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The Stochastic Geometry of non-Gaussian Fields

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Cover Page



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THE STOCHASTIC GEOMETRY OF
NON-GAUSSIAN FIELDS

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The cover shows lines of principal curvature, which indicate the direction along which the curvature of a given surface is maximal (or minimal). At some points the curvature is the same in all directions, so that this direction is not defined. These topological defects are called umbilical points (see chapters 1 and 3). The image on the back represents a Gaussian field.

The design is an adaptation from an image originally published in PNAS 109: 19943-19948 (2012).

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GAUSSIAN FIELDS AND GEOMETRY

Before discussing non-Gaussian fields, we first cover the relevance, definition and characteristics of Gaussian fields. This first part also treats the critical points and umbilical points that will feature in the later chapters. The definitions and notations introduced here will be important for the non-Gaussian fields to follow.

