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## **The holographic glass bead game : from superconductivity to time machines**

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# Chapter 1

## Introduction

*Cognoscibility of the Universe is the basic axiom of beingness of our civilization. Real progress in understanding fundamental principles governing Nature at various scales, from microscopic physics to the dynamical processes in society, can be possible only under one assumption: any conceivable problem we might ever face in our research must be soluble in one way or another. In the novel “The glass bead game”, Hermann Hesse described a surreal world where people have managed to figure out a universal language that allows for establishing precise connections between all branches of human knowledge, and is powerful enough to provide a solution to any problem once it is reformulated in terms of the glass bead game. This fictional concept is a good illustration of what the actual goal of theoretical science should be: not just to solve a problem that seems to be difficult, but rather to find a proper language that automatically makes it almost trivial.*

### 1.1 Preface

A wide class of longstanding open problems in theoretical physics that have proven themselves to be unsolvable with existing methods belongs to the area of strongly interacting quantum systems. The most famous of them include the problem of confinement in quantum chromodynamics, and the question about the physical mechanisms of high Tc superconductivity. After decades spent in efforts to crack these puzzles by standard techniques, it is becoming clear that if we wish to get a handle on strongly correlated quantum dynamics, we should seek a completely new theoretical paradigm.

The problems we are facing in our attempts to analyze physics of strongly interacting systems are caused by two factors - the large value of the coupling constant, and finite fermion density. The first factor manifests itself in the absence of a small parameter that we could use to

construct a perturbative analytic description of a model. If this was the only issue, we still might hope that the equations governing the evolution of a strongly coupled system could be solved at least numerically. This is possible for bosonic models (or models that allow for the effective reformulation in bosonic variables). However, a finite density of quantum fermionic matter leads to an obstacle that may be considered more severe than just the strong correlations. The standard numerical approach to simulation of many-body systems is the (Quantum) Monte Carlo method, which consists of calculating the quantum field theoretical observables by random sampling of the Euclidean phase space configurations, and averaging over them with the Boltzmann distribution. If we are studying a fermionic system, however, the anticommutativity of fermionic operators, along with the finite density, spoils the charge conjugation symmetry of the corresponding Hamiltonian. On the numerical level it leads to the Fermion Sign Problem [1] - uncontrollable sign oscillations of the sampled quantum partition function. This problem is conjectured to be non-polynomially hard [2], and thus it puts very strong limitations even on our ability to perform numerical simulations.

This thesis is dedicated to a new framework, a new language that has demonstrated an unexpected naturalness in dealing with models of many body systems both at strong coupling and finite density. This framework is the holographic correspondence. The holographic principle emerged from attempts to understand the applicability of the laws of quantum theory to black holes in papers by G. 't Hooft [3] and L. Susskind [4]. In its most generic form, the principle states that a theory of quantum gravity in  $D + 1$  dimensions should be equivalent to a quantum field theory in  $D$  dimensions, in the sense that the degrees of freedom of the two theories can be precisely mapped to each other, and all observables of each of the theories are encoded in its dual partner. Phrased this way, the principle seems to be unrelated to real life phenomenological problems. But we can try to revert this statement and ask when it is possible to represent a quantum field theory in dual gravitational form, and what we can gain from it for our understanding of the QFT?

Nowadays we know several concrete examples of holographic dualities [5–8], among which the most developed and understood one relates gravity in spacetimes with a negative cosmological constant (Anti-de-Sitter spacetimes) to conformal field theories, the *AdS/CFT* correspondence. Even before the holographic principle was proposed, in 1986 J. Brown

and M. Henneaux demonstrated that the asymptotic symmetry group of three dimensional anti-de Sitter space coincides with the two dimensional conformal group [9]. But the actual realization of the correspondence was proposed in 1997 by J. Maldacena [5], who seminally conjectured that the  $N = 4$  supersymmetric  $SU(N)$  Yang-Mills theory in  $D = 4$  in the strong coupling limit is exactly dual to type IIB string theory on the  $AdS_5 \times S_5$  background at weak coupling (and vice versa). The concrete rigorous mathematical formulation of the correspondence was developed by S. Gubser, I. Klebanov, and A. Polyakov in [10], and E. Witten [11]. They emphasized an equivalence of partition functions of the two dual theories that provides a way to calculate correlation functions, now known as the GKPW-rule.

We should remark that the idea of weak/strong dualities is not novel in physics. Already in 1941, H. Kramers and G. Wannier discovered that a two dimensional Ising model on a square lattice at small coupling is dual to itself at strong coupling [12]. Since that time, plenty of other dualities in many areas of theoretical physics have been figured out and played a great role in the history of science. Some of them relate only two particular models, some are broader and applicable to large classes of theories. The latter include the Seiberg duality [13] that equates infrared fixed points of  $N = 1$  supersymmetric field theories with different number of flavors and colors, and the particle-vortex duality [14], relating the dynamics of point-like and non-local objects in statistical systems. However, as we will see further, holographic duality has so far demonstrated the largest flexibility and universality in description of various models in completely different areas of theoretical physics.

Originally the  $AdS/CFT$  correspondence was an important string theoretical construction, though unrelated to quantum many-body phenomenology. A key observation was made by G. Policastro, A. Starinets, D. T. Son in [15], and then elaborated on by P. Kovtun, A. Starinets, D. T. Son in [16]. Using the  $AdS/CFT$  correspondence, they calculated the shear viscosity of the  $N = 4$  supersymmetric quark-gluon plasma, and it turned out to be not a value specific for the particular toy model, but a universal quantity equal to  $\hbar/4\pi k_B$ , which they conjectured to be a lower bound for the shear viscosity of an ideal quantum liquid. This calculation and this conjecture were of huge importance since they demonstrated that the holographic correspondence, in principle, can be used to get insights into the physics of experimentally accessible strongly cor-

related field theories. The era of applied holography had begun. The *AdS/CFT* correspondence was successfully applied to plenty of different physical phenomena, from non-equilibrium processes in the quark-gluon plasma [17] and meson spectra in QCD [18], to the transport properties of high temperature superconductors [19, 20] and the evolution of open quantum systems [21]. Holographic quantum chromodynamics and holographic condensed matter theory evolved into broad independent areas of physics [22, 23].

Among other achievements we can mention the following:

- The theory of hydrodynamics has been fully reformulated holographically [24, 25]. That opened a natural and simple way to describe dissipative forces in quantum systems, that is hard to do within the standard field theoretical formalism. In particular, accurate simulations of turbulence in quantum liquids have been performed with the holographic approach [26, 27].
- Transport coefficients of unconventional superconductors have been calculated. The linear temperature dependence of electric resistivity of non-Fermi liquids, that can not be derived within the standard Landau Fermi liquid paradigm, was demonstrated to naturally come out of holographic setups [20, 28], as well as the Hall angle [29].
- A framework for simulations of the non-equilibrium quark-gluon plasma, using advances of numerical relativity, has been developed [30]. This opened a possibility to study real-time processes in the QGP theoretically.
- A language to describe phases of fermionic quantum matter, that does not rely on the paradigm of weakly coupled Fermi liquids, has been suggested [31, 32].

After more than ten years of research, the applied *AdS/CFT* correspondence is still a young and actively developing area of physics. Being a drastically new, and not completely established, language, it attracted both attention and enthusiasm, and justified criticism and skepticism of theoretical physicists. It still remains to be seen whether holographic duality really has the capacity to become The Glass Bead Game of strongly coupled quantum field theory. But we have very good reasons to hope so.

This thesis is organized as follows. In section 1.2 of the introductory chapter we review basic ideas of the holographic principle closely following

the original paper by L. Susskind [4]. In 1.3 we briefly introduce technical aspects of the *AdS/CFT* correspondence, the so called *AdS/CFT* dictionary that translates quantum field theoretical objects to the language of the theory of gravity. In 1.4 we recall the most important steps in the history of the applied “phenomenological” holography. Chapters 2, 3, and 4 are based on research papers and form the core of this thesis.

## 1.2 The holographic principle: the idea.

The idea of the holographic principle is based on the fundamental fact first emphasized by J. Bekenstein [33], that if we wish to account for quantum properties of matter interacting with a black hole, we should unavoidably impose that the black hole must be subject to the laws of thermodynamics. To understand this statement, recall that the black hole event horizon is not a singular surface. A freely falling observer crossing the horizon will not experience anything qualitatively new at this moment. It is always possible to make a simple coordinate change that makes this explicit. Therefore if we wish to construct a self-consistent theory of a quantum field evolving in the background of a black hole, we should properly define it not only outside of the horizon, but also behind it.

