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SUMMARY

Gaussian random fields are ubiquitous in physics. The central limit theorem ensures that a lot of random fields are Gaussian—or approximately so—making them of special theoretical importance and practical interest. Examples in physics include optical speckle fields, the projected gravitational potential and the cosmic microwave background radiation.

Typically though it is the non-Gaussian contributions that are the target of investigation, since these can provide a window on the interesting nonlinear processes that give rise to them.

Gaussian fields have, regardless of their spectra, certain properties in common. Testing whether a given random field has them thus gives a possible way of detecting non-Gaussianity. The focus of this thesis is on quantities pertaining their stochastic geometry, which provide a very low-key and robust measuring tool. Specifically, this thesis is devoted to studying the exact amount in which some geometric quantities of a Gaussian field change when it is perturbed. This does not only provide a way of detecting non-Gaussianity in a given field, but also potentially to determine the type and/or size of the perturbation.

In chapters 2 and 3 we consider first local perturbations, which at each point, are a function only of the original value at that same point. In chapter 2 we look at the relative difference in densities of local maxima and minima. For Gaussian fields these are the same due to symmetry, but for a non-Gaussian field this may no longer be the case. The locality of the perturbation allows for an exact calculation of the imbalance, which does not require the perturbation to be small. One of the features of the result we find is that the imbalance is exponentially small for small perturbations, regardless of its precise form.

In chapter 3 we stick with the local perturbations, but turn to umbilical points. We find a formula for the monstar fraction, up to first order in the perturbation. The result depends on the spectrum of the original Gaussian field and the perturbation, but those contributions are separate. A comparison with the maxima and minima gives two interesting differences: While the maxima-minima imbalance is insensitive to odd perturbations, the mon-

star fraction does not change (up to first order) for even ones. Second, the monstar fraction may change dramatically for small perturbations.

In chapter 4 we go back to the maxima and minima, but this time in a more general setting. A recipe for determining the imbalance up to first order in the perturbation is presented. The method is tested explicitly by applying it to the case of a field that is initially Gaussian, but evolves according to the deterministic Kardar-Parisi-Zhang (KPZ) equation. This equation causes non-Gaussianities to develop which are nonlocal in nature. We find that indeed an imbalance between maxima and minima may result, which we can quantify. An interesting find is that in the case of small nonlocal perturbations, the imbalance need not be exponentially small, as we found for the local case. This means that a nonvanishing imbalance is indicative of either a large non-Gaussianity, or one of nonlocal origin.

In chapter 5 we consider the effects of coarse-graining. This process involves an averaging, which can be used to model an imprecise measurement. It is also a way in which a random field can become nearly Gaussian. In the limit of the coarse-graining scale going to infinity, an imbalance between maxima and minima will converge to zero, at a rate that is inversely proportional to the length scale of the coarse-graining. The precise imbalance depends on the details of both the original field and the coarse-graining function, but in the limit of the aforementioned length scale going to infinity, the contributions of these two can be separated. An interesting fact is that, even though coarse-graining can be seen as a convergence toward Gaussianity, it can actually help to reveal a non-Gaussianity by amplifying the imbalance, which is of particular interest for the case of a locally perturbed Gaussian field.