard Frege's hierarchy of types as a categorization of his logical language on a par with how we have interpreted Aristotle's categories as a categorization of his logical language. Against this proposal it may be objected that what most naturally corresponds to Frege's distinction between object and function in Aristotle's logic is not Aristotle's categories but rather his distinction between what is said of a subject and what is not said of a subject (*Cat* $1^{\text{a}}20$). These characteristics combine with those of being in and not being in a subject to form what is traditionally known as the ontological square, the fourfold distinction Aristotle makes of beings (*ta onta*) in the second chapter of the *Categories*:[29] 1) what is neither said of a subject nor is in a subject; 2) what is said of a subject but is not in a subject; 3) what is said of a subject but is not in a subject; 4) what is not said of a subject but is in a subject.

Aristotle does not provide an explanation of what he means by being said of a subject. In seeking to supply one we should first analyze out the relation of being said of: in the phrase 'said of a subject' the phrase 'subject' may be regarded as a variable bound by an existential quantifier, so that we paraphrase 'x is said of a subject' as $\exists y(xRy)$, with R signifying the relation of being said of. It seems reasonably clear from what Aristotle says about the relation of being said of in the *Categories* (chs. 2, 3, 5), and commentators seem to agree to this, that it is the relation of 'being the genus of' extended so as to encompass individuals. In other words, the relation of being said of is the relation $>$ (or $<$) introduced in section 2.2 of the previous chapter with its field extended to individuals. For general terms s and g, if $s < g$ is true, then g is "predicated in the what it is" of s (*Top* $102^{\text{a}}31$), another way of saying which is to say that s is essentially g (cf. $102^{\text{a}}32$–35). A natural way of including individuals in the field of the relation $<$ is therefore to say that $i < s$ is true if the individual i is essentially s. Socrates, Gabriel, and Jemima are thereby included in

[29] Ammonius *in Cat* 25,12 draws a square. I have not found a square in any earlier source.

the field of the relation $>$, for the one is essentially a man, essentially an animal, etc., while the other is essentially an angel, essentially an animal etc., and the third essentially a cat, essentially an animal, etc. What we in effect have done then is to follow Porphyry in including species among the predicables. A species, according to Porphyry, is namely "what is predicated in the what it is of several items differing in number" (*Isag* 4,12).[30] A genus, according to Porphyry, and according to Aristotle, is what "is predicated in the what it is of several items differing in species" ($102^{\mathrm{a}}31$). In general, therefore, we should say that x is said of y if and only if x is said in the what it is, or predicated essentially, of y. Consequently, x is said of a subject if there is a y such that x is predicated essentially of y. It appears that Aristotle thought that any predicate is predicated essentially of some individual, that is, that for any predicate there is some individual such that the predicate is predicated essentially of it. It follows that x is said of a subject if and only if it is a predicate, what Aristotle elsewhere calls a universal (*ta katholou*, *Int* $17^{\mathrm{a}}3$). Hence, what is said of a subject is a universal. What is not said of a subject is an individual, or a particular.

The other notion underlying the ontological square, that of being in a subject, is explained as follows: "by in a subject I mean what is in something, not as a part, and which cannot exist separately from what it is in" ($1^{\mathrm{a}}24$). There is disagreement in the literature on how to read this passage, related to a disagreement that will be outlined in the next paragraph;[31] but it is clear enough, and the different parties agree, that what is in a subject is in some way dependent on something else. A substance, which we are prone to think of as not thus dependent,[32] should therefore not be in a subject (*Cat* $3^{\mathrm{a}}7$); and indeed, Aristotle thinks that it is only substances which are not in a subject (cf. *Cat* $2^{\mathrm{a}}34$). Assuming the categories, we may therefore say that the characteristic of being in a subject distinguishes non-substances from substances. The categories other than substance, are also traditionally called accidents.[33] Since the characteristic of being said of a subject distinguishes universals from particulars, or in other words, general from individual items, we can therefore give the vertices of the

[30] We speculated in section 2.1.1 of chapter 1 that Aristotle in the *Topics* may not have countenanced singular essential judgements; since the *Categories* assumes that the field of the relation of being said of includes individuals, we now see that he the latter work must have countenanced such judgements.

[31] See e.g. Heinaman (1981, pp. 295–297) for an overview.

[32] E.g. Spinoza's definition (*Ethics* def. I.3): "By substance I understand that which is in itself and is conceived through itself, that is, that which does not need the concept of another thing, form which concept it must be formed."

[33] Through the identification of what is in a subject with accidents they were so-called by Porphyry (*in Cat* 73,22ff., *Isag* 13,5 with Barnes 2003, pp. 230–232); this use of the word 'accident' may have originated with him.

$$
\begin{tikzcd}
\text{gen subst} \arrow[loop above, "\text{said of}"] \arrow[d, "\text{said of}"'] & \text{gen acc} \arrow[loop above, "\text{said of}"] \arrow[l, "\text{is in}"'] \arrow[d, "\text{said of}"] \arrow[dl, "\text{is in}"] \\
\text{ind subst} & \text{ind acc} \arrow[l, "\text{is in}"]
\end{tikzcd}
$$

FIGURE 1. The ontological square

ontological square the following names, more illuminating perhaps than Aristotle's:[34] 1) individual substances, for instance Socrates, Gabriel, and Jemima; 2) general substances, for instance, man, angel, cat, and animal; 3) general accidents, for instance colour and shape; 4) individual accidents. We get a picture as in Figure 1 (the arrows will become important a few paragraphs below).

We did not give any examples of the rubric of individual accidents, as it is contested what it should be taken to encompass. The traditional interpretation, assumed for instance by Porphyry (e.g. *in Cat* 75,36ff.) and Ackrill (1963, pp. 74–75), regards individual accidents as what are variously called abstract particulars, dependent parts, moments, tropes, modes, etc.[35] A hand is an independent part of the body and the legs of a table independent parts of the table: the hand can exist also independently of the body to which it is now affixed and the legs can exist also independently of the table which they now support. This is in contrast to for instance the particular shape of the hand or the particular colour of the table, which cannot exist independently of that hand or those table legs: the particular colour of the table legs would vanish should we use these as firewood; and a gory example would reveal that something similar can be said of the shape of the hand. According to the traditional interpretation, individual accidents are therefore like the colour of the table legs or the shape of the hand; they

[34]Ammonius *ad* loc. (*in Cat* 25,13): "If Aristotle had used these words, what he said would have been clear; but he used other names in pursuit of obscurity." (On Aristotle's pursuit of obscurity Ammonius has the following to say (ibid. 7,10): "Aristotle uses the obscurity of his philosophy as a veil, so that good people may for that reason stretch their minds even more, whereas empty minds that are lost through carelessness will be put to flight by the obscurity when they encounter sentences like these.")

[35]The terms 'dependent part' and 'moment' were employed by Husserl (*LU* III §§ 13, 17); 'abstract particular' is employed by Stout (1923); 'trope' in this sense was introduced by Williams (1953); 'mode' is preferred by Lowe (2006) with a nod to Locke (*Essay* II.xii.4, etc.—cf. Spinoza *Ethics* def. I.5 and perhaps Descartes *Principles* Part I art. 56). There are of course differences in doctrine between these philosophers that influence how they conceive of the relevant entities.

are “non-repeatable” particulars that cannot exist apart from what they are in, or in other words, they are individuated by means of the particular substances they are in. Owen (1965) argued against this traditional interpretation, claiming instead that Aristotle’s particular accidents are lowest species in categories other than substance; examples would be a fully specific shade of red or a particular object of knowledge, like the paradigm for the inflection of a French verb. These are species, hence they can be found in several particular substances, and are therefore not individuated by means of the particular substances they are in. Their individuality consists rather in there not being anything of which they are the genus. Owen’s interpretation, in other words, claims that individual accidents are lowest species that are not said of anything. According to the traditional interpretation, by contrast, lowest species in non-substance categories are said of tropes, the dependent parts of substances, which in their turn are not said of anything.

We do not here have to adjudicate between these opposing interpretations of Aristotle’s particular accidents; let us instead turn to the objection that the appropriate parallel in Aristotle’s logic to Frege’s distinction between function and object is not the categories but rather his distinction between the general and the individual. It must then be emphasized that Frege’s distinction is not simply a distinction between object and function. There is, as we have seen, latent in the ideography of the *Grundgesetze* an infinite hierarchy of functions; and patent in it is the distinction between e.g. unary and binary functions, and between both of these and unary functions of functions of objects. All these various types of function are essentially distinct, a function of one type is as distinct from a function of another type as is a unary function of objects from an object (cf. *Gg* § 21, *FB* pp. 29–31). Frege’s ideography thus stratifies the notion of function; but no such stratification obtains among Aristotle’s genera. There is indeed a hierarchy of genera ordered by the relation of being said of. The relation ‘being said of’, however, differs in important respects from the relation that obtains between a function and any of its arguments. The type of a Fregean function is determined by the type of its argument. This means that the relation of function to argument cannot be transitive: from the fact that a function F may be applied to the function F, and the function F applied to the function f, it does not follow that F may be applied to f; on the contrary it follows that F cannot be applied to f. The relation of being said of, by contrast, is (as we saw in section 2.2 of chapter 1) transitive: “Whenever one thing is predicated of another as of a subject, all things said of what is predicated will be said of the subject also” (*Cat* $1^{\mathrm{b}}10$). Moreover, if g is said of s, then g is predicated not only truly, but essentially of s. No such relation is required to hold between a function and its argument—here the only requirement is that function and argument be of matching types. Hence Frege's hierarchy of functions is quite unlike Aristotle's hierarchy of genera. There is also a difference between Aristotle's individuals and Frege's objects. However we interpret Aristotle's accidental individuals, they are such as "cannot exist separately from what they are in." Fregean objects, by contrast, are "in no need of completion," and could quite naturally be called independent (though we should not identify Frege's notion of unsaturatedness with any form of dependence). I cannot see that there is any place at all for tropes in Frege's order of things.

If 't_1 is t_2' (including quantity and quality) is a well-formed proposition of syllogistics, then so is 't_2 is t_1'. Aristotle's reduction of imperfect moods to perfect ones rests on syllogistic proposition's having this property; even the formulation of Barbara rests on it. This syntactic similarity of terms is not upset if we impose Aristotle's category scheme on his syllogistics. Given the distinctions made in the ontological square between the singular and the general, and the substantial and the accidental, it is to be expected that the same will not be the case if we impose it on Aristotle's syllogistics. A logical syntax reflecting Aristotle's ontological square has recently been proposed by Lowe (2012, pp. 377–382), who in other work has defended the ontological square—assuming the traditional interpretation of individual accidents—as a basic set of kinds of being, or categories (Lowe, 2006). The logical system Lowe proposes is not quite perspicuous in its syntax, and it moreover misrepresents Aristotle's said-of relation. The following proposal is therefore a slight revision of Lowe's system. Corresponding to the four vertices in the ontological square, there must be four sorts of variables.

substance	accident	
a, b	α, β	individual
F, G	Φ, Ψ	general

Unlike what is the case in syllogistics this system distinguishes general from individual names and variables. The role of general names in this system is, however, not like that of functions in Frege's ideography, for a general name may here combine with another general name just as well as with an individual name. Animal, for instance, may combine with man just as well as with Socrates—it stands in one and the same syntactic relation to both man and Socrates. The names and variables are here therefore terms, and the resulting system is a term logic, just as Aristotle's syllogistics. The system departs from syllogistics not only in distinguishing general and individual names, but also in requiring two copulae, one used for the proposition that t_1 is said of t_2, the other used for the proposition that t_1 is in t_2. Just as the four types of variables correspond to the four types of item in the ontological square, so these two types of copulae correspond to the two relations that generate the square. Let us write

$$\begin{array}{lll} t_1 \gg t_2 & \text{for} & t_1 \text{ is said of } t_2 \\ t_1 \succ t_2 & \text{for} & t_1 \text{ is in } t_2 \end{array}$$

Lowe recognizes that the variables of the system are terms, but in his system terms are conjoined by mere concatenation, as in function–argument syntax, and not by means of copulae. A concatenation of terms is, however, not a proposition, but a list of names. At this point our system therefore departs from Lowe's.

With four types of term variables and two copulae we get $4 \times 2 \times 4 = 32$ candidates for basic categorical propositions. The seven arrows in Figure 1 correspond to the seven admissible forms of categorical proposition:

$G \gg F$	a general substance G is said of a general substance F
$F \gg a$	a general substance F is said of an individual substance a
$\Psi \gg \Phi$	a general accident Ψ is said of a general accident Φ
$\Phi \gg \alpha$	a general accident Φ is said of an individual accident α
$\Phi \succ F$	a general accident Φ is in a general substance
$\Phi \succ a$	a general accident Φ is in an individual substance a
$\alpha \succ a$	an individual accident α is in an individual substance a

Lowe does not recognize the two forms $\Psi \gg \Phi$ and $\Phi \gg \alpha$, in which accident terms feature in subject position. But since the said-of relation is just the relation 'being the genus of' extended so as to encompass individuals, be they substantial or accidental individuals, it is clear that propositions whose subject is an accident term must also be recognized.

Of the listed basic propositions we can of course take negations, hence we do get distinctions of quality; but we do not yet have distinctions of quantity. We may provide for distinctions of quantity by introducing the following forms of proposition:

$G \gg \forall F$	G is said of all F
$G \gg \exists F$	G is said of some F
$\Phi \succ \forall F$	Φ is in all F
$\Phi \succ \exists F$	Φ is in some F
$\Psi \gg \forall \Phi$	Ψ is said of all Φ
$\Psi \gg \exists \Phi$	Ψ is said of some Φ

Thus, for each basic proposition with a general term as subject, that subject can be quantified by means of either $\forall$ or $\exists$. In other work, Lowe (cf. his 1989, esp. chs. 8, 9) has emphasized the difference between these quantified propositions and the basic propositions. An instance of a basic proposition, such as $\Phi \succ F$, is a generic proposition, such as 'a raven is black', or 'black $\succ$ raven', in which something is said about the genus of ravens, namely that blackness inheres in it, that it is a disposition of ravens to be black, say. The general proposition 'all ravens are black', by contrast, says something about all (actually existing) ravens, that blackness inheres in each and every one of them, that they are all black. The general proposition thus corresponds to an a-proposition in Aristotle's syllogistics. A generic proposition of the form $\Psi \gg \Phi$, such as 'colour $\gg$ yellow', where the subject is not a substance but an accident, presents no special difficulties: it says merely that colour is a genus of yellow. The corresponding quantified proposition, 'colour $\gg \forall$ yellow', by contrast, does present difficulties of interpretation; for, over which domain do we here quantify? There are two alternatives: (i) all yellow substances in the domain of discourse; (ii) all individual accidents therein falling under the genus of yellow. Alternative (i) yields an ordinary a-proposition of syllogistics, 'colour follows everything yellow'. Alternative (ii) seems to me more natural, however, given that we have now admitted variables ranging over individual accidents. If we settle for alternative (ii), then the truth-conditions for a proposition of the form $A \gg/\succ \forall B$ can be given uniformly: it is true if and only if A is said of/inheres in all individuals that B are said of. If we settle for alternative (i), however, then this rule holds only if the subject is a substance; if the subject is an accident, the rule is rather that $\Psi \gg \forall\Phi$ if Ψ inheres in everything that Φ inheres in; in that case, therefore, truth-conditions cannot be given uniformly. Likewise, if we settle for alternative (ii), then the rule providing the truth-conditions for $A \gg/\succ \forall B$ may be generalized to provide truth-conditions for yet another form of proposition, namely those along the line of 'all mammals are warm-blooded', where one quantifies over all species falling under a genus (cf. Lowe, 2012, p. 380). We could write a formula of this form as

$$\Phi \succ \forall\forall F$$

and similarly for the other quantified propositions above. This formula $\Phi \succ \forall\forall F$ is true if Φ inheres in all species of which F is said. Similar truth-conditions could be given for other formulas of the form $A \gg/\succ \forall\forall B$, though we shall not pursue the matter further here.

Owing to the asymmetries that now have been introduced, both between the general and the singular and between the substantial and the accidental, we shall in any event not be able to develop syllogistics within this new system. Each valid mood of syllogistics gives rise to a set of valid moods in the new system. From the mood of Barbara, for instance, we get such moods as the following.

$$\frac{\begin{array}{c} G \gg \forall F \\ F \gg a \end{array}}{G \gg a} \qquad \frac{\begin{array}{c} \Phi \succ \forall G \\ G \gg \forall F \end{array}}{\Phi \succ \forall F} \qquad \frac{\begin{array}{c} \Psi \gg \forall \Phi \\ \Phi \succ \forall F \end{array}}{\Psi \succ \forall F}$$

And from the third figure mood Darapti, we get the following mood.

(Darapti*)
$$\frac{\begin{array}{c}\Phi \succ \forall G \\ F \gg \forall G\end{array}}{\Phi \succ \exists F}$$

Aristotle demonstrates the validity of Darapti by converting the second premiss, thereby obtaining the first figure mood of Darii (*APr* I.6 $28^{\mathrm{a}}18$–22). That demonstration can be carried over to Darapti*. What we cannot do with Darapti* is to permute the premisses and convert the conclusion. In syllogistics that can be done with any syllogism in Darapti, but it cannot be done with Darapti* since we should then get a syllogism whose conclusion is the nonsense $F \succ \exists\Phi$, namely that the substance F inheres in some accident Φ. From another third figure mood, Disamis, we get the following.

(Disamis*)
$$\frac{\begin{array}{c}\Phi \succ \exists G \\ F \gg \forall G\end{array}}{\Phi \succ \exists F}$$

Aristotle reduces Disamis to Darii by converting the first premiss, permuting the premisses, then converting the conclusion (*APr* I.6 $28^{\mathrm{b}}5$–11).[36] It is clear that this reduction will not work for Disamis*, since conversion of the first premiss as well as of the conclusion results in nonsense.

A logical system that reflects the ontological square will thus not be syllogistics. Each valid mood of syllogistics gives rise to a number of valid moods in such a system, but there is no straightforward way of transforming a valid mood of syllogistics into the corresponding valid moods of the new system, namely such that both premisses and conclusion are well-formed. From Darapti, for instance, we cannot form moods whose major term is a substance and whose minor term is an accident, since that will not yield a well-formed conclusion. Hence, the operation of permuting the premisses and converting the conclusion, which in ordinary syllogistics takes a syllogism in Darapti into another syllogism in Darapti, is not admissible in the new system. The important lesson is that conversion is no longer valid in general, since it now could take a well-formed proposition into nonsense. We saw an instance of this in the attempted reduction of Disamis to Darii. Hence a system of logic that reflects the ontological square upsets the syntactic similarity that is so important to the functioning of syllogistics. In the absence of conversion there is no way of simply reproducing

[36] We may write the reduction as follows
$$\frac{\begin{array}{c}AiB \\ CaB\end{array}}{AiC} \mapsto \frac{\begin{array}{c}BiA \\ CaB\end{array}}{AiC} \mapsto \frac{\begin{array}{c}CaB \\ BiA\end{array}}{AiC} \mapsto \frac{\begin{array}{c}CaB \\ BiA\end{array}}{CiA}$$

Aristotle's syllogistics in the new system, for syllogistics is more than just a set of valid moods, namely also a system for demonstrating the validity of moods by reduction to moods of the first figure. It is clear at once also that a logical syntax reflecting the ontological square, being a term logic, is not predicate logic, hence it does not reflect Frege's type theory.

1.3. Categorial grammar. According to Porphyry and Ammonius terms are divided into categories by virtue of their signification, while they are divided into parts of speech by virtue of such properties that they have as elements of a language (cf. section 4.2.2 of chapter 1). The first division is therefore object directed, while the second is language directed. That the divisions differ is evident from the fact that the number of categories differ from the number of parts of speech. This difference would not be denied by Trendelenburg, although he sought to show the affinity between Aristotle's categories and the parts of speech; for he did not argue that the two divisions are isomorphic. Several modern readers have argued that an isomorphism obtains between the Stoic's list of categories and their list of parts of speech, although we have no ancient source that is explicit on the matter (cf. section 4.2.1 of chapter 1). Frege, in any event, is explicit that an isomorphism obtains between types of object in his hierarchy and types of expressions.[37] A language directed simple type hierarchy was defined by Ajdukiewicz (1935) as a means for determining when a string of words is what Ajdukiewicz called syntactically connected (*syntaktisch konnex*), namely when it is not a piece of word salad, when it has a unitary sense (more on "word salad" in section 3.1 below). Any sentence as well as any single word is syntactically connected, as is any phrase, e.g. the verb phrase 'smells very nice'; but a mere assortment of words, such as 'perhaps horse if will however shine', is in general not syntactically connected.

In Ajdukiewicz's type hierarchy there are two ground types, the type $\mathbf{s}$ of sentences and the type $\mathbf{n}$ of nouns; these correspond in Church's hierarchy to the types o and ι of propositions and individuals respectively. Ajdukiewicz's hierarchy agrees with Frege's in allowing functions with several arguments, while it agrees with Church's in allowing functions to take values in types other than the ground types. The rule for generating function types in Ajdukiewicz's hierarchy is the following: if α and $\beta_1, \ldots, \beta_n$ are types, then $(\alpha : \beta_1 \ldots \beta_n)$ is a type, namely the type of n-ary functions whose i-th argument is of type β_i and whose values are of type α.[38] When mentioning a type the

[37] Cf. the references in footnote 10 above.

[38] Ajdukiewicz uses fractional notation $\frac{\alpha}{\beta_1 \ldots \beta_n}$, but the colon notation is preferred here for reasons of typesetting.

outermost parentheses may be omitted. Thus, $\mathbf{s} : \mathbf{n}$ is the type of functions that take a noun as argument to yield a sentence as its value: it is the type of intransitive verbs. The type of transitive verbs is $\mathbf{s} : \mathbf{nn}$. A function in this sense is a certain expression whose arguments and values are also expressions; it is a linguistic function in the sense of Anscombe and Geach (1961, p. 143). As was noted above (page 70), functions in mathematics are not ordinarily understood as linguistic functions: the function $+$, for instance, is a binary function on numbers and not on numerals. Presumably in order to mark the contrast between linguistic functions and functions in the objective sense common in mathematics Ajdukiewicz used the term *functor* for the elements of the function types of his hierarchy.[39] Assuming that numerals are of type $\mathbf{n}$, the functor $+$, which takes two numerals and yields a numeral, is of type $\mathbf{n} : \mathbf{nn}$. Binary sentence connectives such as 'and' and 'or' are functors of type $\mathbf{s} : \mathbf{ss}$; the adverb 'nice' is a functor of type $(\mathbf{s} : \mathbf{n}) : (\mathbf{s} : \mathbf{n})$, taking a verb phrase and yielding a new verb phrase; while the adverb 'very' is a functor of type $((\mathbf{s} : \mathbf{n}) : (\mathbf{s} : \mathbf{n})) : ((\mathbf{s} : \mathbf{n}) : (\mathbf{s} : \mathbf{n}))$, taking an adverb and yielding a new adverb.

Applying the functor 'very' to the functor 'nice' we obtain a new functor 'very nice' of type $(\mathbf{s} : \mathbf{n}) : (\mathbf{s} : \mathbf{n})$. Applying this functor to the functor 'smells' we obtain yet another functor 'smells very nice', of type $(\mathbf{s} : \mathbf{n})$. This functor can be applied to a noun $\mathbf{n}$, such as 'rosemary' to yield 'rosemary smells very nice' of type $\mathbf{s}$. According to Ajdukiewicz's definition any expression that can be analyzed into functor and argument or arguments in the manner thus illustrated for 'very nice', 'smells very nice', and 'rosemary smells very nice' is syntactically connected. The definition may be given recursively as follows (cf. Ajdukiewicz, 1935, p. 11). Any single word is syntactically connected. A string of words is syntactically connected if it is analyzable into functor and argument or arguments of matching types; in this analysis neither the functor nor any of its arguments has to be a single word, but if it is a phrase, it is itself required to be syntactically connected. Our analysis above shows that the phrases 'very nice' and 'smells very nice' satisfy the definition. The latter phrase is analyzed into the unary functor 'very nice' with argument 'smells', where the functor 'very nice' is analyzed into the unary functor 'very' with the argument 'nice'. By

[39]Both Ajdukiewicz (1935, p. 3) and Tarski (1936, p. 274) attribute the term 'functor' to Kotarbinski. It is well known that this word is employed by Carnap (1934, p. 13), but he may very well have adopted it from the Polish logicians; it is, for instance, found in Tarski and Łukasiewicz (1930), a paper Carnap refers to. Curry (1963, p. 33) uses the suffix *-or* for words indicating kinds of linguistic expression, for instance 'predicator' and 'connector'; Potts (1978, p. 8) also forms the unlovely 'argumentor' and 'valuor'. Geach was an authority on Polish logic, so it is surprising that he did not adopt Kotarbinski's term, using 'linguistic function' instead.

contrast, 'very or' is not syntactically connected, for 'very' and 'or' are not of types such that the one can be applied to the other.

Ajdukiewicz gives an algorithm for testing whether a string of words equipped with types is syntactically connected. This algorithm requires that the string in question be given in so-called Polish notation, namely that a functor always precedes its arguments.[40] The verb phrase 'smells very nice' should accordingly be given as 'very nice smells' and the sentence 'Ann sings and Peter dances' should be given as 'and sings Ann dances Peter'. In order to test the syntactic connection of a string of words in ordinary word order Bar-Hillel (1953) introduced a modified simple type hierarchy, namely a hierarchy of functions that apply to some arguments to the right and to some arguments to the left. In mathematics a function is sometimes written to the left of its argument; thus $a\varphi$ is the function φ applied to the argument a. This, however, is merely a notational convention, useful especially when one is interested in the composition of functions: fg is the function obtained by first applying f and then applying g, according to the order in which f and g occur in 'fg'.[41] In Bar-Hillel's simple type hierarchy, by contrast, it is an essential matter whether a function should be written to the left or to the right of its argument. This makes sense when one considers that the functions in question are functors, for it may well make a difference to syntactic connection whether one word is written to the left or to the right of another. The hierarchy is generated by the following rule: if α, $\beta_1, \ldots, \beta_m, \gamma_1, \ldots, \gamma_n$ are types, then

$$(\alpha : \beta_1 \ldots \beta_m | \gamma_1 \ldots \gamma_n)$$

is a type, the type of functors applying to the left to m arguments of type β_i and applying to the right to n arguments of type γ_i. In this hierarchy the binary sentential connectives 'and' and 'or' are of type $(\mathbf{s} : \mathbf{s}|\mathbf{s})$, for they connect with one sentence to the left and one sentence to the right. An intransitive verb, such as 'smells', is, in English at least, of type $(\mathbf{s} : \mathbf{n}|)$, since in English the subject of a verb stands to the left of the verb. A transitive verb is of type $(\mathbf{s} : \mathbf{n}|\mathbf{n})$ with one argument to the left for the subject and one argument to the right for the object of the verb.

If we require that all functors in Bar-Hillel's hierarchy be unary, then it can given the following more perspicuous presentation, stemming from Lambek (1958). If α and β are types, then (α/β) and $(\beta\backslash\alpha)$ are types, the types of functors applying to the right and left respectively to an argument of type β and yielding an element of type α. The type of intransitive verbs is then written $\mathbf{n}\backslash\mathbf{s}$, indicating that if we place an $\mathbf{n}$ to

[40]This notation stems from Łukasiewicz. It is for instance employed in Tarski and Łukasiewicz (1930).

[41]Dedekind (1894) makes use of this convention, perhaps as one of the first to do so.

the left, then we get an $\mathbf{s}$. The sentential connective 'and', however, which intuitively is a binary functor, belongs in Lambek's hierarchy both to $(\mathbf{s}\backslash\mathbf{s})/\mathbf{s}$ and to $\mathbf{s}\backslash(\mathbf{s}/\mathbf{s})$, and that may seem counterintuitive. That a single word may belong to different types was, however, recognized also by Bar-Hillel (1953, pp. 49–50). In fact, one and the same word 'and' seems *prima facie* to be used not only to conjoin sentences, but also to conjoin verbs, adverbs, adjectives, or indeed almost any pair of words of the same type. Lambek's syntactic calculus, nowadays also known as the Lambek calculus, is a systematic presentation of this "typical ambiguity" of words. The calculus consists of formulas $x \to y$, where x and y are types, whose intended interpretation is that any expression of type x is also of type y. The calculus consists of eight simple rules (three axioms and five rules of inference), all of which are evident by the intended meaning of the formulas (Lambek, 1958, p. 163). By means of these rules one can derive, for instance, the formulas

$$(x\backslash y)/z \rightleftarrows x\backslash(y/z)$$ [42]

$$x/z \to (x/y)/(z/y)$$

$$x \to y/(x\backslash y)$$

The first of these formulas has $(\mathbf{s}\backslash\mathbf{s})/\mathbf{s} \rightleftarrows \mathbf{s}\backslash(\mathbf{s}/\mathbf{s})$ as an instance, whence Lambek's $(\mathbf{s}\backslash\mathbf{s})/\mathbf{s}$ may be equated with Bar-Hillel's $\mathbf{s} : \mathbf{s}|\mathbf{s}$. From the second formula follows what is known as the Geach rule: if $xy \to z$, then $x(y/w) \to z/w$.[43] This rule is used repeatedly by Geach (1970) in his discussion of natural language syntax. The instance $\mathbf{n} \to \mathbf{s}/(\mathbf{n}\backslash\mathbf{s})$ of the third formula is the rule of type raising used when moving from a name such as 'Socrates' to the second order functor taking an intransitive verb and placing it to the right of the name 'Socrates'. The rule is in effect used by Frege at *Gg* § 22, when he notes that $\varphi(2)$ is a second-order function (where '2' of course could be replaced by any other Fregean proper name), and it underlies the treatment

[42]That is, both $(x\backslash y)/z \to x\backslash(y/z)$ and $x\backslash(y/z) \to (x\backslash y)/z$ can be derived.

[43]Using the primitive rules as well as the derived rules listed by Lambek (1958, pp. 163–164), the following simple derivation can be given of the Geach rule from the formula $z/y \to (z/w)/(y/w)$.

$$\frac{\dfrac{\dfrac{\dfrac{xy \to z}{x \to z/y} \quad z/y \to (z/w)/(y/w)}{x \to (z/w)/(y/w)} \quad y/w \to y/w}{x(y/w) \to ((z/w)/(y/w))(y/w)} \quad ((z/w)/(y/w))(y/w) \to z/w}{x(y/w) \to z/w}$$

of singular terms in Montague grammar as second-order functions, hence as being of the same type as quantifiers.[44]

Ajdukiewicz (1935) calls the types of his linguistically directed simple type theory meaning categories (*Bedeutungskategorien*); the doctrine of such meaning categories is nowadays often called categorial grammar.[45] The basic idea of categorial grammar appears to stem from Leśniewski, namely the idea of distinguishing ground types and functor types, and defining functor types inductively in the manner of the simple type hierarchy.[46] Leśniewski (1929, p. 14), Ajdukiewicz (1935, p. 2), and Tarski (1936, p. 335) all refer to Husserl as the inventor of the notion of meaning category. In his Fourth Logical Investigation (*LU* IV) Husserl introduces this term and characterizes it in effect as a maximal class of meanings that are substitutable for each other *salva congruitate* (*LU* IV § 10). Husserl as well as the Polish logicians regarded the relation of being of the same meaning category as an equivalence relation;[47] hence a meaning category is the range of admissible arguments of a suitable propositional functor.[48] It is what we shall call its range of significance, a notion to be discussed extensively in the following sections of this dissertation.

Husserl introduces the notion of a meaning category in the context of a presentation of what he calls pure logical grammar (*reinlogische Grammatik*).[49] Husserl's pure logical grammar has in common with categorial grammar that it is pure, having the presumption of being applicable to any actual language. In doctrine, however, the two differ significantly. The basic difference is that categorial grammar adopts a function–argument syntax, while Husserl's pure grammar adopts a broadly syllogistic syntax. In particular, Husserl assumes that a proposition contains both formal elements and

[44]The explicit type assignment is made at Montague (1973, p. 249). For a useful discussion, see Gamut (1991, pp. 158–165). The assignment of singular terms to the type of quantifiers is criticized by Geach (1970, p. 4).

[45]The term 'categorial grammar' appears to stem from Bar-Hillel et al. (1960).

[46]Leśniewski apparently conceived of the notion of a semantic category in 1922 as a replacement for the simple type hierarchy (cf. Leśniewski, 1929, p. 14). A definition of the hierarchy is not found in Leśniewski's published writings, but see e.g. Luschei (1962, pp. 84–104) and Simons (2011, § 4.1).

[47]Tarski (1936, p. 335) is explicit on the matter, Ajdukiewicz not. For Leśniewski, see his "terminological explanation" *T.E. XXXV* at (Leśniewski, 1929, p. 68) and the ensuing discussion; see also (Luschei, 1962, p. 213). It is clear that Husserl regards meaning categories as disjoint.

[48]Cf. the characterization of meaning categories by Ajdukewicz (ibid., p. 3) and Tarski (loc. cit.).