This was done by S. Hawking [34], and the outcome of his calculation was that quantum effects would lead to evaporation of the black hole. Without getting into formal mathematical details, we can see it from the following reasoning. A classical signal from behind the horizon cannot leave the black hole and reach the outer region. However quantum phenomena are more subtle. Quantum fluctuations can lead to formation of a virtual pair near the horizon. Under normal circumstances these particles would not be observable and would immediately annihilate, since for a very short time they violate the energy conservation law. But if they emerged from a vacuum on the opposite sides of the black hole horizon, the physical picture becomes more complicated. The signature of the space-time alternates across the horizon, and the timelike direction  $\partial_t$  becomes spacelike behind the horizon. Since the energy of a particle is associated with the symmetry of translations in time, this sign change would lead to the fact that the virtual particles on either sides of the horizon will have energies of opposite signs. From the point of view of an external observer the particle behind the horizon has negative energy, and the conservation law is not violated. Thus if the outer particle has speed large enough to

leave the vicinity of the black hole (for light-like particles that's always true), it will not annihilate with its counterpart.

This particle emission due to quantum fluctuations near the horizon is known as Hawking radiation. The radiation takes away energy from the black hole, thus causing its evaporation. Hawking famously demonstrated that this black hole radiation has a black body thermal spectrum with a temperature (in the natural  $k_B = \hbar = c = 1$  unit system):

$$T = \frac{1}{8\pi GM}. \quad (1.1)$$

Because a black hole has a temperature and energy (equal to its mass,  $M = E$ ), it would be natural to define its entropy in accordance with the first law of thermodynamics:

$$dM = dE = TdS. \quad (1.2)$$

supporting Bekenstein's conjecture. So,

$$dM = \frac{1}{8\pi GM}dS, \quad (1.3)$$

and finally for the entropy we get

$$S = 4\pi M^2 G = \frac{\pi R_s^2}{4G} = \frac{A}{4G}, \quad (1.4)$$

where  $R_s = 4GM$  is the Schwarzschild radius, and  $A$  is the area of the event horizon.

The next natural question to ask is what kind of microscopic statistical physics is behind a black hole's thermodynamic properties? This area-law scaling is very puzzling from the perspective of general physics wisdom. We do not have a complete theory of quantum gravity yet, but it would be natural to assume that a unit cell of a quantum gravity phase space must be set by the Planck scale,  $l_P = \sqrt{G}$ . If each Planck cell encodes  $k$  degrees of freedom, then the naive counting tells us that the total number of possible different microstates corresponding to the macroscopic black hole should be proportional to

$$\Gamma_{\text{naive}} = k^{\frac{4\pi R_s^3}{3l_P^3}}. \quad (1.5)$$



The black hole entropy scales then with the volume of the black hole, and not with the area:

$$S_{\text{naive}} = \frac{V}{G^{3/2}} \log k. \quad (1.6)$$

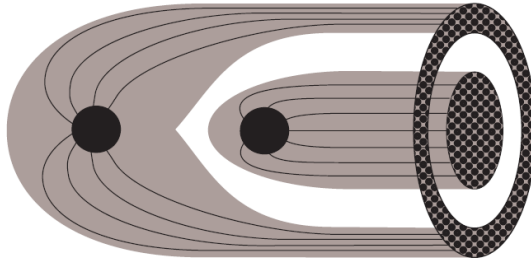
A radical solution to this contradiction has been suggested by G. 't Hooft [3] and L. Susskind [4]. They conjectured that the actual number of states in a theory of quantum gravity should be lower than the naive estimation, and that any quantum state of a subregion of the  $3 + 1$  dimensional spacetime can be completely encoded in a state of its  $2 + 1$  dimensional boundary. In this language, the number of microstates  $n$  inside a spatial volume  $V = \frac{4}{3}\pi R^3$  can not exceed the maximal possible value set by (1.4):

$$n \leq \exp(S) = \exp(\pi R^2 / 4G). \quad (1.7)$$

When this bound, called the entropic bound, is saturated, a black hole forms, and any further increase of the number of microstates is impossible without growing of the black hole volume/area.

Because the number of microstates of a black hole is the maximal for a given volume, if the 't Hooft-Susskind holographic conjecture is correct for a black hole, it should also be correct for any other macroscopic state of a spacetime. In the limit of an infinitely large region, the state of the whole spacetime can be reconstructed from the state of a lower dimensional surface at spatial infinity. From here on, we will call the higher dimensional space the “bulk”, and its encoding boundary surface the “boundary”. An elementary Planckian volume  $\sim l_P^3$  in the bulk then corresponds to an elementary area  $\sim l_P^2$  on the boundary. We will refer to this elementary boundary area as a “pixel”.

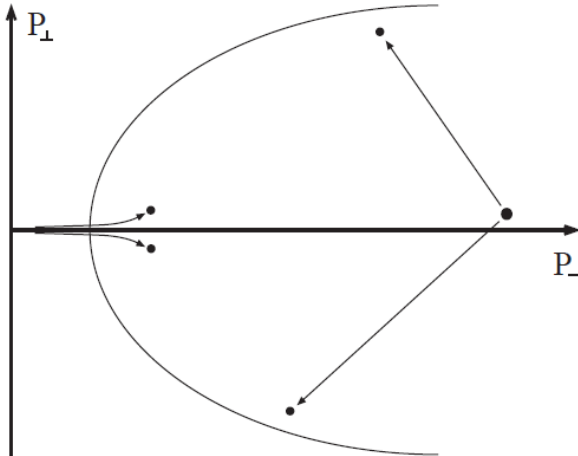
This holographic principle sounds very counterintuitive. A simple counterexample to it appears to be a collection of black holes where one of them is shielded by others from the boundary. Nevertheless configurations of this kind can be holographically projected onto the boundary in an unambiguous manner. If we have a number of black holes in the bulk, their horizons still form a disconnected surface of a finite area, and can be injectively mapped pixel-by-pixel to the boundary. The easiest way to show this is by tracing classical light rays emerging from the horizons. The gravitational field of a black hole has the property that nothing can be hidden in the “shadow” of a black hole. Light rays approaching the black hole from behind would be strongly deflected and reach the observer



**Figure 1.1.** The horizons of two black holes, after the holographic mapping, form a picture on the boundary. Nothing can be hidden behind a black hole, so each Planckian pixel on both of the horizons can be connected with a unique pixel on the boundary without any overlap.

in front of the black hole (see Fig. 1.1). In particular, it can be explicitly calculated that, for an arbitrary distribution of black holes in the bulk, information about the states of their horizon pixels can be unambiguously transmitted to the boundary by light-like geodesics orthogonal to it [4].

What we have discussed so far is related to the physics of black holes. The natural question to ask is whether an arbitrary configuration of gravitational fields *and* matter possesses a holographic description. It is natural to expect that if we have non-gravitational degrees of freedom in the bulk, we can map them onto the boundary only if the dual holographic quantum field theory also contains matter. Here we face a new puzzle. If a point-like particle (we will call it a parton) is located somewhere in the bulk, its projection on the boundary screen at first sight seems to occupy a single boundary pixel, independently of its bulk coordinates. So, how would we encode the parton's distance to the boundary? A possible resolution of this problem has been proposed by L. Susskind [4]. Consider the holographic extra bulk dimension in the momentum representation, i.e. partons with larger values of the holographic coordinate have higher momenta. We must take into account that a parton is a quantum object. If we boost it up to high enough energy, the interplay of quantum and relativistic effects causes particle number non-conservation, and the parton can split into multiple partons (see Fig. 1.2). Thus, in the rest frame of a static observer at the boundary, we see a spatially distributed cloud of apparent partons instead of the original single point-like particle. To estimate the growth of the cloud's transversal area under Lorentzian



**Figure 1.2.** Momentum space representation: the larger is the “holographic” momentum  $P_{\parallel}$ , the stronger is the effect of partonic cloud spreading in the transversal “boundary” directions,  $P_{\perp}$ .

boosts, we can use different models of relativistic partons. In particular, if we treat the parton as a free string in the context of string theory, the dependence of its transversal size on the momentum can be shown to be logarithmic [4] (when the energy of the parton is much lower than the Planckian limit):

$$R_{\perp}^2 \sim l_s^2 \log \frac{P_{\parallel}}{\epsilon}, \quad (1.8)$$

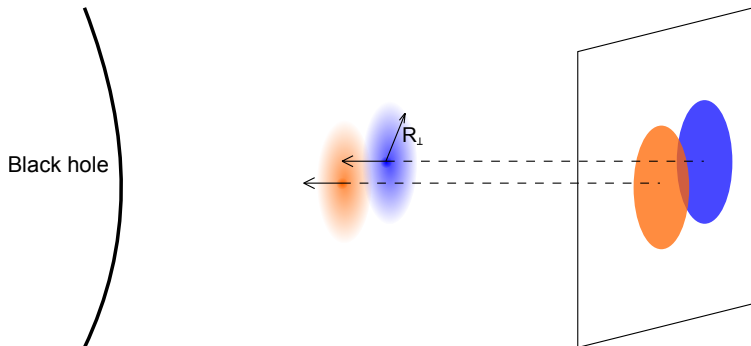
where  $l_s$  is the string length, and  $\epsilon$  is an IR cut-off. Hence information about the holographic momentum of a parton can be stored in the size of its boundary projection.

