The holographic glass bead game : from superconductivity to time machines

Bagrov, A.

Author: Bagrov, Andrey
Title: The holographic glass bead game : from superconductivity to time machines
Issue Date: 2015-09-23

Chapter 5

Discussion

In this thesis we have analyzed three completely different problems using the AdS/CFT correspondence. To conclude, let us discuss how these stories can be embedded in the wider context of the holographic theory of strongly coupled systems, and provide an outlook of possible directions for future research.

5.1 Holographic fermions and superconductivity

Among others, a natural and important goal of studies in the area of applications of gauge/string duality to condensed matter physics is to formulate a complete dynamical holographic theory of an unconventional superconductivity with an explicit fermionic pairing mechanism. However, what could be meant by pairing in holography requires clarification. As a weak/strong duality, the AdS/CFT correspondence helps to describe quantum field theories at strong 't Hooft gauge coupling λ_H by dualizing them to solutions of string theory at small string coupling constant g_s. In real solid state systems, the 't Hooft gauge coupling does not have an unambiguous phenomenological meaning, but we can say that in condensed matter-oriented holographic constructions that λ_H is responsible for the emergence of highly correlated collective excitations out of fundamental degrees of freedom in the field theory. In that sense, it is similar to the Coulomb interaction between fundamental bare electrons in a piece of metal that governs collective dynamics at the very microscopic level. However, if we wish to describe pairing in superconductivity, we should also include in our considerations another channel(s) of interactions, now responsible for coupling between these emergent composite operators, and would play the same role as phonons or spin waves play in real materials.

In Chapter 2 we introduced this explicit pairing channel via a Yukawa-like term in the bulk action that couples fermions to the order parameter. We have demonstrated that this leads to the opening of a superconducting gap in the system and to condensation of the scalar order parameter. It is natural to make the dual bulk scalar field dynamical, and the strength of the interactions in this setup is then effectively controlled by the ratio of the Yukawa coupling and scaling dimension of the scalar: λ_Y/Δ_ϕ. In the regime of weak coupling, $\lambda_Y/\Delta_\phi \ll 1$, the boundary field theory exhibits properties of the conventional BCS superconductivity, while upon increasing the coupling it undergoes the BEC/BCS crossover.

However, this result should be considered as a very first step towards the ultimate goal. Our model should be further improved in the following ways.

- Because we were originally interested in a holographic reformulation of the Bardeen-Cooper-Schrieffer theory of superconductivity, we studied the superconducting instability in a model of a holographic Fermi-liquid, i.e. in a model with a sharp spectrum of fermionic quasi-particles. But if we are interested in an unconventional SC, we should take into account that fermionic degrees of freedom can be of a critical nature. So, to generalize our model, one can try to remove the IR cut-off and analyze the pairing of "unparticle" fermions.

- We have studied only the $T = 0$ ground state of the theory. On the other hand, it would be interesting to understand properties of the fermionic superconductivity at finite temperature and to study its thermal phase diagram.

- Our model accounts for interactions between fermions, the bulk gauge field, and the scalar order parameter, but we did not take into account the backreaction of the bulk fermions on the background metric. This could be important if one really wants to go beyond the probe limit and study superconductivity in a strongly correlated system. A first attempt to construct a setup with backreacting fermions (though without pairing and a dynamical scalar field) has been performed in [1, 2], and it is clear that to generalize it to a fully interacting case would be a very difficult numerical challenge.

- It would be interesting to analyze the transport properties of the model and to study the fermionic corrections to the electric conductivity. Also, it might be important to know how the model would behave once we introduce a lattice.

- We implemented the pairing while staying ignorant about its physical mechanism. Thus it would be interesting to formulate a more detailed "microscopic" theory where this interaction is caused by a non-trivial dynamical field.

5.2 Holography and the non-equilibrium quark-gluon plasma

Another important area of research in contemporary science is the physics of the quark-gluon plasma. When QCD matter is heated up to a very high temperature, $T \gtrsim 170\text{MeV} \sim 10^{12}K$, it undergoes a phase transition, and quarks and gluons, normally bounded within hadrons, deconfine: they start behaving as independent unbounded entities. Such an extremal and unusual state of matter opens room for studying properties of quantum matter that are inaccessible under normal circumstances. Experimentally the QGP can be produced in high energy collisions of heavy ions, like Pb and Au. A large number of constituent nucleons is crucial for creating a many-body state of matter.

The underlying Lagrangian for the quark-gluon plasma is just the well-known Lagrangian of QCD, but it is of little use: the experimentally produced QGP is a strongly coupled and highly non-equilibrium state of matter [3], and standard mathematical methods of quantum field theory are not applicable here. One can think about a holographic description of quark-gluon plasma formation in high energy heavy ions collisions. On the dual gravitational side, the relativistic ions are represented by gravitational shock waves. The collision leads to mutual stopping of the waves, and their kinetic energy transforms into rest energy, causing creation of a black hole. In the boundary field theory, this effect can be interpreted as the formation of a thermal deconfined state of matter, the QGP.