[49]This is a main topic of *LU* IV, especially of §§ 10–14. More detailed accounts of Husserl's ideas can be found in his lecture notes, especially Husserl (1996), but also Husserl (2001) and Husserl (2003). Material from these lecture notes was published in a compressed form in Husserl (1929, Beilage 1). The sometimes significant additions to *LU* IV §§ 10–13 in the second edition of the work also stem from these lecture notes. I am not aware of any secondary literature on Husserl's ideas concerning pure logical grammar taking into account all the material that has now been published.

material elements,[50] and as I shall argue one paragraph below, such a distinction has no place in categorial grammar. Husserl's meaning categories collect the material elements of propositions. In categorial grammar there is an infinite hierarchy of meaning categories. For Husserl, by contrast, the number of meaning categories is fixed. He mentions the following four: categories of propositional meanings, of nominal meanings, of adjectival meanings, and of (binary) relational meanings. Forms of meaning are for Husserl closely related to syncategorems.[51] Each of the connectives, for instance, being a syncategorem, is associated with a form of meaning. Thus, there are forms of meaning $A\ \mathit{and}\ B$ and $\mathit{if}\ A\ \mathit{then}\ B$, where A and B are place holders for propositions. There is a form $S\ \mathit{is}\ p$ where S is a nominal and p an adjectival meaning; a form Sp, which is the qualification of S by p, for instance 'white snow'; and a form $\mathit{this}\ S$, an instance of which is 'this house'. The two latter forms thus result in nominal meanings when their "open places" are filled. Since the set of meaning categories is fixed, something else must account for the variety of linguistic structure. In Husserl's pure grammar this is accounted for by recursive rules for constructing forms of meaning. The forms of meaning we have listed so far belong to the base clause of the recursion. For each of these it is laid down what category the meanings must belong to that fill its "open places" as well as which category the result of filling the form by suitable meanings belongs to. New forms of meaning can be constructed on the basis of this knowledge. Thus, since the result of filling $A\ \mathit{and}\ B$ with propositions is another proposition, there is therefore a form $(A\ \mathit{and}\ B)\ \mathit{and}\ C$, as well as forms $\mathit{if}\ (A\ \mathit{and}\ B)\ \mathit{then}\ C$ and $\mathit{if}\ A\ \mathit{then}\ (B\ \mathit{and}\ C)$. Likewise there is a form $\mathit{this}\ S\ \mathit{is}\ p$. The primary task of pure logical grammar is to find the set of meaning categories, the set of basic forms of meaning, and the set of rules for constructing forms of meaning.[52]

While the distinction between categorematic and syncategorematic parts of speech is basic to Husserl's conception of pure grammar, it is obliterated in categorial grammar. What traditional logic would treat as syncategorems are in categorial grammar instead treated as functors of one or more arguments of suitable type. There is thus a distinction between words of ground type and words of functor type, but this is not at all like the distinction between categorematic and syncategorematic words.[53]

[50] Cf. e.g. *LU* p. 329: "Alles in allem erkennen wir, [...] daß jede konkrete Bedeutung ein Ineinander von Stoffen und Formen ist."

[51] There appears to have been a general interest in syncategorems in the Brentano school; see Schmit (1992, pp. 38–41).

[52] See especially *LU* pp. 337–338, and also Husserl (1900, p. 245). It is clear from the lecture notes cited in footnote 49 that pure logical grammar in fact has a wider task than this, that the logical structure of language is not exhausted by meaning categories and forms of meaning; but we shall not go into that here.

[53] I therefore disagree with Bocheński (1949, p. 266), who equates functors of functors with syncategorems; and with Leclercq (2011, p. 184), who equates functors quite generally with syncategorems.

Although all syncategorematic words may be of functor type, not all categorematic words are of ground type; in fact, by the type raising rule $y \to z/(y\backslash z)$ any word of ground type is also of functor type. Moreover, functors are indeed words of a type, they belong to the lexicon of the language, hence they have meaning by themselves, and not only in the company of other words. Syncategorems, by contrast, were conceived as words that have meaning only in the company of other words. Syncategorems are also words that cannot by themselves function as terms in a syllogistic proposition; *a fortiori*, no predicate is a syncategorem; but a predicate is a functor. The general point, already mentioned in section 4.3 of chapter 1, is that the notion of a syncategorem makes sense only against the background of traditional logic and grammar. With the distinction between categorematic and syncategorematic words obliterated, so is the distinction between the form and the matter of a proposition obliterated in function–argument syntax. We cannot point to a word in a proposition of function–argument syntax and say "that is a formal element" or "that is a material element." If hylomorphic terminology is applicable at all here, then we should have to say of any element, be it of ground type or functor type, that it is a form–matter composite rather than just a formal element or just a material element.

In categorial grammar there is only one basic mode of syntactic construction: functor applied to argument or arguments. In traditional grammar, by contrast, each syncategorem is associated with its own mode of syntactic construction; hence there are as many modes of syntactic construction as there are syncategorems. This can be seen as following from two characteristics of syncategorems: (i) that they have no meaning by themselves, in modern terms, that they do not belong to the lexicon of the language; (ii) that, in the words of Ammonius (*in Int* 13,1–3), they "are useful for combining and constructing the parts of speech with one another." The latter was, for instance, the characteristic mark of syncategorems according to Locke. With this in mind one sees that the account of grammar in Quine (1986, pp. 15–30) is quite traditional, for Quine there treats the connectives and the quantifiers as syncategorems; indeed, he calls them *particles*, employing Locke's term for syncategorems. Quine does regard a formula of the form Fx to be obtained by applying the functor F to the argument x. But the formula $\neg Fx$, for instance, is for him not obtained by applying a functor $\neg$ of type $(\mathbf{s} : \mathbf{s})$ to the formula Fx of type $\mathbf{s}$. Rather, $\neg$ is, as Quine says, "incidental to the negation construction" (ibid. p. 28), viz. the construction that from a formula Fx constructs a new formula $\neg Fx$. The other connectives and the quantifiers are likewise incidental to constructions. Thereby each connective and quantifier is associated with a mode of sentence construction different from the basic mode of applying a functor to an argument. Also Montague (1973) treats the connectives and the quantifiers of the "fragment" of English that he studies as incidental to constructions, hence as not belonging to the lexicon.[54] Montague is, however, not as a consequent in this treatment of the connectives and quantifiers as Quine is, for Montague applies it only to the object language. By Montague's so-called translation function an English sentence of the form A *and* B is translated into a formula $\varphi \wedge \psi$ (ibid. p. 262). This formula is in turn interpreted in a so-called intensional type hierarchy, a simple type hierarchy with three ground types, of individuals, of truth-values, and of "possible worlds."[55] In this type hierarchy the connectives and the quantifiers are interpreted as objects. The connective $\wedge$ for instance is interpreted as an object of type $(t)(t)t$, where t is the type of truth values; and the universal quantifier over a type α is interpreted as an object of type $((\alpha)t)t$. In their interpretation, therefore, the connectives and quantifiers are for Montague not syncategorematic.

Objects in a simple type hierarchy combine with each other without the need for any "glue" from outside the hierarchy. This contrasts with the terms of syllogistics, which do not by themselves combine to form propositions, but need a certain form, expressed by one or more syncategorematic words, in order so to combine. Let us say that a category scheme is syntactic if it determines the modes of combination of the members of the categories into some new item. Simple type theory constitutes a syntactic category scheme, for a function of type τ is defined precisely by the type of the item or items with which an item of τ may combine in order to form a new item. In fact, all the category schemes that we shall consider in what follows that define a type as the range of significance of a propositional function are syntactic. The type of an object is there namely determined by which predicates it may combine with, and the type of a predicate is determined by which objects it may combine with. Aristotle's category scheme, by contrast, is not syntactic. A term may combine with any other term so as to form a syllogistic proposition. This is the principle of syntactic similarity, which we saw to be essential to the workings of Aristotle's syllogistics. Ryle (1938), comparing Aristotle's category scheme with that provided by type theory, criticized the former precisely for its not be syntactic. According to Ryle, laying down the categories and describing the syntax of a language are intimately related activities. To treat the category of terms independently of the syntax of propositions is therefore to treat as separate what are really two sides of the same thing; "it is," in Ryle's memorable words (ibid. pp. 195–196), "to treat as freely shuffleable counters

[54] See Montague's syntactic rules (ibid. pp. 251–251). Gamut (1991, p. 154) fittingly calls Montague's manner of proceeding *syncategorematic introduction* of the connectives and quantifiers.
[55] See Gamut (1991, ch. 5). Montague in addition has a ground type of time instances.

factors the determinate roles of which in the combination into which they can enter are just what constitute their types."

2. Ranges of significance

In speaking of Frege's hierarchy of types, we are using the word 'type' in a sense deriving from Russell, and corresponding roughly to the sense of the word recorded in the *OED* as "kind, class or order as distinguished by a particular characteristic." This is a relatively recent use of the word—the earliest occurrence recorded by the *OED* is dated 1854—and certainly not the only one current in philosophy.[56] Whewell (1840, Bk. VIII ch. ii sec. iii art. 10) defines a type as "an example of any class... which is considered as eminently possessing the characters of the class." This is not a type in Russell's sense, but rather what nowadays is more commonly called a prototype. There is, secondly, the sense of 'type' as contrasted with 'token', stemming from Pierce, and still current in philosophy today;[57] and, thirdly, a closely related sense of the word used by mathematicians when they speak of order types, or more generally of isomorphism types: 'type' then has the sense of the abstract shape or form of something.[58] In the *Principles of Mathematics* (1903) Russell in fact employs 'type' in both the first and third of these other senses. He speaks of Socrates as the type of humanity (§ 15), indicating that he serves as a typical or generic example of a man;[59] and of 'x is a man' as the type of a certain class of propositions, namely the class of those propositions obtained by substituting x by a suitable constant term (§ 22). But in the technical sense of that work a type is the range of significance of a propositional function (§ 497). It is this notion of a type as a range of significance that will occupy us in the current section.

2.1. Russell's type theories. The *Principles* contains nothing more than the sketch of a type theory, "put forward tentatively, as affording a possible solution of the contradiction" (§ 497), namely the Zermelo–Russell paradox.[60] A more developed doctrine of types is found in the paper *Mathematical logic as based on the theory of types* (1908) and in the first volume of the *Principia Mathematica* (1910). But here as well a type is defined as the range of significance of a propositional function (1908,

[56]See e.g. Heyde (1941) and Lessing (1998).
[57]See Wetzel (2014).
[58]The notion of order type appears to stem from Cantor (1895, p. 497). See also (Dedekind, 1890, p. 275), who calls the natural numbers the "abstract type" of simply infinite systems.
[59]*PoM* § 15: "For example, when it is said that 'Socrates is a man, therefore Socrates is mortal' Socrates is *felt* as a variable: he is a type of humanity, and one feels that any other man would have done as well."
[60]Regarding Zermelo, see a note recording an oral communication of his to Husserl published in (Husserl, 1979, p. 399).

p. 236; *PM* p. 161); and again the motivation is the need to avoid "the contradictions and paradoxes which have infected logic and the theory of aggregates" (*PM* p. vii). Types being ranges of significance, the doctrine of types should show that statements on which these contradictions and paradoxes rest are in fact nonsense. I will not say much pertaining to the paradoxes here. What interests me in the following is rather the idea of a range of significance. We shall see that, contrary to what Russell says, one cannot take his types as being defined as ranges of significance of propositional functions. Within Russell's type theory types are ranges of significances; but they cannot be that by definition.

Before considering the doctrine of types in the *Principles* and in the *Principia* a note should be made about Russell's notions of proposition and propositional function. In the *Principles* a proposition is defined as "anything that is true or that is false" (§ 13). A propositional function is a function all of whose values are propositions, or in Russell's language "ϕx is a propositional function if, for every value of x, ϕx is a proposition" (§ 22). The definition of a proposition leaves it open what it is that is true or that is false, and obscurity at this point remains in Russell's discussion; it is in particular not clear whether he takes a proposition to be linguistic or ontic in character, whether, that is, he takes a proposition to be something apt for expressing or rather as something apt for being expressed.[61] A propositional function is in any event something which it is natural to view as linguistic in character: since Russell defines a type as the range of significance of a propositional function, he of course assumes that propositional functions are the sort of things that can be or can fail to be significant, and it is primarily linguistic items which are of that sort.[62] In the *Principia* a propositional function is, accordingly, understood to be "an expression containing an undetermined constituent, i.e. a variable, or several such constituents" (p. 92). The *value* of a propositional function for a given argument may then be regarded either as the linguistic item that results upon substituting the argument viewed linguistically for the variable in the propositional function—in which case a propositional function would be a sentential functor in the sense of Ajdukiewicz. Or else the value must be regarded as whatever one takes to be the signification of the value of this functor, for instance a proposition in the ontic sense.

[61] *PoM* § 13 speaks of propositions as expressions, whereas *PoM* § 51 speaks of them as ontic, namely as containing the entities signified by words. *PM* p. 44 distinguishes propositions as expressed from propositions as expressing (Russell now views the former as "a false abstraction"). In the *Introduction to Mathematical Philosophy* (1919a, p. 155) Russell holds that a proposition is "primarily a form of words which expresses what is either true or false."

[62] Cf. *PM* p. 48 fn.: "Significance is a property of signs."

That the types in the theory sketched in the *Principles* cannot in general be taken to be ranges of significance seems clear from inspection of Russell's text. There is a type of individuals which contains, besides "simple individuals," such things as "persons, tables, chairs, apples, etc." (§ 497); but Russell does not point to any propositional function whose range of significance is constituted by this varied lot; and it is difficult to think of any function that would do the job. The same must be said about the type of sets of individuals. Russell claims that (loc. cit.)

> 'Brown and Jones' is an object of this type [sc., of sets], and will in general not yield a significant proposition if substituted for 'Brown' in any true or false proposition of which 'Brown' is a constituent.

That a substitution of 'Brown' for 'Brown and Jones' yields nonsense may be true for propositions whose predicate is collective in the manner of 'make a good team'; but it is a matter of linguistic accident, namely grammatical number, that such substitutions in general result in nonsense. We cannot, however, take the purely grammatical fact that the number of the verb matches 'Brown' but not 'Brown and Jones' as an indication of the logical fact that Brown and the set of Brown and Jones belong to different types. A logical notion such as the notion of a type cannot depend on a surface grammatical notion such as the notion of number. In the cited passage Russell must therefore be assuming a technical criterion of significance, namely that of belonging to different types. The type of sets is not defined as the range of significance of a certain propositional function; rather, that individuals and sets are of different types is employed as a criterion of significance. Just as no propositional function is provided in the case of sets, so none are provided for the further types in the hierarchy of sets of sets, sets of sets of sets, etc. As far as I have seen, it is only for the type of all sets and for the type of propositions that propositional functions are provided: the type of all sets is the range of significance of the propositional function 'x has a number' (§ 497), while the type of propositions is the range of significance of the propositional function 'x is true or false'.

To the extent that we grasp the type structure sketched in the *Principles*, we grasp it therefore quite independently of the notion of a range of significance. It is a structure like that of the hierarchy of simple types, although the elements of the types are sets and relations rather than functions. Besides the type of individuals, there is a type of sets of individuals, a type of sets of sets of individuals, and for any n a type of sets of n-tuples of individuals, etc. That the structure is only like that of the hierarchy of simple types, and not quite the same, is seen by Russell's assumption that types may be included in each other, and indeed that cumulative sums of types may themselves be types; in particular, and as already noted, Russell holds that there is a type of all sets. The universe of types is, moreover, not exhausted by this modified simple type hierarchy, for the types of numbers and the types of propositions are taken to fall outside of it. Russell's reasons for so taking them are obscure, but in the case of propositions they may have to do with an antinomy pertaining to propositions, and closely related to the original Contradiction. Russell notes that we have to admit sets of propositions, "for we often wish to assert the logical product" of such a set (§ 500). Consider, therefore, the set

$$w := \{q \in \text{prop} \mid \exists m \subset \text{prop}[q = (\forall p \in m)(p \text{ is true}) \wedge q \notin m]\}$$

And consider the proposition

$$r := (\forall p \in w)(p \text{ is true})$$

Then $r \in w$ if and only if $r \notin w$, a contradiction. Russell suggests a possible repair but finds it wanting and concludes that this contradiction "is probably not soluble" by the doctrine of types as conceived in the *Principles*.[63]

Following a number of other attempts at solving the original Contradiction and related antinomies, Russell (together with Whitehead) settles in the *Principia Mathematica* on what is now called the ramified theory of types.[64] As in the *Principles* a type is here defined as a range of significance, but the type structure is considerably more complex. In the course of a debate with Poincaré, Russell had become convinced that a theory capable of solving the antinomies must comply with what he called the vicious-circle principle (see esp. Russell, 1906a, p. 205). In one of its formulations this principle says that "whatever involves all of a collection cannot be one of the collection" (1908, p. 225; *PM* p. 37). In a more technical formulation it says that "any expression containing an apparent variable must not be in the range of that variable, i.e. must belong to a different type" (*PM* p. 161; cf. 1906a, p. 204). As Ramsey (1925a, p. 356) in effect remarks, the ramified hierarchy can be seen as the natural outcome of the vicious-circle principle in this technical formulation together with the principle that a function must be of a type, or an order, higher than that of each of its arguments. The latter principle was accepted by Frege and was later

[63]Cf. the letter to Frege on September 29, 1902: "Mein Vorschlag über logische Typen scheint mir jetzt unfähig das zu leisten was ich davon hoffte," whereupon the above reasoning follows.

[64]For an overview of the former attempts, see Russell (1906b) with Urquhart (1988); for the so-called substitutional theory in particular, see Russell (1906c) with Hylton (1980). It is worth remarking that the latter does not comprise a type theory, even though Russell does speak of types of so-called matrices—these matrices are not entities and do not belong to domains of quantification.

dissociated from the ramified hierarchy by Chwistek (1921, p. 342–343; 1922, p. 241) and independently by Ramsey (loc. cit.). It motivates the hierarchy of simple types.

No rigorous exposition of the ramified hierarchy can be found in the *Principia*, but an elegant definition was provided by Church (1976a).[65] There are two ground types, one of individuals ι and one of propositions; and there is a hierarchy of functions (of functions of functions of. . .) from individuals into the ground type of propositions, that is, a hierarchy of propositional functions.[66] Russell seems moreover to have conceived of a ramified hierarchy of propositions, in which one branch is derived from the hierarchy of functions, while another branch arises through quantification over types of propositions. The full extent of this hierarchy is, it seems to me, not fully taken account of in Church's exposition, but we shall not worry about that here.[67] Following Church, we define the ramified hierarchy of propositional functions inductively as follows: ι is a type, the type of individuals; and if $\alpha_1, \ldots \alpha_m$ are types and $n \in N$, then $(\alpha_1, \ldots, \alpha_m)/n$ is a type, the type of propositional functions of variables of types $\alpha_1, \ldots \alpha_m$ and of level n; by metonymy we shall also say that the type $(\alpha_1, \ldots, \alpha_m)/n$ has level n. Thus on top of the simple type structure there is here a stratification into levels. The notion of level is not expressly employed in the *Principia*, but is introduced by Church mainly as an auxiliary notion for giving a rigorous definition of the notion of order, which is employed in the *Principia*, playing in fact a crucial role in the sketch given there of the ramified hierarchy. Before giving Church's definition of the notion of order it seems wise to consider some examples.

Let us begin with 'Socrates is a man', which (let us assume, though see *PM* pp. 45, 50) is what Russell would call an elementary proposition (*PM*, pp. 91–92). By substituting into this a variable for 'Socrates' we obtain a propositional function 'x is a man', or man(x), which is what Russell would call an elementary propositional function (loc. cit.), a function of individuals of lowest complexity. In Church's notation

[65] For the case of unary propositional functions there is an alternative definition, assumed for instance by Myhill (1974). Each function has a certain order (a notion to be defined below), and a type is here defined as a sequence of orders: the type of a function of individuals is a sequence of two elements, of a function of function of individuals a sequence of three elements, etc.

[66] Functions to individuals, or so-called descriptive functions more generally, are derivative for Russell (*PM* p. 15), explained (officially at *PM* $*30.01$) in terms of the definite description operator; on this, see Hylton (1994).

[67] For the first dimension, see *PM* pp. 54–55 and also the title of *PM* $*9$: "Extensions of the theory of deduction from lower to higher types of propositions." This notion of type of proposition seems to be captured by Church's types $()/n$. It is this notion of type that motivates the doctrine of the typical ambiguity of the propositional connectives (*PM* pp. 46–47), reflected in the definition in $*9.01$ of $\neg\forall x\varphi(x)$ in terms of $\exists x$ and the negation of the elementary proposition $\varphi(x)$. The second dimension is what Copi (1971, p. 80) calls the hierarchy of propositions; it seems to be presupposed by Russell's discussion of the Liar.

it is a function of type $(\iota)/1$, a function of individuals of level 1. To obtain functions of higher level, consider the function of x defined by

$$\forall f^{(\iota)/1}(\forall y(\mathrm{man}(y) \supset fy) \supset fx)$$

This function is true of x if every function of type $(\iota)/1$ that is true of every man is also true of x. Like man(x), this is a function of individuals, but its definition involves quantification over functions of type $(\iota)/1$, so in accordance with the vicious-circle principle it cannot itself be of this type; it is of type $(\iota)/2$. In general, the function of individuals x defined by

$$\forall f^{(\iota)/n}(\forall y(\mathrm{man}(y) \supset fy) \supset fx)$$

is of type $(\iota)/n{+}1$. The vicious-circle principle requires in general that if the definition of a function of individuals involves quantification over functions of individuals of level n, then it must itself be of level at least $n + 1$.

Consider now what happens if we define a function of individuals by quantifying not over functions of individuals, but over functions of functions of individuals. There is an infinite range of such functions, so for concreteness let us consider functions of the type $((\iota)/1)/1$. Quantifying over this type we can define, for instance, the following function $\Phi(x)$ of individuals.

$$\exists g^{(\iota)/1}[gx \wedge \forall F^{((\iota)/1)/1}(F(\mathrm{man}) \supset Fg)]$$

This function Φ is true of an individual x if there is a propositional function g of type $(\iota)/1$ such that gx is true and Fg is true for every F of type $((\iota)/1)/1$ that is true of the function man(y). Then Φ is a function of type $(\iota)/k$ for some k; but which k? A function of individuals defined by quantification over $(\iota)/1$ is of type $(\iota)/2$. The function Φ, however, involves quantification over $((\iota)/1)/1$; thus it presupposes a collection, namely $((\iota)/1)/1$, that in turn presupposes the collection $(\iota)/1$. It should therefore be of a level higher than that of functions involving quantification only over the latter type, that is, we should have $k > 2$. It is in fact of type $(\iota)/3$, so $k = 3$. Hence quantification over $((\iota)/1)/1$ as well as quantification over $(\iota)/2$ forces the level of the defined function to be at least 3. We see that, in intuitive terms, moving up one level in the simple type hierarchy has the same effect on the level as moving one stage further in the stratification of the simple type of functions of individuals. It is this interplay between the levels in the simple type hierarchy on the one hand and the stages in the stratification of each such level on the other hand that is accounted for by Russell's notion of order. No definition of this notion is found in the *Principia*, but illuminating examples of functions of order two are provided: these include not only functions of functions of individuals, but also functions of individuals defined by means of quantification over other functions of individuals (*PM* pp. 50–53, 163–164). By means of the notion of level Church gives a general definition of the notion of order, in accordance, apparently, with everything Russell says; as a function ρ on types it is defined as follows.

$$\begin{array}{rcl} \rho(\iota) & := & 0 \\ \rho((\alpha_1, \ldots, \alpha_m)/n) & := & n + \max\{\rho(\alpha_1), \ldots, \rho(\alpha_m)\}. \end{array}$$

By metonymy we shall also speak of the order of a function.

With the notion of order in hand we are able to assign types to defined functions (*PM* p. 53):

> If the highest order of a variable occurring in a function, whether as argument or as apparent variable, is a function of the nth order, then the function in which it occurs is of the $n+1$th order.

That is to say, when assigning a type τ to a function defined by some formula φ it is required that the order $\rho(\tau)$ of τ should exceed the orders of all the variables occurring bound or free in φ (and it should be no less than the order of all constants occurring in φ). When finding the appropriate *level* of τ the main point of consideration will therefore be the orders of the various types involved. From the definition of order one sees that the level of a type is equal to its order minus the maximum of the orders of the types of the arguments. Hence, the level of a function f is the "distance" between the order of the bound variables and constants occurring in the definition of f and the maximum order of the arguments of f. What Russell called a predicative function (*PM* p. 53) is therefore just a function of level 1; it is a function "of the lowest order compatible with its having the arguments it has."

Let us consider some further examples. When we pass to functions of functions of individuals we need to carry with us the stratification of the type of functions of individuals into the types $(i)/n$. Among the resulting types there will again be stratification, yielding types $((i)/n)/m$. This double stratification needs in its turn to be carried along into the type of functions of functions of functions of individuals, etc. Besides these complications there are those involved in taking into account types of n-ary functions for $n > 1$. We restrict ourselves here to unary functions of unary functions of individuals. A standard example of such a function—which we have already met in our discussion of Frege—is, for a given individual a, the function fa. In the ramified hierarchy there is an infinite tower of such functions, corresponding to the tower of levels. That is, for each n there is the function $f^{(i)/n}a$ of type $((\iota)/n)/1$. As these functions do not involve quantification over functions, they are all of level 1. The same is true of a function of individuals defined by quantification over individuals, as for instance $\forall x f^{(i)/n}x$. What about a function involving quantification over functions of individuals? Let R be a relation of type $(\iota, \iota)/1$ and a an individual, and define the function $\Psi(g^{(\iota)/1})$ as follows:

$$ga \wedge \forall f^{(\iota)/1}[fa \wedge \forall x, y(fx \wedge Rxy \supset fy) \supset \forall x(gx \supset fx)]$$

Then $\Psi(g)$ is true if and only if g is true precisely of the R-descendants of a. The function Ψ is again of level 1, i.e. it has the type $((\iota)/1)/1$, for its definition quantifies only over $(\iota)/1$. Hence the definition of Ψ, itself of type $((\iota)/1)/1$, is analogous to the definition of a function of type $(\iota)/1$ that involves quantification over ι. What distinguishes Ψ from such functions is that these also have order equal to 1, while Ψ is of order 2. To get a higher level we need something like the function $\chi(g^{(\iota)/1})$,

$$\exists F^{((\iota)/1)/1}(F(\text{man}) \wedge Fg),$$

in which we quantify over $((\iota)/1)/1$. In accordance with Russell's principle for type assignment just cited, the function χ is of order 3, since the order of the variable F being quantified over is 2. The type of the argument of g is $(\iota)/1$, hence the type of g itself must be $((\iota)/1)/2$, since only then will its order be 3.

There are well-known problems, both of a philosophical and of a more exegetical nature, pertaining to the ramified hierarchy and the vicious-circle principle that motivates it. One may, for instance, ask how the principle is best formulated; whether it is valid; whether it is in fact needed for the solution of the paradoxes; and whether it is compatible with Russell's realism.[68] There are also well-known, and perhaps more serious, technical problems. Russell was aware that it will not in general be possible to carry out proof by induction in ramified type theory unless one adopts what he calls the axiom of reducibility (e.g. Russell, 1908, p. 242). This axiom, if that is what it is, says that any propositional function is extensionally equivalent to a predicative propositional function, that is, to a function of level 1. In Appendix B to the second edition of *Principia* Russell purports to show the contrary, but a gap in this proof was pointed out by Gödel (1944, pp. 145–146); and Myhill (1974) shows that it is in fact not possible to justify proof by induction in general in ramified type theory without the axiom of reducibility. The construction of arithmetic within ramified type theory would therefore seem to require the axiom of reducibility. It is, however, difficult to see the justification of this axiom: why should there always exist a predicative

[68] For a classic discussion of the first two questions, see Gödel (1944). Ramsey (1925a, p. 356) famously argued that the antinomies pertaining to logical notions are solved already by the simple theory of types. For the final question, see e.g. Goldfarb (1989).

function of the kind required? Indeed, if one accepts that there is such a baroque structure as the ramified hierarchy it is curious that one should also accept that this hierarchy in effect collapses to the simple type hierarchy (i.e. the hierarchy consisting of level 1 functions of individuals, level 1 functions of level 1 functions of individuals, etc.).

Let us now at last consider the claim that a type in the sense of *Principia* is defined as the range of significance of a propositional function. The claim is false. For, if a is an individual, then there is a propositional function fa, and the range of significance of this propositional function consists of all functions of individuals. By Russell's own doctrine of ramified types, fa is significant so long as f is a function of individuals. In ramified type theory there is, however, no type of all functions of individuals; instead there is an infinite family of types $(\iota)/n$, stratifying the functions of individuals into various levels. Thus we have found something that in accordance with the ramified theory of types is the range of significance of a propositional function, but that is not a ramified type. What consideration of the range of significance of the propositional function fa does yield is the simple type of functions of individuals. When Russell and Whitehead explain at *Principia* pp. 47–48 "why a given function requires arguments of a certain type" they seem in fact to have only simple types in mind. The principle presupposed throughout this discussion seems to be that a function needs to differ in type from its arguments; that principle, however, leads most naturally to the simple theory of types, where the type of a propositional function is fully determined by the types of its arguments. There is thus very little in the idea of a type as the range of significance of a propositional function that should find its most natural realization in the ramified hierarchy. With the ramified hierarchy already in place, however, one can of course lay it down that any variable is to have a type as its range of values. It will then follow that a well-formed propositional function, since it contains a variable, has a type as its range of significance. Instead of the propositional function fa we then have $f^{\tau}\alpha$, in which the variable f is restricted to the type τ. This function does not straddle ramified types as the function fa considered above does; its range of significance is simply the type τ. Hence, given the ramified hierarchy, it will follow that a type is the range of significance of a propositional function; but the notion of a range of significance of a propositional function will not by itself give us the ramified hierarchy.

2.2. Sommers's type theory. There is thus a range of significance of functions of individuals that is not a ramified type. One can argue that the dual problem pertains to the type of individuals: within the universe of individuals—which is equally undifferentiated in simple and ramified type theory—there are plenty of ranges of significance, which, being only proper parts of the type of individuals, are therefore not themselves types. The argument appeals to propositional functions of ordinary language: Socrates, but not the *Illiad*, is in the range of significance of 'x has an aquiline nose', while the *Illiad*, but not Socrates, is in the range of significance of 'x is divided into 24 chapters'; and John, but not Saturday, is in the range of significance of 'x loves Mary', while Saturday, but not John, is in the range of significance of 'x follows Thursday'. Such examples can be multiplied to show that there is a broad variety of ranges of significance of individuals. Ryle (1938, p. 203) must therefore have assumed a novel notion of type, one different from the ramified as well as the simple notion, when he laid it down that two items a and b are of different types if there is a propositional function φx such that φa is significant while φb is absurd. It is this new notion of type, developed especially by Sommers (1963), that will now occupy us.

One may ask, however, how novel Ryle's notion of type in fact is, whether it is not in fact Russellian. Russell (1924) defines

> A and B are of the same logical type if, and only if, given any fact of which A is a constituent, there is a corresponding fact which has B as a constituent, which either results by substituting B for A, or is a negation of what so results.

And *Princinpia* $*9.14$, slightly reformulated, says likewise that

> a is of the same type as b if and only if φa is significant if φb is significant, for any propositional function φ.

Ryle (1938, p. 203) denies the if-direction of this proposition, namely that a and b are of the *same* type if the one can be substituted for the other in any proposition without loss of significance, holding only the only if-direction, namely that a and b are of *different* types if there is a function φ such that φa is significant while φb is not. His reason for denying the former is, however, not a good reason. It is that 'I' but not 'the writer of this paper' lies in the range of significance of 'x never wrote a paper'; but 'I' and 'the writer of this paper' belong to the same type, for they have one and the same signification (for Ryle the elements of categories are linguistic); hence there is a propositional function φ and items a and b of the same type such that φa is significant while φb is absurd. This argument fails because 'the writer of this paper never wrote a paper' is not absurd in the manner of 'the *Illiad* has an aquiline nose', but merely a plain falsehood in the manner of 'the King of Norway is not a king'. Thus Ryle agrees with the only if-direction of *Principia* $*9.14$, and he does not offer a valid reason for denying the if-direction. This increases the suspicion that Ryle's notion of type is Russell's.

Ryle does, however, not understand significance as Russell does, and that of course makes a difference to a definition of type as a range of significance. *Principia* $*9.14$ is a primitive proposition, and is presumably meant to gain its evidence from $*9.131$, the definition of sameness of type. The latter is an inductive definition in whose basic clause it is simply laid down that all individuals are of the same type (and that all elementary functions, i.e.—assuming the type theory—functions of level 1, are of the same type, and likewise with propositions).[69] The primitive proposition $*9.14$ must therefore assume a notion of significance such that any function of individuals can be significantly applied to any individual, which is just what Ryle denies. With no indication to the contrary in Russell (1924), I would say that the same notion of significance—or of "being a fact"—is assumed there, namely such that any function of individuals can be significantly applied to any individual. Hence Russell, in the *Principia* as well as in the quoted passage, operates with a notion of significance that differs from Ryle's, whence with a notion of sameness of type that differs from Ryle's, thence with a notion of type that differs from Ryle's. Ryle's notion of type is, in other words, not Russellian.[70]

The range of significance of a propositional function φx is the range of things a such that φa is either true or false. This is one of the ways in which the notion of range of significance is explained by Russell (*PM* pp. 161, 400). It is the primary way of explaining the notion in Sommers (1963), a paper investigating the mathematical properties of ranges of significance. Sommers in fact defines as many as four notions of type, two notions of types of individuals and two notions of types of what he calls predicates, what in our terminology are propositional functions of individuals. For a propositional function φ, let $|\varphi|$, the "absolute value" of φ, be the range of significance of φ. Naively employing set-theoretical notation, we can define these various notions of type as follows.

α-type	For any propositional function φ, the type $|\varphi|$, i.e. $\{a : a \in |\varphi|\}$
A-type	For any α-type τ, the type $\{\varphi : |\varphi| = \tau\}$
B-type	For any individual a, the type $\{\varphi : a \in |\varphi|\}$
β-type	For any B-type τ, the type $\bigcap\{|\varphi| : \varphi \in \tau\}$

[69] This definition is not quite adequate, for it contains no clause saying when two functions of functions (of whichever types) have the same type; one possible revision would be to substitute 'predicative' for 'elementary' in clause (2) of the definition.

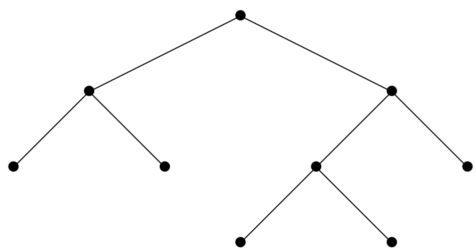
[70] The first to have understood types in the manner of Ryle (1938) may have been Carnap (1928, §§ 29–31) with his notion of *Gegenstandsphähre*, described by Carnap (loc. cit.) as "Russellian types applied to non-logical concepts."