This picture can be rephrased in terms of the coordinate representation. Consider a parton falling into a black hole in the bulk. The further away it is from the boundary, and closer to the horizon, the larger are its momentum and transversal spreading. In the rest frame of a static observer, the transversal size of the parton grows in time as [4]

$$R_{\perp}^2 \sim l_s^2 \frac{t}{4GM}, \quad (1.9)$$

when the energy is much smaller than the Planckian limit, and

$$R_{\perp}^2 \sim l_s^2 e^{\frac{t}{4GM}}, \quad (1.10)$$



**Figure 1.3.** Overlapping partonic clouds and their holographic projection. Transversal size grows in time as the parton approaches the black hole.

in the ultra-relativistic transplanckian regime. So the distance to the screen is encoded in this quantity.

The last point to be discussed is how to describe holographically a many-particle bulk state. When a number of partons are boosted by the gravitational forces to very high energies, their images on the screen start overlapping, as shown on Fig. 1.3. As they approach the horizon, the overlap becomes stronger, and at some point the partons lose their identity. Information about their bulk state can not be any more recovered just from local probes of the holographic boundary. In the dual boundary language, the pixels on the screen start getting entangled, and in order to reconstruct the full bulk state we have to analyze non-local correlations. If the boost is extremely strong, we might expect a situation where the partonic clouds spread over the whole horizon, forming infinitely thin identical shells (as is clear from (1.10), this would happen in a finite time). If it happens, partons will become holographically indistinguishable, and even the non-local correlations would not provide us with enough information to reconstruct the bulk state. Nevertheless, the holographic principle still can be saved. As was argued in [4], the apparent partons forming the cloud are not subject to Lorentz contraction. Therefore, when approaching the black hole horizon, each partonic cloud occupies a unique shell of a finite radial thickness, and interpenetration of the two clouds is impossible (we could say they behave as an incompressible liquid). As a result, different partons have different holographic radial positions, and the corresponding pattern imprinted on the holographic screen is unique and unambiguous.

That concludes the generic discussion of the ideas behind the holo-

graphic principle, and we can proceed further to its concrete constructive realisation - the Anti-de-Sitter/Conformal Field Theory correspondence (*AdS/CFT*).

## 1.3 The AdS/CFT dictionary

In this section we provide a brief introduction to technical aspects of the AdS/CFT correspondence. We start with reviewing basic facts about anti-de Sitter spacetime, and provide a holographic interpretation of its geometric features. Then we introduce the holographic prescription for a quantum field theory partition function (the Gubser-Klebanov-Polyakov-Witten formula), and demonstrate how it can be used both to describe collective thermodynamic properties of the QFT and to calculate microscopic correlation functions.

### 1.3.1 Anti-de Sitter spacetime

Anti-de Sitter space is a maximally symmetric space of a constant negative curvature, which can be thought of as a one sheet hyperbolic surface embedded into  $\mathbb{R}^{(2,d)}$  space. In physics it naturally appears as a vacuum solution to the Einstein-Hilbert equations with negative cosmological constant. The embedding into the higher-dimensional flat space is given by the equation

$$-X_0^2 - X_{d+1}^2 + \sum_{i=0}^d X_i^2 = -L^2. \quad (1.11)$$

By construction, this surface is a homogeneous and isotropic Riemannian manifold with the  $SO(2,d)$  isometry group.  $L$  here is the *AdS* radius, and hereinafter we set it to be  $L = 1$ . The induced metric on this surface can be obtained by parameterizing these  $d + 2$  dimensional coordinates in terms of  $d + 1$  independent variables:

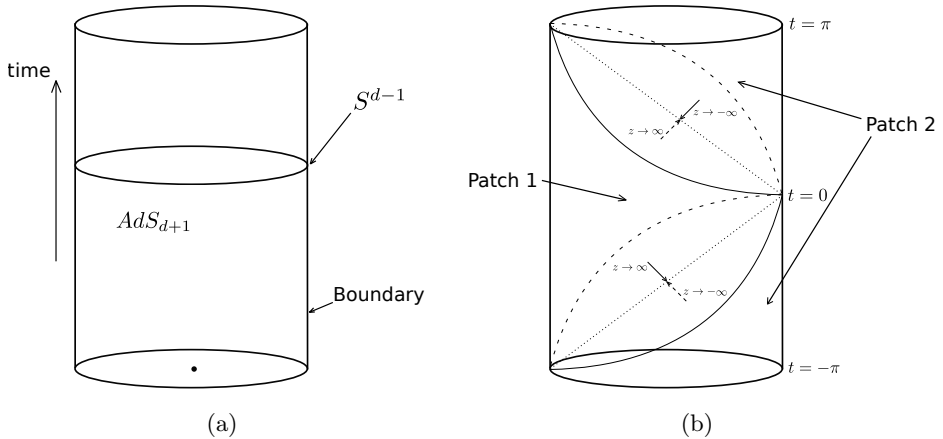
$$X_0 = \sqrt{1+r^2} \cos(t), \quad (1.12)$$

$$X_{d+1} = \sqrt{1+r^2} \sin(t), \quad (1.13)$$

$$X_i = r\Omega_i, \quad (1.14)$$

where  $\Omega_i$  are coordinates on a  $d$ -dimensional sphere. The induced metric on the hypersurface is the

$$ds^2 = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_{d-1}^2. \quad (1.15)$$



**Figure 1.4.** (a): Penrose diagram of the universal cover of the anti-de Sitter spacetime. (b): Global (unwrapped!) AdS consists of two Poincare charts separated from each other by Poincare horizons  $z \rightarrow \pm\infty$ . Time is periodic, so time slices  $t = -\pi$  and  $t = \pi$  are identified.

This set of coordinates is called global, and covers the whole hyperboloid. Note that time is periodic in these coordinates. In order to avoid closed time-like curves, the time axis can be unwrapped to make  $t \in (-\infty, \infty)$ . We will always consider this universal cover of global AdS.

The topology of the universal cover of AdS is easily recognized as a cylinder (see Fig. 1.4a): the axis of the cylinder ( $r = 0$ ) is codirected with the time axis, the boundary of the cylinder depicts spatial infinity  $r = \infty$ , and each  $t = \text{const}$  slice of the boundary has the topology of a sphere  $S^{d-1}$ . The boundary of the cylinder is a visual representation of the so called conformal boundary of AdS - the surface where the quantum field theory that holographically encodes gravity in AdS is defined.

For our future purposes we also introduce a different coordinate system, which is more convenient for many applications - the Poincare chart

coordinates:

$$X_0 = \frac{z}{2} \left( 1 + \frac{1}{z^2} (1 + \vec{x}^2 - t^2) \right), \quad (1.16)$$

$$X_i = \frac{x_i}{z}, \quad (1.17)$$

$$X_{d+1} = \frac{z}{2} \left( 1 - \frac{1}{z^2} (1 - \vec{x}^2 - t^2) \right). \quad (1.18)$$

The *AdS* metric in these coordinates has an explicitly conformal form:

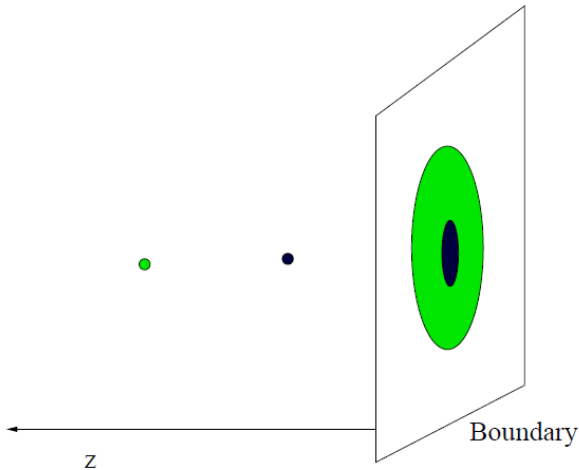
$$ds^2 = \frac{1}{z^2} \left( -dt^2 + dz^2 + \sum_{i=1}^{d-1} dx_i^2 \right). \quad (1.19)$$

These coordinates cover only half of the cylinder (as in Fig. 1.4(b)), and describe a case where the conformal boundary is a Minkowski  $\mathbb{R}^{1,d-1}$  spacetime (as opposed to  $\mathbb{R} \times S^{d-1}$  in the global case) located at  $z = 0$ .

### 1.3.2 Holographic interpretation of *AdS* spacetime

According to the holographic principle, the *AdS* gravity should have a dual quantum field theoretical partner on the boundary of *AdS*. We have already announced that the dual theory is a conformal field theory. The reason for this is rooted in the fact that the isometry group  $SO(2, d)$  of  $(d + 1)$ -dimensional anti-de Sitter spacetime coincides with the conformal group of  $d$ -dimensional Minkowski spacetime, which includes  $SO(1, d)$  isometry transformations, dilatations and special conformal transformations (radial inversions). Hence it is natural to expect that the symmetries of two sides of the duality should match, and the boundary field theory is actually conformal.