In Chapter 3 we have considered this model of colliding shock waves at non-zero chemical potential. Although in real experiments on heavy ion collisions the chemical potential is negligible, and all corresponding holographic models do not take it into account, it might be interesting to see if introducing non-zero charge density would lead to qualitatively correct phase diagram. In particular, it is expected in QCD that at higher density of hadronic matter, the temperature of the deconfinement phase transition becomes smaller, i.e. less energy is needed to produce the quark-gluon plasma. Surprisingly, in the simplest model of gravitational plane waves we discovered an opposite tendency: larger chemical potential suppresses formation of QGP. It might indicate that already the chargeless models commonly used in holographic computations might require some modifications to be correct.

However, to really test our observation a further extensive analysis is required. In our calculations we limited ourselves by an analytic estimate of the size of the formed black hole based on a critical trapped surface calculation, that can be performed without simulations of the bulk gravitational field after the shock wave collision. But in order to fully understand the effect of non-zero chemical potential, complete real-time numerical simulations are required. With the advances in non-equilibrium numerical holography [4] this may be done in the near future.

5.3 Theoretical aspects of time travelling

Finally, in Chapter 4 we have applied the AdS/CFT correspondence to get an insight into a more fundamental issue in gravitational physics, and analyzed the behaviour of a two-point Green's function of a quantum field theory in a space with causality violation, by dualizing it to a space-time with closed time-like curves. In this case, the main advantage of the AdS/CFT was not that it is a weak/strong duality, but rather that it is a classical/quantum duality. Therefore the non-causal quantum dynamics could be mapped onto classical geometry, and the problem drastically simplifies. But our calculation leaves many open questions.

First of all, we considered a time machine solution to classical General Relativity in three dimensions. Is it possible to find a string theory embedding for this solution? Would this embedding preserve the closed time-like curves, or do holographic screens appear, restoring the causality, like happens for Gödel spacetimes in string theory [5, 6]?

In our model we relied on the geodesic approximation to avoid solving the field equations in a topologically non-trivial causality-violating background. On the other hand, because three dimensional gravity is purely topological and does not have propagating degrees of freedom, any solution is locally isometric to an empty AdS_3 and often can be represented as its factorization over some symmetry group. So we can not exclude that it is possible to represent the DeDeo-Gott solution that we considered in this form. Then an exact solution to a wave equation on this background could be easily generated.

Another issue is the field theoretical interpretation of the result. In the bulk we have a very simple configuration of two conical defects, so we might expect that it corresponds to a decent quantum state in the dual field theory. We do not know what that state is, but can try to speculate. In the case of three dimensions, bulk geodesics do not only contribute to the Green's functions, but also define the entanglement entropy of boundary regions, as conjectured by Ryu and Takayanagi [7]. Usually only equal-time entanglement is well-defined. However recently it has been proposed [8] to simulate CTC quantum mechanically by entangling a qubit to an older version of itself. Thus it could be possible that in our setup we deal with a field theoretical version of this non-equal time entanglement. Whether this conjecture is correct is a question for future investigation.

In this work, by solving several completely different problems, we tried to demonstrate that the holographic correspondence is a paradigm that has the power to provide intuition on totally diverse phenomenological concepts and systems by mapping them onto the same set of resonantly connected mathematical structures. A long way is ahead, a lot of problems remain to be addressed, and we still do not understand much about borders of the applicability of holography. If the boldest formulation of the holographic principle is true, and any quantum theory has a dual, the correspondence may very well become the Glass Bead Game of theoretical physics, in the original, literal meaning that Herman Hesse gave to this concept.

Bibliography

[1] A. Allais, J. McGreevy and S. J. Suh, "A quantum electron star," Phys. Rev. Lett. **108** (2012) 231602 [arXiv:1202.5308 [hep-th]].

[2] A. Allais and J. McGreevy, "How to construct a gravitating quantum electron star," Phys. Rev. D **88** (2013) 6, 066006 [arXiv:1306.6075 [hep-th]].

[3] J. Adams *et al.* [STAR Collaboration], "Experimental and theoretical challenges in the search for the quark gluon plasma: The STAR Collaboration's critical assessment of the evidence from RHIC collisions," Nucl. Phys. A **757** (2005) 102 [nucl-ex/0501009].

[4] P. M. Chesler and L. G. Yaffe, "Numerical solution of gravitational dynamics in asymptotically anti-de Sitter spacetimes," JHEP **1407** (2014) 086 [arXiv:1309.1439 [hep-th]].

[5] E. K. Boyda, S. Ganguli, P. Horava and U. Varadarajan, "Holographic protection of chronology in universes of the Godel type," Phys. Rev. D **67** (2003) 106003 [hep-th/0212087].

[6] E. G. Gimon and P. Horava, "Over-rotating black holes, Godel holography and the hypertube," [hep-th/0405019].

[7] S. Ryu and T. Takayanagi, "Holographic derivation of entanglement entropy from AdS/CFT," Phys. Rev. Lett. **96** (2006) 181602 [hep-th/0603001].

[8] M. Ringbauer, M. A. Broome, C. R. Myers, A. G. White and T. C. Ralph, "Experimental simulation of closed timelike curves," Nature Commun. **5** (2014) 4145 [arXiv:1501.05014 [quant-ph]].