Thus, α-types and β-types are types of individuals, while A-types and B-types are types of propositional functions of individuals. An α-type is simply the range of significance of a propositional function. An A-type is the type of propositional functions whose range of significance coincides with some α-type. We may think of an A-type as the intensional correlate of the α-type involved in its definition, namely as the range of properties having the given α-type as their extension.[71] A B-type is the type of propositional functions having a given individual in their respective ranges of significance. A β-type, finally, is the type of individuals contained in the range of significance of all propositional functions of some B-type. Further explication of the notions of B-type and β-type will be offered in a short while.

2.2.1. *The law of categorial inclusion.* Apart from these various definitions the most important part of Sommers's theory of types is what he calls "the law of categorial inclusion," which is a law governing the inclusion relation between α-types.[72] Again employing set-theoretical notation, it can be stated as follows.

> If σ and τ are α-types, then $\sigma \cap \tau \neq \emptyset$ implies either $\sigma \subset \tau$ or $\tau \subset \sigma$.

This law forces α-types to be ordered through the inclusion relation in a tree-like structure, as in



Here we think of each node as representing an α-type; the type corresponding to a node is included (*qua* range of significance) in all nodes above it, and—according to the law of categorial inclusion—is disjunct to all nodes not above or below it. A path in a tree is a maximal linearly ordered subset of it. One sees that B-types correspond to paths in the tree of α-types. A β-type is formed by taking the intersection over B-types, and will therefore correspond to the lowest node in a path, if it exists. In other words, if there is a lowest node in a path, i.e. an α-type with no type below it,

[71]We are then employing 'intension' roughly in the sense of Carnap (1947); cf. footnote 144 of chapter 1.
[72]This "law" was first introduced in Sommers (1959, p. 172ff.).

then that α-type is a β-type; if there is no lowest node, then we can obtain one by adding the relevant β-type at the bottom of the path.[73]

In light of our discussion in section 2 of chapter 1 it is natural to ask what the relation is between the tree of α-types and the tree of genera and species. A little reflection shows that neither is Aristotle's tree a tree of ranges of significance, nor is Sommers's tree a tree of genera and species. There are nodes in Aristotle's tree that do not occur in a tree of ranges of significance. The term 'white', for instance, is a species of 'colour' (e.g. *Top* 123$^{\text{b}}$26), and so occurs in Aristotle's tree, namely under 'colour' and in the division of quality; but there is no range of significance of all the white things in the world. This points to another difference, namely in the kind of individuals that fall under the nodes in the tree. A node in the tree of α-types is a range of significance such as |white|, and this consists of all individuals of which 'white' could be predicated. A node in the tree of species and genera, on the other hand, is a general term, and if this term falls outside the category of substance, then the individuals that fall under it are the individual accidents discussed in section 1.2 above. We mentioned two conceptions of individual accidents: as dependent parts of individual substances, and as lowest species, which may have instances in several individual substances; but on neither conception is it the case that the term 'white' includes mice and men, the sort of individuals that fall under |white|. Next one may note a difference in how nodes in the two trees are related. In the tree of genera and species nodes are related by an intensional link—we have $s < g$ if g is essentially predicated of s—while in the tree of α-types nodes are connected by the extensional link of inclusion. The extension of a term s is indeed included in the extension of its genus g (*Top* 121$^{\text{b}}$3–4), but that is not the reason why the one is placed under the other in the tree; in the tree of α-types, by contrast, it is only inclusion that plays a role in placing one node above another. There is, finally, a fundamental difference in the information the trees contain about the possibility of combining terms P and Q in a proposition. The terms that may be combined with P in a proposition are precisely those that belong to a path in the tree of α-types to which P belongs; for then, and only then, do we have $|P| \cap |Q| \neq \emptyset$, which is a necessary condition for combining P and Q in a predication. In the tree of genera and species, by contrast, no such property obtains, since, for instance, no non-substance belongs to a path to which substances belong, while it is assumed that any term can be predicated of some

[73]One may well wonder what the purpose is of introducing the notion of a β-type. Sommers (1963, pp. 330–331) claims that the relation $\sim_\alpha$ of being of the same α-type is not transitive, and he introduces β-types to obtain a transitive relation $\sim_\beta$. If the law of categorial inclusion holds it is, however, easy to see that $\sim_\alpha$ is transitive: if a and b are of α-type σ, while b and c are of α-type τ, then $\sigma \cap \tau \neq \emptyset$, whence either $\sigma \subset \tau$ or $\tau \subset \sigma$, and so each of a, b, c is in the α-type $\sigma \cup \tau$, in particular we have $a \sim_\alpha c$.

substance. For example, in the tree of α-types the ranges of significance |white| and |man| belong to a common path; but in the tree of genera and species the quality 'white' and the substance 'man' belong to different paths.

One can ask of the tree of α-types, just as was asked of the tree of genera and species, whether it has various order-theoretic properties. We saw in section 2.3 of ch. 1 that Aristotle failed at proving the existence of lowest genera. Sommers's argument to the similar conclusion that there are $\subset$-minimal α-types fails as well. He claims (ibid. p. 332):

> it is evident that for any language of finite vocabulary—and every natural language is finite in that sense—the number of α-types is [...] is finite.

It is, firstly, not the case that every natural language has a finite vocabulary. English, for instance, contains infinitely many numerals. And, secondly, even with a finite vocabulary one can define infinitely many predicates (e.g. $x > 0$, $x > s0$, $x > ss0$, $x > sss0$, etc.), and so possibly have infinitely many ranges of significance. Indeed, in both the simple and the ramified type theory one defines an infinite hierarchy of types by means of a finite vocabulary. In particular, it may well be false, as it is in the simple and the ramified type theory, that there are only finitely many types defined in a language of finite vocabulary. But if a type is the range of significance of a propositional function it nevertheless seems reasonable to assume that for any type τ for which there is a type σ with $\sigma \subset \tau$, there is a $\subset$-minimal type $\rho \subset \tau$, that is, to assume that we cannot go on forever making finer and finer ranges of significance. It is easy enough to define an infinitely $\subset$-descending chain of sets of natural numbers (the example in brackets above will do), but these are not ranges of significance. It is indeed difficult to see how one could construct an infinitely descending chain of ranges of significance.

Assuming the finitude of α-types Sommers moreover claims that the law of categorial inclusion entails that there is a universal α-type, one which includes all others (ibid. p. 355). But again this does not follow from the premisses. What does follow is that there must be $\subset$-maximal α-types—types that have no other types above them—but not that there is a single such type, a greatest α-type. The argument thus rests on the faulty inference from the existence in a partial order of a maximal element to the existence therein of a greatest element.[74] Since the finitude of α-types is in fact merely an assumption, it is thus also merely an assumption that there are

[74] A maximal element of a partial order $(A, <)$ is an element m such that there is no $a \in A$ with $m < a$. A greatest element in $(A, <)$ is an element such that $m < a$ for all $m \neq a$.

$\subset$-maximal α-types. But again, if a type is the range of significance of a propositional function this nevertheless seems a reasonable assumption; that is, it seems reasonable to assume that for any type τ for which there is a type σ with $\sigma \supset \tau$, there is a $\subset$-maximal type $\rho \supset \tau$.

Sommers maintains that the law of categorial inclusion is a theorem. Its purported proof runs, as I understand it, as follows (ibid. p. 362). Let P be a predicate whose range of significance is σ, and Q a predicate whose range of significance is τ. If $\sigma \cap \tau \neq \emptyset$, then it is a significant, though perhaps not a true, proposition that some P is Q. Sommers holds that the domain of discourse of this proposition is $\sigma \cap \tau$, that is, it consists precisely of those individuals of which it makes sense to predicate both P and Q. That is reasonable enough, for thus the domain of discourse is rendered as the maximal range of individuals of which it makes sense to say that it is "a P which is Q." I have more difficulty with what follows. For it is now claimed that the proposition 'some P is Q' (or all, or none, or most, etc.) is *about* either σ or τ. Sommers is thus offering an explication of the ordinary notion of 'about': a proposition involving two predicates is about everything in the range of significance of at least one of the predicates. Granting the explication for the moment, only one further assumption is needed, namely that whatever a proposition is about is included in the domain of discourse. Putting the pieces together we find then namely that either σ or τ, being the range of what the proposition is about, is included in $\sigma \cap \tau$, the domain of discourse; but this is just to say that either $\sigma \subset \tau$ or else $\tau \subset \sigma$.

But should we grant the suggested explication of 'about'; should we grant, for instance, that the proposition 'some men are snubnosed' is about the cat Jemima? As Jemima is in the range of significance of 'x is a man', that follows from the explication; it also follows that the proposition is not about the number 7. But why should the proposition 'some men are snubnosed' not be about the number 7 when it is about Jemima? It is hard to tell, and Sommers does not tell us. He tells us merely that what is explicated is one sense of 'about'. That must, in any event, be a non-standard sense.[75] Without further elucidation it is therefore difficult to grant the second premiss of the argument, where the word 'about' is a key element, namely the premiss that a proposition 'some P is Q' is about either $|P|$ or $|Q|$. Not understanding what the relevant sense of 'about' amounts to, neither do we understand what this

[75]Goodman (1961, p. 12) holds that "a statement absolutely about any class or classes is absolutely about each Boolean function of them," hence that 'all crows are black' is about the class of non-crows. But it is about the *class* of them, not about each of them; so even though Jemima is a non-crow, 'all crows are black' is not, on Goodman's account, about Jemima, but only about a certain class of things to which Jemima belongs.

premiss amounts to, hence neither can we grant the inference to the law of categorial inclusion. Sommers's purported proof of this law is therefore not conclusive.[76]

Another proof of the law of categorial inclusion is offered in Sommers (1971, p. 35). Also this proof seems to turn on obscurity as to precisely what an ordinary English word means, in this case the particle 'not'. Sommers (ibid. pp. 21–23) distinguishes two senses in which one can deny that a is P: firstly, one can say that a is in the range of significance, but not in the range of truth of P; secondly, one can say that a is outside the range of truth of P, be it within or without the range of significance of P.[77] Following a similar distinction drawn by Mannoury (1934, p. 333) the first sort of negation is sometimes called choice negation and the latter sort is called exclusion negation. Let us write 'a is $\neg P$' to indicate choice negation and 'a is $\complement P$' to indicate exclusion negation. Following Sommers, let us, for current purposes, say that P is a category term if its truth range coincides with its range of significance. Then it is obvious that if P is a category term, then 'a is $\neg P$' is either false or meaningless. Sommers now considers the following schema, which we may call the $\neg$-schema (pronounced 'neg-schema')

$$(\text{some } x \text{ is } y) \Rightarrow (\text{every } x \text{ is } y) \vee (\text{some } x \text{ is } \neg y) \vee (\text{every } y \text{ is } x) \vee (\text{some } y \text{ is } \neg x)$$

This would seem to be universally valid. If we now substitute category terms P and Q for x and y in the this schema such that 'some P is Q' is significant, then we see that the second and the fourth disjunct are false. Whence we get

If some P is Q, then every P is Q or every Q is P,

which is another formulation of the law of categorial inclusion. Hence we appear to have a proof of this law. But using the other notion of negation we have an alternative

[76] Both Nelson (1964, pp. 520–521) and De Sousa (1966, pp. 43–47) criticize Sommers's argument, but on different grounds; responses to these can be found in Sommers (1964) and Massie (1967).

[77] Aristotle *APr* I.46 urges that we must distinguish between asserting 'S is non-P' and denying that S is P; the denial that S is P entails that S is non-P, but the assertion that S is non-P does apparently not in general entail the denial that S is P ($51^{\text{b}}36$–$52^{\text{a}}14$). The point appears to be that you may for instance neither like nor dislike the relish of pineapple, but be indifferent to it, in which case it is true to deny that you like the relish of pineapple, but not to assert that you dislike it. Notice that if we express the denial of 'S is P' as '$\sim(S$ is $P)$', then Aristotle's distinction corresponds to Sommers's distinction only if $\sim$ is such that applied to a category mistake it yields the value true; that is, only if, $\sim$ in the significance logic of Goddard and Routley (1973) has the truth-table

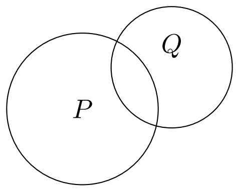
p	$\sim p$
t	f
n	t
f	t

This is the connective Goddard and Routley (ibid. p. 280) denote by '$\neg$'.

formulation of the schema, which we may call the C-schema:

$$(\text{some } x \text{ is } y) \Rightarrow (\text{every } x \text{ is } y) \vee (\text{some } x \text{ is } \mathsf{C}y) \vee (\text{every } y \text{ is } x) \vee (\text{some } y \text{ is } \mathsf{C}x).$$

This would also seem to be valid, in fact it follows from the $\neg$-schema, since 'some x is $\neg y$' entails 'some x is $\mathsf{C}y$'. If we substitute category terms P and Q for x and y in the C-schema, however, the law of categorial inclusion does not result, for it may well be true that some P is $\mathsf{C}Q$. We must therefore ask whether the $\neg$-schema is in fact universally valid; it may well be that it is only the weaker C-schema that is valid. Suppose there are category terms P and Q related as in this Venn diagram:



Substitution of P and Q into the C-schema yields a truth, since both 'some P is $\mathsf{C}Q$' and 'some Q is $\mathsf{C}P$' are true. But the substitution of P and Q into the $\neg$-schema yields a falsity, as one easily checks (the antecedent is true while each of the disjuncts in the consequent is false). The terms P and Q also falsify the law of categorial inclusion. Hence we have reason to accept the $\neg$-schema only if we have reason to accept the law of categorial inclusion. This attempted proof is therefore circular.

There is, finally, an indirect argument for the law of categorial inclusion presented in Sommers (1959). Its conclusion is that the law of categorial inclusion is "inviolable" (ibid. p. 175), that it cannot be disproved. A counterexample to the rule would have to present two predicates P and Q such that P can be meaningfully predicated of some but not all $|Q|$'s and Q meaningfully predicated of some but not all $|P|$'s. One could for instance suggest that 'expensive' can be predicated of some but not every thing which is |hard|, and 'hard' predicated of some but not every thing which is |expensive|: both can be predicated of chairs, while 'hard' can be predicated of problems but not of restaurants, and 'expensive' can be predicated of restaurants but not of chairs. The category terms |hard| and |expensive| would therefore seem to provide a counterexample to the law of categorial inclusion, for we have $|\text{hard}| \cap |\text{expensive}| \neq \emptyset$, but neither $|\text{hard}| \subset |\text{expensive}|$ nor $|\text{expensive}| \subset |\text{hard}|$. Sommers's argument is now that what such an instance shows is that one of the two predicates is homonymous. It is indeed not unreasonable to say that 'hard' does not mean the same in its application to problems as it does in its application to chairs. When we split the word into 'hard_1' and 'hard_2', with $|\text{hard}_1| \cap |\text{hard}_2| = \emptyset$, we shall have

$|\text{chair}| \subset |\text{hard}_1| \subset |\text{expensive}|$, whence the law of categorial inclusion is not violated after all. This strategy is available in all cases: one can always insist that a proposed counterexample to the law of categorial inclusion instead of disproving the law only shows that one of the predicates involved is homonymous. One could claim that the strategy would be unsuccessful if it forced us to make judgements of homonymy where our ordinary sense of language recognizes none. To this Sommers would most likely reply that our ordinary sense of language is often an inadequate standard in such judgements:[78] we may well feel that 'hard' is homonymous, but what does our ordinary sense of language tell us about 'being'? Sommers insists that we need a theory to guide us in deciding such matters.

2.2.2. *Types and homonymy.* Assuming the law of categorial inclusion Sommers (1965) converts the above strategy into a method for showing that a term is homonymous. Such methods had been presented already by Aristotle in *Topics* I.15. One of these methods involves what is one of the few obvious applications of the categories in the *Topics.* Aristotle says that one must consider the term in question with respect to the categories, "to see whether these are the same in all cases" ($102^{\mathrm{a}}3$–4). It is not clear from the text precisely what it is that Aristotle wants us to check: the category or categories of the term itself, or rather the category or categories of the subjects of which the term in question may be predicated?[79] The latter method certainly seems the most promising, for to apply the former method one would already have to know that the term signifies items of different categories, that it in effect exhibits what Russell called systematic or typical ambiguity (e.g. *PM* pp. 64–65). Knowing that would, however, seem to amount to knowledge that the term is in fact homonymous, which is what the method was meant to tell us.[80] In the famous examples of the homonymy of 'being' with respect to the categories (*Met* Δ.7 $1017^{\mathrm{a}}22$–27) it seems

[78] Cf. e.g. the discussion at Sommers (1963, pp. 338–343).

[79] Two recent books on homonymy in Aristotle appear to assume opposite answers: Shields (1999, pp. 52, 54–56) seems to read the passage as telling us to check the category of the term itself (or, closer to Shields's language, whether it signifies things of different categories in different predications); while Ward (2008, pp. 65–66) clearly assumes the second reading, that the method consists in checking whether the term can be applied to subjects of varying categories. In the course of the relevant passage, *Top* $107^{\mathrm{a}}2$–12, it seems in fact that Aristotle slides from the method of the first reading to that of the second reading. The term considered is 'good'. And according to the first part of the passage, 'good' as applied to food does not belong to the same category as it does when applied to a man: in the second case it signifies a quality, while in the first case it signifies what is productive of pleasure (*to poiētikon hēdonēs*), which presumably is a doing (*poiein*). Here it is thus the predicate 'good' that shifts category, while the subject in each case is a substance (I assume that food is indeed a substance). According to the latter part of the passage, however, 'good' is said to mean different things when applied to items of different categories, in particular to times and quantities. Here it is thus the subject that shifts category.

[80] Though see *Top* $152^{\mathrm{a}}38$–39, where it is clear that Aristotle asks us to check the category of the term in question, thus to apply the former method.

in any event to be the second method that is assumed: since being is said of items of different categories, 'being' is homonymous; it does, for instance, not mean the same to say of a substance that it has being as what it means to say of a quantity that it has being.

Thus Aristotle formulates, albeit ambiguously, in the *Topics*, and appears to rely in his argument for the homonymy of 'being' in *Metaphysics* Δ, on the rule that, if a term is predicated of two subjects belonging to different categories, then the term is homonymous. Sommers notes that the corresponding rule for types as ranges of significance has undesirable consequences.[81] It does have some desirable consequences, for instance that 'hard' is homonymous in its application to chairs and problems: chairs and problems belong to different types, so hard means one thing in its application to the one and another thing in its application to the other.[82] But the rule would also deem 'lasted an hour' homonymous in its application to headaches and lectures, assuming, as seems reasonable, that these are of different types; and that is an undesirable consequence, since I can very well say that my headache lasted as long as the lecture. In place of the rule apparently assumed by Aristotle Sommers (1965, pp. 264–266) therefore proposes the following:

> If there are three things a, b, and c such that P can be predicated of a and b, but not of c, while Q can be predicated of b and c, but not of a, then P does not mean the same when predicated of a as it does when predicated of b, or else Q does not mean the same when predicated of b as it does when predicated of c.

The validity of this rule follows from the law of categorial inclusion, as the three individuals a, b, and c would otherwise mess up the tree structure that the law entails. Namely, if the antecedent of the proposed rule obtains, but not the consequent, then we should have both $|P| \nsubseteq |Q|$ and $|Q| \nsubseteq |P|$, but also $b \in |P| \cap |Q|$, contradicting the law of categorial inclusion; hence, if the antecedent obtains, then so must the consequent obtain. The rule is in accordance with the examples just considered. The predicate 'hard' will be deemed homonymous over chairs and problems, as one sees by letting a be a chair and b and c be problems, and setting $P =$ hard and $Q =$ problem.

[81]Similar undesirable consequences of this rule were noted by Black (1944, pp. 237–240). The rule thus applied to ranges of significance is stated and accepted by Carnap (1928, § 30).

[82]This instance of homonymy may also be established by means of one of Aristotle's methods from *Top* I.15 ($107^{\mathrm{b}}13$–18), noting that we cannot say that a problem is harder than a chair.

On the other hand, since we presumably have

$$|\text{headache}| \subseteq |\text{lasts an hour}|$$
$$|\text{lecture}| \subseteq |\text{lasts an hour}|$$
$$|\text{headache}| \cap |\text{lecture}| = \emptyset$$

there will be no way of applying the rule to show that 'lasts an hour' is homonymous over headaches and lectures.

The consequent of Sommers's homonymy rule states that either the predicate P does not mean the same when predicated of a as it does when predicated of b, or else Q does not mean the same when predicated of b as it does when predicated of c. Here it is assumed that b is univocal in the two predications 'b is P' and 'b is Q'. If we drop this assumption a third disjunct must be added to the consequent: or b does not mean the same when P is predicated of it as it does when Q is predicated of it.[83] It is with this disjunct added that Sommers draws one of the more interesting consequences from his rule, namely the thesis that certain entities of ordinary life are what one may call categorially composite: they straddle several categories, and so must be composites of several mono-categorial entities. Switzerland is both mountainous and democratic; but while it makes sense to say of the Moon that it is mountainous, it does not make sense to say of it that it is democratic; and while it make sense to say of Viking society that it was democratic, it does not make sense to say of it that it was mountainous; since neither 'democratic' nor 'mountainous' appear to be homonymous in this argument, it follows that 'Switzerland' must be homonymous. In fact, in one of its applications 'Switzerland' must fall under the type |democratic| and in another of its applications it must fall under the type |mountainous|. In objectual terms, Switzerland is the kind of thing that is both |democratic| and |mountainous|. Since these are disjunct α-types, it follows that the type of Switzerland is composite; it is a composite of a geographical region and a society and perhaps other things besides, and therefore belong to several categories. A similar argument of more philosophical consequence concerns the type of men.[84] Bob is 1.80 m tall and worried; but while it makes sense to say of the Eiffel Tower that it is 1.80 tall, it does not make sense to say of it that is worried; and while it makes sense to say of a mind, or someone's mind, that it is worried, it does not make sense to say of it that it is 1.80 m tall; since neither '1.80 m tall' nor 'worried' appear to be homonymous in this argument, it follows that 'Bob' must be homonymous. Bob is the kind of thing that is both |1.80 m tall| and |worried|; and since there are things, e.g. the Eiffel Tower, that are |1.80 m tall| but

[83] The rule in that form is implicitly assumed at Sommers (1964, p. 524).
[84] Both arguments are found in Sommers (1971).

not |worried|, and things, e.g. Bob's mind, that are |worried| but not |1.80 m tall|, it follows from the law of categorial inclusion that $|\text{worried}| \cap |1.80 \text{ m tall}| = \emptyset$. Whence Bob, like any other man, straddles types, he is the composite of a mind and a body, or, perhaps better, of a |mind| and a |body|.

2.2.3. *Sommers and simple types.* There are two conspicuous contrasts distinguishing the structure of Sommers's types from the simple type hierarchy. Firstly, in Sommers's type theory the domain of individuals is divided into several types; there is not just the one universe of individuals, but a collection of habitats known as α-types. Secondly, in Sommers's theory the only simple types distinguished are individuals and predicates; there are for instance no relations and no second-order predicates such as the quantifiers. Where the simple theory of types is occupied with building the tower of predicates and relations, Sommers's theory is occupied with cultivating the ground of individuals. It is natural to ask whether the efforts of the two theories may be combined. The answer is yes: their joint effort is just a simple type structure built over all α-types as ground types. Instead of the one type ι of individuals in simple type theory there are then the ground types $\iota_1, \ldots, \iota_n$. As α-types, these need not be mutually exclusive, but they do satisfy the law of categorial inclusion; that is to say, any two ι_k's are either disjoint or else one is included in the other. Let us assume that there is a type o of propositions (which may or may not be one of the ι_k's). Following Frege, we may regard functions of type $(\iota_k)o$ as predicates whose range of significance is ι_k. We call this type hierarchy, where the type ι has been split up into several ι_k's, a many-sorted simple type hierarchy.

Let us recall Sommers's notion of an A-type. An A-type is the type of predicates whose range of significance is some α-type τ; in set-theoretical notation it is for any α-type ι_k, the type $\{\varphi : |\varphi| = \iota_k\}$. A-types are thus just the types $(\iota_k)o$. The definition of A-types is the only step Sommers's theory makes into the simple type hierarchy. In particular, Sommers offers no account of relations. That is clearly a lacuna in the theory, since relations, at least binary ones, are just as prominent in natural language as are predicates, and it was Sommers's stated aim to articulate the theory of types that governs natural language (e.g. Sommers, 1963, p. 327). That relations have ranges of significance just as predicates do is clear from the fact that while 'John admires the rainbow' is a fine sentence, 'the rainbow admires John' is nonsense. Since a binary relation has two argument places and these may have different ranges of significance, we must distinguish the left and the right range of significance of a relation: the range of arguments that are admitted in the first argument place and the range of arguments that are admitted in the second argument place.[85] It is natural to assume that any left or right range of significance coincides with an α-type. Binary relations are thus of type $(\iota_j)(\iota_k)o$. Since α-types, and therefore the left and right ranges of significance of relations, stand in relations of inclusion and disjointedness, one can classify relations accordingly: if ι is the left and ι' the right domain of a relation we have either $\iota = \iota'$, $\iota \subsetneq \iota'$, $\iota' \subsetneq \iota$, or $\iota \cap \iota' = \emptyset$.[86] All of this can be generalized to k-ary relations of individuals.

2.3. Ranges of significance and categories. Neither Frege nor Russell associated types with Aristotelian categories. The first to do so quite explicitly was Ryle,[87] who opened his paper *Categories* (1938) with the announcement that

> Doctrines of categories and theories of types are explorations in the same field.

In the opening pages of his paper Ryle explains how Aristotle's categories may be viewed as the value ranges of a certain propositional function, namely 'Socrates is x'. He does so by appealing to the account of the origin of the categories considered at the the end of the first chapter above, according to which a category collects the range of appropriate answers to a 'wh'-question that may be asked of a primary substance, typically a man. As noted by Cohen (1929), and as spelled out in more detail by Carnap (1934, § 76), such a 'wh'-question may be viewed as a propositional function, and the range of appropriate answers to it as the range of significance of that propositional function. Thus, an appropriate answer to the question, Where is Socrates?, is an item in the range of significance of the function 'Socrates is x_{where}', and an appropriate answer to the question, What is Socrates?, is an item in the range of significance of 'Socrates is x_{what}'; moreover, the resulting propositions are true precisely for the correct answers to the respective questions. Hence, to the various questions, Socrates is what? Socrates is of what quality? Socrates is of what quantity? Socrates is where? Socrates is when? Socrates is doing what?, we may assign the propositional functions, 'Socrates is x_{what}', 'Socrates is x_{quality}', 'Socrates is x_{quantity}', 'Socrates is x_{where}', 'Socrates is x_{when}', 'Socrates is x_{doing}'. When carried out for

[85] Hence we assume that the relations are what Goddard (1966, p. 155) called homogeneous: if R has ι_j as left range of significance and ι_k as right range of significance then Rab is significant for any $a \in \iota_j$ and any $b \in \iota_k$.

[86] The classification is developed by Goddard (1966, pp. 157–162).

[87] The qualification "quite explicitly" is required, for the association is also made, though more implicitly, by Leśniewski (1929, p. 14). Both Leśniewski (cf. ibid.) and Ryle (cf. Ryle, 1971a, pp. 8–9) had studied Husserl's Fourth Logical Investigation (esp. its § 10), where categories, or rather meaning categories, are connected with the notions of sense and nonsense (see section 1.3 above), and so indirectly with the notion of range of significance.

all the categories this correspondence yields altogether ten propositional functions, and so ten types. We noted in section 5.2.2 of chapter 1 that the correspondence between questions and categories breaks down for the category of relatives; but that fact should not prevent us from appreciating the underlying idea that the categories are ranges of appropriate answers to questions: one could concoct a question such as 'Socrates is what relative to something else?' as the question whose range of appropriate answers constitute the category of relatives. And assuming this idea, assuming that any category collects the range of appropriate answers to a certain question, it follows that Aristotle's categories are also the ranges of significance of certain special propositional functions.

At this point Ryle criticizes Aristotle (cf. Ryle, 1938, pp. 195–196). It is not only the ranges of significance of these special propositional functions that are categories: any propositional function, or any "sentence frame," gives rise to a category. Not only propositional functions of the form 'Socrates is x_{where}' and 'Socrates is x_{doing}' must be taken into account, but also such propositional functions as 'I am the man who x' and 'x implies that tomorrow is Tuesday'. In the range of significance of the first is a phrase such as 'visited The Hague yesterday', which we may take to fall into the Aristotelian category of doing; but in the range of significance of the second there is a phrase such as 'today's being Saturday', for which there is no place in Aristotle's category scheme, as it is not a thing said without combination (cf. our discussion towards end of section 1 of chapter 1). This suggests that Ryle's set of categories is more numerous than that of Aristotle. According to Ryle, the number of categories is in fact indefinite:

> Scholasticism is the belief in some decalogue of categories, but I know of no grounds for this belief.[88]
>
> The truth is that there are not just two or just ten different logical *métiers* open to the terms or concepts we employ in ordinary and technical discourse, there are indefinitely many such different *métiers* and indefinitely many dimensions of these differences.[89]

There are indefinitely many types in each of the simple and ramified hierarchies; but that is an organized indefiniteness, controlled by an inductive definition. The realm of Rylean types, on the other hand, is open-ended in the sense that it cannot be circumscribed by a definition. One may start, for instance, with Aristotle's categories, but find that one needs a *métier* for 'today's being Saturday'. Next one may find

[88]Ryle (1938, p. 200).
[89]Ryle (1954, p. 10)

that the range of significance of the propositional function 'x likes a Gershwin tune' coincides neither with any of Aristotle's *métiers* nor with that containing 'today's being Saturday', hence a new *métier* would have to be introduced. And thus it would continue.

A question then arises of how this account fits with the idea that categories, however one explains them, are concepts of the highest generality. Smart (1953) asked whether Ryle's notion of difference of type, "if pushed to the limit, may not show every expression to be of a different logical category from every other," that is, whether every entity has its own category. For the realm of numbers Smart pointed to propositional functions of the form $\frac{m}{n-x} = \frac{k}{l}$, which are undefined for $n = x$. On Ryle's definition of a category difference these propositional functions would show that each number belongs to a separate category. One may object to this example that it exploits the rather vague and wobbly character of the notion of significance. We might feel that putting $x = n$ yields nonsense of a very different order from what results by putting $x =$ Julius Caesar; hence it may not be so clear what the range of significance of $\frac{m}{n-x} = \frac{k}{l}$ includes and what it excludes. But there are other examples that appear to show that categories on Ryle's account can get quite specific, perhaps too specific. There is a range of significance containing only literary works, namely 'x is divided into 24 chapters'; the range of significance of 'x has 24 floors' includes only buildings; and one can think of many other equally specific ranges of significance. The question is whether we would want to call these categories, since they all can be taken to fall under a higher concept. Both books and buildings fall under the concept of a cultural object, say; and both fall in the range of significance of 'x took a long while to complete'. Would anyone who followed Fowler's guidance not to use the word 'category' unless he is prepared to state "(1) that he does not mean *class*, & (2) that he knows the difference between the two"[90] maintain that there is a category of literary works or a category of buildings?

2.3.1. *Westerhoff on categories.* Anyone who defines categories as ranges of significance but does not regard all ranges of significance as categories faces what Westerhoff (2005, p. 35) calls the cut-off point problem: the problem of specifying what characterizes those ranges of significance that deserve the title of category. Westerhoff responds to this problem against the background of his own account of categories, an outline of which may be inserted here.

In the opening of his book Westerhoff announces that he will deal only with the "metaontological" issues of the nature and structure of the categories, and offer no

[90]See *A Dictionary of Modern English Usage*, lemma 'category'.

special theory of categories (p. 20); but his metaontological considerations are conducted within a framework that takes states of affairs and sets as primitives, and the constituents of states of affairs, whatever they be, as derived; and a relatively large part of the book is devoted to developing this special ontological theory. Westerhoff's more general theory of categories is the following (cf. esp. § 48). There is a domain of individuals (or, constituents of states of affairs). This domain is partitioned into ranges of significance. Ranges of significance go together in groups to form wider ranges of significance, namely the union of the group. These wider ranges of significance in their turn go together in groups to form even wider ranges of significance; and so on upwards. Since the initial ranges are disjoint and new ranges are obtained by taking unions, the resulting structure is that of a tree satisfying Sommers's law of categorial inclusion. The tree has only finitely long paths, and it may or may not have a unique top node. The cut-off point problem can now be vividly formulated as follows. Walking along a path in the tree that starts at a most specific range of significance we shall after a certin number of steps reach a stage after which all nodes in the path are categories—what is this critical stage? To say that categories are found only at the end of the path, that is, at top nodes of the tree, solves the cut-off point problem; but Westerhoff wishes to countenance the possibility that some categories involve others—in extensional terms, that some categories are included in others.[91]

Westerhoff proposes to solve the cut-off point problem by saying that categories are those ranges of significance from which all others can be "constructed" (ibid. § 41). As instances of construction he mentions the definition of rational numbers as equivalence classes of ordered pairs of natural numbers, and the definition of events as triples of individuals, properties, and time-instances (§ 49). Westerhoff does not, however, offer anything like a general account of what a construction is; and what he does say about the notion does not suffice to settle the following questions. (1) What is the category of a construct? There needs to be some general account saying that if a range of significance is constructed in such and such a way, then it belongs to such and such a category. Westerhoff deals with this question only for the construct of triples of individuals, properties, and time-instances, saying that it belongs to the category of sets (p. 122–123); this is so, presumably, since triples themselves are sets. If all constructs are set-theoretical, the precedent of this example would rule that everything is a set, a tenet that trivializes the theory of categories. If, on the other hand, some constructs are not set-theoretical, the example of events, the only example provided, does not tell us how to assign categories to them. (2) How do we know that

[91]As is the case, for instance, in the theory of categories defended by Chisholm (1996).

a construct is a range of significance? It is assumed that we have a tree of ranges of significance, a tree in which all ranges of significance find their place. A construction is the construction of a node in this tree, taking various higher nodes and yielding a lower node. With no clear conception of what a construction is, neither can we be convinced that a construct is a node in the tree, i.e. a range of significance. It is for instance not clear in which sense a triple of individuals, properties, and time-events is a range of significance. (3) Even if the two foregoing questions were successfully met, and we had been shown that a construct is a range of significance and had been offered some general account of which category a construct belongs to, there would still be the question of whether the constructed range of significance coincided with the target of the construction. There is a range of significance of events (p. 121); triples of individuals, properties, and time-instances are meant to construct this range; but how do we know that the construction succeeds, that the constructed range of significance coincides with the range of events? Indeed, how do we know that the category of the triples, namely the category of sets, includes the range of events? It is not evident that it does. (4) There are also questions one could ask regarding the constructors, in particular which ranges of significance they belong to. Westerhoff seems to assume that there is a range of significance of sets, while set-formation is a constructor. He also takes mereological fusion to be a constructor (p. 125); is there then, by analogy with the case of sets, a corresponding category of mereological sums?