Finally, let us discuss how quantum properties of the boundary theory can be stored in the classical geometry. In the holographic picture we have an extra emergent “radial” coordinate  $r$  (or  $z$ ), which should have some interpretation in the dual boundary terms. To gain some intuition, consider a classical point-like particle in the bulk of  $AdS_{d+1}$ . What is it dual to on the boundary? As we will show in the subsequent section, a state of the bulk corresponds to a state of the boundary theory. So, if an empty *AdS* is dual to a ground state, it is natural to expect that this configuration with a point-like particle is dual to an excited state in the corresponding CFT. In conformal field theory, excitations are massless, otherwise



**Figure 1.5.** The size-to-distance holographic relation.

an explicit scale  $m$  would spoil the conformal symmetry. Unlike massive particles, massless conformal excitations are non-local and characterized not only by their position in space, but also by their size [35]. So both the particle in the bulk and the conformal excited state in the boundary can be described by a  $(d + 1)$ -vector: the  $d$ -vector of the transversal location of the bulk particle encodes for the transversal location of the conformal state, and the extra radial coordinate of the bulk particle encodes for the size of the conformal state (see Fig.1.5). The further the particle is from the boundary, the larger it is. We can see that this interpretation is in perfect agreement with Susskind’s vision laid out in the previous section.

In the momentum representation, the spatial size of the excitation corresponds to its inverse energy. Thus the extra holographic coordinate has meaning of energy scale in the boundary field theory: the boundary  $z = 0$  corresponds to the UV limit, while  $z \rightarrow \infty$  is the IR. The corresponding variation along this direction  $\partial_z$  thus geometrically encodes a renormalization group flow, where the  $z = 0$  surface corresponds to the ultraviolet fixed point of the theory.

The boundary field theory can be thought of as defined not only at  $z = 0$ , but at any  $z = \text{const}$  surface. In the case of pure  $AdS$  all  $z = \text{const}$  slices obviously are equivalent, and the RG flow is trivial as should it be in a completely scale invariant theory. However, as we will see later, the



*AdS/CFT* correspondence is defined not only for pure *AdS*, but also for any solution of General Relativity that possesses an asymptotically *AdS* form. The strict conformal invariance then may be broken, and we find a non-trivial renormalization flow connecting UV and IR fixed points.

To understand how the spectrum of states of the field theory can be mapped onto the bulk gravity, consider a massive particle evolving in *AdS* with zero transversal momentum. Its Lagrangian has form

$$\mathcal{L} = g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = -\frac{1}{z^2} \dot{t}^2 + \frac{1}{z^2} \dot{z}^2 = -1. \quad (1.20)$$

The particle has two conserved momenta:

$$E = \frac{\dot{t}}{z^2}, \quad J = \frac{\dot{z}}{z^2}, \quad (1.21)$$

and its radial evolution equation is

$$\dot{z}^2 = -z^2 + E^2 z^4. \quad (1.22)$$

At the boundary, the  $z^2$ -term is leading, and for any value of energy  $E$  the r.h.s. is negative. This means that the particle experiences an (infinitely) strong repulsive potential as it approaches the boundary:

$$V(z) \sim \sqrt{-g_{00}} \sim \frac{1}{z}, \quad z \rightarrow 0. \quad (1.23)$$

Thus if instead of a particle we consider a field in the bulk, this repulsive potential will act as an infinitely high quantum well. Posing appropriate boundary conditions at both sides of the well, we obtain a well-defined spectrum of quantized bulk wavefunctions that dualizes to the spectrum in the boundary field theory. In other words, classical boundary conditions in the bulk define the quantization rules in the dual theory.

### 1.3.3 The Gubser-Klebanov-Polyakov-Witten rule

So far we have discussed only the general ideas of *AdS/CFT* holography. Now we define a precise constructive correspondence that allows for concrete calculations of observables.

We want to be able to calculate correlation functions of gauge invariant operators  $\mathcal{O}_i$  in the boundary field theory. On the QFT side, a correlation

function of  $n$  operators in Euclidean signature can be defined in terms of a generating functional depending on sources  $J_i$ :

$$e^{-W[J_i]} = \langle e^{J_i \mathcal{O}^i} \rangle_{QFT}, \quad (1.24)$$

where the averaging on the r.h.s. means path integral evaluation. The  $n$ -point correlation function is then just given by an  $n$ -th order functional derivative:

$$\langle \mathcal{O}^{i_1} \dots \mathcal{O}^{i_n} \rangle = \frac{\partial^n}{\partial J_{i_1} \dots \partial J_{i_n}} e^{-W[J_i]} \Big|_{J=0}. \quad (1.25)$$

The holographic correspondence states that these operators should be dual to fields  $\phi_i$  in the bulk. More specifically, the mathematical formalism of the *AdS/CFT* correspondence is based on the formula figured out by S. Gubser, I. Klebanov, and A. Polyakov [10], and E. Witten [11] that resides in the fact that the partition functions of the bulk and boundary theories are identical:

$$e^{-W[J_I]} = Z_{bulk} |_{\phi_I(z=0)=J_I}, \quad (1.26)$$

where the boundary value of the bulk field plays the role of the source in the boundary field theory.

To emphasize the calculational power of this law, let us focus on the original version of the *AdS/CFT* correspondence that equates the  $\mathcal{N} = 4$  Super Yang-Mills theory on the boundary and the IIB string theory on  $AdS_5 \times S_5$  in the bulk. Super Yang-Mills is a  $SU(N)$  gauge theory with the following Lagrangian:

$$L_{SYM} = -\frac{1}{4g_{YM}^2} \text{Tr} \left( F_{\mu\nu} F^{\mu\nu} + D_\mu \Phi^I D^\mu \Phi^I + \bar{\psi}^i \gamma^\mu D_\mu \psi^i + [\Phi^I, \Phi^J]^2 + \dots \right). \quad (1.27)$$

Dots are for interaction terms required by the condition of maximal supersymmetry, and the trace is taken over the gauge indices.  $F_{\mu\nu}$  is the gauge field strength. The fields  $\Phi^I$  ( $I = 1, \dots, 6$ ) are scalars, and  $\psi^i$  ( $i = 1, \dots, 4$ ) are fermions, all in the adjoint representation of the gauge group.

This theory is completely defined by two parameters: the rank of the gauge group  $N$ , and the coupling constant  $g_{YM}$ .<sup>1</sup> On the other hand, the string theory also contains only two defining parameters - the string coupling constant  $g_s$ , and the curvature scale  $L/l_s$  the theory lives on,

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<sup>1</sup>In a generic case, the theory also has a  $\theta$ -term  $\frac{\theta}{8\pi^2} \int \text{Tr} F \wedge F$ , but this is not relevant for the discussion here.

given in terms of the inverse string length. According to 't Hooft we can redefine the perturbative expansion in terms of  $\lambda = g_{YM}^2 N$ . Later on, this will allow us to take a smooth limit  $N \rightarrow \infty$  while keeping the coupling constant finite.

Without getting into detailed derivations, we just quote that these constants dualize according to the relations:

$$4\pi g_s = \frac{\lambda}{N}, \quad \frac{L}{l_s} = \lambda^{1/4}. \quad (1.28)$$

Here is where the weak/strong nature of the duality manifests itself. If we are interested in the strongly coupled regime of the gauge quantum field theory,  $\lambda \gg 1$ , we should take  $L \gg l_s$ . In other words it means that on the string theory side of the duality we consider only large scale dynamics, stringy corrections to the geometry are negligible, and the low energy supergravity limit of the string theory is valid. If we also wish to avoid taking into account quantum gravity effects and keep  $g_s$  small, we need to stay at the large  $N$  limit of the gauge theory,  $\lambda/N \ll 1$ . From a field theoretical point of view precisely this  $N \rightarrow \infty$  limit with fixed  $\lambda \ll 1$  corresponds to the contributions of all planar Feynman diagrams, i.e. those which can be drawn on a topologically trivial surface [36].

In the strongly correlated regime  $\lambda \gg 1$  the diagrammatic expansion is not well defined, but on the dual side the supergravity approximation is at work, and in Euclidean signature we have

$$Z_{string} = Z_{gravity} = e^{-S_{gravity}} \Rightarrow W[J_I] = S_{gravity}[\phi_I(z=0) = J_I]. \quad (1.29)$$

When the rank of the boundary gauge group is large,  $N \gg 1$ , the gravitational action can be evaluated just on the classical solution to the equations of motion. Thus we dualize states in the strongly coupled large  $N$  boundary quantum field theory to solutions of the classical supergravity theory.