The idea that categories are the concepts from which all other concepts can be constructed is in itself quite reasonable. This was perhaps what Kant had in mind when he called his categories *Stammbegriffe* (KrV A81/B107; cf. p. 53 above). It was presumably what Husserl had in mind when he characterized what he called categories as the primitive concepts of various ontologies.[92] And those who are concerned with "reducing" certain notions to other notions may reasonably call the notions not reducible in their theory the categories of the theory. As the unanswered questions in the previous paragraph suggests, however, it is difficult to combine this idea with the the idea that categories are ranges of significance, and even more so if the ranges of significance are to exhibit a rigid tree-structure. What lacks in Westerhoff's theory, in particular, is a more detailed account of the notion of construction. From what he says about this notion it is not clear whether constructs can be assigned a place at all in the tree-structure of ranges of significance; and provided that they can, whether they then are assigned the right place. Until a more detailed account has

[92]See Husserl (1900, § 67), *LU* III § 12, Ideen §§ 10, 16. For more on Husserl's categories, see section 6.2 below.

been provided one cannot say that Westerhoff has succeeded in solving the cut-off point problem.

3. Category mistakes

As already Ryle (1938) acknowledged in ending his paper with the question, "But what are the tests of absurdity?" the definition of types as ranges of significance rests quite heavily on the notion of significance. In this section it is argued that it rests too heavily on that notion; one cannot build a rigorous theory of types simply on the basis of intuitions of significance. It is further argued that the solution is not to scrap the notion of the range of significance of a predicate altogether; that notion is still needed, and attempts to do away with it are unsuccessful. We first consider "the field of nonsense" and situate category mistakes within this field.

3.1. A taxonomy of nonsense. A radical sort of nonsense is what one may call articulate noise, noise that is apt for being transcribed by means of an alphabet. This is the stuff that so-called sound poetry is built of. 'Fümms bö wö tää zää Uu/ pögiff kwii Ee', the first strophe of Kurt Schwitters's famous *Ursonate*,[93] is an instance of articulate noise. Articulate noise is, I think, also what the ancients illustrated by the string *blituri*.[94] We call this articulate noise, and not mere noise, since for vocal sound to be classified as nonsense it must bear traces of language, and, as already Aristotle noted, articulation is a characteristic that sets language apart from growls, hisses, bleats, etc.[95] There are probably other factors besides articulation that must be in place for noise to count as nonsense; in particular, the context in which something we would call nonsense occurs must be such that one expects to find the expression of sense. Nonsense is not the mere absence of sense, but its absence in contexts where one expects to find sense. We should not count as nonsense the recitation of a certain melody, even though what is then uttered is the gibberish dam–shubi–dam–ooh–ooh…

Articulate noise should be distinguished from the sort of nonsense manifested in *Jabberwocky.* 'The slithy toves did gyre and gimble in the wabe' is not a phrase of

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[93] This strophe, save a few vowels, in fact originates in one of Raoul Hausmann's "poster poems." Renditions of the *Ursonate* as well as of Hausmann's sound poetry can be found at
`http://www.ubu.com/sound/` See Scholz (1989) for an instructive study of the genre.

[94] Ammonius (*in Int* 31.14) remarks that *blituri* is both meaningless and articulate. The illustration may stem from the Stoics (*DL* VII.57). Other strings used for the same purpose were *knax* and *skindapsos*; see Blank (1996, p. 148) and Meier-Oeser (1996).

[95] See in particular the distinction between *psofos*, *phōnē*, and *dialektos* at *Historia Animalium* IV.9 ($535^{a}29$–$536^{b}24$), on which see Ax (1978); and, furthermore, the connection between articulation and being significant by convention alluded to at *Int* $16^{a}26$–29, on which see e.g. Ammonius *in Int* 29.31–31.33.

English, since the words 'slithy', 'toves', and 'wabe' are not words of English but Carroll's own invention.[96] Yet since they accord with English phonology they are "potential" English words;[97] whence one can meaningfully ask whether something is a happy translation of these words, or of the whole of Carroll's poem, into another language, for the translations should be potential (but not actual) words of the target language.[98] By contrast, it is not clear what it would mean to translate the *Ursonate* into English or any other language; and it is a fact that translators tend to leave *blituri*, as well as other instances of articulate noise such as 'Jubjub' from the *Jabberwocky*, untranslated. Translatability could therefore be said to be a characteristic that distinguishes *Jabberwocky*-style nonsense from articulate noise.

What no doubt contributes to the translatability of the nonsense words of *Jabberwocky* is their manifestation of a grammatical role. In general, grammatical roles are gleaned not only from the morphology of the words, but also from the syntax of the sentence in which they occur—which of course in turn may be deciphered on the basis of the morphology of the words, so there is a two-way interaction here. 'Toves', for instance, can be seen to have the role of grammatical subject, whence its translation must have the form of a plural noun in the target language, and not that of a third person singular verb. Grammatical roles are obvious in Carnap's 'Piroten karulieren elatisch' (Carnap, 1934, § 1), another instance of *Jabberwocky*-style nonsense: neither word in this sentence is German, but from the morphology of the words and the syntax of the sentence it is clear that 'Piroten' is a plural noun, that 'karulieren' is a plural present verb, and that 'elatisch' is an adverb (though I cannot quite say why this cannot be the object of the verb; perhaps it has to do with the lack of articles); whence this sentence has quite appropriately been translated into English as 'Pirots carulize elatically'.

The source of the nonsensicality of articulate noise and *Jabberwocky*-style nonsense is one or more words that themselves are nonsense. Carnap (1932, p. 220) distinguishes nonsense having that source from nonsense whose source is syntax. A sentence may contain only genuine words but nevertheless fail to make sense owing

[96]Carroll in his *Mischmasch* glossed 'gyre' as 'to scratch like a dog', but there is in fact an English word 'gyre', stemming from the Greek *gyros*, a ring or circle, whence Humpty Dumpty's explanation 'to go round and round like a gyroscope'. 'Gimble' is explained by Humpty Dumpty as 'to make holes like a gimblet', and so is close, if not identical, in meaning to the verb 'gimlet', "to pierce as or with a gimlet" (*OED*). Thus *Le Jaseroque* renders these words as *gyrer* and *vriller*, and Scott's *Der Jammerwoch* renders them as *wirren* and *wimmeln*, all of which are actual words. Translations of *Jabberwocky* can be found at `http://www76.pair.com/keithlim/jabberwocky/translations/index.html`.

[97]For this notion, and its relevance to *Jabberwocky*, see Lyons (1968, pp. 119–120).

[98]It would be misleading if the translated word was an actual word of the target language; for that reason Scott's 'schlicht' may not be the happiest translation of 'slithy'.

to how these words have been put together. Carnap distinguishes two cases of such syntactic nonsense (ibid. p. 227), what we shall call word salad and category mistakes respectively.[99]

An example of word salad is Carnap's 'Caesar is and', Husserl's 'king but or similar and' (*LU* IV § 14),[100] or indeed most strings of words arbitrarily strung together, as 'tree car however green however the the'.[101] For a string to qualify as word salad the elements it strings together should indeed be words. Hence a piece of articulate noise, although it could very well be called a salad, is not word salad, since its elements are not words. The notion of word salad is intuitively clear, but it is not straightforward how one should characterize it. Drawing upon Ajdukiewicz's definition of syntactic connection, I suggest the following characterization.[102] Word salad of a language L are strings of words of L that do not admit of an analysis into function and argument or arguments. As we saw in section 1.1 above, function and argument are more general notions than predicate and subject, so general in fact that any sentence, not only those of subject–predicate form, can be analyzed into function and argument or arguments. Hence there are no sentences that are not thus analyzable, as was noted by Frege (1892a, pp. 204–205). Not being analyzable into function and argument is therefore a necessary condition for a string of English words to be word salad. It is not a sufficient condition for being a *significant* sentence of English, since category mistakes are both thus analyzable and non-significant (or so I shall maintain); but it does not disturb one's ordinary sense of language to define a sentence of English as a string of English words that can be analyzed into function and argument. Not being thus analyzable is thus both a necessary and a sufficient condition on a string of words of L for being a piece of word salad of L.

[99]The term 'word salad', or its German twin *Wortsalat*, appears to have originated in psychiatry; the *OED* records an English use from 1904. For 'category mistake', see below.

[100]What Husserl called *Unsinn* is what we are calling word salad. He contrasted this with *Widersinn*, countersense, the instances of which are a priori falsehoods such as 'all triangles have four vertices' and 'some circles are square' and descriptions known a priori not to have any reference, such as 'square circle' or 'wooden iron'; see esp. *LU* IV § 12. As far as I know, it is only in the *Formal and Transcendental Logic* (1929) that Husserl recognizes the phenomenon of category mistake; it is clear from *LU* IV § 10 (p. 327) that Husserl regards category mistakes neither as nonsense nor as countersense.

[101]According to Magidor (2009b) it is a "dogma" that word salad of a kind with 'Caesar is and' (obtained from a sentence by substituting one expression with another expression of a different grammatical category) is meaningless. But her arguments are all destructive, of the type 'such and such semantic theories fail to provide a reason why word salad is meaningless'. Without a constructive account of what meaning these instances of word salad could have, Magidor's arguments, to the extent that they are successful, rather show that there is something unsound about the semantic theories she considers (or perhaps that it is a presupposition of these theories that word salad is meaningless).

[102]Cf. Ajdukiewicz (1935, p. 11).

Analyzability into function and argument shows the string to enjoy what we may call linguistic unity. Linguistic unity is the unity of a linguistic item that gives it the character of a sentence.[103] Having this character does not imply that the thing is a significant sentence, as the case of category mistakes shows. A string may, in fact, have this character yet fail to be a sentence, namely on account of its containing nonsense words. Thus, I would claim that the following string of nonsense enjoys linguistic unity:

$$\overbrace{\text{mome raths}}^{\text{argument}}\ \overbrace{\text{outgribe}}^{\text{function}}$$

There is no function in the objective sense signified by 'outgribe', but nevertheless it seems alright to assign to this word the role of function, and to 'mome raths' the role of argument, in this piece of nonsense. That articulate noise looks like salad is then to be explained by the fact that it is not analyzable into function and argument; since *Jabberwocky*-style is so analyzable, this is another point of difference between these sorts of nonsense.

The term 'category mistake' was introduced by Ryle in his book *The Concept of Mind* (1949).[104] The idea is present in some earlier works of Ryle, but it is only in that book that it is given the name 'category mistake'.[105] Ryle glosses the term as the allocation of concepts to logical types to which they do not belong (ibid. p. 17), but the idea is officially explained by means of a number of examples (ibid. p. 16). It is clear from those examples, as well as from the use Ryle makes of the notion, that a category mistake for him is an opinion arising from a misapprehension of categorial distinctions, or in Ryle's own words, "the practice of operating with an idea as if it belonged to a certain category," when in fact it belongs to another (Ryle, 1945b). Thus, if I were to give an inventory of my wallet and I counted not only the coins in it but also the obverses and the reverses, then I would be committing a category mistake, for I would then be taking the piece of money to be of the same category as the engravings. Likewise Ryle argues in his book that we commit a category mistake if we think that by describing a certain behaviour as intelligent or a certain action as volitional we are signaling the occurrence of such and such states of mind assumed to be separate, or indeed of a kind different, from that behaviour or action. In this dissertation the term 'category mistake' will have a more special meaning. A category mistake for us is a sentence of subject–predicate form whose subject is

[103] Cf. the so-called problem of the unity of proposition; for a detailed discussion and a non-Fregean solution, see Gaskin (2008).

[104] Perhaps owing to the wide appeal of this book, the term is, as one sees by searching for it on the internet, now in wide use also outside philosophy.

[105] Ryle (1945a) employs instead 'type-mistake' and Ryle (1945b) 'type-confusion'.

not in the range of significance of the predicate, as in Carnap's example 'Caesar is a prime number' (Carnap, 1932, p. 227).[106] This is a more special meaning of the term 'category mistake', for if I commit myself to the truth of e.g. 'Caesar is a prime number', then I commit a category mistake in Ryle's sense, but as the example with the coin shows, category mistakes in Ryle's sense need not involve a category mistake in the sense assumed here. We may restrict the scope of our category mistakes to sentences with a singular subject, for we are interested in the notion only to the extent that it provides a criterion for deciding whether a certain item belongs to a certain range of significance; that is, we are interested in the notion only to the extent that it helps us in deciding questions of the form 'is "s is P" significant?', where s is a singular subject.[107]

What we here call category mistakes are known in the literature under various other appellations. Carnap (1928, § 30), in what appears to be the first definition of the notion, calls them "confusions of spheres" (*Sphärenvermengungen*);[108] Husserl (1929, § 90) does not give these sentences a special name, but he does take them up for discussion, describing them as "contensively senseless" (*inhaltlich sinnlos*);[109] Ryle himself used 'type trespasses' (Ryle, 1938, p. 200); Drange (1966), in the first monograph on the topic, called them type crossings; Lappin (1981), in another monograph on the topic, called them, following Thomason (1972), sortally incorrect sentences. But 'category mistake' appears to be what philosophers nowadays would call such sentences; they are so called by Magidor (2013), the only study of the topic in recent

[106]According to "the alternative view" of nonsense sketched by Diamond (1981), who also calls this the Frege–Wittgenstein view of nonsense, there is no or very little difference between category mistakes and *Jabberwocky*-style nonsense: for if a string of words has the unity of a sentence and is nevertheless nonsense, then none of these words are significant in the context of that string. In what follows I adopt what Diamond calls the natural view of nonsense, according to which a nonsignificant sentence may be composed of significant parts.

[107]The question of how to extend the notion to other forms of sentence is considered by Drange (1966, pp. 93–107). In the setting of formal logic this is the question of how to assign the value 'insignificant' (besides 'true' and 'false') to well-formed formulas; for an extensive treatment of which see Goddard and Routley (1973, Part II); a particular solution is given in Lappin (1981, p. 69), though note that the rule given there for quantified formulas is not in accordance with the rule given for the boolean formulas (according to Lappin's rules, for $A \vee B$ to be insignificant it is sufficient that A or B is insignificant, while for $(\exists x \in C)Ax$ to be insignificant it is necessary that $A[c/x]$ is insignificant for all $c \in C$; for finite domains these conditions contradict each other).

[108]It appears from a remark of Ewing (1937, p. 360) that the phenomenon of category mistakes was well-known to Oxonians in the 1920's: "the most usual example cited lately at Cambridge is 'quadratic equations go to race-meetings', the example in my days at Oxford was 'Virtue is a fire shovel'." Ewing completed his studies at Oxford in 1923 and went elsewhere afterwards (cf. e.g. `http://archiveshub.ac.uk/data/gb133-ace`). Negations of category mistakes were given by Hegel (*Wissenschaft der Logik* Band II, p. 98) and Bosanquet (1911, Bk. I p. 282) as examples of so-called infinite judgements; Hegel describes them as true but *widersinnig* (is that an autological predicate?), Bosanquet describes them as unmeaning.

[109]For an instructive discussion, see De Palma (2008).

years; and this is not a recent corruption, for the term 'category mistake' occurs in this sense in Sommers (1963, p. 329), and can indeed be found as early as in Wang (1954, p. 259).

3.2. Types and category mistakes. Within each sort of nonsense distinguished above one might discern various degrees of nonsensicality. A balanced piece of articulate noise like 'jolifanta bambla o falli bambla' is somewhat less forbidding than the consonant soup of 'hjckrrh'. This line, the first line of Hugo Ball's *Karawane*, comes in fact rather close to *Jabberwocky*-style nonsense. The lesson would seem to be that as articulate noise gets less and less forbidding, and approaches the phonology of a familiar language, it may take on the character of *Jabberwocky*-style nonsense. Differing degrees of nonsensicality recognized within the class of word salad are what Chomsky (1964) and Katz (1964) called "degrees of grammaticalness." Carnap's example 'Caesar is and' is less of a jumble than the melange of 'tree car however green however the the', hence it has a higher degree of grammaticalness than the latter. A yet higher degree of grammaticalness is enjoyed by 'Socrates is identical' (TLP 5.473) or 'Draco frightens', strings that become sentences simply by appending a phrase to them; while innocent grammatical errors such as 'Caesar swim' and 'man crosses river' has a still higher degree.

That also category mistakes admit of degrees of nonsensicality is of more relevance for our purposes. There are clear examples of category mistakes such as 'the number 5 is malodorous', 'my headache is blue' and 'the cat Jemima is divided into 24 chapters'. But it is not clear what one should say about, for instance, 'my brother is pregnant' or 'this sonnet is divided into 24 chapters'. It seems that one cannot settle whether these are category mistakes without having made a theoretical decision regarding the ranges of significance of 'pregnant' and 'divided into 24 chapters'. The existence of such indefinite cases of category mistakes does not entail that the concept is vacuous. The existence of clear cases such as 'the number 5 is malodorous' is sufficient to show its reality. That a category mistake can be made sense of by a metaphorical reading proves nothing to the contrary, precisely because the required reading is metaphorical and not literal.[110] What the existence of indefinite cases does entail is that one cannot employ a naive conception of category mistakes for the definition of a type as the range of significance of a predicate of English. An indefinite case is precisely the case of a predicate whose range of significance is not a well-defined range. This problem is recognized by Westerhoff (2005, p. 204), but he deals with it simply by making the

[110] See Lappin (1981, ch. 8) and Magidor (2013, pp. 66–74) for some remarks on the relevance of category mistakes to the theory of metaphor.

assumption to the contrary, namely that we do in fact know for any sentence whether it is a category mistake or not. A similar assumption is made by Lappin (1981, p. 94). He argues that an ideal speaker of the language—the existence of which, according to Lappin, in any case is assumed in the theory of syntax—is able to decide all borderline cases. But the indefiniteness of a predicate's range of significance does not disappear simply by assuming, as Westerhoff does, that it does not exist. And the claim that the borderlines may be definite to an ideal speaker is of little help to the ordinary man who aims at recovering the type theory implicit in English.

The conclusion must be that a theory of types cannot depend exclusively on the notion of a category mistake. We see then the use for constraints such as the law of categorial inclusion: they might guide the type theorist where the notion of a category mistake leaves him in the dark. A theory of types in the tradition of Ryle and Sommers is thus best regarded not as the uncovering of the type theory implicit in English, as a systematization of the knowledge that supports judgements of category mistakenness; rather it must be seen as a refinement of this knowledge, as a regimentation of the notion of a range of significance that we possess as part of our understanding of the language. The type theory must accord with that understanding as far as it goes and must supplement it in cases outside its reach. If on the basis of that understanding one judges 'the number 5 is malodorous' to be a category mistake, then the type theory should allot numbers and malodorous things to different types. If on the same basis we are not able to determine whether 'my brother is pregnant' is a category mistake, then the theory, when fully developed, should determine it for us.

3.2.1. *Frege and Quine on the universal range of significance.* A type theory in this sense is thus a development of type distinctions already found in natural language. There is a venerable tradition, comprising both Frege and Quine, of dealing with such type distinctions among predicates by, in effect, erasing them altogether. Frege insisted that each predicate—as is clear from *Gg* II § 57 he has in mind predicates of exact sciences—must be defined for all objects (*FB* pp. 19–20). For example the predicate 'prime' should be defined not only for the integers, but for the Sun and for all other heavenly bodies, indeed for all objects whatsoever. Frege's stated reason for requiring this is that otherwise "it would be impossible to lay down logical laws about them," i.e. about the predicates (loc. cit.). The point, I take it, is that if F is not defined for a, then neither is $\neg F$ defined for a, whence neither can we say that the law of non-contradiction holds for F, as for instance $\neg(Fa \wedge \neg Fa)$ will then not be true. This argument rests on what I think is an unfounded assumption, namely that a logical law should have no presuppositions. For according to a defender of type distinctions among predicates the correct statement of the law of non-contradiction includes the statement of the presupposition that F is a predicate and that a is in the range of significance of F. With this presupposition added the law of non-contradiction will hold even if predicates are not assumed to have the universal range of significance. It is this presupposition, therefore, that has to be the point of Frege's attack. It will, however, be difficult for him to attack it, for one can insist that even Frege makes a presupposition of this kind: in his statement of the law of non-contradiction it is presupposed that F is a predicate and that F is defined for a, namely that a is an object. One can insist, that is to say, that the presupposition that a defender of type distinctions believes should accompany the statement of logical laws is in fact made also by Frege in his statement of logical laws; one may suggest, for instance, that this presupposition is part of the notational conventions of Frege's ideography.

The requirement on a predicate of being defined for all objects is for Frege closely related, if not identical, to the requirement that a predicate should have what he calls sharp boundaries (*scharfe Grenzen*). In the second volume of the *Grundgesetze* this requirement is called the "principle of completeness" of definitions (*Gg* II §§ 56–65): the definition of a predicate (or relation, or a function of any kind) should cover all cases. In that discussion Frege claims that this requirement is just what is laid down in the law of the excluded middle (*Gg* II § 56): "a given object Δ either falls under the concept Φ or it does not fall under it: *tertium non datur*." A defender of type distinctions may agree, but remark that there are two ways in which an object could fail to fall under a concept: (1) the predicate applied to the object yields a false proposition, or (2) it fails to yield a proposition at all, that is, the sentence is a category mistake. The defender of type distinctions could, in other words, insist that the proper formulation of Frege's principle of completeness is given by the statement of the law of the excluded middle with the presupposition added that Δ lies in the range of significance of Φ. He would accordingly say that for a predicate to have sharp boundaries it is only required that it be definitely determined for which arguments the predicate is true, for which it is false, and for which it is undefined. Thus the predicate 'prime' has sharp boundaries even though it is not defined for the Sun.

Frege's main concern in the cited passage (*Gg* II §§ 56–65) is to argue against the legitimacy of what he calls piecemeal defining (*stückweises Definieren*). We should not define addition first for the natural numbers, then for the integers, then for the rational numbers, then for the real numbers, then for the complex numbers, then for etc., but at once for all objects in general. It is clear that to someone who accepts Frege's principle of completeness piecemeal definition is faulty; but Frege appears also to want the purported fault of piecemeal defining to support the principle of completeness.

Someone arguing against Frege's principle of completeness should therefore try to show that piecemeal defining is quite alright. It is quite alright. Frege is correct to note, what perhaps few had noted before him, that in piecemeal definition each piece in effect defines a novel operation, and so to avoid confusion these should each be given a different designation (*Gg* II § 57). But anyone who is not already committed to the law of completeness would certainly say that Frege is mistaken in holding that the procedure of piecemeal defining is faulty. What is required in the definition of a function or predicate is that it has a definite domain; but that domain need not be the universal domain. That is what the defender of type distinctions would say and that, I think, is what a mathematician reflecting on the methodology of his science would say. For there are important generalizations in mathematics that could not be made if one followed Frege's strictures. In algebraic number theory the notion of a prime number is defined for rings in general, not only for the integers.[111] The definition generalizes the notion of being prime in the integers, so that characteristics that hold of these also hold of prime numbers in the more general rings. This general notion is, however, one which depends on the underlying ring, hence a number that is prime in one ring may not be prime in another ring; the number 2, for instance, is prime in the integers, but it is not prime in the so-called Gaussian integers, i.e. in the ring of numbers of the form $a + bi$ for integers a, b.[112] Hence there is no way of defining the unary predicate 'prime' for all numbers such that what one then obtains is the "correct" generalization of what the predicate means for the integers (though one could perhaps define a relation whose first relatum is the ring in question).[113]

The deeper reason for Frege's requirement that a predicate be defined for all objects is, however, not to be found in his conception of the logical laws nor in his ideas about mathematical methodology, but in the architecture of his ideography, namely that it encapsulates a simple theory of types with a single domain of individuals. We saw in section 1.1 that each symbol of this ideography—excepting the judgement stroke—is assigned a type. The unification that this involved—that e.g. negation is just a unary function of objects and universal quantification just a unary function of unary functions of objects—was clearly important to Frege. But on Frege's typing scheme there is only one type of unary predicates of objects. Hence any predicate of the ideography, if it is to count as a predicate of that ideography at all, must be defined

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[111] The definition is: $\alpha \in J$ is prime iff ($\alpha \nmid \mu$ and $\alpha \nmid \nu$) $\Rightarrow \alpha \nmid \mu\nu$ for all $\mu, \nu \in J$. The generalization of Euclid's definition of prime number (*Elements* VII def. 11) is taken to define another notion: $\alpha \in J$ is irreducible iff any $\mu \in J$ such that $\mu \nmid \alpha$ is either $= \alpha$ or a unit in J. The point to be made here goes through also for the concept of irreducibility.

[112] This follows from the identity $(1 + i)(1 - i) = 2$.

[113] On the notion of "correct" or fruitful generalization in mathematics, see Tappenden (2008).

for all objects. Thus the principle of completeness of definition follows naturally from the simple type structure assumed by Frege's ideography. In *Gg* § 29 Frege lays it down that the name of a function of objects has a signification (*Bedeutung*) if and only if the proper name arising from it by filling its argument places with a proper name has a signification whenever this proper name itself has a signification: F has a signification if and only if Fa has a signification whenever a has a signification. Since all signs of the ideography must have a signification—"Rechtmäßig gebildete Namen müssen immer etwas bedeuten" (*Gg* § 28; cf. TLP 5.4733)—this dictate is therefore just the requirement that a predicate be defined for all objects.

In practice the principle of completeness means that most, and perhaps all, functions will be defined for a range of what Quine (1960, p. 229)—apparently following a usage established in electrical engineering—called "don't-cares," namely objects lying outside what one naturally would count as the range of significance of the function. Frege and, as far as I know, everyone who has followed Frege's or Quine's lead in these matters say very little about which values the function should be assigned for these don't-care arguments. This silence, as well as Quine's epithet, could lead one to think that no care is required in this assignment. As Brady and Routley (1973) point out, that would be a mistaken opinion. A first problem has to do with defined, or composite predicates; for if F is composite, then the truth value assigned to Fa has to accord with the definition of F. To take a simple case, if F is $\neg G$, then Fa has to be given the opposite value of Ga, for otherwise we get a contradiction. A second problem is that there might be cases that we should, for philosophical reasons, need to care about. Namely, if F is some philosophically interesting predicate, say a mental predicate, and it is disputed whether Fa is true, false, or neither, then it cannot be a matter of arbitrary choice which value we assign to Fa. A third problem, not mentioned by Brady and Routley, is the following. Someone committed to Frege's principle of completeness has to say that category mistakes are either true or false. There is a tradition going back at least to Ewing (1937), and defended among others by Prior (1954), Quine (1960, p. 229), Drange (1966), Lambert (1968), and most recently by Magidor (2013), that simple category mistakes, i.e. such as do not involve negation, are false. Perhaps they are false in a special way,[114] but at least they are false. This view of simple category mistakes can, however, not be combined with the requirement that a function be defined for all objects. For let s stand for the Sun and m for the Moon, and suppose that we have defined $s + m = a$, whatever a may be. Then the sentence $s + m = a$ is a true simple category mistake. That is,

[114] Pap (1960, p. 53) suggests that they are synthetic a priori falsehoods.

the requirement that addition, or any other function, should be defined for all objects forces one to accept that there are true simple category mistakes: if $f(a)$ is assigned the value b, then $f(a) = b$ is true.

It is thus not a trivial matter to develop a coherent and consistent position that assumes all predicates to have the universal range of significance. Some care is needed in assigning truth-values to category mistakes; and if the scope of the position is extended to include the requirement that functions in general should have the universal domain of definition, then it may be difficult to explain why category mistakes have the truth-values they do—for why are some categorially mistaken sentences of the form $x + y = z$ false and others true? It may nevertheless be a logically possible position. However, the artificiality attaching to the procedure of assigning values to a function for don't-care arguments suggests rather than contests the existence of non-trivial ranges of significance. And as Sundholm (1999, p. 22) remarks, the recognition of don't-care arguments of a function is in effect the recognition of a non-trivial range of significance, namely as the range of arguments that we do care about.

The recent work of Magidor (2013) illustrates how difficult it is to avoid the notion of range of significance. Magidor does not make any explicit statement about ranges of significance in her book, but the claim that all predicates have the universal range of significance is implicit in its main theses, namely that category mistakes are meaningful, that they have a truth-value, and that their deviance is to be explained as an instance of presupposition failure.[115] The range of significance of a predicate is just the range of objects of which the predicate is true or false. Hence, if all category mistakes have a truth-value, as Magidor holds, then all predicates must have the universal range of significance. Yet it seems to me that non-trivial ranges of significance feature quite prominently in the positive part of Magidor's theory, that accounting for the deviance of category mistakes. According to Magidor, there is for any predicate P a predicate P^* such that an utterance of a sentence of the form 's is P' presupposes that s is P^* (Magidor, 2013, ch. 5); the sentence 's is P' is a category mistake if this presupposition fails. Magidor offers no general account of the relation of the predicate P to the predicate P^*, but in the examples she gives P^* is always the range of significance of P.[116] Thus she relates 'coloured' to 'green', 'natural number'

[115]Ewing (1937) seems to me to defend precisely the same theses: he is explicit that category mistakes are meaningful and that they have a truth-value (ibid. pp. 360ff.); and he compares (ibid. p. 361) the queerness of category mistakes to that of the sentence 'I did not commit more than six murders last week', whose queerness I think a philosopher today would account for as stemming from presupposition failure (namely failure of the presupposition that I did commit a murder). See also the following footnote 116.

[116] In that case, Magidor's account is anticipated by Pap (1960). Pap says for instance that (ibid. p. 48) "the statement 'a is red' may be said to presuppose in the same sense [sc. in the sense in which 'the King of

to 'prime', and 'something that can be pregnant' to 'pregnant' (ibid. pp. 140–146). That P^* is the range of significance of P is indeed entailed by Magidor's account of category mistakes, for it is precisely when a is not in the range of significance of P that 'a is P' is a category mistake. It appears that ranges of significance have entered through the backdoor into Magidor's theory.

4. Type predications

Our discussion of category mistakes leads in this section to the topic of type predications: judgements in which an entity is assigned to a type. Some philosophers have found such judgements problematic. Frege's problem of the concept horse, for instance, is in its purest form a problem of making sense of type predications; and Wittgenstein in the *Tractatus* held that what one attempts to say in a type predication can only be shown. The examination of type predications occasions our introduction of Martin-Löf's constructive type theory, which will play an important role in the rest of this dissertation. Subsection 4.1 deals with the predication of a type in the sense of Ryle and Sommers, subsection 4.2 with the predication of a simple type.

4.1. Ryle–Sommers type predications. Magidor's arguments for the thesis that category mistakes have a truth-value are indirect. She finds problems both with the contrary thesis as such (Magidor, 2013, pp. 83–91) and with defences of it given in the literature (ibid. pp. 91–109). Quine's main argument for the same thesis is an appeal to the "simplicity of theory" that results from accepting it (Quine, 1960, p. 229). The considerations at the end of the previous section show that it may not at all be a simple matter to assign truth-values to category mistakes; and indirect arguments of the sort offered by Magidor carries little conviction so long as they are not accompanied by any indications of which truth-values at least some category mistakes do in fact have.[117] This is to my mind a case where we cannot rest content with abstract arguments. We need to be shown for certain concrete cases of category mistakes why they are true or false. In the absence of such concrete proofs I am inclined to regard the general considerations of Magidor and others meant to show that category mistakes have a truth-value as warranting an inference by modus tollens rather than one by modus ponens. The strongest argument for the thesis that category

Switzerland is a pipe smoker' presupposes that there is a king of Switzerland] that a is visible or spatially extended" (cf. ibid. p. 54). Like Magidor, Pap holds that category mistakes are false.

[117] A similar objection can be made against Magidor's arguments that category mistakes are meaningful (cf. the title of Magidor 2009a, reproduced as ch. 3 in Magidor 2013): these are all indirect and carries little conviction so long as they are not accompanied by any indication of what category mistakes in fact mean.

mistakes have a truth-value is therefore not any of these general considerations, but rather the more mundane piece of reasoning presented by Prior (1954, p. 159). He constructs a syllogism from the conclusion of which it may be inferred that 'Virtue is not square' is false, as follows.[118]

$$\begin{array}{ll} & \text{What is square has some shape} \\ & \text{Virtue has no shape} \\ \hline \therefore & \text{Virtue is not square} \end{array}$$

Since the premisses are true, so is the conclusion; hence by the T-schema and the definition of 'is false' as 'is not true' we have

'Virtue is square' is false

This argument does not rely on any peculiarities of virtue or squareness, and may be adapted to other category mistakes.