In this thesis we will be studying physical problems that can not be described by a simple supersymmetric Yang-Mills theory, but these key ingredients will be there. What the large  $N$  limit corresponds to in, for instance, condensed matter phenomenology is not completely clear. Roughly speaking, it describes a kind of mean field theory of a many-body system, where a self-consistent approximation of the collective dynamics is governed by the bulk classical action. Of course it would be nice to overcome this limitation, but we might hope that the most interesting properties of the physical systems of interest survive in this limit.

### 1.3.4 AdS/CFT and thermodynamics

Now we are going to use *AdS/CFT* to study strongly coupled physics and discuss the basic elements of the holographic Glass Bead Game.

All physical processes involve energy dynamics, so the first thing to be shown is how it can be described in holography. The classical supergravity theory includes General Relativity as a universal subsector. So we can ignore the supersymmetric nature of the full *AdS/CFT* construction and stick to the Einstein-Hilbert theory with a negative cosmological constant in the bulk and, in general, some matter:

$$S = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g} \left( R + \frac{d(d-1)}{L^2} + 2\kappa^2 \mathcal{L}_M \right). \quad (1.30)$$

Following the GKPW rule (1.26) and the large-N limit (1.29) the bulk dynamical field - the metric - gains the following interpretation: fluctuations of the bulk metric sources energy currents on the boundary, and thus *the boundary asymptotics of the metric dualizes to the boundary energy-momentum tensor*. This is not to be confused with the bulk energy-momentum tensor.

Another “bead” to be defined is the notion of thermal matter in holography. It can easily be demonstrated that a black hole metric with *AdS* asymptotics corresponds to a thermal state of the dual field theory with temperature equal to the Hawking temperature of the black hole. Consider the simplest black hole solution - the anti de Sitter-Schwarzschild metric. In Poincare coordinates it takes the form

$$ds^2 = \frac{1}{z^2} \left( -f(z)dt^2 + \frac{dz^2}{f(z)} + dx^i dx^i \right), \quad (1.31)$$

where the emblackening factor

$$f(z) = 1 - \left( \frac{z}{z_H} \right)^d. \quad (1.32)$$

Here,  $z_H$  is the location of the horizon. Because in coordinates of the Poincare chart the metric is transversally symmetric along the boundary directions, sometimes it’s called a planar black hole.

We can perform a Euclidean continuation of this metric to imaginary time  $t \rightarrow i\tau$ :

$$ds_E^2 = \frac{1}{z^2} \left( f(z)d\tau^2 + \frac{dz^2}{f(z)} + dx^i dx^i \right). \quad (1.33)$$

It should be emphasized that the Euclidean continued metric does not solve the Euclidean theory automatically. The imaginary time coordinate is periodic, and it can be shown [37] that (1.33) is a saddle point to the *vacuum* Euclidean version of (1.30), only if we fix the period as<sup>2</sup>:

$$\tau \sim \tau + \frac{4\pi}{|f'(z_H)|} = \tau + \frac{4\pi z_H}{d}. \quad (1.34)$$

To understand meaning of this fact in the dual field theory language, we can perform a near-boundary expansion of the metric ( $z \rightarrow 0$ ):

$$g_{\mu\nu}(z) = \frac{1}{z^2} g_{(0)\mu\nu} + \dots. \quad (1.35)$$

The expansion coefficient  $g_{(0)\mu\nu}$  has a natural interpretation as the background metric the boundary field theory evolves in. Clearly, for any asymptotically AdS spacetime it is

$$g_{(0)\mu\nu} = d\tau^2 + dx^i dx^i, \quad \tau \in [0, 4\pi z_H/d) \quad (1.36)$$

so the dual quantum field theory is also defined on a space with periodic imaginary time. Hence, it is in a thermal state of temperature  $T = \frac{d}{4\pi z_H}$  which precisely coincides with the Hawking temperature of the bulk black hole.

Here we have made a standard Wick rotation to periodic imaginary time to make the connection between thermal properties of both sides of the correspondence more clear. However a remarkable property of the *AdS/CFT* holography is that the temperature is now an inherent property of the Lorentzian bulk - it is given by the Hawking temperature. It can be shown by a direct calculation [40] that once we have rotated back to the Lorentzian signature, the field theory still is in a thermal state. Therefore we can work with our bulk objects in real time and at finite temperature simultaneously. This is one of the biggest technical advantages of the holographic language over standard field theory machinery.

The next thermodynamical holographic dictionary entry to be defined is the finite charge density or the chemical potential. Again, the *AdS/CFT* correspondence is capable of including it in a very simple, natural manner. Let's add the electromagnetic Maxwell term to our action:

$$S_M = -\frac{1}{4} \int_0^\infty dz \int d^d x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}, \quad (1.37)$$

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<sup>2</sup>Otherwise the metric would contain an apparent unphysical conical defect [37].

where  $F_{\mu\nu}$  is the  $U(1)$  gauge field strength tensor in the bulk. Integrating it by parts we get

$$S_M = \int_0^\infty dz \int d^d x \sqrt{-g} A_\nu \partial_\mu F^{\mu\nu} + \frac{1}{2} \int d^d x \sqrt{-g} F_{\mu\nu} A^\mu n^\nu|_{z \rightarrow 0}, \quad (1.38)$$

where  $n^\nu$  is a unit vector orthogonal to the boundary.

Consider then a solution to the equations of motion of the gauge field  $A_\mu$ . We can deduce that the radial-directed bulk electric field is

$$\frac{\partial \mathcal{L}_{EM}}{\partial(\partial_z A_0)} = \frac{\partial(F_{\mu\nu} F^{\mu\nu})}{\partial(\partial_z A_0)} = E_z, \quad (1.39)$$

and the field  $A_\mu$  approaches its boundary value as

$$A_\mu(\vec{x}, z) = A_\mu^{(0)}(\vec{x}) + A_\mu^{(1)}(\vec{x}) z^{d-2} + \dots. \quad (1.40)$$

Substituting this function into the electromagnetic boundary action (second term in (1.38)) we can see that the leading and subleading expansion coefficients are coupled:

$$S_b = - \int d^d x \sqrt{-g_{(0)}} A_\mu^{(0)}(\vec{x}) A^{(1)\mu}(\vec{x}) + \dots, \quad (1.41)$$

thus they should be regarded as holographic dictionary entries for conjugate operators in the boundary field theory. Knowing the asymptotic behaviour (1.40), we can derive that the subleading term  $A_0^{(1)}$  is the radial-directed bulk electric field  $E_z$  evaluated at the boundary:  $A_0^{(1)}(\vec{x}) = E_z(\vec{x})$ , which in turn is equal to the surface charge density. So holographically we identify the subleading term with the negative<sup>3</sup> boundary charge density,  $A_0^{(1)} = \rho$ , and its conjugate  $A_0^{(0)}$  then can be recognized as the chemical potential,  $A_0^{(0)}|_{z \rightarrow 0} = \mu$ .

In a similar way we can interpret the spatial transversal components of the gauge field. The spatial subleading expansion coefficients  $A_i^{(1)}$  are dual to the current  $J_i$ , while their sources  $A_i^{(0)}$  are dual to spatial components of the global  $U(1)$  boundary field. Note that interestingly the bulk  $U(1)$  gauge field dualizes to the global  $U(1)$  current on the boundary. That's another basic property of the *AdS/CFT* correspondence: bulk

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<sup>3</sup>Due to the overall minus sign in (1.41)

gauge symmetries holographically encodes for global symmetries in the field theory. That's not only true for the discussed example of  $U(1)$ , but rather a generic property of the correspondence.

Temperature and chemical potential are two important thermodynamical quantities. But it is trivial to show that the GKPW rule provides a complete description of the thermodynamics of the boundary quantum field theory. By definition the free energy  $F$  of a QFT is the logarithm of its partition function:

$$e^{-\beta F} = \langle Z \rangle_{QFT}, \quad (1.42)$$

where  $\beta = 1/T$ . Through *AdS/CFT*, the right hand side of this identity is equal to the bulk string theory partition function, or, in the large  $N$  limit, to the classical exponent evaluated at the gravity saddle point. Thus the free energy is simply

$$F = TS_{grav}, \quad (1.43)$$

where the gravity action also contains a boundary term which fixes the boundary conditions for the bulk fields in accordance with (1.29). All other thermodynamical potentials can be derived from  $F$ .

### 1.3.5 Correlation functions from holography

One of the most important aspects of *AdS/CFT* is that we can also extract much more detailed microscopic information. In principle we can compute the full set of correlation functions of the theory. Let us demonstrate the holographic techniques on the two-point Green's function, one of the most physically interesting and simple to calculate quantities. For simplicity, we firstly reproduce the standard result for a Green's function in a conformal field theory in Euclidean signature. After that we will comment on how to use the power of holography to obtain Green's functions at strong coupling directly in the real time representation.

Consider a massive scalar field action:

$$S = \frac{1}{2} \int dz d^d x \sqrt{g} \left( \partial_\mu \phi \partial^\mu \phi + m^2 \phi \right), \quad (1.44)$$

with the corresponding equations of motion:

$$\frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} g^{\mu\nu} \partial_\nu \phi(z, \vec{x})) - m^2 \phi(z, \vec{x}) = 0. \quad (1.45)$$

A solution to this equation in the Euclidean empty  $AdS$  space behaves near the boundary as

$$\phi(z, \vec{x}) \simeq z^{\Delta_-} \phi_{(0)}(\vec{x}) + z^{\Delta_+} \phi_{(1)}(\vec{x}) + \dots, \quad \text{when } z \rightarrow 0, \quad (1.46)$$

where

$$\Delta_{\pm} = \frac{d}{2} \pm \frac{1}{2} \sqrt{d^2 + 4m^2}. \quad (1.47)$$

We already know that the leading term  $\phi_{(0)}(\vec{x})$  is a source for a dual boundary operator,  $\int d^d x \phi_{(0)}(\vec{x}) \mathcal{O}(\vec{x})$ .

The Green's function of the differential operator (1.45) can be found exactly:

$$G(0, \vec{x}; z, \vec{x}') = \frac{z^{\Delta_+}}{(z^2 + |\vec{x} - \vec{x}'|^2)^{\Delta_+}}, \quad (1.48)$$

so the solution (1.46) has a representation:

$$\phi(z, \vec{x}) = \int d^d \vec{x}' \frac{z^{\Delta_+}}{(z^2 + |\vec{x} - \vec{x}'|^2)^{\Delta_+}} \phi_0(\vec{x}'). \quad (1.49)$$

Having the solution, we can substitute it into the action, and evaluate the functional derivative with respect to  $\phi_0(\vec{x})$  to apply the GKPW rule. The bulk action variation around the solution is zero by definition, so, as before when we discussed the gauge field, all non-trivial structures are contained in the surface boundary term:

$$\delta S(\phi) = \int d^d \vec{x}' d\Sigma^\mu \partial_\mu \phi \delta \phi, \quad (1.50)$$

where  $d\Sigma^\mu$  is the boundary area element. To apply the GKPW rule (1.25), (1.26) we need to express this variation in terms of the boundary data. Varying the boundary action is a subtle procedure that requires accurate treatment of near boundary divergences. Without touching on related technical issues, we just quote the result [38]:

$$\delta \phi = z^{\Delta_-} \delta \phi_0, \quad (1.51)$$

$$\delta S(\phi) = \int d^d \vec{x} d^d \vec{x}' \frac{\phi_0(\vec{x}) \delta \phi_0(\vec{x}')}{|\vec{x} - \vec{x}'|^{2\Delta_+}}. \quad (1.52)$$

Taking the first functional derivative, for the boundary vacuum expectation value sourced by  $\phi_0(\vec{x})$  we get

$$\langle \mathcal{O}(\vec{x}) \rangle = \frac{\delta(-S(\phi))}{\delta \phi_0(\vec{x})} = - \int d^d \vec{x}' \frac{\phi_0(\vec{x}')}{|\vec{x} - \vec{x}'|^{2\Delta_+}} \quad (1.53)$$



Repeating the procedure we arrive at the correct result for the Euclidean 2-point conformal Green's function:

$$\langle \mathcal{O}(\vec{x}) \mathcal{O}(\vec{x}') \rangle = \frac{\delta(-S(\phi))}{\delta\phi_0(\vec{x})\delta\phi_0(\vec{x}')} = \frac{\text{const}}{|\vec{x} - \vec{x}'|^{2\Delta_+}}. \quad (1.54)$$

Clearly  $\Delta_+$  is the conformal dimension of the operator  $\mathcal{O}$  in the boundary field theory. We obtain a new holographic dictionary entry: *the mass of the bulk field corresponds to the conformal dimension of the dual boundary operator.*

In many cases, the Euclidean correlator can be then analytically continued to Lorentzian signature. However sometimes it is really necessary to have the Lorentzian Green's function right away. In particular, we need it when dealing with systems out of equilibrium, like a superconductor quenched by an external pulse, or the quark gluon plasma that exists for a tiny fraction of a second after the moment of a heavy ion collision. In such time dependent cases, Euclideanization usually can not be performed because the state of the theory is different at different time instants, and the notion of global-in-time Wick rotation is ill-defined. Remarkably, the *AdS/CFT* correspondence provides a generic prescription for real-time response functions. But in order to derive it, we need to resolve two issues.

One of the issues with real time QFT is rooted in the fact that unlike the Euclidean case, we can have a multitude of different Green's functions (retarded, advanced, Feynman, Wightman). On the holographic side this is reflected in the fact that there is no unique choice of boundary conditions for the bulk field in the infrared.

For time-like bulk excitations there are two linearly independent solutions to the equations of motion possessing the same near-boundary regular asymptotics. On the other hand, near the Poincare horizon they behave as

$$\phi(z) \sim e^{\pm i q z}, \quad z \rightarrow \infty, \quad \text{where } q = \sqrt{\omega^2 - \vec{k}^2}. \quad (1.55)$$

As proven in [39], for the retarded Green function  $G_R$  we have to impose infalling boundary conditions in the IR region:

$$\phi(z) \sim e^{-i q z}. \quad (1.56)$$

This choice is intuitively reasonable. Infallingness means that wave fronts of the bulk field move towards the (black hole or Poincaré) horizon and

disappear behind it. Thus it is in an agreement with the fact that  $G_R$  describes causal propagation of an excitation.

The second problem, apart from the ambiguity of the boundary conditions, is that in the Lorentzian signature the two-point Green's function can not be calculated as a second functional derivative of the action. For instance, the boundary action of a 4-dimensional scalar field theory evaluated on the classical solution is

$$S_{bnd} = \int \frac{d^4k}{(2\pi)^4} \phi_0(-k) \mathcal{F}(k, z) \phi_0(k) \Big|_{z=z_B}^{z=z_H}, \quad (1.57)$$

where  $\mathcal{F}(k, z)$  is a certain *real* function. Therefore the object naively anticipated to be the retarded Green function is

$$-(\mathcal{F}(k, z) + \mathcal{F}(-k, z)) \Big|_{z=z_B}^{z=z_H}, \quad (1.58)$$

and it is completely real. So this can not be the correct  $G_R$ .

In [40] it has been shown that we have to neglect contributions coming from the horizon and contributions of negative momenta, and the proper definition for the retarded propagator in holography is

$$G_R(k) = -2\mathcal{F}(k, z_B). \quad (1.59)$$

This function has both real and imaginary parts, and was proven to reproduce the correct real-time Schwinger-Keldysh formalism in [39].

The prescription (1.59) together with (1.56) works for any spacetime metric with *AdS* asymptotics in an arbitrary dimension, giving us a powerful universal tool for studying real time physics in quantum field theories at finite temperature and charge density, with different field content, in non-trivial external fields, in spatially modulated lattice backgrounds etc.

To summarize the aforementioned technical elements of the *AdS/CFT* correspondence, let us provide the holographic dictionary in the form of a short table.

Bulk $AdS$ gravity	Boundary QFT
Extra dimension	Renormalization group
Hawking temperature of a black hole	Temperature
Maxwell $U(1)$ gauge field	Chemical potential and charge density
Field	Operator
Mass of a field	Conformal dimension of a dual operator
Space-time metric	Energy-momentum tensor
Quantum numbers	Quantum numbers

## 1.4 The correspondence as a method

In this section we survey several holographic models of real life physical systems. By no means is this exposition exhaustive. The only goal is to demonstrate that the  $AdS/CFT$  correspondence is a really universal language that can be applied to a variety of problems in theoretical physics. We start with a brief review of the very first paper on applied holography, dedicated to the minimal shear viscosity of the quark-gluon plasma, and then we focus on condensed matter theory applications of the  $AdS/CFT$  correspondence - models of holographic superconductors and non-Fermi liquids.

### 1.4.1 Minimal viscosity

When dealing with translationally invariant “planar” black hole solutions in  $AdS$ , we may ask a question about the physics of small hydrodynamical (long wavelength) fluctuations of the horizon and their boundary field theory interpretation. It turns out that hydrodynamical response functions of the boundary theory can be precisely encoded in the dual long wavelength dynamics of gravity theories with horizons, providing a route to yet another incarnation of applied holography - the fluid/gravity correspondence [25].<sup>4</sup>

A particular system of interest in this context is the quark gluon plasma formed in heavy ion collisions [42]. The QGP is a strongly-coupled

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<sup>4</sup>Some subtleties arise if we wish to study high-frequency fluctuations in the boundary field theory [41], but in the low-frequency limit the mapping is unambiguous.

system that exhibits the behaviour of a nearly perfect quantum liquid, and its experimentally measurable properties are defined by the hydrodynamical transport coefficients, especially by the shear viscosity that measures the strength of transversal momentum transport in a liquid. So it is natural to start our discussion of phenomenological applications of the *AdS/CFT* correspondence with the result on minimal shear viscosity obtained in [15, 16].