What may be dubbed Prior's paradigm is a syllogism in Camestres with a singular minor term, followed by one application of the T-schema:

$$\begin{array}{ll} & \text{Every } P \text{ is } Q \\ & s \text{ is not } Q \\ \hline \therefore & s \text{ is not } P \\ \hline \therefore & \text{`}s \text{ is } P\text{' is false} \end{array}$$

The paradigm is applied by Drange (1966, p. 24) to show the falsity of three quite randomly chosen category mistakes: 'the theory of relativity is blue', 'smells are loud', and 'moral perfection is a prime number'. Here, then, is a direct argument of the kind required to the conclusion that certain category mistakes are false. The argument is faulty, however.

Prior points out the difference between a proof that virtue is not square and a proof that the Sun is not square: while the latter rests on the fact that the Sun has a non-square shape, the former rests on the fact that virtue has no shape at all; it is thus the difference between having another shape and having no shape at all.[119] Prior thereby himself provides the clue to a diagnosis of his argument, as nicely spelled out by Goddard (1970). We must distinguish two sorts of negation. There is firstly choice negation (cf. p. 110 above), which places the subject in the complement of P

[118]A piece of reasoning of this form is implicit also in Ewing (1937, p. 360). Prior's as well as two other syllogisms are extensively discussed by Drange (1966, pp. 19–36). The sentence 'Virtue is not square' is taken from Bosanquet (1911, Bk. 1 p. 282), who cites it in his discussion of so-called infinite judgements (cf. footnote 108 above).

[119]There is also a difference in the mood of the syllogism; here it is Cesare with a singular minor premiss:

$$\begin{array}{ll} & \text{What is circular is not square.} \\ & \text{The Sun is circular.} \\ \hline \therefore & \text{The Sun is not square} \end{array}$$

but inside its range of significance, $|P|$; as above we denote this by $\neg P$. There is secondly what we may call categorial negation, which places the subject in $\complement|P|$, i.e. in the amorphous lump of individuals outside the range of significance of P; let us denote this by $-\!\neg P$. Categorial negation is not the same as exclusion negation (cf. p. 110): while exclusion negation places the subject in $\complement P$, categorial negation places it in $\complement|P|$. Categorial negation may therefore be glossed as 'is not the sort of thing that can be': to assert that s is $-\!\neg P$ is to assert that s is not the sort of thing that can be P.[120]

The validity of Prior's syllogism presupposes that the same negation is employed in the minor premiss and in the conclusion. For, if 'every P is Q' is significant, then we must have, as we in fact do have in Prior's syllogism, that $|P| = |Q|$. Hence, from the premisses that every P is Q while s is $\neg Q$ we can therefore only infer that s is $\neg P$; and from the premisses that every P is Q while s is $-\!\neg Q$ we can only infer that s is $-\!\neg P$. The diagnosis of Prior's argument is then that the inference from the validity of the syllogism to the conclusion that a certain category mistake is false presupposes that categorial negation is employed in the minor premiss while choice negation is employed in the conclusion. The argument to the conclusion that 'virtue is square' is false is therefore faulty. In order to see that this is the correct diagnosis, suppose first that choice negation is employed in the minor premiss of Prior's syllogism. Then virtue is assumed to fall in the range of significance of 'shape'; but that assumption is just as much in need of argument as is the thesis that virtue falls in the range of significance of 'square'. In this case the argument to the conclusion that 'virtue is square' is false therefore begs the question. Suppose next that categorial negation is employed in the conclusion. The conclusion then says that 'virtue' falls outside the range of significance of 'square'; but from that assertion we cannot infer that 'virtue is square' is false. Rather, we can infer that it is neither true nor false, for the range of significance of a predicate is the range of subjects for which it is true or false. In this case Prior's syllogism therefore establishes that 'virtue is square' *does not* have a truth-value, the opposite of what it was meant to establish. So the argument fails to establish that 'virtue is square' is false.

[120] In the significance logic of Goddard and Routley (1973) categorial negation, viewed as a sentential—"propositional" makes little sense here—function, will have the truth-table

p	$-\!\neg p$
t	f
n	t
f	f

In significance logic this connective would be glossed as 'it is not significant that'. It is what Goddard and Routley would call a non-standard negation (cf. ibid. pp. 277–282).

More goes on in Prior's syllogism than just the employment of non-standard sorts of negation. For its major premiss is not any old a-predication, but a predication that lays down the range of significance of the major term P; and its minor premiss is not any old singular e-predication, but one that places the minor term s outside this range of significance. The middle term of Prior's syllogism is therefore a term whose range of truth is equal to its range of significance. This is what in section 2.2.1 above we called, following Sommers, a category term; we shall also call it a type predicate.[121] Predicating a type predicate of an object is, at least on the Ryle–Sommers account of types, to assign it to a range of significance, hence to say that such and such predicates can be significantly predicated of it. And type predicates are employed in laying down a type hierarchy, for they serve as names of the ground types. A type predication is a predication of the form 's is τ' where τ is a type predicate. Prior's syllogism is thus a syllogism of type predications, and our diagnosis of it suggests that type predications may have "a different logic" from ordinary predications. At least there is the difference that to negate an ordinary predication is usually to employ choice negation, while to negate a type predication, as is done in the minor premiss of Prior's syllogism, must be done by employing categorial negation.

Prompted by observations along these lines, Goddard (1966) sketches a logical system that incorporates type predications. To a many-sorted predicate logic let us add a symbol ι_k for each sort, or ground type. These type symbols have the syntax of unary predicate symbols with the fundamental difference from the latter that $t \in \iota_k$ is well-formed for any individual term, while Pt is well-formed only if P and t have matching types. To negate a type predication one must employ categorial negation $\neg\!\!\neg t \in \iota_k$. This predication is true if and only if t is not of type ι_k. If Pt is well-formed, then t falls in the range of significance of P, whence $\neg\!\!\neg Pt$ is false whenever it is well-formed; restricted to ordinary predications Pt, categorial negation is thus just the falsity connective. If type predications are to form more than a mere appendix to many-sorted logic, there needs to be interaction between the two forms of predication, i.e. between $t \in \iota_k$ and Pt. As there is no relation in this system that has the whole universe of individuals as its left or right range of significance, the only way such interaction can be effected is through connectives and quantifiers. It is quite straightforward to add connectives to this system connecting categorial and ordinary predications; but we shall immediately run into problems when considering quantifiers in whose scope we should want to place both forms of predication: for that would have to be quantification over the whole domain of individuals, and such

[121]'Category predicate' is less happy, for it looks like a pleonasm.

quantifiers cannot meaningfully apply to predicates restricted to a sort. There can, for instance, be no way of forming a formula such as $\forall x(x \in \iota_k \supset \neg Px)$ where x is to range over all individuals, for then Px would not be well-formed. Hence, for there to be any interesting interaction between the two forms of predication in the current system, we need to follow Goddard in loosening the formation rules of its predicate logical segment: we need to admit Pt as well-formed even when P and t do not have matching types.

To incorporate type predications into a standard system of many-sorted predicate logic we shall thus have to accept that there are well-formed formulas that are not significant. This, however, goes against a fundamental presupposition of predicate logic and takes us into the realm of so-called significance logic, as developed by Goddard and Routley (1973).[122] Erecting a system of significance logic is by no means a trivial matter, and faces both conceptual and technical problems. (1) Since formulas can be well-formed yet non-significant, the predicate and individual variables must be regarded as schematic letters, and quantification must therefore be substitutional. (2) Since a formula may be well-formed but unmeaning, consequence requires that also singular terms may be well-formed but unmeaning, a decision that has further repercussions for the treatment of quantification (cf. Goddard and Routley, 1973, ch. 3). (3) And, to take one last complicating factor, since it is not sentences in the abstract that are non-significant, but only sentences as used on a certain occasion—'Friday is in bed' would be significant in the context of *Robinson Crusoe*—it is necessary to take context into account.[123] All such issues need to be clarified before one can begin to consider the role of type predications in significance logic. Instead of going into all of these preliminaries, let us consider a system in which the distinction between type predications and ordinary predications is made form the start, namely Martin-Löf's constructive type theory.

4.1.1. *Sets in constructive type theory.* The two notions of type that have mainly occupied us in this chapter both have their pendants in constructive type theory. What are called *sets* in constructive type theory may be taken to correspond to Ryle–Sommers types, while what is called *types* there may be taken to correspond simple types.[124] The role played by the one individual domain in simple type theory is

[122]The book develops ideas found in earlier papers of its authors. A projected second volume was, as far as I can see, never published.

[123]Contexts are included in what Goddard and Routley call context logic; here it is non-trivial even to state what the well-formed formulas are (one distinguishes semi-wff, partial wff, wfc, wffcl, and wffsl), but its basic form of judgement is: expression e in context c yields the statement α. For more details, see Goddard and Routley (1973, ch. 2).

[124]This use of the terms 'set' and 'type' follows Martin-Löf, and is found in Nordström et al. (1990), Ranta (1995), and Nordström et al. (2000). In homotopy type theory these terms have a different use. What we

in constructive type theory taken over by sets; these are thus the ground types. The higher types of constructive type theory are generated by a rule which generalizes the rule by which higher types in simple type theory are generated. We shall look at this rule in more detail below (section 4.2.4); here our concern is with the theory of sets, the theory presented in Martin-Löf (1984). A set is in constructive type theory defined by laying down what its elements of canonical form are and when two elements of canonical form are equal. Instead of 'element of canonical form' we often say 'canonical element'. An element of a set A is a method that when executed yields a canonical element of A. Elements of a set A are equal if they evaluate to equal canonical elements of A.

To define a set one must therefore say how its elements are *formed*; that is typically achieved by means of an inductive definition. This definition does, however, not lay down all forms whatsoever that an element of the set may have, but only the canonical forms. The set N of natural numbers, for instance, is defined in the following way:

$$0 : N \qquad 0 = 0 : N \qquad \frac{n : N}{s(n) : N} \qquad \frac{n = m : N}{s(n) = s(m) : N}$$

These rules lay down the canonical form of natural numbers, and so do not by themselves sanction a judgement such as $2 + 2 : N$, since $2 + 2$ is neither of the form 0 nor of the form $s(n)$. On the basis of the definition of addition and that of 2 in terms of 0 and s, however, we can carry out a reduction of $2 + 2$ to the canonical form $s(s(s(s(0))))$. This is of course just what it means to compute $2 + 2$: when a schoolboy is asked to compute $2 + 2$ what he is asked to do is to reduce this number to canonical form (which for him happens to be 4 and not $s(s(s(s(0)))))$. Since this reduction depends only on the meaning of the symbols out of which $2+2$ is composed, the number $2 + 2$ may itself be regarded as a method, namely a method that when executed yields the number $s(s(s(s(0))))$ of canonical form.[125] That $2 + 2$ is an N is thus to say that it is a method, in accordance with the general account of what an element of a set is. That $2 + 2$ and $3 + 1$ are the same N is to say that they evaluate to the same canonical form. In addition to the basic sets N and N_k, the canonical

call sets are there called types (cf. The Univalent Foundations Program, 2013, ch. 1); while what is there called a set is a type in their sense of a special kind (ibid. p. 107).

[125] In fact, as is clear from the third rule above, already $s(3)$ is canonical provided $3 : N$. Hence, the evaluation halts already when $s(3)$ is reached. I here use the example $s(s(s(s(0))))$ as it is more intuitive. A formulation of constructive type theory in which the evaluation halts only with $s(s(s(s(0))))$ is found in Granström (2011, chs. 3–5). In computer science jargon the mode of evaluation to canonical form in standard formulations of constructive type theory is "lazy" (Martin-Löf, 1984, p. 9), while in Granström's formulation it is "eager."

sets of k members for each natural number k, the theory of sets in constructive type theory defines operators for forming new sets from old. These include an operator Π for forming the cartesian product of a family of sets over a set and an operator Σ for forming the disjoint union of a family of sets, and many others besides. In each case, what the canonical elements of a set thus formed are, as well as what equal canonical elements of the set thus formed are, is laid down in a so-called introduction rule. The introduction rule for a set former is paired with an elimination rule as well as an equality rule; the general form of the two latter rules need not be explained here.[126]

The notion of set thus explained is quite different from each of the conceptions of set that we may call the combinatorial conception, the dichotomy conception, and the iterative conception, respectively.[127] Although these conceptions of set are all closely tied to each other, it is possible, and may be useful, to distinguish them. By the combinatorial conception of set I have in mind what underlies the definition of set found in for instance Dedekind (1888, art. 2), Husserl (1891, p. 74), and Cantor (1895, p. 481). According to this conception a set collects together a certain plurality of objects already "given," and does so without adding anything apart from the mere collection of them.[128] Dedekind suggests that the plurality of individuals thus collected is given in a list, but that, of course, is possible only when the resulting set is finite. Since infinite sets are needed in mathematics, the combinatorial conception is therefore often replaced by the dichotomy conception. Whence Dedekind (loc. cit.), after having explained the notion of set according to the combinatorial conception, says that a set S is "completely determined when it is determined for each thing whether it is an element of S or not." This assumes the dichotomy conception of sets, according to which a certain condition exists that defines the set and thereby divides the universe of individuals in two: those individuals satisfying the condition, hence belonging to the set, and those individuals not satisfying the condition, hence not belonging to the set.[129] If no restrictions are laid down on what this condition may be, then, as

[126] For general remarks on these rules, see Martin-Löf (1984, pp. 24–25) and Nordström et al. (1990, p. 35).

[127] For a comparison of set in the sense of constructive type theory with other notions of set, see also the interesting discussion of Granström (2011, pp. 53–63).

[128] Dedekind (1888, art. 2): "Es kommt sehr häufig vor, daß verschiedene Dinge $a, b, c \ldots$ aus irgendeiner Veranlassung unter einem gemeinsamen Gesichtspunkte aufgefaßt, im Geiste zusammengestellt werden, und man sagt dann, daß sie ein System S bilden; man nennt die Dinge $a, b, c \ldots$ die Elemente des Systems S, sie sind enthalten in S; umgekehrt besteht S aus diesen Elementen."

Husserl (1891, p. 74): "Ein Inbegriff entsteht, indem ein einheitliches Interesse und in und mit ihm zugleich ein einheitliches Bemerken verschiedene Inhalte für sich heraushebt und umfaßt."

Cantor (1895, p. 481): "Unter einer 'Menge' verstehen wir jede Zusammenfassung M von bestimmten wohlunterschiedenen Objecten m unserer Anschauung oder unseres Denken (welche die 'Elemente' von M gennant werden) zu einem Ganzen."

[129] The name 'dichotomy concept' stems from Wang (1974, p. 187); see also Gödel (1947, pp. 518–519).

is well known, contradiction ensues. In particular, if we accept the formation of sets of sets and allow the membership relation to occur in the defining condition without any restrictions, then we can form the condition

$$x \in r \Leftrightarrow x \notin x \tag{D}$$

which would define a set r, for which we should have

$$r \in r \Leftrightarrow r \notin r \tag{R}$$

If, however, we require that a set be formed only "after" all its elements have been formed, then we cannot instantiate r in (D) to get (R), hence we should avoid at least this particular antinomy. The iterative conception of set is often motivated by precisely this observation.[130] It thus emphasizes the component of the combinatorial conception that a set should be formed from individuals *already* given or formed. The iterative conception places no restrictions on how sets may be formed from ones already formed, hence at any stage we may form all sets whose elements may be sets formed so far as well as other individuals that are not sets. In practice this yields the cumulative hierarchy as first defined by Zermelo (1930, p. 36) by iterating the power set operation along the ordinals, taking unions at the limit stages, and beginning from some domain, possibly empty, of non-sets.

Neither of these conceptions of set agree with that of constructive type theory. The basic reason for this is the emphasis in each of these conceptions on the priority of the elements of a set to the set itself. According to the combinatorial conception a set is defined by collecting together, and regarding as one, certain objects given in a list or the like. According to the dichotomy conception we have before us the universe of individuals, which we bisect by means of a condition, thereby defining a set. That a set is posterior in some sense to its elements is the motivating idea of the iterative conception of set. So in each case the set is posterior to its elements. These elements, however, are individuals of a certain kind or sort: they are numbers, or polynomials, or triangles, or thoughts, or trees, or tables, or what have you; they may even be sets themselves. But in any event they are all *something*, for there is an answer to the question of what they are. Each element of a set may be an individual of a different kind—the notion of set is formal in the sense that its elements may be as heterogenous as you wish—but it is still an individual of some kind or sort, still an individual that belongs to a certain domain. It is this notion of domain of individual, or kind or sort of individual, rather than the notion of a collection of any individuals whatsoever, that the notion of set in constructive type theory is meant to capture.

[130] So, for instance, in Boolos (1971) and Shoenfield (1977).

That this notion of set is more fundamental than the notion of iterative set, i.e. a set on the iterative conception, is vividly illustrated by Zermelo's discussion (ibid.). For him a set is a member of a domain of iterative sets (*Mengenbereich*), and the cumulative hierarchy affords a way of stratifying this domain. Here it is the notion of a domain of sets, rather than the sets themselves, that corresponds to the notion of set in constructive type theory. In his characterization of domains of sets Zermelo makes crucial use of ordinals, and these belong to a domain that does not coincide with any domain of sets. Thus, in addition to domains of iterative sets Zermelo assumes the notion of a domain of ordinals. In constructive type theory Zermelo's domains of ordinals correspond to so-called well-ordering sets (cf. Martin-Löf, 1984, pp. 79–86); it is also possible to define there a set corresponding to a domain of iterative sets (cf. Aczel, 1978, pp. 61–62).

It is a basic tenet of constructive type theory that judgements must be distinguished from propositions. Judgements feature in inferences, and a judgement is what one demonstrates in a mathematical proof. A proposition, on the other hand, is what is asserted as true in a judgement. It is, moreover, on propositions and propositional functions, but not on judgements, that the logical connectives operate.[131] This is thus the distinction Frege drew in the *Begriffsschrift* between judgement and judgeable content, but which tends to be ignored, or at least not properly recognized, by most present-day philosophical logicians.[132] The notion of judgement is more fundamental than that of proposition, and a presentation of constructive type theory must begin by presenting the form of judgements it assumes. For our present purposes three forms of judgement are relevant (more forms of judgement will be presented later):

$$A : \mathsf{set} \qquad a : A \qquad A \ \text{true}$$

The first two forms of judgements have already been explained above when it was explained (1) that a set A is defined by laying down how its canonical elements are formed and how equal canonical elements are formed; (2) that an element a of a set A is a method that when executed yields a canonical element of A. Another basic tenet of constructive type theory is the identification of the type of propositions with the type of sets,[133]

$$\mathsf{prop} = \mathsf{set}$$

[131]For this point, see esp. Martin-Löf (1996).
[132]See the works cited in footnote 2 above.
[133]This is a development of the so-called Curry-Howard correspondence (Howard, 1980).

Hence 'A : prop' is another way of writing the judgement 'A : set'. A proposition is identified with the set of its canonical proofs, or proof objects. Accordingly, a proof of a proposition is a method for obtaining a canonical proof. A proposition is true if it is inhabited, i.e. if it has a proof. These ideas are reflected in the third form of judgement 'A true' above, a form of judgement that is written $\vdash A$ in Frege's ideography. It is explained by saying that the following rule of inference determines its meaning

$$\frac{a : A}{A \text{ true}}$$

In passing from $a : A$ to 'A true' some information is left out: in asserting 'A true' we assert that we have a proof of A, but we do not make this proof manifest, as we do in asserting $a : A$. Another way of writing the judgement 'A true' is 'A exists'.

Viewing sets in constructive type theory as Ryle–Sommers types, the judgement

$$a : A$$

should be viewed as a type predication. Type predications are thus among the basic forms of judgement in constructive type theory. The same holds for ordinary predications. For, given a set A, propositional functions may be defined over it, i.e. functions P whose value Pa for any argument a in A is a proposition. The judgement

$$Pa \text{ true}$$

to the effect that the propositional function, or predicate, P holds of an a in the set A is thus an ordinary predication. Hence the difference between type predications and ordinary predications corresponds to the difference of the forms of judgement $a : A$ and 'Pa true'.

From the explanation of the form of judgement 'A true' it is clear that any judgement of this form is preceded by a judgement of the form $a : A$. In particular, an ordinary predication 'Pa true' is preceded by a type predication $p : Pa$ in which a proof object p of the proposition Pa is made manifest. Thus already the meaning explanation of ordinary predications shows that they are epistemically posterior to type predications. For present purposes another asymmetry between ordinary predications and type predications is of more interest: the ordinary predication 'Pa true', in which P is a propositional function over the set A, presupposes the type predication $a : A$. Indeed, we cannot make the judgement 'Pa : prop' unless we know that a belongs to the range of significance of P. In order to appreciate this asymmetry between 'Pa true' and $a : A$ it is instructive to consider a propositional function that is true for every element of A. Given any set A and elements a, b in A constructive type theory allows the formation of a proposition $I(A, a, b)$, which is true if a and b

are equal elements of A, that is, if they evaluate to equal canonical elements. This proposition $I(A, a, b)$ must be distinguished from the more fundamental *judgement* $a = b : A$ (cf. Martin-Löf, 1984, pp. 59–64). An I-set, as I shall call it, has the following introduction rule

$$\frac{a : A}{r(A, a) : I(A, a, a)}$$

Thus, the proposition $I(A, a, a)$ is inhabited, hence true, for any element of A. Given a set A we may therefore define a propositional function over A

$$Px \equiv I(A, x, x)$$

that is true for every element of A. Then both of the following inferences are valid.

$$(1)\ \frac{a : A}{Pa \text{ true}} \qquad (2)\ \frac{Pa \text{ true}}{a : A}$$

These two inferences are justified in fundamentally different ways, however, and that difference reflects the difference between the type predication $a : A$ and the ordinary predication 'Pa true'. The inference (1) is valid in virtue of the definition of P and the introduction rule for I-sets. This inference thus draws its justification from how P has been defined. The inference (2), by contrast, is valid for any propositional function P defined over A. It is justified as follows. Assume that the premiss 'Pa true' is known. Then the presuppositions of the premiss must be known; but the judgement $a : A$ is one of these presuppositions; whence $a : A$ is known. The only fact used about P in this justification is that A is its domain of definition. The judgement 'Pa true' then has $a : A$ as a presupposition. Since we must know that this presupposition is fulfilled before making the judgement '$Pa : \mathsf{prop}$', *a fortiori* before making the judgement 'Pa true', the conclusion of (2) is therefore just the repetition of a judgement that will already have been made, assuming that we know the premiss of the inference. The lesson is that, even though the type predication $a : A$ and the ordinary predication 'Pa true', with P defined as above, are equiassertable, the former is a presupposition of the latter. In constructive type theory the order of conceptual priority between a propositional function and its range of significance is therefore the reverse of what it is in Ryle–Sommers type theory. In the latter a propositional function is given in the abstract, so to say, and its range of significance is determined on the basis of judgements of significance, perhaps with the help of certain formal constraints such as the law of categorial inclusion. In constructive type theory, by contrast, one must, in defining a propositional function, first specify its range of significance.

While the type predication $a : A$ is a basic form of judgement in constructive type theory, there is no corresponding negative judgement to the effect that a is not an element of the set A. This can be seen as following from what Martin-Löf has called the doctrine of types, namely the doctrine that an object is always an object of a certain type.[134] One does not conceive of a universe of bare objects that only subsequently is divided into types according to the nature of the object. Rather, an object is introduced with its type, and therefore cannot be thought apart from it. Hence we may regard the type of an object as its nature, or essence, and there is no separating the object from its essence to ask whether it perhaps has another essence, i.e. whether it perhaps belongs to another type. Since there are no negative type predications, neither are there predications to the effect that two types are unequal, for that would require a witness of their inequality; *a fortiori* there are no predications that two types are disjoint. Since Prior's syllogism trades in the negations of type predications, there is no way of formalizing it in constructive type theory. In general it is required for the formalization of a syllogism in constructive type theory that the three terms A, B, and C be propositional functions defined over the same domain D, in particular that A, B and C have the same range of significance.[135] Constructive type theory is thus a system in which type predications can be made, but according to which category mistakes are not well-formed, whence non-significant.

4.2. Simple type predications. Category mistakes motivated our treatment of predications of types in the sense of Ryle and Sommers; our treatment of predications of simple types is motivated by Frege's problem of the concept horse.

4.2.1. *The problem of the concept horse.* There are various ways of formulating Frege's problem of the concept horse.[136] According to one formulation, the problem is that of accounting in general for propositions in which a phrase of the form 'the function f' serves as subject. Since, according to some of the things Frege says (esp. *GLA* §§ 51, 66), 'the function f' is a proper name, it cannot be used to signify a function, for what signifies a function must be unsaturated, just as the function itself is unsaturated; but a proper name is by definition an expression that is not

[134] Cf. Martin-Löf (1975, p. 76).

[135] The mood of Barbara is formalized as follows

$$\begin{array}{r} f : (\forall y : (\Sigma x : D)A(x))B(p(y)) \\ g : (\forall y : (\Sigma x : D)B(x))C(p(y)) \\ \hline \therefore \quad \lambda y.\mathrm{ap}(g, \langle p(y), \mathrm{ap}(f, y)\rangle) : (\forall y : (\Sigma x : D)A(x))C(p(y)) \end{array}$$

Here the domain D is made explicit. It is reasonable to assume that D is non-empty. This assumption does, however, not validate the inference from $f : (\forall y : (\Sigma x : D)A(x))B(p(y))$ to the judgement that $(\Sigma x : D)A(x)$ is non-empty ("existential import"), since A might not hold of any D, in which case f is a function with an empty domain.

[136] As emphasized by Proops (2013).

unsaturated.[137] Whence it appears that we cannot make a function f the topic of a proposition to say of it that it has such and such properties. That would namely require use of an expression such as 'the function f' as subject; with 'the function f' as subject, however, we shall not succeed in talking about a function, but shall find ourselves instead talking about what must be an object. Whence anything we might wish to say about the function f that would require making it the topic of the proposition we cannot in fact say. Whence, perhaps, Frege's famous words that "my expression taken quite literally miss my thought" (*BG* p. 204).

The problem of the concept horse according to this formulation is thus the problem of making a function the topic of a proposition; or in other words, the problem of expressing a thought the only way of expressing which would be by means of a sentence whose subject is a phrase of the form 'the function f'. Against this formulation of the problem one may appeal to what is known in the literature as Frege's Context Principle: "never to ask for the meaning of a word in isolation, but only in the context of a proposition" (*GLA* p. x). For it may be that when considering a subject phrase of the form 'the function f' not in isolation, but in its sentential context, we should find no "awkwardness of language" (*sprachliche Härte*, *BG* p. 196) that stands in the way of what we wish to say.

One may also appeal to the distinction between the surface grammatical form of a sentence and its ideographical form. That these come apart is evident in the case of quantified propositions, a point of which Frege must have been well aware. In the surface grammatical form of a quantified proposition a predicate is said of a quantified subject; so this is parallel to the surface grammatical form of the singular proposition. In the ideographical rendering of a quantified proposition, however, a second-level function 'every' is applied to a first-level function, and that is not parallel to the ideographical rendering of a singular proposition. Thus, on the surface level of 'every horse is mortal' the predicate 'is mortal' is said of the quantified subject 'every horse'; but on the ideographical level the second-level function 'every' is applied to the first-level function 'if x is a horse, then x is mortal'. Surface grammar and ideography come apart also in the case of propositions of the form now under discussion, namely propositions whose subject appears to be of the form 'the function f'. The quite ordinary mathematical sentence 'the sine function is periodic' is a case in point. In formalizing this sentence in ideographical notation one feels no inclination to render

[137] Cf. footnote 10 above and the text footnoted there. For the relevant definition of proper name, see *SB* p. 27.

'the sine function' as a Fregean proper name. The formalization would rather be

$$(\exists r \in \mathbb{R})(\forall x \in \mathbb{R}) \sin(x+r) = \sin(x),$$

where 'sin' is a functional expression and not a proper name. There is an analysis of this formula into the function

$$\Phi(f) \equiv (\exists r \in \mathbb{R})(\forall x \in \mathbb{R}) f(x+r) = f(x)$$

applied to the argument sin. That analysis corresponds well to the grammatical analysis of the sentence 'the sine function is periodic' into the subject 'the sine function' and the predicate 'is periodic'. But the function Φ is a second-level function and sin still a function of objects.

Another case in point is 'Euler's totient function is multiplicative' or 'the Ackermann function is not primitive recursive' of in fact most sentences of mathematics whose subject has the form 'the function f'. In each case the function that is mentioned in the subject phrase of the natural language formulation of the theorem occurs in the ideography together with an argument, hence it does not occur as an object. The real problem of the concept horse is therefore not to account for phrases of the form 'the function f' as they occur in subject position in mathematical prose, for such phrases in general "disappear on analysis." The real problem of the concept horse is rather to account for propositions where 'is a function' is the predicate: to account for type predications, that is.[138] If anything else is suggested by Frege's writings—which it presumably is by the *Grundlagen der Arithmetik* and *Über Begriff und Gegenstand*, a paper responding to a criticism of the *Grundlagen*—then that rather points to a tension in Frege's thinking. For there is no reason why he should not have accepted our analysis of 'the sine function is periodic' and sentences of a similar surface form.

As there are infinitely many types of function in the simple type hierarchy, so there are also infinitely many type predicates (it is worth repeating a point that is often sinned against in the literature, namely that 'is a function' is not a type predicate according to Frege, since there are infinitely many sorts of function, each as distinct from any other as unary functions of objects are from objects; cf. *BG* pp. 26–27 and *GG* p. 37). But let us concentrate here on the type of unary functions of objects;

[138]A similar observation is made by Carnap (1934, §§ 76–77) when he distinguishes two modes of employment of what he calls universal words: in the first mode of employment the universal word serves merely to show the sort of a variable or other sign, and so can be done away with provided the sign wears its type on its sleeve, and that is merely a matter of convention; in the second mode of employment the universal word occurs as a non-qualifying part of the proposition and cannot be thus done away with; Carnap notes that a type predication is the simplest case of this second mode of employment and proceeds to account for such predications (we shall discuss his account in section 4.2.3 below).

the considerations made should generalize to all other types. The problem of the concept horse is thus that of explaining how one can say of a thing that it is a unary function of objects. If a solution to this problem requires that a predication of the sort 'f is a function' be rendered in Frege's ideography, then I think no solution can be found. One suggestion is to define a first-order predicate that is true precisely of the courses-of-values of functions of objects and let this predicate be a proxy for the predicate 'is a first-order function'.[139] Another suggestion is to define 'is a function' as a second-order predicate that is true of all unary functions of objects. An example of such a predicate is

$$\Phi(f) \equiv \forall x \exists y (f(x) = y),$$

where the quantifiers range over all individuals.[140] Both of these suggestions supply a predicate of the ideography that may be read as 'is a function'. But both suggestions are problematic. The first is problematic for the reason that much of Frege's doctrine of courses-of-values is problematic, namely in that it opens the door to the Zermelo–Russell paradox, requiring as it would appear to do an injective function from concepts into objects.[141] There is a more fundamental problem that confronts both suggestions. To understand the predicates here being defined—the first-order proxy predicate 'is a course-of-values of a first-order function' or the second-order predicate true of all first-order functions—one needs to understand the ideography. To understand the ideography, however, one needs to understand what a function is, and therefore also to understand type predications of the form 'f is a function'. The definition of the proxy predicate 'is a course-of-values of a first-order function' lays down that it is a function; so this definition presupposes a grasp of the type predicate 'is a function'. Likewise, the definition of the second-level function true of all first-order functions lays down that it is a second-level function, and therefore presupposes a grasp of the type

[139] For discussion of this suggestion in the case where the predicate is 'is a concept', see Parsons (1986, pp. 452–455) and Proops (2013, pp. 77–84). One may recognize the suggestion at *BG* p. 197, but at the place in question Frege refers not to *GG* § 3, where the notion of course-of-values is introduced, but to *GLA* p. x, which is just a comment on the third of Frege's three *Grundsätze* in that work, namely "Der Unterschied zwischen Begriff und Gegenstand ist im Auge zu behalten." I am not certain whether the proxy predicate 'is a first-order function' is definable in Frege's ideography. According to Frege's definition of his application function, $\xi \frown \zeta$, the value of $\Delta \frown \Gamma$ is to be the course-of-values $\dot{\varepsilon}(\varepsilon \neq \varepsilon)$ for all Δ when Γ is not a course-of-values (*Gg* § 34); but it will not do to define 'is a first-order function' as $\forall \mathfrak{a}(\mathfrak{a} \frown \xi \neq \dot{\varepsilon}(\varepsilon \neq \varepsilon))$, for this will yield the value False also for the course-of-values of the function whose value is $\dot{\varepsilon}(\varepsilon \neq \varepsilon)$ for any object. The deeper problem here would seem to be that we lack clear application criteria for the notion of being a course-of-values. Notice, however, that the proxy predicate 'is a concept' is easily definable in Frege's ideography, namely as $\forall \mathfrak{a}(\mathfrak{a} \frown \xi = -(\mathfrak{a} \frown \xi))$, where '$-$' is Frege's horizontal stroke (*Gg* § 4).

[140] This proposal is developed by Dummett (1973, pp. 211–217). Frege appears to defend it in a letter to Russell of 29/06/1902: "Der Begriff der Function muss ja ein Begriff zweiter Stufe sein, während er in der Sprache immer als ein Begriff erster Stufe erscheint."

[141] As is emphasized by Parsons (1986, pp. 454–455).

predicate 'is a second-level function', which in its turn would seem to presuppose a grasp of the type predicate 'is a function'. The definition of each of these two predicates thus presupposes that we already grasp the type predicate the definition is meant to account for. An objection closely related to this one is the following. For each type τ there is a function of type $(\tau)o$ that is true of every object of type τ; but that function must not be confused with the type τ itself. The function of individuals $\lambda x.x = x$, for instance, cannot be identified with the type ι of individuals. This is an instance of the more general point that a type cannot be identified with an element of some type. But that, in effect, is what we must do if we are to define the type predicate 'is a function' in Frege's ideography.