In a field theory, the shear viscosity can be calculated within the Kubo formalism, which relates it to equilibrium two-point correlation functions:

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{2\omega} \int dt d^3 \vec{x} e^{i\omega t} \langle [T_{xy}(t, \vec{x}), T_{x,y}(0, \vec{0})] \rangle. \quad (1.60)$$

To compute the correlator in *AdS/CFT*, consider a simple thermal field theory whose dual is the 5-dimensional Schwarzschild black hole in Poincare coordinates:

$$ds^2 = \frac{1}{z^2} \left( - \left( 1 - \frac{z^4}{z_H^4} \right) dt^2 + dx^2 + dy^2 + d\xi^2 + \frac{1}{1 - \frac{z^4}{z_H^4}} dz^2 \right). \quad (1.61)$$

As we have discussed in the previous section, according to the *AdS/CFT* correspondence the boundary energy-momentum tensor  $T_{\mu\nu}$  is dual to metric perturbations  $h_{\mu\nu} = g_{\mu\nu} - g_{\mu\nu}^{(0)}$  in the bulk. From this perspective, the correlator (1.60) corresponds to the graviton absorption rate by the planar black hole:

$$\sigma(\omega) = -\frac{2\kappa^2}{\omega} \text{Im} G^R(\omega) = \frac{\kappa^2}{\omega} \int dt d^3 \vec{x} e^{i\omega t} \langle [T_{xy}(t, \vec{x}), T_{x,y}(0, \vec{0})] \rangle. \quad (1.62)$$

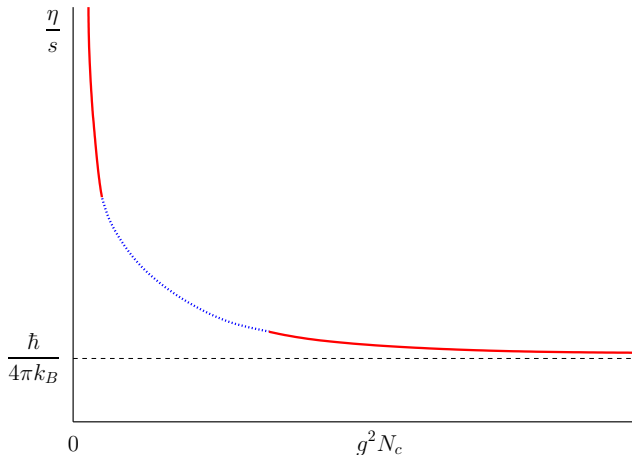
Thus we see that

$$\eta = \frac{\sigma(0)}{2\kappa^2}, \quad (1.63)$$

where  $\kappa^2 = 8\pi G$  is the gravitational Newton constant (1.30). Note that the planar black hole has an infinite horizon area, so instead of the absolute Bekenstein entropy we can define the entropy per unit area:

$$s = \frac{a}{4G} = \frac{2\pi a}{\kappa^2}. \quad (1.64)$$

To calculate the quantity (1.63), we can assume that the metric perturbations are orthogonal to the ‘‘holographic’’ direction, and have only



**Figure 1.6.** Dependence of the  $\eta/s$  ratio on the 't Hooft coupling constant.

transversal  $xy$ -polarization. Then on the linearized level, the equation of motion for  $h_y^x$  simplifies and takes the form of the Klein-Gordon equation for a massless scalar:

$$\square h_y^x = 0. \quad (1.65)$$

This allows a shortcut to the answer - we can apply the theorem of universality of low frequency scalar field absorption by the black hole [43] and claim

$$\sigma(0) = a = \frac{\kappa^2}{2\pi} s. \quad (1.66)$$

So we arrive at a simple parameter-independent result

$$\frac{\eta}{s} = \frac{\sigma(0)}{2\kappa^2} = \frac{1}{4\pi}, \quad (1.67)$$

or, with restored units:

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}. \quad (1.68)$$

The result is remarkable in at least two fundamental aspects. Here we have sketched the calculation for a simple planar 5-dimensional  $AdS$ -Schwarzschild black hole, but this result was also obtained in many different theories within the complete supergravity context, including models dual to various brane configurations [44], and models with non-relativistic Schroedinger symmetry [45]. In this regard it is very universal.

This leads to a conjecture that  $\frac{\eta}{s} = \frac{1}{4\pi}$  is the universal minimal bound on the possible values of shear viscosity of quantum liquids, Fig. 1.6. For a long time this conjecture was widely believed to be true, but finally it was figured out that under certain circumstances, e.g. in the presence of matter fields in the fundamental representation, this bound can be violated [46]. Still, the shear viscosity of the quark gluon plasma measured in experiments on high energy ion collisions [42] turned out to be very close to the original  $\frac{1}{4\pi}$  value. Historically, this was the first manifestation of the surprising applicability of holographic duality to real life physics. And this ignited the applied holography revolution.

### 1.4.2 The holographic superconductor

One of the first applications of the *AdS/CFT* correspondence to condensed matter physics was the formulation of a Landau-Ginzburg-like scalar order parameter theory of superconductivity in holographic terms [47].

This set up is particularly elegant. Again, consider a planar Schwarzschild black hole in four dimensions:

$$ds^2 = - \left( r^2 - \frac{M}{r} \right) dt^2 + \frac{dr^2}{r^2 - \frac{M}{r}} + r^2(dx^2 + dy^2). \quad (1.69)$$

The boundary in these coordinates is at  $r \rightarrow \infty$ . Following the GKPW rule we encode the superconducting order parameter charged under a global  $U(1)$  current in a bulk complex charged scalar field coupled to the local gauge  $U(1)$  field:

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \right. \quad (1.70)$$

$$\left. - (\partial_\mu \Psi - iA_\mu \Psi) (\partial^\mu \Psi^* + iA^\mu \Psi^*) \right) + 2\Psi\Psi^*, \quad (1.71)$$

where the mass of the bulk scalar field is taken to be  $m^2 = -2$  for simplicity.

The action in the bulk is just the abelian  $U(1)$  Higgs model. Higgsing the gauge  $U(1)$  symmetry in the bulk corresponds to the spontaneous global  $U(1)$  symmetry breaking, opening room for the Landau-Ginzburg phase transition in the boundary field theory.

We now study the system at finite chemical potential,  $A_0|_{r \rightarrow \infty} = \mu$ . Assuming time independence, homogeneity and isotropy, we impose the

bulk gauge field to have only  $A_0 = \Phi$  non-vanishing component, and both  $\Psi(r)$  and  $\Phi(r)$  to be functions of only the holographic radial coordinate. Then we arrive at the equations of motion:

$$\begin{aligned} \Psi'' + \left( \frac{f'}{f} + \frac{2}{r} \right) \Psi' + \frac{\Phi^2}{f^2} \Psi + \frac{2}{f} \Psi &= 0, \\ \Phi'' + \frac{2}{r} \Phi' - \frac{2\Psi^2}{f} \Phi &= 0. \end{aligned} \tag{1.72}$$

Solving them in the background (1.69), we obtain the near boundary asymptotics:

$$\Psi = \frac{\Psi_1}{r} + \frac{\Psi_2}{r^2} + \dots, \text{ as } r \rightarrow \infty, \tag{1.73}$$

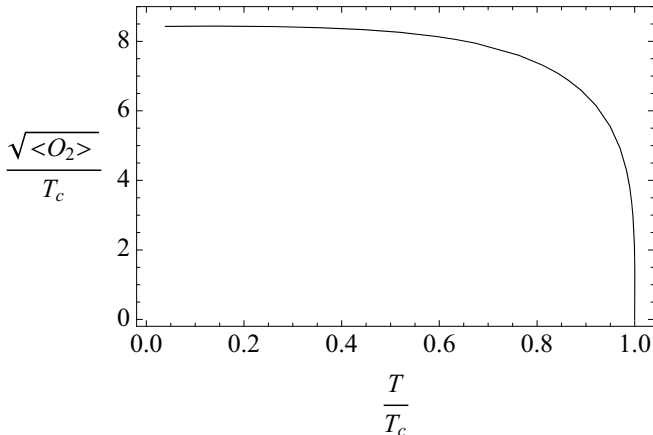
$$\Phi = \mu - \frac{\rho}{r} + \dots. \tag{1.74}$$

As we already know, according to the *AdS/CFT* dictionary the leading coefficient in the near boundary expansion corresponds to the source of a dual operator, and the subleading one corresponds to the expectation value. Thus we can proceed with the following strategy:

- As we are interested in *condensation* of the order parameter, i.e. in formation of the vev in absence of the source, we fix  $\Psi_1 = 0$ .
- We want to study the thermal phase transition between disordered and superconducting phases, and demonstrate that the system undergoes a second order phase transition as the temperature approaches a critical value  $T_c$ . In the holographic setting we can do this by varying the Hawking temperature of the bulk black hole.
- In the superconducting phase we analyze the electric conductivity of the field theory by considering perturbations of spatial components of the gauge field, and explore whether it indeed exhibits the characteristic gap associated with order parameter condensation.