4.2.2. *Tractarian silence.* Hence, there can be no predicate of Frege's ideography that faithfully renders the predicate 'is a function'. In face of this fact one position to take is that of Wittgenstein in the *Tractatus*.[142] Wittgenstein held that something's falling under what he called a *formal concept* cannot be asserted by means of propositions, but is rather shown in propositions (TLP 4.122, 4.124, 4.126).[143] As examples of formal concepts Wittgenstein mentions object, complex, fact, function, and number (TLP 4.1272). These examples together with the various properties of formal concepts that we shall now discuss make it natural to identify formal concepts as the types or categories of the Tractarian system. To say that an object falls under a concept is in Frege's ideography to say that a certain function yields the value true when applied to that object; in Russell's ideography it is to say that a certain propositional function ϕx yields a true proposition when that object is assigned as value to the variable x. Thus we may say that a genuine concept, such as the concept of man, is represented by a certain function, the propositional function 'x is a man', say. Wittgenstein denies that formal concepts are genuine concepts represented by functions; instead they are pseudo-concepts (*Scheinbegriffe*), and represented by variables (TLP 4.126–4.1272).[144] In particular, there is no propositional function representing the formal concept of an object, a concept rather represented by a variable, like the 'x' in 'x is a man'. An entity's falling under a formal concept is therefore not asserted by the combination of a propositional function and the entity in question, but is shown by the fact that the entity is a possible value of a variable of a given sort;

[142]Several remarks made in this section and the next echo ones made by Klein (2004).
[143]Wittgenstein distinguishes what he calls formal concepts from what he calls formal properties and relations. The latter are said to be the marks (*Merkmale*) of the former (TLP 4.126), but it is a moot point what the difference is between the two notions. I shall not distinguish the two.
[144]The notion of variable assumed in the *Tractatus* (TLP 3.311–3.317, 4.1271–4.1273) is obscure, if not confused. I shall not attempt to clarify it here, but take for granted the understanding of variables articulated for instance by Church (1956, pp. 9–15).

or equivalently, that a variable of a given sort is what one must replace the symbol of the entity by in order to obtain a propositional function from a proposition in which that name occurs.[145]

Regarding the problem of the concept horse Wittgenstein could therefore have said that the predicate 'is a function' is a pseudo-predicate. What we wish to say by means of it cannot be said; something's being a function is rather shown in a symbol. The type of a symbol is a characteristic feature (*charakteristischer Zug*) common to all symbols of that type (TLP 4.126). This is not to say that a function symbol is just whatever can be substituted by or for an f or a g or primed versions of these, for a symbol's being of a certain type is not merely a question of typography. Rather, the type of a symbol is shown in the use that is made of the symbol in the ideography, in its "logico-syntactic employment" (TLP 3.326–3.327). In general, then, the Tractarian approach to type predications is to say that they are nonsensical (TLP 4.124), in effect that there are no such predications. What one attempts to say in a type predication is rather something that is shown in the symbolism itself, something that comes in of the logico-syntactic employment of the symbols.

Although the Tractarian view of type predications is a possible reaction to Frege's concept horse problem, it seems not to have been conceived as such by Wittgenstein. His ground for holding the thesis that type predications are nonsenical rather has to do with the so-called picture theory of meaning (TLP 4.01, etc.).[146] To say that something falls under a formal concept would namely amount to saying something about the logical form of the propositions in which it occurs, and that, according to Wittgenstein, is not possible. For a picture to be a picture of a piece of reality—be it a true or a false picture—the picture and the piece of reality need to have something in common. What a picture shares with what it depicts Wittgenstein calls a form of representation (*Form der Abbildung*) (TLP 2.16–2.17). A picture can, however, not depict its own form of representation, for that would mean transgressing what makes it a picture in the first place (2.172–2.174). Logical form is, according to Wittgenstein, a form of representation. Logical form is in fact the most general form of representation that there is, for it is present in any picture. It is a minimal condition on a picture that it shares the logical form of what it depicts; by contrast, the picture need not share, for instance, the spatial form of what it depicts (2.18–2.182). A proposition (*Satz*) is a picture, and shares the logical form of what it depicts (e.g. 4.03 or 3 with 3.1). But since logical form is a form of representation, a proposition cannot depict its logical form, for again, that would conflict with the conditions for the possibility

[145]Husserl called this the process of formalization; more on that in section 6.2 below.
[146]For a sympathetic discussion of the picture theory of meaning, see Stenius (1960, esp. ch. VII).

of the proposition's being a picture in the first place (4.12–4.121). Since a proposition p that purports to say that an entity e falls under a certain formal concept purports to say something about the logical form of the propositions in which a symbol of the entity e occurs, and since one such symbol would occur in the very proposition p, there can in fact be no such proposition.

That there is a close relationship between formal concepts and what Wittgenstein calls logical form is not stated by him in so many words. That he held there to be a close relationship, however, seems clear when we consider the theses describing the notion of a formal concept in their context. With numbers $4.12x$ these theses must be regarded as developments of thesis 4.12, which is precisely the thesis that applies TLP 2.172 to propositions, saying that a proposition cannot depict its own logical form. Thesis 4.121 adds that a proposition *shows* its logical form, while formal concepts are introduced in TLP 4.122 with the proviso that an object's falling under a formal concept cannot be said by, but is only shown in, propositions.

4.2.3. *The Russell–Carnap reduction.* A different reaction to the problem of incorporating type predications into an ideography is to say with Russell (1919b, p. 267) that "the theory of types is really a theory of symbols, not of things." According to Russell (1924, p. 332) all symbols are of the same logical type: they are all "classes of series of noises or shapes," by which he means classes of equiform utterances or inscriptions (cf. Russell, 1944, p. 692). This is presumably not the only way in which symbols can be treated as being of a single type—what is required is only that they be a sort of general object whose instances are utterances or inscriptions—but for concreteness let us concentrate on this account. What we say in a type predication such as 'the sine function is a function of real numbers' is therefore that the class of symbols equiform with 'sin' belongs to the class of function-symbols, and this is just an ordinary predication Pt. Thus it appears that we can do away with type predications altogether, as they are reducible to ordinary predications over the domain of equiformity classes of inscriptions or utterances.

Russell seems to have been led to a view like this through consideration of how to present the type theory without violating the doctrine of types, "a point which formerly troubled me a good deal" (Russell, 1944, p. 692). An especially clear illustration of the difficulty this raises is offered by *PM* $*9.14$:

> $*$**9.14** If "ϕx" is significant, then if x is of the same type as a, "ϕa" is significant, and vice versa.

This "primitive proposition," as it is called in *Principia*, lays down an important tenet of the doctrine of types, namely that types are ranges of significance of propositional functions. But for it to have the import it is meant to have the variables x and a must cross all types, and that, according to the doctrine of types, is not possible, as the range of a variable is restricted to a single type. Hence *PM* $*9.14$, in laying down an important part of the doctrine of types, violates that very doctrine, whence the whole thing looks like a self-contradiction. It was as a way out of this predicament that Russell came to view the theory of types as a theory of symbols, for on that view the variables x and a would range over symbols, all of which are of one type, and thus the theory could be presented without self-contradiction. The reduction of type predications to ordinary predications over symbols follows as a corollary.

In the *Logical Syntax of Language* (1934) Carnap deals with type predications in quite the same way, though I think motivated by considerations rather different from Russell's. Carnap views type predications as belonging to what he calls the logic of science (*Wissenschaftslogik*). The logic of science is, according to Carnap, what philosophy becomes when it has been purged of all unscientific aspects: all questions of philosophy are logical questions concerning sciences (ibid. § 72). Type predications will therefore belong to the logic of science, since they classify the objects studied in the various sciences. The logic of science is, according to Carnap, "the syntax of the language of science" (ibid. § 73). Syntax here means "nothing but the combinatorics, or if one prefers, the geometry of finite discrete sequential structures of a certain kind" (ibid. § 2). The syntax of the language of science is therefore the study of the structure of science and scientific propositions considered as a certain sort of mathematical objects. Carnap describes syntax as the purely *formal* study of these things, by which he means that it does not take meaning into account, but considers only the sorts and sequences of certain figures, certain "discrete sequential structures" (ibid. § 1). This use of the word 'formal', as indicating a concern only with the visible surface of a sign, is quite novel in the history of logic,[147] and not in line with how the word is used elsewhere in this dissertation.

As a part of the syntax of the language of science type predications concern symbols. A type predication thus has the form: such and such a symbol belongs to such and such a class of symbols; for instance, " '5' is a numeral," " 'is a horse' is a property-word" (ibid. § 77). Carnap remarks (loc. cit.) that, so conceived, type predications, as well as closely related predications such as 'no property is an object', can be made without violating the doctrine of types. But it is not this observation that motivates Carnap's conception of the logic of science as the study of syntax, and consequently his treatment of type predicates as ranging over symbols. The

[147] I am uncertain where it originates; it corresponds well to the second sense of 'formal' marked out by Frege (1886).

motivation is rather Carnap's belief in the virtues of syntax in his sense. Syntax allows for precisely formulated and definitely answerable questions, and helps one avoid the grip of pseudo-problems. The identification of the logic of science, hence also of philosophy, with the syntax of the language of science is therefore of benefit to the former. The identification is of benefit to the doctrine of type predications in particular, for according to Carnap type predications form a major source of pseudo-problems. Problems concerning the nature of numbers, the nature of time and space, of the physical and the mental, and the nature of universals are all pseudo-problems raised by an improper understanding of type predications. They are resolved only by dealing with type predications in Carnap's way, namely as being solely concerned with the visible surface of signs (ibid. § 80).

We must now ask whether this Russell–Carnap reduction of type predications to ordinary predications is a satisfactory treatment of the former. To my mind it is not. One could firstly question whether it is in fact true that type theory is a theory of symbols in Russell's sense. For granted that all symbols are of the same type—and that must be granted if the reduction is to have any effect—then it would seem that the theory of types is precisely not a theory of symbols, since there are no type distinctions among symbols, while a theory of types must distinguish between types. The theory of symbols cannot comprise a theory of types, but at most a theory of the single type of symbols. It will not do to counter that types enter when we consider the meaning of these symbols: for then one grants that the actual theory of types is not a theory of symbols, but a theory of the meanings of symbols. A second and more fundamental objection to the reduction is that it simply does not remove the need for type predications. This is evident from how we presented the reduction above: type predications are reducible to ordinary predications over the domain of equiformity classes of inscriptions or utterances. In this very formulation we commit ourselves to the distinction between symbols and predicates over symbols, and that is a distinction of type. If we do not opt for Tractarian silence, we shall therefore see the need, when carrying out the Russell–Carnap reduction, to say that such and such is a predicate over symbols, that is, we shall see the need to make type predications. Hence the Russell–Carnap reduction does not in fact dispose of type predications, as they are required in the statement of the reduction. One could counter that here there is certainly a confusion of object language and metalanguage: we have removed type predications from the object language, and that is all that is required. In order to reply to this we must first consider the relation of type theory to the idea of an object language.

As is the case with any scientific investigation, the study of type theory as an object language in the sense of a mathematical object of the type of symbols necessarily takes place in a language. Following Curry (1963, pp. 28–29) we may call that language the U language, where 'U' does not stand for 'universal', but indicates that the U language is the language being used.[148] An ideography in the tradition of Frege and Russell should be seen, not as an object of mathematical study, but as a way of making precise a fragment of the U language, namely the language being used in mathematics (and perhaps other sciences, as Frege suggested, *Bs* pp. xii–xiii; but let us concentrate on mathematics here). The U language is to a greater or lesser extent imprecise; in most areas of language use this imprecision is perhaps only a benefit, but for a logical analysis of arithmetic Frege realized that it was an obstacle that can be overcome only by the construction of an ideography (*Bs* p. x). Frege's ideography was thus not introduced as an object for mathematical study, but as an addition to the language in which mathematics is studied,[149] as a means of making this language more precise. As such it may be compared to the introduction of technical terminology with explicit meaning explanations that we find in all sciences. Like the U language itself, an ideography should be regarded as open: it can always be extended with new primitives if that is required and justified. An ideography in Frege's spirit can therefore not be identified with an inductively defined set of strings. Given a definite portion of the ideography we can of course choose to consider that as a mathematical object, and study its various properties along the lines of model theory and proof theory. We do, however, not thereby transcend the U language, for this objectified ideography will still have to be studied in the U language. By its definition there is no way out of the U language.

In the works that have occupied us above type theory is generally assumed to be an addition to the U language, and not an object of mathematical study on a par with, say, groups or manifolds. That is the case with Frege, with *Principia*, with Ryle, Sommers, Goddard, and with constructive type theory. Type theory is here regarded as an addition to the U language along the lines of Frege's ideography and not as a mathematical object. The addition to the U language of a type theory is here thus a way of making precise the various type distinctions that are made in the U language. This conception of type theory is more fundamental than the conception of it as an object language, for in presenting type theory as an object language we have to make use of typical distinctions in the U language. Namely, we have to make use of the distinction between between the type of symbols and the type of predicates

[148] The term was first introduced in Curry (1950).
[149] For this point, cf. Sundholm (2001, 2002).

over symbols, and that distinction is not captured by the object language. Although we may be able to "model" some definitely circumscribed part of the U language in a type-free theory, we cannot infer that there are no type distinctions in the U language.

The reply to the contention recorded above that nothing more is required than an account of type predications in the object language is therefore that more is in fact required.[150] What we want is to make explicit the type distinctions made in the U language, and our arguments show that this is not achieved with the Russell–Carnap reduction. It is a further question whether type predications in the U language in fact can be made precise in an ideography. Wittgenstein's point in the *Tractatus* is precisely that they cannot: in an ideography formal concepts are represented by variables and not by propositional functions (TLP 4.1272). Indeed, there could perhaps be inherent conceptual or technical difficulties in representing type predications in an ideography. But there is an ideography in the tradition of Frege in which type predications can be made, and which thus would seem to show that the contrary is the case, viz. constructive type theory.[151] We have already seen how predications of the form $a : A$, where A is a set, are explained in constructive type theory (section 4.1.1); we shall now look at how predications of the form $a : \alpha$, where α is a type are explained.

4.2.4. *Types in constructive type theory.* As was noted at the beginning of section 4.1.1 above, what are called types in constructive type theory may quite well be compared to simple types, in contrast to what are there called sets, which may be compared to Ryle–Sommers types. If α is a type of constructive type theory and a is an object of α, then the judgement $a : \alpha$, i.e. a type predication, can be made in the theory, and not merely in the metalanguage. There is thus in this theory not only the form of judgement $a : A$, where A is a set, but also the more general form of judgement that something is an element of a type (that it is more general will emerge presently). The explanation of the simple type predication $a : \alpha$ is given together with the explanation of what a type is; that explanation is given together with the explanation of what the rules of type formation are;[152] but first some preliminaries. (i) A type α of constructive type theory is explained by saying what its elements are and what equal elements of the type are. This explanation is in general effected by explaining a pair of judgments

150 I take Geach (1976, pp. 58–61) to be saying much the same thing. For another, more Tractarian, response to the Russell–Carnap reduction, see Stenius (1960, pp. 182–188).

151 On the place of constructive type theory in the tradition after Frege, see e.g. Sundholm (2012b).

152 The development of the rules of type formation postdate the lectures published in Martin-Löf (1984), and so are not included in that book. The rules are presented and discussed in Martin-Löf (1994, pp. 91–92) and in Martin-Löf (2013). A presentation of the whole theory that takes types into account is found in Nordström et al. (2000).

$$a : \alpha \quad (a \text{ is an element of type } \alpha)$$
$$a = a' : \alpha \quad (a \text{ and } a' \text{ are equal elements of type } \alpha)$$

By explaining the first judgement, one explains what an element of α is, and by explaining the second judgement, one explains what equal elements of α are. (ii) As we shall see in more detail below, not only type predications, but also predications with an even more general predicate can be made within constructive type theory; in particular, one can also make the judgement $\alpha : \mathsf{type}$ in the theory, and this is not a type predication, since type is not itself a type. (iii) An important feature of constructive type theory that distinguishes it from Frege's ideography is the incorporation of hypothetical judgements. In particular there is a hypothetical form of judgement

$$\alpha : \mathsf{type}\ [x : \beta],$$

read "α is a type provided x is a β." Let us write $\alpha\langle b/x\rangle$ for the result of substituting b for x in α. This hypothetical form of judgement is explained by saying that

$\alpha\langle b/x\rangle$ is a type provided $b : \beta$, and
$\alpha\langle b/x\rangle$ and $\alpha\langle b'/x\rangle$ are equal types provided $b = b' : \beta$

If these judgements are valid, then we may think of α as a family of types over β. Hypothetical judgements of the form

$$a : \alpha\ [x : \beta], \qquad \text{(Hypo)}$$

read "a is an α provided x is a β," are explained similarly, namely by saying that

$a\langle b/x\rangle : \alpha\langle b/x\rangle$ provided $b : \beta$
$a\langle b/x\rangle = a\langle b'/x\rangle : \alpha\langle b/x\rangle$ provided $b = b' : \beta$.

The rules of type formation in constructive type theory are the following.

$$\mathsf{set} : \mathsf{type} \qquad \frac{A : \mathsf{set}}{A : \mathsf{type}} \qquad \frac{\beta : \mathsf{type} \qquad \alpha : \mathsf{type}\ [x : \beta]}{(x : \beta)\alpha : \mathsf{type}}$$

We justify a rule of type formation by justifying its conclusion on the assumption that we know the premisses of the rule. A judgement of the form $\alpha : \mathsf{type}$ is justified by showing that α is indeed a type, i.e. by saying what an object of type α is and what equal objects of type α are. Let us justify each of the three rules of type formation in turn. The first rule $\mathsf{set} : \mathsf{type}$ says that there is a ground type set of sets. To justify this rule we need to say what a set is and when two sets are equal. It was explained in section 4.1.1 above what a set is. Two sets are equal if a canonical element of the one is a canonical element of the other, and vice versa.

The second rule says that any set A is itself a ground type. Instead of the one type of individuals of most formulations of type theory, there is here therefore one type of individuals for each set. We justify the second rule by recalling the explanation of what an element of a set is and what equal elements of a set are. An element of a set is a method that when executed yields a canonical element of the set; elements are equal if they evaluate to the same canonical element. Recall that $\mathsf{prop} = \mathsf{set}$. Hence the two first rules of type formation can also be written as follows:

$$\mathsf{prop} : \mathsf{type} \qquad \frac{A : \mathsf{prop}}{A : \mathsf{type}}$$

According to the first rule there is a type of propositions; what a proposition is and when propositions are equal is given by the explanation of what a set is and when sets are equal. The second rule now says that any proposition A is also a ground type.

The third rule of type formation allows the formation of so-called dependent function types: if β is a type, and α is a family of types over β, then we may form the type $(x : \beta)\alpha$. If $\alpha\langle b/x\rangle = \alpha\langle b'/x\rangle$ for any b and b' in β, then we put $(\beta)\alpha = (x : \beta)\alpha$. This identification is in accordance with our default notation for function types $(\beta)\alpha$. A function type $(\beta)\alpha$ of the simple type hierarchy may thus be identified with a type $(x : \beta)\alpha$ in the dependent type hierarchy provided α is a constant family over β. From this we see that the dependent type hierarchy is a generalization of the simple type hierarchy.

To justify the third rule of type formation we must explain the following pair of judgements, relying on the assumption that we know the premisses of the rule, namely that we know $\beta : \mathsf{type}$ and $\alpha : \mathsf{type}\ [x : \beta]$.

(1) $f : (x : \beta)\alpha$
(2) $f = g : (x : \beta)\alpha$

Judgement (1) means that

(1*) $f(b) : \alpha\langle b/x\rangle$ whenever $b : \beta$,

and that

(1**) $f(b) = f(b') : \alpha\langle b/x\rangle$ whenever $b = b' : \beta$.

Here $f(b)$ is the application of the function f to the element b, where the notion of application is primitive. We shall say more about this primitive notion of application in the next section 4.2.5; it is not to be identified with substitution. We then know what the judgements (1*) and (1**) mean, since we know $\beta : \mathsf{type}$ and $\alpha : \mathsf{type}\ [x : \beta]$. Judgement (2) means that

$$f(b) = g(b) : \alpha\langle b/x\rangle \text{ whenever } b : \beta.$$

Again we know what this judgement means because of our knowledge of the premisses of the rule. In rule form we can gather the explanations of judgements (1) and (2) into the following two rules

$$\frac{f : (x : \beta)\alpha \qquad b : \beta}{f(b) : \alpha\langle b/x\rangle} \qquad \frac{f = g : (x : \beta)\alpha \qquad b = b' : \beta}{f(b) = g(b') : \alpha\langle b/x\rangle}$$

These meaning explanations may have seemed tedious, but they are our answer to the challenge that type predications cannot be made. By explaining the form of judgement

$$f : (x : \beta)\alpha$$

we explain what it means to assert that f is a unary function of a certain type, precisely what the problem of the concept horse requires us to explain. There is no *proposition* Pf where P is a predicate 'is a function'; in other words, there is no propositional function 'x is a function'. But there is a *judgement* $f : (\beta)\alpha$ assigning f to a type of functions. Frege did distinguish judgements and propositions in this sense. But he recognized only one form of judgement $\vdash A$; he did not recognize type predications as a form of judgement. To recognize and explain type predications is, however, just what is needed in order to account for the problem of the concept horse.

Following Martin-Löf, let us call what stands to the left of the colon in a judgement of constructive type theory the subject of the judgement, and what stands to the right its predicate.[153] As appears from the foregoing discussion the predicate of a judgement need not be a type. Firstly, in the first rule of type formation the predicate is type, and type is not itself a type. Secondly, the predicate of a hypothetical judgement is not a type. Thus

$$\mathsf{type}\ [x : \beta]$$
$$\alpha\ [x : \beta]$$

are both genuine predicates (provided β is a type), namely of the judgements

$$\alpha : \mathsf{type}\ [x : \beta]$$
$$a : \alpha\ [x : \beta]$$

but they are not types, for they are not constructed by means of the type formation rules. Hence we must distinguish between predications of the form $a : \alpha$ where α is a type, and predications of the form $a : \mathscr{C}$, where $\mathscr{C}$ is any predicate. Martin-Löf suggests to call a category anything that can be the predicate of a judgement of constructive type theory. If we recall section 1.2 of chapter 1 above, we see that this is in line with one meaning of the Greek word *katēgoria* as employed by Aristotle; but that it is not quite in line with Aristotle's designation of what we call his categories, for these Aristotle called, not *katēgoriai*, but rather *genē tōn katēgoriōn*, classes of

[153]For this and for the interpretation of categories in constructive type theory reported below, see for instance Martin-Löf (2013, Lecture 1).

predicates; so a system of Aristotelian categories is not primarily a system of predicates, but a system of classes of predicates. Given the type-theoretical framework Martin-Löf's designation is, however, a natural one, since the notion of a category is now the most general notion of a concept: certainly more general than the notion of a set, but also more general than the notion of a type. We have namely that any set is a type and any type is a category, while not all categories are types and not all types are sets.

4.2.5. *Notions of function.* Elements of dependent function types $(x : \beta)\alpha$ conform to one notion of function; we shall call elements of function types *mappings*.[154] Another notion of function is that of a *dependent object*; a dependent object is an object of the category $\alpha\ [x : \beta]$. What it means to be an element of the category $\alpha\ [x : \beta]$ is explained by explaining the two forms of judgement

(3) $a : \alpha\ [x : \beta]$
(4) $a = a' : \alpha\ [x : \beta]$

The form of judgement (3) is the form of judgement (Hypo) explained on page 154 above. Judgement (4) is explained by saying that

$$a\langle b/x\rangle = a'\langle b/x\rangle : \alpha\langle b/x\rangle \text{ provided } b : \beta$$

Thus we explain both what a mapping is and what a dependent object is by saying how it is applied to elements in its domain. We explained the judgement $f : (x : \beta)\alpha$ in part by laying it down that the following rule is valid

$$\frac{f : (x : \beta)\alpha \qquad b : \beta}{f(b) : \alpha\langle b/x\rangle}$$

A mapping f of type $(x : \beta)\alpha$ is thus something that when applied to any element b of β yields an element $f(b)$ of type $\alpha\langle b/x\rangle$. The notion of application proper to mappings is a primitive notion. It is part of the explanation of elements of function types that they are such things that can be applied to certain things to yield values (recall our discussion of Fregean functions in section 1.1). From the explanation of the judgement $a : \alpha\ [x : \beta]$ it follows that this rule is valid:

$$\frac{a : \alpha\ [x : \beta] \qquad b : \beta}{a\langle b/x\rangle : \alpha\langle b/x\rangle}$$

A dependent object a is thus something that when an element b of β is substituted for x in it, we get an element $a\langle b/x\rangle$ of type $\alpha\langle b/x\rangle$. Substitution is also a primitive notion, but it does not coincide with the notion of application proper to mappings.

[154] The distinction between three notions of function drawn in what follows stems from Martin-Löf; for an account in print, cf. Sundholm (2012a, pp. 953–954).

Substitution is in a sense more general than application, since it is explained for all objects—or expressions, if one prefers—of the theory, including types themselves. Substitution is usually dealt with in the metalanguage of a theory, but it has been shown that it can be made a part of constructive type theory itself.[155] With substitution included in the theory an expression such as $a\langle b/x\rangle$ will be a proper part of the theory and not a shorthand notation in the metalanguage for "the result of substituting b for x in a."

Given a dependent object a, we may form a mapping $(x)a$ by abstraction:

(Abstr) $$\frac{a : \alpha\ [x : \beta]}{(x)a : (x : \beta)\alpha}$$

Let us consider two examples. (i) Addition of natural numbers is defined in constructive type theory by means of the the recursion operator R introduced in the elimination rule for N.[156] Hence we may define a dependent object $2n$ as follows

$$2n \equiv n + n : N\ [n : N]$$

From the dependent object $2n$, of the category $N\ [n : N]$, we may form the mapping $(n)2n$ of type $(N)N$ by abstraction. (ii) There is a binary operator $+$ on set by means of which the disjoint union of two sets are formed. Hence we may define a dependent object $2X$ as follows

$$2X \equiv X + X : \mathsf{set}\ [X : \mathsf{set}]$$

From the dependent object $2X$, of the category $\mathsf{set}\ [X : \mathsf{set}]$, we may form the mapping $(X)2X$ of the type $(\mathsf{set})\mathsf{set}$ by abstraction. To obtain a mapping from the dependent object

$$X + Y : \mathsf{set}\ [X : \mathsf{set}, Y : \mathsf{set}]$$

two rounds of abstraction are needed. First we form the dependent object

$$(Y)X + Y : (\mathsf{set})\mathsf{set}\ [X : \mathsf{set}]$$

and then we form the mapping

$$(X)(Y)X + Y : (\mathsf{set})(\mathsf{set})\mathsf{set}.$$

This mapping is of course equal to the mapping $+$ itself.[157]

Neither a mapping nor a dependent object is an individual, i.e. an element of a set. Both of these notions of function must therefore be distinguished from the

[155]For the technical details of this, see Tasistro (1993).

[156]Cf. Martin-Löf (1984, pp. 71–74).

[157]This follows by the rule of η-conversion:

$$(x)(f(x)) = f$$

notion of function as an iterative set of a certain kind, namely a set of ordered pairs $f \subseteq X \times Y$ such that for all $x \in X$ there is at most one $y \in Y$ with $\langle x, y \rangle \in f$; and from the notion of function as a Fregean course-of-values. Within their respective frameworks both sets of ordered pairs and Fregean courses-of-values are individuals. Here is therefore a third notion of function. In constructive type theory a function of this kind is an element of the set $\Pi(A, B)$. Such a Π-set, as I shall call it, is formed by the following rule

$$\frac{A : \mathsf{set} \qquad B : (A)\mathsf{set}}{\Pi(A, B) : \mathsf{set}}$$

Hence, given a set A and a mapping B from A into set, we can form the set $\Pi(A, B)$. The introduction rule for Π-sets is the following

$$\frac{A : \mathsf{set} \qquad B : (A)\mathsf{set} \qquad f : (x : A)B(x)}{\lambda(f) : \Pi(A, B)}$$

Let us go through this rule in detail. We are given a set A and a mapping B from the set A into set. This means that

$$B(x) : \mathsf{set}\ [x : A]$$

In other words, the result of applying B to x is a set $B(x)$ whenever x is an element of A. From the rules of type formation it follows that

$$(x : A)B(x) : \mathsf{type}$$

We are, finally, given an element f of this type $(x : A)B(x)$. The introduction rule for Π-sets allows us to form the individual $\lambda(f)$, a canonical element of the set $\Pi(A, B)$. We may think of it as an individual coding the mapping f. Objects that evaluate to the form $\lambda(f)$ for some mapping f are thus the pendants in constructive type theory of functions in the sense of set theory and of Fregean courses-of-values. We must now ask how such an object is applied to an argument.

In the definition of a function as a set of ordered pairs of a certain kind it is not said what it means to apply such a set to an argument. It is therefore misleading then to write $f(x)$ without any further explanation. This is a piece of notation that must be explained. In axiomatic set theory that explanation is given by means of a contextual definition of the term $f(x)$. If φ is a formula satisfying

$$T \vdash (\forall x \in a)(\exists! y \in b)\varphi(x, y)$$

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where x does not occur free in f. The mapping to the left of the equality sign here is obtained as follows. One starts with a mapping f of type $(x : \alpha)\beta$. Then $f(x) : \beta\ [x : \alpha]$. Hence, by abstraction, $(x)(f(x)) : (x : \alpha)\beta$. The rule of η-conversion states that this mapping $(x)(f(x))$ is equal to f.

for certain sets a and b, then one defines

$$(f(x) = y) \equiv (x \in a) \wedge (y \in b) \wedge \varphi(x, y)$$

It is thus the equation $f(x) = y$ that is defined, and not the term $f(x)$ itself. An expression of the form $\psi(f(x))$ is then understood to be of the form $\exists y(f(x) = y \wedge \psi(y))$. This treatment of the term '$f(x)$' may well suffice for the purely mathematical purposes of axiomatic set theory, but it is questionable whether it suffices as a piece of conceptual analysis. For if this manner of dealing with functional expressions in any way reflects our own dealings with such expressions, it should be the case that we do not understand expressions like $2 + 2$ outside the context of an equation; but no such case has been made, nor, I think, can one be made. If I say that $2 + 2$ is an even number what I mean to say is that $2 + 2$ is an even number and not that there is some number n such that $2 + 2 = n$ and n is an even number. Neither is it convincing to claim that the proposed analysis somehow improves on our ordinary understanding of functional expressions, which if left unfettered may lead us into philosophical problems like those pertaining to the golden mountain or the king of France. For those problems arise from the fact that a meaningful expression has no reference; but the expression '$f(x)$' is meaningful only if $f(x)$ exists. So there appears to be no reason other than technical convenience for analyzing functional expressions away in the manner of set theory.

A better alternative, therefore, is to make use of an application operator $\mathsf{ap}(f, x)$ and define $f(x) \equiv \mathsf{ap}(f, x)$. A regress argument shows that this operator cannot itself be construed as a set of ordered pairs, or indeed as an individual of any kind whatsoever.[158] For if ap is an individual then we shall have the problem of accounting for the term '$\mathsf{ap}(f, x)$', which cannot be accounted for solely on the basis of how ap is explained, namely as an individual of a certain kind. The application operator ap can therefore not be an individual. Hence, Frege's application operator $\xi \frown \zeta$ is a binary function in his sense, in particular it is not itself an object in Frege's sense (*Gg* § 34). In constructive type theory ap is a mapping, i.e. an element of a dependent function type. Given an element c of the set $\Pi(A, B)$ and an element a of the set A, we justify the judgement $\mathsf{ap}(c, a) : B(a)$ as follows. When we evaluate c to canonical form we get something of the form $\lambda(f)$, where f is a mapping of type $(x : A)B(x)$. We define

[158] The argument closely resembles Bradley's regress argument. It is found e.g. in Dummett (1973, pp. 251–252), and in a somewhat different form in Frege's *BG* pp. 204–205.

$\mathsf{ap}(c, a)$ to be $f(a)$. Hence the following two rules are valid:[159]

$$\frac{c : \Pi(A, B) \qquad a : A}{\mathsf{ap}(c, a) : B(a)} \qquad\qquad \frac{f : (x : A)B(x) \qquad a : A}{\mathsf{ap}(\lambda(f), a) = f(a) : B(a)}$$

It should now be clear that the notion of mapping is more fundamental than the notion of an element of a Π-set. Firstly, in laying down what a canonical element of a Π-set is we appeal to a certain mapping: the canonical element $\lambda(f)$ is formed from the mapping f given in the premiss of the introduction rule. Secondly, in order to say how an element of a Π-set is applied to an individual we must appeal both to the ap-mapping and to the mapping f from which the canonical element $\lambda(f)$ is formed. Since ap is a mapping, *its* application to the arguments c and a is primitive, and not to be explained by means of any further notions.

Being a mapping, ap belongs to a dependent function type. It may be instructive to see which type this is. The ap-mapping takes an element of $\Pi(A, B)$ and an element a of A and yields an element of the set $B(a)$. The set A, the mapping B of type $(A)\mathsf{set}$, and the set $\Pi(A, B)$ must therefore all be included in the typing of ap. Using the letters X and Y in place of A and B one sees that

$$\mathsf{ap} : (X : \mathsf{set})(Y : (X)\mathsf{set})(\Pi(X, Y))(x : X)B(x)$$

The mapping ap is thus in fact a quarternary function taking a set X, a mapping Y in $(X)\mathsf{set}$, an element of $\Pi(X, Y)$, and an element x of X, and yielding an element of $B(x)$. Note the use of the dependent function type here: the type of each of the three last arguments of ap depends on at least one previous argument. Hence, if the first argument of ap is some set A, then the type of the second argument must be $(A)\mathsf{set}$; if the second argument is some mapping B of type $(A)\mathsf{set}$, then the type of the third argument must be $\Pi(A, B)$; the type of the fourth argument must be A. Similar considerations apply to the λ-operator, used for constructing the canonical elements of Π-sets. It is in fact a ternary function, taking a set A, a mapping B of type $(A)\mathsf{set}$, and an element f of type $(x : A)B(x)$, and yielding an element $\lambda(A, B, f)$ of type $\Pi(A, B)$. It is therefore typed as follows

$$\lambda : (X : \mathsf{set})(Y : (X)\mathsf{set})((x : X)Y(x))\Pi(X, Y)$$

For convenience we shall omit the two first arguments both of λ and of ap.

The λ-operator takes a mapping f and yields an individual $\lambda(f)$. It may therefore be regarded as a nominalization operator, taking a higher-order entity and yielding a first-order entity. From the typing of λ one sees that it is applicable to the mapping

[159] These are the rules of Π-elimination and Π-equality presented by Martin-Löf (1984, p. 28–29). A more general form of these rules are found in (ibid., Preface) and Nordström et al. (1990, pp. 51–52).

f only if the domain of f is a set, that is, only if f is of type $(x : A)B(x)$, where A is a set. It is natural to ask whether there can be nominalization also if the domain of f is not a set, if it for instance is set itself. There cannot be; at least there cannot be any set of the form "$\Pi(\mathsf{set}, B)$" explained along the lines of Π-sets. For assume that there were such a set. The canonical forms of its elements would be $\lambda(f)$, where f would be a mapping of the type $(X : \mathsf{set})B(X)$, where B is a mapping of type $(\mathsf{set})\mathsf{set}$. The domain of the mapping f would thus be set. Hence if $\Pi(\mathsf{set}, B)$ itself were a set, it would be included in the domain of f. But then an arbitrary canonical element $\lambda(f)$ of $\Pi(\mathsf{set}, B)$ would be the nominalization of a mapping f whose domain would include $\Pi(\mathsf{set}, B)$ itself. Hence the set $\Pi(\mathsf{set}, B)$ would have to be explained in terms of itself. For in defining a set we must lay down how its canonical elements are formed, and canonical elements of $\Pi(\mathsf{set}, B)$ would be formed from mappings f whose domain would include the set $\Pi(\mathsf{set}, B)$. We should therefore not be able to give a non-circular definition of $\Pi(\mathsf{set}, B)$. Hence it cannot be admitted as a set in constructive type theory.

5. Sortals, identity, and generality

For something to serve as a basic domain in logic it has not only to be a range of significance: it must also be a domain of quantification. Quantification, however, requires individuation. In this section we first consider concepts that serve individuation; thereafter we consider the notion of a domain of generality, of which the notion of a domain of quantification is a special case.

5.1. Sortals. The distinction drawn in constructive type theory between a set and a propositional function defined over that set corresponds closely to the distinction drawn by Strawson (1959, p. 168) in an ontological turn of phrase between sortal and characterizing universals, and by Geach (1962, p. 39) in a more linguistic turn of phrase between substantival and adjectival general terms. In later literature what Strawson calls sortal universals and Geach substantival general terms have also been called sortal concepts, sortal predicates, etc., or simply sortals;[160] for the most part of what follows we shall employ the latter term. A sortal universal, or substantival general term, A comes equipped with what Geach calls a criterion of identity,[161] "that

[160]The adjective 'sortal' was coined by Locke, *Essay* III.iii.15 on the pattern of how 'general' is formed from 'genus' (Locke used 'sort' as an English equivalent of both 'genus' (III.iii.15) and 'species' (III.iii.12)). The earliest use of the noun 'sortal' I have seen is in Wallace (1965, p. 10).

[161]This phrase occurs in Austin's translation of Frege's *Grundlagen*, namely of Frege's 'Kennzeichen für die Gleichheit' (*GLA* § 62). It is unclear to me whether Frege with this phrase intended what later philosophers have intended with the term 'criterion of identity' (note, for instance, the difference between the criterion

in accordance with which we judge as to identity" (Geach, 1962, p. 39), or in Strawson's terminology "a principle for distinguishing and counting individual particulars" (Strawson, 1959, p. 168). A characterizing universal, or adjectival general term, by contrast, presupposes such a criterion, since its role is to supply a "principle of grouping" particulars already distinguished. Thus, 'apple' is a sortal universal, say, since it provides a principle for distinguishing and counting those individual particulars we call apples, while 'red' is a characterizing universal, supplying for instance a principle for grouping apples into red and non-red apples.

A set A in constructive type theory comes equipped with a criterion identity: that is what one lays down in explaining what equal canonical elements of A are. The definition of a propositional function over a set presupposes the definition of that set, hence in particular it presupposes its criterion of identity. Sets thus have the role of sortals while propositional functions over sets have the role of characterizing universals. There is a further point of similarity between sortals and sets. As emphasized by Wiggins (1967, 2001) a sortal provides an answer to the question of what an individual is. Confronted with an individual and the question, What is it?, any appropriate answer is a sortal. Likewise it is a characteristic of sets in constructive type theory, in contrast to for instance iterative sets, that they provide such answers, being domains of individuals.

Dummett (1973) pairs the notion of criterion of identity with the notion of criterion of application, that which determines when it is correct to apply a predicate to an individual. In constructive type theory the criterion of application associated with a set is contained in the explanation of how the canonical elements of that set are formed. Recall the four introduction rules for the set N of natural numbers, stated on page 137 above. Two of these give the criterion of application of the set N:

$$0 : N \qquad \frac{n : N}{s(n) : N}$$

The other two give its criterion of identity:

$$0 = 0 : N \qquad \frac{n = m : N}{s(n) = s(m) : N}$$

As these rules illustrate, the criterion of application of a set A is conceptually prior to its criterion of identity: in stating that a is the same A as b I presuppose that a is an A and that b is an A. Dummett (1973, p. 75) holds that some adjectival terms are not associated with any criterion of identity, though they are associated with a criterion of application. The adjectival term 'red', for instance, provides no criterion

for being awarded a *cum laude* and a 'Kennzeichen' of having been awarded it—'criterion' apparently has this ambiguity, as noted by Geach 1973, p. 288).

of identity, but when we say that something is red we seem to rely on a criterion of application. Adjectival terms in general would therefore seem to bring out the conceptual priority of criteria of application over criteria of identity, since they are associated with criteria of application but not with criteria of identity.

It is doubtful, however, whether this analysis of adjectival terms is correct. Let us recall the distinction between type predications and ordinary predications: a type predication is one to the effect that something is an object of a certain type, while an ordinary predication is one to the effect that a certain object whose type has already been laid down has a certain property, that two objects whose types already have been laid down stand in a certain relation, etc. From the example of the set N we see that something's satisfying the criteria of application or identity of a sortal is expressed in a type predication, $a : A$ or $a = b : A$. That an individual satisfies an adjectival term, that for instance a number is even or an apple is red, is, by contrast, expressed in an ordinary predication 'Pa true'. The sense in which the apple can be said to satisfy a purported criterion of application associated with the adjective 'red' is thus quite different from the sense in which it is said to satisfy the criterion of application associated with the sortal 'apple': the one case is expressed by means of a type predication and the other by means of an ordinary predication. It may help to consider a predicate P defined over A and true for every element of A. Even though the judgement 'Pa true' as well as the judgement $a : A$ in some sense expresses the A-hood of a, only the latter is a judgement to the effect that a satisfies the criterion of application associated with A. The former judgement 'Pa true' presupposes that a satisfies this criterion of application, and *a fortiori* is not a judgement to the effect that it does satisfy it. It is thus only on the strength of an equivocation of type predications and ordinary predications that we may talk about criteria of application associated with adjectives.

Dummett offers an account of categories in terms of the notion of criterion of identity.[162] The account, presented at (Dummett, 1973, pp. 75–76), starts from the assumption that two sortals may be associated with the same criterion of identity. If A and B are two sortals associated with the same criterion of identity, then it is further assumed that there is a sortal C such that both anything which is an A is a C and anything which is a B is a C. Writing $A \sim B$ to denote that A and B are associated with the same criterion of identity and making intuitive use of the $\subset$-sign, this assumption can be expressed as follows:

[162] Similar accounts of categories are given by Hale and Wright (2001, p. 389) and by Linnebo (2005, p. 215).

If $A \sim B$, then there is a C such that $A \subset C$ and $B \subset C$.

Dummett finally assumes that among all sortals associated with the same criterion of identity there is a most general sortal, namely a sortal C such that if A is associated with the same criterion of identity as C, then anything which is an A is a C. This assumption can be expressed as follows:

For any A there is a C such that $A \sim C$, and $B \subset C$ whenever $B \sim C$.

Dummett identifies a category with such a C, what is sometimes called an "ultimate sortal."[163] Categories are thus identified with the most general sortals, sortals that are not subordinate to any other sortals.

From Dummett's account of categories it seems as if the criterion of identity of a sortal A is conceptually prior to the category C under which A falls: for the category C is defined as the most general sortal with the same criterion of identity as A. The actual order of conceptual priority is, however, the reverse. The criterion of identity associated with A is, namely, the same as that associated with C; but the criterion of identity associated with C presupposes the criterion of application associated with C. The criterion of identity associated with A, being the same as that associated with C, therefore also presupposes the criterion of application associated with C. Hence, both the criterion of application and the criterion of identity associated with a category, whence also the category itself, is conceptually prior to the sortals subordinate to it. The picture can therefore not be as suggested by Dummett's definition, namely that we obtain a category by considering more and more general sortals. The picture must rather be that we start with a category and obtain more and more specific sortals by adding characteristics, thereby altering the criterion of application while keeping the criterion of identity constant. We must thus regard the category as supplying the criterion of identity of these sortals. The resulting structure is presumably a tree structure with categories at top nodes; it is therefore similar to the structure of a tree of genera and species with categories at top nodes. The adding of characteristics to a sortal in order to obtain a more specific sortal corresponds in the traditional theory of genus and species to the adding of a differentia to a genus.

Both sortals and Ryle–Sommers types correspond to sets in constructive type theory: sortals by being associated with criteria of application and identity, and by providing answers to the question of what an individual is; Ryle–Sommers types by being the ranges of significance of predicates of individuals. That sortals and Ryle–Sommers types thus in a way complement each other appears, however, to have passed without much notice in the literature. In the works of Ryle, Sommers,

[163] For instance by Stevenson (1975, p. 195), Griffin (1977, e.g. p. 77), and Westerhoff (2005, p. 59–63).

Goddard, and others on the notion of range of significance it has not been asked whether such ranges are associated with criteria of identity and application. And for instance in the logic of sortals developed by Stevenson (1975) the notion of range of significance is altogether ignored. Stevenson's logical system contains sortal terms in addition to ordinary predicates, but these predicates are not restricted in their range of significance to sortals, but defined over the whole universe of individuals. In constructive type theory, by contrast, sets are associated with criteria of application and identity, and they serve as the ranges of significance of predicates. The notion of set in constructive type theory therefore absorbs both the notion of sortal and the notion of type as a range of significance.

5.1.1. *Identity.* The doctrine of restricted identity may be described roughly as the doctrine that identity is *restricted* to a sortal. Some such doctrine seems a natural consequence of the doctrine that sortals are associated with criteria of identity: if the criterion identity associated with S differs from that associated S' one would feel inclined to say that what identity means in S is different from what identity means in S'. This intuition cannot be satisfactorily dealt with in simple type theory. A restricted identity relation will need to be ternary, $x =_S y$, with a third argument for the sortal S, and that is straightforward enough. The type of this ternary identity relation in the simple type hierarchy, however, will be $((\iota)o)(\iota)(\iota)o$, taking a propositional function S and two individuals, and yielding a proposition; and that is problematic. For $a =_S b$ will then be a proposition no matter which individuals a and b are. In particular, relying on simple type theory one is forced to say that $a =_S b$ is a proposition even if either a or b is not an S.[164] But if either a or b is not an S, then the criterion of identity associated with S does not apply, hence it would seem that the relation $x =_S y$ is undefined for the pair of a and b: which criterion of identity would one be relying on in judging either that $a =_S b$ is false or that it is true? Not the criterion associated with S, for that does not apply to a, say; and, by the meaning of $x =_S y$, not the criterion associated with some other sortal S'. If in this case no criterion is needed, then one might wonder why a criterion is needed when judging whether $a =_S b$ where a is an S and b is an S. We seem forced to say that $a =_S b$ is meaningless if either a or b is not an S. This means that identity is of dependent type, since the type of the x- and the y-argument in $x =_S y$ depends on which S is chosen. In order to make sense of the doctrine of restricted identity we therefore need a dependent type hierarchy.

[164]From the the relevant clause in the definition of satisfaction in Stevenson's system it appears that the formula $x =_S y$ is false if x or y are not assigned elements of S (cf. Stevenson, 1975, p. 198).

The doctrine is made good sense of by the notion of propositional identity in constructive type theory, that is, the notion of identity encapsulated by I-sets (cf. p. 142 above). The ternary identity relation I is a mapping of the dependent function type $(X : \mathsf{set})(X)(X)\mathsf{prop}$. Identity is thus a mapping that yields the proposition $I(A, a, b)$ for any set A and elements a and b of A. The binary relation of identity on a set A, the relation $(x)(y)I(A, x, y)$, is of type $(A)(A)\mathsf{prop}$, hence it is undefined for individuals not in the set A. Constructive type theory thus supports what Griffin (1977, p. 97), taking a clue from Dummett (1973, p. 550), calls the function thesis: identity is a function from sortals to relations. Dummett (loc. cit.) adds to this that the sense of the identity relation associated with a given sortal is uniformly related to this sortal. Such is precisely the case in constructive type theory, where the introduction rule of the set $I(A, a, a)$ is uniform in A and a, as follows:

$$\frac{A : \mathsf{set} \qquad a : A}{r(A, a) : I(A, a, a)}$$

Identity is thus absolute: there is only one identity relation, namely the mapping I. The doctrine of *restricted* identity must therefore be distinguished from the doctrine of *relative* identity, known from the philosophical literature.[165] According to the doctrine of relative identity there may be two identity relations defined over one and the same sortal: the individuals a and b, falling under the same sortal, may be identical A's but non-identical B's. This doctrine is therefore not consistent with the function thesis,[166] hence, neither is it consistent with the notion of identity encapsulated by the I-sets of constructive type theory.

5.2. Quantification and generality. A domain of quantification is the domain over which a quantifier ranges, and which must be appealed to in giving the truth-conditions of propositions in which the quantifier occurs. The domain of quantification is often indicated in English by a noun phrase following the determiner 'every', 'any', 'some', 'a', etc., as in 'every bottle is empty' or 'some books were stolen'. Even when this is not the case, as in 'I have tried everything' or 'everyone is on board', where the determiner 'every' has been glued to the dummy nouns 'thing' and 'one', the

[165] Geach defends both restricted and relative identity (cf. his 1962, pp. 38–40, 157, 1972, pp. 238–249, and 1973); Wiggins (2001) defends restricted identity, but rejects relative identity (cf. esp. pp. 21–76); while Griffin (1977) holds the more idiosyncratic position that rejects restricted identity but defends relative identity.

[166] *Pace* Griffin (1977, p. 98), who claims that "the function thesis plainly derives its plausibility from (R)," i.e. from the doctrine of relative identity.

context usually supplies a domain.[167] In ordinary predicate logic a quantifier has the form $\forall x$ or $\exists x$ for some variable x. Here the domain of quantification is therefore not made explicit; that a quantifier of predicate logic nevertheless is associated with a domain is a basic assumption of model theory, where the domain of quantification is just the domain of the model (or one of its domains if the language is many-sorted). Constructive type theory differs from ordinary predicate logic by including reference to the domain of quantification in the syntactic form of the quantifier: the universal quantifier is there written $(\forall x : A)$, where A is a set.

According to the analysis of the syntax of quantified propositions provided by Frege in the *Grundgesetze* and followed by most logicians since,[168] such a proposition is obtained by applying a quantifier to a propositional function: given a propositional function Px we may form the quantified proposition $\forall x Px$. From this we see that the domain of the quantifier in $\forall x Px$ cannot exceed the range of significance of the propositional function Px. For, by definition it is only for elements a in the range of significance $|P|$ of Px that Pa is a proposition; hence it is only the elements in $|P|$ that one may draw upon in explaining the truth-conditions of $\forall x Px$. On the other hand, since the range of significance $|P|$ consists of the range of values for which Px is either true or false, it is clear that they must all be appealed to in explaining the truth-conditions of $\forall x Px$. Hence the quantifier in $\forall x Px$ must be taken to range over all the values in the range of significance of Px. We conclude—as already Russell (1908, p. 234) did on the basis of a similar argument—that the domain of quantification of a quantified proposition coincides with a range of significance.

This connection between domains of quantification and ranges of significance appears to be ignored in most of the literature on the question of absolute generality,[169] the question of whether it is possible to quantify over "absolutely everything." An answer to this question depends on an answer to the question of whether there is a universal range of significance: only if there is a universal range of significance is there a universal domain of quantification. In the literature discussing the question of absolute generality it appears to be generally assumed that there are propositional functions with the universal range of significance. In particular, it appears to be assumed by all parties of the debate that $x = x$ is such a function. To be more precise, it appears to be assumed by all parties of the debate that if the notion of

[167]The context plays a role also in utterances of 'every bottle is empty', for the noun 'bottle' does not by itself determine a domain of quantification; see Stanley and Gendler Szabó (2000) for a discussion of this phenomenon of so-called "quantifier domain restriction."
[168]Geach (1962, pp. 178–180) is perhaps an exception.
[169]For instance Williamson (2003), Glanzberg (2004), and the anthology of Rayo and Uzquiano (2006).

"absolutely everything" makes sense, then there is a propositional function $x = x$ ranging over that domain; the main point of contention is then whether the notion of "absolutely everything" does in fact make sense, and if so, whether it supplies a domain of quantification. That assumption, however, requires argument, and our previous discussion of ranges of significance and of identity shows that convincing arguments are hard to find. Our considerations in sections 2–4 regarding the ranges of significance of propositional functions apply of course as well to the propositional function $x = x$. In the case of identity the considerations of the previous section 5.1.1 apply in addition. If one accepts the division of individuals into sortals and views these as associated with criteria of identity, then one seems committed to a doctrine of restricted identity. But according to such a doctrine the range of significance of the propositional function $x =_S y$ must be the sortal S and not the whole universe.

Thus, any domain of quantification is a range of significance; is any range of significance a domain of quantification? Recall the rule of formation of Π-sets:

$$\frac{A : \mathsf{set} \qquad B : (A)\mathsf{set}}{\Pi(A, B) : \mathsf{set}}$$

Since $\mathsf{prop} = \mathsf{set}$ this rule can also be written

$$\frac{A : \mathsf{set} \qquad B : (A)\mathsf{prop}}{\Pi(A, B) : \mathsf{prop}}$$

To assert that the proposition $\Pi(A, B)$ thus formed is true is to assert that it is inhabited. We may assert that provided we have found a member c of $\Pi(A, B)$. This object c has the property that given any element a of the set A, $\mathsf{ap}(c, a)$ is an element, i.e. a proof object, of the proposition $B(a)$. Following the so-called Brouwer–Heyting–Kolmogorov interpretation of the logical constants, c may therefore be regarded as a proof object of the proposition $(\forall x : A)B$.[170] Hence we may define

$$(\forall x : A)B = \Pi(A, B) : \mathsf{prop}$$

As $\Pi(A, B)$ can be formed for any set A and any propositional function $B : (A)\mathsf{prop}$, it follows that any set is a domain of quantification, namely the domain being quantified over in the proposition $(\forall x : A)B$. Let us reserve the term 'quantification' for this notion of generality, that is, the notion of generality thus captured by Π-sets. We shall soon see that constructive type theory also knows other notions of generality.

[170] A standard formulation of the Brouwer–Heyting–Kolmogorov interpretation is found in Troelstra and van Dalen (1988, pp. 9–10); see Sundholm (1983) for its philosophy and history.

Let A be a set. Then $(x : A)I(A, x, x)$ is a dependent function type. By the introduction rule for I-sets,

$$\frac{A : \mathsf{set} \qquad a : A}{r(A, a) : I(A, a, a)}$$

there is a mapping $(x)r(A, x)$ of type $(x : A)I(A, x, x)$ taking an element a of A and yielding a proof object $r(A, a)$ of $I(A, a, a)$. The introduction rule for Π-sets then gives us

$$\lambda((x)r(A, x)) : (\forall x : A)I(A, x, x)$$

By omitting the proof object $\lambda((x)r(A, x))$ we obtain the judgement that every element of A is equal to itself:

$$(\forall x : A)I(A, x, x) \ \text{ true}$$

Here A is a given set. The law of identity states, however, not that every element of some particular set is equal to itself, but that any element of any set whatsoever is equal to itself. The law thus involves generality over sets.

We saw above that there is no set of the of the form $\Pi(\mathsf{set}, B)$. Hence there is no proposition of the form $(\forall x : \mathsf{set})B$, where B is a propositional function over set. In particular there is no proposition of the form "$(\forall X : \mathsf{set})(\forall x : X)I(X, x, x)$," which would express the law of identity. But consider the following judgement

$$(X)(x)r(X, x) : (X : \mathsf{set})(x : X)I(X, x, x)$$

It can be derived in a chain of reasoning similar to the one just carried out where we let X be an arbitrary set on which we abstract in a final step.[171] The mapping $(X)(x)r(X, x)$ thus asserted to exist takes a set X and an element x of X, and yields an element, i.e. a proof, $r(X, x)$ of the proposition $I(X, x, x)$. The judgement therefore asserts that the proposition $I(X, x, x)$ is inhabited for any set X and element x of X; that is, it asserts the law of identity. By demonstrating that the type

$$(X : \mathsf{set})(x : X)I(X, x, x)$$

[171] Here is a formalized version of the reasoning:

$$\cfrac{\cfrac{\cfrac{X : \mathsf{set}\ [X : \mathsf{set}] \qquad x : X\ [x : X]}{r(X, x) : I(X, x, x)\ [X : \mathsf{set}, x : X]}}{(x)r(X, x) : (x : X)I(X, x, x)\ [X : \mathsf{set}]}}{(X)(x)r(X, x) : (X : \mathsf{set})(x : X)I(X, x, x)}$$

The first inference uses I-introduction, and the next two uses (Abstr), given on page 158 above.

is inhabited, we have thus been able to demonstrate a judgement in which there is generality over set. Let us call a type of the form

$$(x_1 : \alpha_1) \dots (x_n : \alpha_n) B(x_1, \dots, x_n)$$

where $B(x_1, \dots, x_n)$ is a proposition provided $x_1 : \alpha_1, \dots, x_n : \alpha_n$, a generality type.[172] The types $\alpha_1, \dots, \alpha_n$ will be called the domains of the generality type. A judgement of the form

$$b : (x_1 : \alpha_1) \dots (x_n : \alpha_n) B(x_1, \dots, x_n)$$

will be called a type general judgement. Here b may be regarded as a verification object of the generality type $(x_1 : \alpha_1) \dots (x_n : \alpha_n) B(x_1, \dots, x_n)$. It takes any elements $a_1, \dots, a_n$ of $\alpha_1, \dots, \alpha_n$ respectively and yields a proof object $b(a_1, \dots, a_n)$ of the proposition $B(a_1, \dots, a_n)$. We call b a verification object and reserve the term 'proof object' for individuals, namely for the verification objects of propositions.

We can now answer our question. A propositional function is an element of a type $(\alpha)\mathsf{prop}$ where α can be any type. Hence, any type may serve as a range of significance. A range of significance is of course the range of significance of some propositional function. Hence, any range of significance is a type. Moreover, any type is the domain of some generality type; hence any range of significance is the domain of a generality type. However, not every range of significance is a domain of quantification; the type set, for instance, is not. In fact, the type α is a domain of quantification if and only if α is a set. Hence, although every range of significance is the domain of a generality type, not every range of significance is a domain of quantification.

This, then, answers our question; but the answer raises a new question. The introduction rule for Π-sets allows us to form an object $\lambda(f) : \Pi(A, B)$ from the mapping $f : (x : A)B(x)$. Hence, the rule allows us to pass from the generality type $(x : A)B(x)$ to the quantified proposition $\lambda(f) : \Pi(A, B)$, and from the verification object f, which is not an individual, to the proof object $\lambda(f)$, which is an individual. Since the type general judgement $f : (x : A)B(x)$, occurring as a premiss of the rule,

[172] Generality types stand to universally quantified propositions roughly as what Sundholm (1997, p. 206–208) calls closed consequences stand to implications. A closed consequence is a type

$$(A_1) \dots (A_n) B$$

where $A_1, \dots, A_n, B$ all are propositions. The corresponding implication is

$$A_1 \supset (\dots \supset (A_n \supset B \underbrace{) \dots)}_{n-1}$$

The verification object of a closed consequence is a mapping, while the verification object of an implication is an individual, namely an element of a Π-set.

is already a judgement of generality, the question arises what the point is of a rule for forming Π-sets and an operator λ for nominalizing verification objects f of type $(x : A)B(x)$. Is it not sufficient to have a rule for forming generality types $(x : \alpha)B(x)$ and a rule of abstraction for forming verification objects of type general judgements?

It is not sufficient if we are to incorporate predicate logic into constructive type theory. For that requires identifying the operators of predicate logic—the connectives and the quantifiers—with elements in the dependent type hierarchy. Only by identifying these operators with elements of the hierarchy can we honour the basic tenet of type theory that all symbols of the language be assigned a type. But there is no element in the dependent type hierarchy corresponding to the construction of the generality type $(x : \alpha)B$ from a type α and a propositional function B over α. When we pass from a propositional function over a type to a generality type we do not apply an operator to the former so as to obtain the latter. The formation of a generality type is rather an instance of dependent function type formation, which is a primitive of the theory, just as the application of a mapping to an argument is a primitive. By contrast, there is an element of the dependent type hierarchy by which a quantified proposition is formed from a set and a propositional function over that set: that is the mapping Π. It, and therefore the quantifier ∀, is a mapping of type $(X : \mathsf{set})(Y : (X)\mathsf{prop})\mathsf{prop}$. Only quantified propositions, thus, and not generality types are formed by means of an operator of the type hierarchy. We must therefore recognize both notions of generality: quantified propositions and generality types.

The type set, hence also prop, may be the domain of a generality type, but not of a quantified proposition. Generality types therefore provide for a notion of higher-order generality. It differs from higher-order quantification in predicate logic. The result of applying a higher-order quantifier to a propositional function is a new proposition, to which, therefore, the propositional connectives are applicable. A generality type is, however, not a proposition, but a dependent function type. The notion of generality provided for by generality types also differs from what is sometimes called schematic generality.[173] This is the kind of generality involved in the assertion of axiom schemes, such as induction in arithmetic or replacement in set theory. A generality type, namely, involves binding, which schematic generality does not. This comes out clearly when we consider negation and denial. One cannot deny a judgement of schematic generality by another another judgement of schematic generality.

[173]That difference is utilized in Sundholm (2013) in order to transform the so-called Kripke schema into a principle.

One can, for instance, not deny the axiom of replacement

$$\forall a[(\forall x \in a)\exists! y\, \varphi(x,y) \supset \exists b\, (\forall x \in a)(\exists y \in b)\varphi(x,y)]$$

where the variables range of iterative sets, by asserting the following schema:

$$\neg\forall a[(\forall x \in a)\exists! y\, \varphi(x,y) \supset \exists b\, (\forall x \in a)(\exists y \in b)\varphi(x,y)]$$

A generality type, by contrast, may occur within the scope of a negation. Let us see how. The negation $\neg A$ of a proposition A can be defined as $A \supset \bot$, where $\bot$ is a false propositional constant. In constructive type theory the set N_0 is such a constant, hence we may define

$$\bot = N_0 : \mathsf{set}$$

This set has no introduction rule, hence it is empty, and therefore false as a proposition. The implication $A \supset B$ is treated in constructive type theory as a special case of a Π-set. Assume that $C : (A)\mathsf{set}$ is a mapping satisfying $C(x) = B$ for every x in A. Then a proof object of $\Pi(A, C)$ is an object d with the property that $\mathsf{ap}(d, a)$ is an element of $C(a) = B$ provided a is an element of A. Hence, in accordance with the Brouwer–Heyting–Kolmogorov interpretation of the logical constants, we may define $A \supset B = \Pi(A, C) : \mathsf{prop}$. A proof object of $A \supset B$ is thus a lambda term $\lambda(f)$ derived from a mapping $f : (A)B$.

By setting $B = \bot$ in the above we see that $\neg A = \Pi(A, C)$, where C is a propositional function over A such that $C(a) = \bot$ for every a in A. A proof object of $\neg A$ is therefore a lambda term $\lambda(f)$ derived from a mapping $f : (A)\bot$. To assert that such a mapping exists, that is, to make the judgement

$$f : (A)\bot$$

is to deny that A is true, for that is to assert that any proof object of A can be mapped to a proof object of $\bot$, while there are no proof objects of $\bot$. In general, if α is a type, then $f : (\alpha)\bot$ in effect asserts that α is empty. Hence we may think of

$$(\alpha)\bot$$

as the negation of the type α. If α here is a generality type, $(\alpha)\bot$ is its negation. Let us consider an example, namely the negation of the generality type

$$(X : \mathsf{set})(x : X)(y : X)I(X, x, y)$$

A verification object of this type would be a mapping taking any set X and any two elements x and y of X to a proof object of the proposition $I(X, x, y)$, the proposition that x and y are equal. Assume the generality type to be inhabited; that is, assume the judgement

(J) $$z : (X : \mathsf{set})(x : X)(y : X)I(X, x, y)$$

Thus we are assuming that for any set X all pairs of elements x, y of that set are equal. If we apply the z provided by this assumption (J) to the set N and its elements 0 and $s(0)$ we obtain a proof object $z(N, 0, s(0))$ of $I(N, 0, s(0))$. Now, a judgement of the form

$$c : \neg I(N, 0, s(0))$$

can be demonstrated if one assumes the existence of what is called a *universe* in constructive type theory.[174] Applying this proof object c of $\neg I(N, 0, s(0))$ to $z(N, 0, s(0))$ of $I(N, 0, s(0))$ we obtain an element $\mathsf{ap}(c, z(N, 0, s(0)))$ of $\perp$. Thus we have demonstrated the hypothetical judgement

$$\mathsf{ap}\big(c, z(N, 0, s(0))\big) : \perp \ [z : (X : \mathsf{set})(x : X)(y : X)I(X, x, y)]$$

By the rule (Abstr) of abstraction, given on page 158 above, we can infer

$$(z)\mathsf{ap}\big(c, z(N, 0, s(0))\big) : \big((X : \mathsf{set})(x : X)(y : X)I(X, x, y)\big)\perp$$

The formalized version of this demonstration may be relegated to a footnote.[175] The judgement demonstrated asserts that a certain mapping exists from the generality type $(X : \mathsf{set})(x : X)(y : X)I(X, x, y)$ into the set $\perp$. Hence it sanctions a judgement that can be expressed in scare quotes as follows:

$$\text{“}\neg(\forall X : \mathsf{set})(\forall x : X)(\forall y : X)I(X, x, y) \ \text{ true”}$$

6. Formal and material categories

A recurring theme of this chapter has been the distinction between ground types and higher types. In this final section we consider a related distinction, namely that between what I shall call, employing Husserlian terminology, formal and material

[174] On universes, see Martin-Löf (1984, pp. 87–91); the demonstration of $c : \neg I(N, 0, s(0))$ is given ibid. p. 91. That the assumption of one universe is necessary for this demonstration was proved by Smith (1988).

[175] We recall that z officially is a unary mapping, yielding a new unary mapping when applied to a set X, etc. As in the main text, we let c be a proof object of $\neg I(N, 0, s(0))$. We abbreviate the judgement $z : (X : \mathsf{set})(x : X)(y : X)I(X, x, y)$ by J. The demonstration is then as follows.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{J}\ [\mathrm{J}] \qquad N : \mathsf{set}}{z(N) : (x : N)(y : N)I(N, x, y)\ [\mathrm{J}]} \qquad 0 : N}{z(N)(0) : (y : N)I(N, 0, y)\ [\mathrm{J}]} \qquad s(0) : N}{z(N)(0)(s(0)) : I(N, 0, s(0))\ [\mathrm{J}]} \qquad c : \neg I(N, 0, s(0))}{\mathsf{ap}\big(c, z(N)(0)(s(0))\big) : \perp\ [\mathrm{J}]}}{(z)\mathsf{ap}\big(c, z(N)(0)(s(0))\big) : \big((X : \mathsf{set})(x : X)(y : X)I(X, x, y)\big)\perp}$$

categories. Formal categories may be thought of as topic-neutral, while material categories are topical. In the words of Ryle (1954, p. 116), who coined the term 'topic-neutral', material categories provide "the fat and the lean," whereas formal categories provide the "joints and tendons" of thought. The distinction will here be discussed in the context of the works of Frege, Husserl, and Carnap.

6.1. Frege's three realms. Frege in his *Der Gedanke* (1918) distinguished between three realms (*Reiche*) or worlds: the outer world (*Außenwelt*), the inner world (*Innenwelt*), and the third realm (*drittes Reich*).[176] The first is the realm of things that we access through perception, containing for instance stones, trees, and houses. The second is the realm of what Frege calls ideas (*Vorstellungen*), which include both intentional experiences, such as perceptions, imaginations, and remembrances; and non-intentional experiences, such as sensations and moods. To the third realm belong what Frege calls thoughts. Thoughts differ from things in the first realm in that we can have no perceptual awareness of them; and from things in the second realm in not being dependent on any subject for their existence: in Frege's words, thoughts do not need a "carrier" (*Ged* p. 69). Frege had made a similar trifold division already in the *Grundlagen*, namely between the spatial or physical; the subjective, or the world of ideas; and what is objective yet not perceived by the senses. Frege appears to argue that numbers, since they belong to neither of the first two, must belong to the third of these realms (*GLA* §§ 21–27). As objects of a similar status he also mentions the axis of the earth, the centre of mass of the solar system, and the equator. To the third realm one should perhaps also count "logical objects" such as the truth-values and courses-of-values,[177] and the primitive functions of the ideography.[178] In fact, at *Gg* p. xviii Frege allocates concepts in general to what he there calls the domain of the objective but non-real (*ein Gebiet des Objectiven, Nichtwirklichen*), which may be another variant of the third realm.

The division into three realms deserves the title of a categorial division, for it purports to be a most general division of entities in a general sense of 'entity'. The question therefore arises how this division relates to Frege's simple type hierarchy.[179] On the basis of Frege's writings one can only speculate on an answer; but three

[176] On the history of the notion of *Drittes Reich*, see Gabriel (1992). Many of the references to Frege's works in what follows I have found in Künne (2010, pp. 486–506, 514–541).

[177] For the characterization of these as logical objects, see *WB* p. 121. Numbers are characterized as logical objects at *Gg* II § 74.

[178] The negation function and the identity relation are both said to "belong to logic" at *UGG*2 p. 428.

[179] The *Nachlass* piece known as *Aufzeichnungen für Ludwig Darmstaedter* (*NS* pp. 271–277), dated 1919, shows that Frege also at the time when *Der Gedanke* was published accepted the distinction between first- and second-order functions.

readings suggest themselves. According to the first the three realms divide the domain of Fregean objects, while functions do not belong to any realm. Instead of one ground type there would thus be three, corresponding to the three realms. Employing 1, 2, and 3 as names for the first, second, and third realm respectively the resulting type structure could be defined as follows: 1, 2, and 3 are types; if $\tau_1, \ldots \tau_n$ are types, then $(\tau_1 \ldots \tau_n)1$, $(\tau_1 \ldots \tau_n)2$, and $(\tau_1 \ldots \tau_n)3$ are types, namely the types of n-ary functions from $\tau_1, \ldots, \tau_n$ into 1, 2, and 3 respectively. The three realms effect a partition of the ground type of objects also on the second and third reading; but unlike the first reading these readings assign functions to realms. According to the second reading functions belong to the third realm. The third realm is thus taken to encompass all types of functions as well as a certain section of the type of objects. The third ground type is therefore no longer to be equated with the third realm but only with the objects of the third realm, 3_o say. According to the third reading the realm of a function is the same as the realm of its arguments. This reading therefore requires that all n-ary functions for $n > 1$ be homogenous, namely that all the arguments of a function belong to the same realm. In the literature Künne (2010, p. 505) defends a version of the first reading, Burge (1992) assumes the second reading, while Dummett (1982, pp. 120–123) defends a version of the third reading.

All the three readings hold that the ground type of objects is partitioned into three ground types. This conflicts with the doctrine of the *Grundgesetze*, where functions are required to be defined for all objects, for instance addition to be defined for the Sun (*FB* p. 20; cf. section 3.2 above). But there are indications that the Frege of the *Grundlagen* accepted such a partition. He says for instance that "spatial predicates are not applicable to ideas" (*GLA* § 61, cf. § 48); thus, not that spatial predicates are false of ideas, but that they are not applicable to them. This can only be the case if ideas and spatial predicates are not of matching types. Frege moreover appears to regard 'blue idea', 'salty concept' and 'chewy judgement' as category mistakes (§§ 24, 3). This must be explained by the fact that with these expressions one purports to apply a first realm predicate to a second or third realm object. In the *Grundgesetze* (p. xxi), by contrast, Frege remarks that to say of an idea that it is green is false, hence not nonsensical. We have seen that this is a common account of category mistakes in later writers, and the one Frege is forced to with his acceptance of a universal domain of individuals.

In light of these tensions and in light of the motley assortment of entities the third realm must be taken to include when one juxtaposes the *Grundlagen*, the *Grundgesetze*, and *Der Gedanke* it seems preferable to regard the various trifold divisions of these works as distinct, serving distinct theoretical frameworks. It may for instance be argued that the third realm of *Der Gedanke* is the realm of senses. Indeed, in the contemporaneous *Die Verneinung* (1919) Frege speaks of "the realm of thoughts and parts of thoughts" (*Vern* p. 155). Thus he seems to assume that parts of thoughts just as thoughts themselves are denizens of the third realm; and these are all and only the senses of expressions. But if the third realm is the realm of senses, then it does not include logical objects, since logical objects are not Fregean senses; for the same reason the realm of senses does then not include numbers. In an unpublished piece from 1919 Frege indeed explicitly denies that objects and functions are parts of thoughts (*NS* p. 274),[180] whence he implicitly denies that objects and functions inhabit the realm of thoughts and parts of thoughts. The realm of senses cannot therefore be the realm of the objective but non-real in the *Grundlagen* and the *Grundgestze*.

It is indeed doubtful whether everything Frege regards as objective but non-real in the *Grundlagen* can reasonably be said to belong to one and the same realm: does the equator and the number 14, for instance, belong to the same realm? They are both non-real and objective, but their commonalities would seem to stop there. It can moreover be questioned whether senses and objects that are not senses, such as numbers, can be taken to belong to the same realm. It seems odd, for instance, that the sense of a number expression should belong to the same realm as the number designated by that expression, for the two are "constituted" in quite different ways, the one as the object of an act, the other as what provides the direction of that act. For Frege, however, this may not have seemed odd, for he appears to have viewed thoughts not as providing the direction of acts, but rather as being the objects of acts of thinking. To think, according to Frege, is to grasp a thought (*Ged* p. 62); and one grasps a thought in an act directed towards that thought (*Ged* p. 75).[181] In such an act a non-sensible capacity, which by operating upon material provided to it by the senses discloses the first realm, operates without such material, thereby disclosing the third realm.[182] With each realm we may therefore associate a mode of access to that realm. The first realm is indeed characterized by Frege as the realm of things that can be perceived by the senses (*Ged* p. 66) or as the world of sensibly perceptible things (*Ged* p. 75). Frege does not seem to have a term for the perception

[180] *Aufzeichnungen* (*NS* p. 274): "Doch sind Gegenstand und Begriff nicht Bestandteile dieses Gendankens. Die Bestandteile des Gedankens weisen aber in eigentümlicher Weise auf Gegenstand und Begriff hin."

[181] "Obgleich zum Bewußtseinsinhalte des Denkenden der Gedanke nicht gehört, muß doch in dem Bewußtsein etwas auf den Gedanken hinzielen." Cf. the commentary of Künne (2010, pp. 514–518) *ad loc.*

[182] The capacity in question may be what Frege calls 'reason' in *GLA* §§ 26, 105. Frege's account of perception fits the "content-apprehension model" that Husserl assumed for all acts in the *Logical Investigations* (cf. Klev, 2013). In Kant's terminology thoughts would be *noumena*, "Dinge, die bloß Gegenstände des Vestandes sind" (KrV A249).

of one's ideas, what Locke called *reflection*,[183] but that he recognizes such perception must be implicit in his speaking of perception *by the senses*, viz. presumably as a contrast to inner perception: otherwise the phrase 'perception by the senses' would be a pleonasm, and 'perception' would suffice.[184] The third realm is accessed by the act by virtue of which we grasp thoughts.

6.2. Husserl on regions and formal categories. Frege's pair of notions of type and realm bears resemblance to Husserl's pair of notions of formal and material category. A formal category may be described as topic-neutral, while material categories, or regions, may be described as providing the most general topics with respect to which formal categories are neutral. Such a characterization also fits the type-theoretical notions of function and ground type respectively. For, although each function is defined over a single type, and in that sense is not topic-neutral, the notion of function itself is topic-neutral in the sense that given any two types α and β there is a type $(\alpha)\beta$ of functions from α to β. The ground types in a type hierarchy provide topics in the sense that all other types are types of functions of functions of... individuals of some ground type; that is, all other types are ultimately grounded in one or more ground types. Objects in the simple type hierarchy are thus either individuals or derived from individuals by means of the "formal category" of 'function of'. We have already seen (in section 1.3 above) that Husserl does not adopt a type hierarchy; but he does think that within each region various entities may be derived by means of formal categories from the individuals of that region (Ideen I § 10). As examples of formal categories Husserl lists property, relation, state of affairs, set, number, part and whole, and others (loc. cit.). These apply in every region and give rise to the "higher types" of entities of a region, for instance properties of physical things and states of affairs involving physical things. These higher type entities are equally part of the region (cf. the third reading on page 176 above, according to which the realm of a function is the same as the realm of its arguments); a region thus consists of individuals and categorial derivations of those individuals.

It is a fundamental philosophical problem, according to Husserl, both to find out what regions there are (Ideen I § 17) and to understand the nature of a given region and the interdependence of various regions on each other (ibid. § 152). In order properly to understand the region of consciousness one needs to carry out the so-called phenomenological reduction; only after that has been done does one have

[183]*Essay* II.i.4. Reflection is for Locke contrasted with sensation, by means of which we access the first realm.
[184]This point is due to Künne (2010, pp. 488–489).

before oneself this region in its purity (ibid. § 33). Other regions, which not being consciousness are regions of transcendent beings (ibid. §§ 41–46), raise the problem of constitution, namely the problem of describing precisely how objects of that region present themselves to consciousness, how they are constituted in consciousness (ibid. §§ 149–153; Ideen II *passim*). Such is for instance the case with the region of nature, or physical thing (Ideen II §§ 12–18). Husserl is not quite definitive on what regions there are other than nature and consciousness, but he clearly does assume that there are other and that it is a task of phenomenology to find them and to describe them. Other regions Husserl mentions in the three books known as the *Ideas*[185] include the region of the body, the region of living beings (animals), the region of persons, and the region of society and culture ("*die Geistige Welt*").[186] Husserl's set of regions thus offers a refinement of Frege's first two realms. Fregean senses would for Husserl either belong to the region of consciousness, this being the region of so-called noemata (Ideen I § 88), which at least some readers have regarded as the Husserlian pendant to Fregean senses;[187] or to the formal category of meanings, which is the domain of pure logic, including pure grammar (cf. section 1.3 above). Numbers, as well as sets, do in any event constitute formal categories according to Husserl. In general it appears that Frege's third realm in the Husserlian scheme should be distributed over the region of consciousness and various formal categories.

In the first chapter of the *Ideas* Husserl gives a definition of the notion of a region.[188] Put concisely, a region is a highest genus under which essentially independent entities fall. This definition relies on Husserl's doctrine of so-called essences (*Wesen*), which we may think of as a sort of objectified concepts (Ideen I §§ 1–7). Essences stand in relations of genus and species (ibid. § 12). Moreover, some essences are dependent and others are independent, namely if and only if a particular standing under that essence is dependent or independent, respectively (ibid. § 15; *LU* III); an individual is a particular standing under an independent essence. Regions are highest

[185] Cf. the remark on these works on p. xiv above.

[186] In lecture notes from 1917/18 Husserl speaks of a region of valued being ("*Wertsein*") that should be coordinated with the "value-free" regions of nature, consciousness, etc. (Husserl, 1996, p. 297). It seems to me to fit better with Husserl's doctrine, both that of regions and that of axiology, that "axiological categories" should stand, not coordinated to regions, but rather as the formal categories stand to regions: they are applicable in all regions, transforming whatever they apply to into valued objects of various sorts.

[187] The *locus classicus* for this interpretation is Føllesdal (1969). The noema involves several components that we would presumably not find in Fregean senses, in particular "thetic character" and saturation (Ideen I §§ 132–133); the pendant of Fregean senses is thus rather what Husserl calls noematic sense (ibid. §§ 129–130). Husserl was anxious to distinguish the pure region of consciousness from the domain of study of psychology, so this reading of Fregean senses need not be a piece of psychologism. Woodruff Smith (1995, p. 331) expressly links Frege's third realm and Husserl's noemata.

[188] For an instructive discussion of this definition, see Stone (2000, pp. 97–131).

independent essences (Ideen I § 16). Husserl in fact distinguishes the notion of a region and the notion of a material category. A region gives rise to what Husserl calls a regional ontology, an a priori science of the region in question. A material category is a primitive concept of a regional ontology (loc. cit.). Husserl seems also to have assumed that any highest genus which is not itself a region is a dependent part of a region. Spatial form (*Raumgestalt*), for instance, is a dependent genus; and it, as well as any higher genus it may fall under, is a dependent part of the region of nature, since any physical thing has a spatial form.

Husserl's definition of regions does not play any significant role in how he actually deals with this notion. Instead of discussing that definition in detail, it is therefore more instructive to consider some of the characteristics Husserl in his employment of the notion takes regions to have. We shall see that characteristics that we have considered earlier in this dissertation, such as being the range of significance of a propositional function, being the range of a quantifier, and being associated with criteria of application and identity are not among these. A first characteristic Husserl takes regions to have is related to the idea that regions are highest genera. A region is namely said to prescribe a rule for how we may vary an individual of that region in imagination so that it still remains an individual, in other words, so that we still have a unitary course of experience of an individual (Ideen I §§ 142, 149, 150; Ideen III § 7). Less general concepts, for instance the concept of a diamond, may also be regarded as prescribing a rule for the course of experience, for instance that the thing posited as a diamond does not bounce back in our hands in the manner of a bouncy ball when we let it fall to the ground. But we can still imagine a continuous transformation of the diamond into a bouncy ball without our experience falling apart into a series of disconnected appearances; in fact we can imagine the diamond continuously transformed into any other physical thing, namely so long as we remain inside the region of nature. Husserl's discussion of imaginative variation in the cited paragraphs rely on the region in question being that of nature, but he appears to have thought that the discussion would generalize to other regions.

A second characteristic of regions concerns "access," a topic we have already discussed in connection with Frege's realms. For each region there is an original mode of awareness of entities of that region (Ideen I § 138). In the case of nature that is ordinary perception (e.g. loc. cit.). This is an original mode of awareness in contrast for instance to remembrances and imaginings of things in nature (ibid. § 99), in which the object is not present there before us. In the case of consciousness the original mode of awareness is reflection (ibid. §§ 77–78); this, however, is not naive reflection, but rather what one may call phenomenological reflection, involving the phenomenological reduction, for only when this is carried out can pure consciousness be disclosed (ibid. § 50). One becomes aware of the body in what Husserl calls bodily apprehension (Ideen II § 36; Ideen III § 2). The individuals in the region of the body are localized sensations, namely sensations localized in the body. In order to apprehend these sensations a special form of apprehension is required, since usually the sensations are apprehended as things in the outer world. I touch this table; I then speak of the sensations in my fingers on the basis of bodily sensations, but of the surface of the table on the basis of a perceptive apprehension. It is in the former sort of apprehension, which according to Husserl also applies for the other senses, that I am aware of the body as a region. We are aware of animals and persons by forms of empathy (*Einfühlung*) (e.g. Ideen I § 151). Thus we speak of "seeing" sorrow or joy in our fellow human beings, and to some extent also in other living beings. It appears that for Husserl also the region of culture is accessed through a form of empathy. The formal categories crosscut the regions, hence the proper mode of awareness of formal categories do not conflict with the proper modes of awareness of regions. Rather, Husserl held that any intuition can be categorially formed so that we can see not only the individuals of a region, but also its higher type entities, notably states of affairs. This is Husserl's doctrine of categorial intuition, developed in *LU* VI §§ 40–66. 'Intuition' here designates an act in which the object is given with a certain amount of liveliness and saturation, and contrasts with what Husserl calls signitive acts; any act of original awareness of a region is an intuition (e.g. *LU* VI §§ 14, 25–27). Categorial forms of intuition are therefore the intuitional counterparts to forms of meaning in Husserl's sense (cf. section 1.3 above); and categorially formed intuitions are the counterparts to complex meanings (ibid. §§ 62–63).

A third characteristic of regions concern their relation to sciences. A region gives rise to several "ontologies," that is, to a priori sciences of concepts that compose the region (Ideen I §§ 9, 16). In the case of nature, there is for instance geometry as the ontology of space, and there should likewise be ontologies of time and of matter, since space, time, and matter are all involved in the constitution of physical objects. As a matter of fact, most ontologies have not been developed in any systematic fashion, but such developments are in principle possible. Any empirical science studies objects of some domain, and that domain stands under a region. An empirical science accordingly assumes the results of the ontologies associated with their respective region (ibid. §§ 8,9); thus, natural science assumes the results of geometry, or more generally the results of the ontologies of nature. Formal ontology and formal logic, the a priori sciences of the formal categories, are assumed by all sciences, whatever their domain, for formal categories apply in all regions. This concludes our discussion of the three characteristics of regions.

The topic-neutrality characteristic of formal categories may be regarded as a certain notion of generality: formal categories apply across all regions. The regions themselves enjoy another sort of generality, namely that pertaining to highest genera. Husserl does not draw attention to these distinct notions of generality, but he does make a related distinction between what he calls formalization and generalization (Ideen I § 13; *LU* III § 24). Formalization, which Husserl also calls pure categorial abstraction (*LU* VI § 60), is what we carry out when passing from an entity to its formal category purely as a form or schema. In order to achieve this we replace the "material elements" of the entity by "empty forms" (*Leerformen*). This is, for instance, the operation we carry out when passing from a presentation of geometry in which the primitive terms have their intuitive, geometrical meaning to a presentation in the style of Hilbert (1899), in which the primitive terms are replaced by variables and the theory becomes "schematic."[189] The reverse operation, in which the empty forms are "filled," Husserl calls de-formalization (*Entformalisierung*) or substantivazation (*Versachlichung*). In generalization we pass from the essence of an entity to a genus of it; continued generalization leads ultimately to a highest genus, which is a region in case the entity in question is independent. The reverse operation of generalization is specialization.

Since the formal categories crosscut the regions, both generalization and formalization make sense for any entity; in other words, any entity has both a formal and a material category. This reflects the fact that any entity is made up of both formal and material elements. Since no distinction between form and matter is made in function–argument syntax, neither does it come equipped at the outset with a set of formal and material categories. As has already been suggested above, there are nevertheless natural candidates for serving these roles in the simple type hierarchy: the notions of individual and function may serve as formal categories, while the ground types may serve as material categories. With the distinction between formal and material categories in place we can also make sense of the notions of formalization and generalization. Formalization in the simple type hierarchy is applicable to any entity and yields either of the two formal categories. Generalization applies only to

[189]Cf. the following remark of Hilbert from lecture notes dated 1894 (Hallett and Majer, 2004, p. 104):

> Unsere Theorie liefert nur das Schema der Begriffe, die durch die unabänderliche Gesetze der Logik mit einander verknüpft sind. Es bleibt dem menschlichen Verstande überlassen, wie er dieses Schema auf die Erscheinung anwendet, wie er es mit Stoff anfüllt.

Hilbert makes a similar remark in a letter to Frege (*WB* p. 69).

individuals, or rather, to sorts of individuals; or at least it will apply to sorts when one has superimposed a genus/species hierarchy on each ground type, for instance in the manner envisaged by Dummett (cf. p. 164ff. above). A ground type is then a highest genus, or an ultimate sortal, that divides into more and more specific sortals. Generalization means moving upward in this hierarchy, and specialization downwards.

According to a traditional interpretation Aristotle's categories are highest genera (cf. chapter 1 section 2). Kant's categories, by contrast, are described as forms of thought (cf. chapter 1 section 5.1) Here we could thus speak of material and formal categories respectively. It would be mistaken, however, to think that Husserl with his distinction of material and formal categories synthesizes the doctrines of Aristotle and Kant. Firstly, none of Aristotle's categories could be regarded as Husserlian material categories. In the greater scheme of things Aristotle's primary substances correspond to Husserlian individuals;[190] hence the Aristotelian category of substance splits into all the various Husserlian regions, since these are precisely the highest genera under which individuals fall. The Aristotelian category of quality would presumably split into various material categories associated with the regions of nature and consciousness. Number is in the Aristotelian category of quantity, but for Husserl it is a formal category. Secondly, the role categories play in Kant's philosophy differs from the role formal categories play in Husserl's philosophy.[191] For Kant the categories are concepts of "pure synthesis" (KrV A78/B104). For Husserl, as well, the formal categories are concepts of synthesis. The synthesis corresponding to Husserl's formal categories, however, has a much narrower scope than the synthesis corresponding to Kant's categories. The latter is involved in any act; in order, for instance, to perceive objects at all the mind must synthesize a "manifold of intuition," namely by bringing it under the categories. According to Kant, the unity of an object does not lie in the object itself, to be extracted from it by perception, but is an achievement of the understanding, by virtue of which the object has unity in the first place.[192] The synthesis corresponding to Husserl's formal categories, by contrast, is involved only in acts whose objects are higher-level, such as sets and states of affairs. The mind is in general not active in bringing about the unity of objects. The colour and the spatial form of this table, for instance, are not connected by the mind, but are given to it already connected. Likewise, the many "snapshots" I make of the table as I regard it from different sides

[190]Stone (2000, p. 129) accepts this identification.

[191]On this point, see De Palma (2010).

[192]E.g. KrV B134–135: "Verbindung liegt aber nicht in den Gegenständen, und kann von ihnen nicht etwa durch Wahrnehmung entlehnt und in den Verstand dadurch allererst aufgenommen werden, sondern ist allein eine Verrichtung des Verstandes, der selbst nichts weiter ist, als das Vermögen, a priori zu verbinden, und das Mannigfaltige gegebener Vorstellungen unter Einheit der Apperzeption zu bringen." Cf. B129–130.

are not synthesized in the way higher-level objects are synthesized from other objects. We may think of the latter as an active form of synthesis, while the former is passive (*LU* VI § 47; Ideen II § 9). Kant does not have this distinction between active and passive synthesis.[193] He therefore regards all experience as categorially formed; for Husserl, by contrast, only the experience of higher-level objects is categorially formed.

6.3. Formal and material categories in Carnap's *Aufbau*. We suggested that in the simple type hierarchy the ground types are material categories and the notions of individual and function are formal categories. In *Der logische Aufbau der Welt* (1928) Carnap suggests a different view: material categories form strata in a simple type hierarchy. Thus, the ground type or types, as well as perhaps some functions of ground type, and functions thereof, etc. constitute one material category. Moving higher up in the hierarchy we find another material category; and so on. The aim of this section is to get a more precise picture of this idea.

The type hierarchy Carnap assumes has the structure he defines in his *Abriß der Logistik* (1929).[194] There is one ground type; if $\alpha_1, \ldots, \alpha_n$ are types, then $(\alpha_1, \ldots, \alpha_n)$ is a type, namely the type of n-ary relations whose i-th argument is of type α_i. The programme of the *Aufbau* is to show that all entities whatsoever, all entities in the "world" referred to in the title of the book, are elements of a certain simple type hierarchy of this structure (e.g. §§ 1, 26). The ground type of the hierarchy consists of what Carnap calls basic experiences (*Elementarerlebnisse*), namely a subject's experiences "in their totality and complete unity" (§ 67).[195] In addition Carnap assumes as given one non-logical constant relation over the type of elementary experiences. This is the relation holding between basic experiences x and y if the reproduction of x in recollection is similar to y (§ 78). On this basis, Carnap argues, one should be able to define classes and relations playing the role of "rational reconstructions" (§§ 100, 143) of all other denizens of the world, that is, one should be able to construct the world.[196]

The world is for Carnap divided into various kinds of objects (*Gegenstandsarten*) (§§ 17–25). The kinds that play a role in Carnap's construction are, in Husserlian terminology, the regions of consciousness, nature, other subjects, and culture. These regions stand in a strict order of epistemic priority: consciousness is epistemically

[193] Husserl criticized Kant for this already in (Husserl, 1891, p. 41).
[194] Cf. footnote 21 above with the text footnoted there.
[195] That they are the experiences of a subject, also called by Carnap the *Ausgangssubjekt* (§ 66), is seen only after the constitution of the world has been completed; in themselves the basic experiences are subjectless (§ 65).
[196] The construction becomes a *logical* construction only with "the elimination of the primitive relation" (§§ 153–155).

prior to nature; nature is epistemically prior to other subjects; the region of other subjects is epistemically prior to culture (§ 58). Carnap requires that his construction of the world conform to this epistemic order (loc. cit.). That is the reason why he chooses a section of the region of consciousness as the ground type of the construction (§ 60). The construction might succeed as well with a ground type of nature, but such a construction would not respect the order of epistemic priority among the regions (§ 59). That the construction as a whole conforms to this order is, however, not secured simply by the choice of ground type; in addition at least the following two requirements must be met.

(i) It need not, and usually will not, be the case that all elements of a type are used in the construction of the world. Let us write $u(\alpha)$, the use of α, for those elements of type α employed in the construction. The first requirement says that all elements of $u(\alpha)$ belong to the same region. This is a requirement of purity: no type should be separated by two regions.

(ii) Given requirement (i), the epistemic priority among the four regions induces a partial order on types: $\beta \leq \alpha$ if and only if no element of $u(\alpha)$ is epistemically prior to an element of $u(\beta)$. For instance, if $u(\beta)$ falls in the region of nature and $u(\alpha)$ in the region of other subjects, then we have $\beta \leq \alpha$. If $u(\alpha)$ falls in the region of consciousness, however, then we have $\beta \not\leq \alpha$. Let us call the trace of a type the set of all types involved in its construction from the ground type. With ι as name of the ground type, the trace $\mathrm{tr}(\alpha)$ of a type α can be defined inductively as follows:

$$\begin{array}{rcl} \mathrm{tr}(\iota) & := & \emptyset \\ \mathrm{tr}((\alpha_1,\ldots,\alpha_n)) & := & \bigcup\{\{\alpha_1,\ldots,\alpha_n\},\mathrm{tr}(\alpha_1),\ldots,\mathrm{tr}(\alpha_n)\} \end{array}$$

Employing this definition we find for instance that

$$\mathrm{tr}\big((((\iota),\iota),\iota)\big) = \{((\iota),\iota),(\iota),\iota\}$$

The second requirement says that we should have $\beta \leq \alpha$ for all types β in $\mathrm{tr}(\alpha)$. Thus, if $\beta \in \mathrm{tr}(\alpha)$, then the objects in $u(\beta)$ are not epistemically prior to the objects in $u(\alpha)$.

Carnap does not spell out these requirements in his book. He expressly states that the converse of (i) is false: not all objects of the same kind belong to one and the same type (§ 29). One may perhaps assume that he would have done the same about (i) itself if he thought it to be false as well; and he does say that elements of the region of culture are always of a type different from elements of consciousness (§§ 56, 151). Requirement (ii) would seem to be a minimal condition on the construction if it is to reflect the epistemic priority among regions. It is a mathematical expression

	Type hierarchy	Regions
	$\vdots$	?
level $k + l + m + n$	—	
	$\vdots$	Culture
level $k + l + m$	—	
	$\vdots$	Other subjects
level $k + l$	—	
	$\vdots$	Nature
level k	—	
	$\vdots$	Consciousness
level 0 (ground domain)	*Elementarerlebnisse*	

FIGURE 2. Carnap's construction of the world?

of the idea that in the construction of an element a we shall not need to refer to an element to which a, in view of its region, is epistemically prior.

That Carnap entertained a picture of the construction of the world in which the two requirements are met is suggested by what he says about levels of constitution (§ 41). Constitution is another word for definition (§ 38),[197] and the level of constitution of an entity is the level in the type hierarchy at which a rational reconstruction of the entity is defined. Carnap had defined the notion of level in a simple type hierarchy in the *Abriß* (p. 32).[198] The level ℓ of the ground type is 0; the level of a higher type $(\alpha_1, \ldots, \alpha_n)$ is defined by

$$\ell\big((\alpha_1, \ldots, \alpha_n)\big) := \max\{\ell(\alpha_1), \ldots, \ell(\alpha_n)\} + 1$$

In the picture Carnap appears to entertain the regions respect levels: it holds that

$$\text{if } \ell(\beta) < \ell(\alpha), \text{ then } \beta \leq \alpha.$$

We thus have a situation as in Figure 2. The first few levels make up the region of consciousness. The levels after consciousness make up the region of nature, the

[197]It has been argued by Mayer (1991, pp. 292–295) and Rosado Haddock (2008, pp. 42–48) that Carnap takes over Husserl's notion of constitution. But to define a rational reconstruction of an object (Carnapian constitution) is quite different from describing how the object presents itself to consciousness (Husserlian constitution). Carnap is in fact explicit that constitution need only preserve the logical, and not the cognitive, value of propositions regarding the object (*Aufbau* § 50).

[198]Carnap's notion of level does not coincide with Frege's notion of level (*Gg* §§ 21–24), which applies to an n-ary function only if all its arguments are of the same level (otherwise the function is said to be *ungleichstufig*). Neither does it coincide with the notion of order in the *Principia* (*PM* pp. 52–55), which is designed for the ramified hierarchy (Church's definition of this notion of order is given on page 101 above).

following levels make up the region of other subjects, and finally comes the levels of culture. Whereas each region thus takes up only finitely many levels, the type hierarchy continues into infinity; at levels above those making up the region of culture one can therefore imagine a region that is yet to be discovered.

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Samenvatting

Zoals de titel van dit proefschrift aanduidt vormt het begrip logische syntaxis de leidraad voor dit onderzoek over categorieën. Het wordt betoogd dat zowel Aristoteles' als Kants categorieën gelieerd zijn aan de traditionele logische syntaxis, die 'S is P' als grondvorm aanneemt; en dat typentheorie gelieerd is aan de functie–argument syntaxis, die $f(a)$ als grondvorm aanneemt. Het proefschrift is opgedeeld in twee hoofdstukken, het eerste verbonden door de traditionele logische syntaxis, het tweede door de functie–argument syntaxis.

In het eerste hoofdstuk worden een aantal traditionele categorieënleren bestudeerd, met name de leer van Aristoteles, die van Kant en de grammaticale leer van woordsoorten. Aristoteles' categorieënleer vormt de kern van onze beschouwing. Zowel de leer van Kant als de grammaticale leer van woordsoorten worden vanuit deze kern benaderd.

In de eerste afdeling van het eerste hoofdstuk vragen wij naar de aard van de entiteiten die onder Aristoteles' categorieën vallen. Volgens onze interpretatie zijn dat termen in de zin van de syllogistiek. Uitgangspunt van de tweede afdeling is de overgeleverde interpretatie van Aristoteles' categorieën als de hoogste genera. Het begrip genus maakt deel uit van de vier of vijf zogeheten predikabilieën, wiens leer hier ook wordt besproken. In tegenstelling tot de traditie beweren wij dat Aristoteles' categorieën geen hoogste genera zijn, maar dat zij veeleer als louter klassen van termen moeten worden beschouwd. De vraag of de entiteiten die onder Aristoteles' categorieën vallen linguïstische of ontische wezens zijn wordt in afdeling drie benaderd door een debat over dezelfde vraag uit de late Oudheid. In een verwant hedendaags debat beweren enkele vooraanstaande Aristoteles-geleerden dat er een verschil bestaat tussen de lijst van categorieën in hoofdstuk 4 van het boek *Categorieën* en die van hoofdstuk I.9 van de *Topica*: de eerste is ontisch van karakter, de tweede linguïstisch. Wij geven ons bezwaar tegen deze lezing. In de vierde afdeling wordt de leer van de woordsoorten historisch voorgesteld en de samenhang (of de afwezigheid daarvan) tussen woordsoorten aan de ene kant en zowel Stoische als Aristotelische categorieën aan de andere kant besproken. In de traditionele grammatica en logica wordt het verschil gemaakt tussen de zogeheten categorematische en syncategorematische woordsoorten. De verscheidene karakteristieken van de syncategorematica worden in dezelfde afdeling uiteengezet en vormen in de volgende afdeling het uitgangspunt van onze lezing van Kants categorieën. Hun relatie tot de Aristotelische categorieën is namelijk vergelijkbaar met die tussen de syncategorematica en de categorematica. Kants beroemde kritiek op Aristoteles' categorieënlijst luidt dat die zonder enige principes waren "bijeengeraapt." Aan het eind van afdeling vijf bespreken wij een aantal pogingen van Aristoteles-commentatoren om een dergelijk principe te vinden.

In het tweede hoofdstuk wordt de typentheorie in haar diverse verschijningen bestudeerd: de simpele en vertakte typenhïerarchie, typen als significantiebereiken van propositionele functies, en typen in de zin van Martin-Löfs constructieve typentheorie.

In de eerste afdeling wordt de simpele typenhïerarchie uiteengezet, zowel haar logisch-ontologische uitwerking, die naar Frege teruggaat, als haar grammaticale uitwerking, die naar Ajdukiewicz teruggaat. Hier wordt ook de relatie tussen de simpele typenhïerarchie en het zogeheten ontologische vierkant besproken. In afdeling twee stellen wij aan de hand van voorbeelden de vertakte typenhïerarchie voor en voeren wij vervolgens het zeer belangrijke begrip significantiebereik in. De interpretatie van typen als significantiebereiken wordt uitvoerig besproken. Uit de discussie in afdeling drie over het significantiebegrip wordt echter duidelijk dat dit begrip te vaag is om als grondslag te dienen voor een typentheorie. Niettemin heeft men het begrip significantiebereik nodig: geen functie geeft voor alle individueën een waarde. Wat wij een typenpredikatie zullen noemen is een oordeel wiens predikaat een categorie of type is. Wij bespreken in afdeling vier meerdere hardnekkige filosofische problemen die met typenpredikaties verbonden zijn. Ook betogen wij dat deze problemen in de constructieve typentheorie een oplossing vinden. In de twee laatste afdelingen worden enkele begrippen die met het typenbegrip nauw verwant zijn ter sprake gebracht. Het eerste is het begrip soort (*sortal* in het Engels), dat in hedendaagse discussies over identiteit een belangrijke rol speelt. In dit verband worden ook de begrippen algemeenheid en kwantificatie besproken. Uitgangspunt voor de laatste afdeling is Husserls onderscheiding tussen formele en materiele categorieën. Dit onderscheid wordt vergeleken met verwante onderscheidingen bij Frege en Carnap, en zijn verband met het typenbegrip besproken.

Curriculum vitae

Ansten Mørch Klev was born on 7 October 1982 in Lørenskog, Norway. He grew up and got his primary and secondary education in Fredrikstad, Norway. He did civil service at a primary school in the same town (2001–2002). He holds a BSc in mathematics from the University of Oslo (2005), a MSc in logic from the University of Amsterdam (2007), and a MA in philosophy from Leiden University (2009). In the academic year 2009–2010 he studied philosophy at McGill University. From September 2010 to September 2014 he has been a doctoral fellow at the Institute for Philosophy at Leiden University.