We begin with the analysis of the order parameter condensation. To make our notation consistent with [47], we normalize the order parameter as

$$\langle \mathcal{O}_2 \rangle = \sqrt{2} \Psi_2. \tag{1.75}$$



**Figure 1.7.** Thermal phase transition between superconducting and normal phases of a holographic superconductor.

Its dependence on the temperature can be deduced numerically, and can be perfectly fit with the following law (see also Fig.1.7):

$$\langle \mathcal{O}_2 \rangle = const \cdot T_c \left( 1 - \frac{T}{T_c} \right)^{\frac{1}{2}}. \quad (1.76)$$

One can notice that this is completely consistent with the Landau-Ginzburg critical exponent of the second order thermal phase transition in a superconductor.

Next we have to analyze the transport properties of the order parameter and make sure that it really superconducts. Let us recall that in *AdS/CFT*, the boundary electromagnetic current and its source - electric field - are encoded in the near boundary expansion of the spatial bulk gauge field components:

$$A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r} + \dots, \quad (1.77)$$

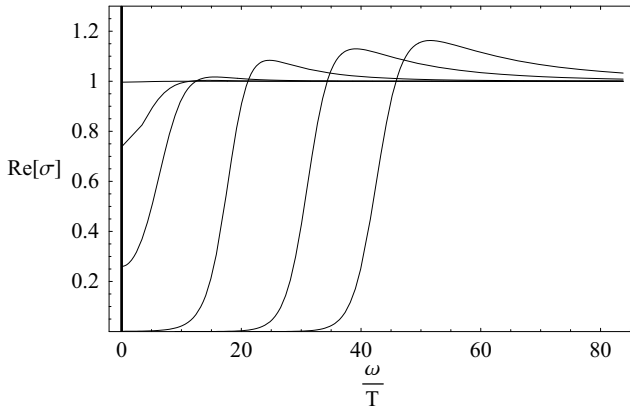
$$A_x^{bnd} = A_x^{(0)}, \quad \langle J_x \rangle = A_x^{(1)}. \quad (1.78)$$

Consider a small, constant frequency perturbation of the gauge field along the  $x$  direction:

$$A_x = A_x(r) e^{-i\omega t}. \quad (1.79)$$

Then we can rewrite the Ohm law as





**Figure 1.8.** Electric conductivity of the holographic superconductor. The D.C. conductivity in the ordered phase is delta function-like, and the A.C. conductivity exhibits a characteristic gap which gradually increases as we lower the temperature .

$$\sigma(\omega) = \frac{\langle J_x \rangle}{E_x} = -\frac{\langle J_x \rangle}{\partial_t A_x} = -\frac{iA_x^{(1)}}{\omega A_x^{(0)}} \quad (1.80)$$

Thus, once we know the solution to the linearized equation of motion for the gauge field, we can read off the conductivity from the near boundary behaviour of  $A_x$ .

The e.o.m. is

$$A_x'' + \frac{f'}{f} A_x' + \left( \frac{\omega^2}{f^2} - \frac{2\Psi^2}{f} \right) A_x = 0. \quad (1.81)$$

Solving this equation of motion with infalling boundary conditions near the horizon, we can show that the A.C. conductivity has a gap that gradually increases when the temperature is lowered, and the density of the condensate increases. This gap is qualitatively similar to the gap in a conventional Bardeen-Cooper-Schrieffer superconductor [48], but contra to the standard case, the conductivity is never strictly zero at low frequencies. Rather, we have a soft “algebraic” gap,  $\sigma(\omega) \sim \omega^p$ , where  $p > 0$ . The D.C. conductivity in the ordered phase exhibits a delta function-like peak at  $\omega = 0$  which persists upon increment of the temperature (Fig. 1.8). The peak might be misinterpreted as yet another signature of superconductivity, but we should be careful here. The considered model is

translationally invariant, so there is no source of momentum dissipation like an atomic lattice or disorder, and the peak is there even in the non-SC phase. Therefore, strictly speaking, we can not judge whether it is a superconductor or a perfect conductor from this consideration. However, in follow-up papers it was shown that the conductivity peak in the condensed phase is not destroyed by translational symmetry breaking [49], so the system actually exhibits superconductivity.

Ideologically, this holographic superconductor is very similar to the superconductor of the Landau-Ginzburg theory. However, it differs in two crucial aspects. First of all, holographically it is possible to describe a scalar order parameter of an arbitrary scaling dimension. Microscopically the scalar is formed of fermionic pairs, so in a weakly interacting system the scaling dimension of the order parameter is just twice that of the fermionic operator dimension,  $\Delta_s = 2\Delta_f$ . At strong coupling it might be renormalized due to strong non-perturbative interactions, and holography can naturally capture this. Secondly, holographic superconductivity emerges from a critical rather than quasi-particle system. Criticality (in other words, emergent conformal invariance in many-body systems) is commonly believed to underlie the physics of many unconventional phases of strongly correlated systems, like high-Tc superconductors and strange metals [50]. In this regard, *AdS/CFT* is able to explore a whole new range of physical systems, which are inaccessible by perturbative quantum field theory.

### 1.4.3 AdS/CFT and fermionic matter

In the previous subsection, we have demonstrated how the holographic correspondence can be applied to describe a superconducting phase transition. The next step to be done is to go beyond the order parameter level and take into account the fermionic nature of strongly correlated systems.<sup>5</sup>

The only really well understood phase of fermionic matter is the weakly interacting Fermi liquid [51], and the associated BCS superconductor [52]. The key assumption of the standard Landau theory of the Fermi liquid is the existence of coherent long-living quasiparticle excitations near the Fermi surface. In field theoretical terms, this means that the corresponding fermionic Green function has a sharp pole at a well-defined Fermi-

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<sup>5</sup>This will also be an important motivation for Chapter 2 of this thesis.

momentum  $k_F$ . However at strong coupling we should not expect the quasi-particle picture to be universal. Strong interactions can cause complicated emergent phenomena that manifest themselves in non-trivial reconstruction (or even destruction) of the Fermi surface, modification of the dispersion relations, exotic transport properties etc.

As we discussed earlier, the conventional analytic and numerical methods fail to provide us with a universal tool to study such systems. Is *AdS/CFT* capable of bringing us a better understanding of strongly coupled fermionic matter at finite charge density, and of providing a mathematical description of phases of quantum matter beyond the conventional Fermi liquid picture? This issue has been addressed independently in Leiden and MIT in 2009 [31, 32], and the result of these studies was a holographic model that clearly exhibits properties of a finite density state of fermionic matter that is not a Fermi liquid; we will call such states non-Fermi liquids.

Consider a 4-dimensional Einstein-Hilbert-Maxwell theory (as in the example of the scalar holographic superconductor), but add to the action a fermionic term<sup>6</sup>:

$$S_\psi = \int d^{d+1}x \sqrt{-g} i \left( \bar{\psi} \Gamma^M D_M \psi - m \bar{\psi} \psi \right), \quad (1.83)$$

where the covariant derivative is

$$D_M = \partial_M + \frac{1}{4} \omega_{abM} \left[ \Gamma^a, \Gamma^b \right] - ie A_M. \quad (1.84)$$

Here  $\omega_{abM}$  is the spin connection, and  $e$  is the fermionic electric charge. As we discussed in Sec. 1.3.4, finite charge density at the boundary dualizes to the electric field in the bulk, so in order to account for it we will put the bulk fermion in the background of a charged Reissner-Nordström black hole (and for simplicity treat the fermionic field as a probe):

$$ds^2 = r^2 \left( -f(r) dt^2 + d\vec{x}^2 \right) + \frac{1}{r^2} \frac{dr^2}{f(r)}, \quad f(r) = 1 + \frac{Q^2}{r^4} - \frac{1+Q^2}{r^3}. \quad (1.85)$$

---

<sup>6</sup>In what follows the capital latin indicies denote coordinates in the bulk, the greek boundary coordinates, and the small latin ones coordinates on a flat tangent bundle. Indices on the gamma matrices always correspond to the tangent space. We choose the gamma matrices basis as

$$\Gamma^r = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \Gamma^r = \begin{pmatrix} 0 & \sigma^\mu \\ \sigma^\mu & 0 \end{pmatrix}, \quad (1.82)$$

where  $\sigma^\mu$  are the Pauli matrices.

Here  $Q$  is the dimensionless charge of the black hole, and the electric field is

$$A_0 = \mu \left(1 - \frac{1}{r}\right) \quad \mu = g_F Q, \quad (1.86)$$

where  $g_F$  is the coupling constant of the Maxwell gauge field.

It is convenient to split the four-component fermionic field into two two-component eigenfunctions of the projector:

$$(1 + \Gamma^r) \psi_{\pm} = \pm \psi_{\pm}, \quad (1.87)$$

Fourier transform it along the boundary directions, and rescale

$$\psi_{\pm} (-gg^{rr})^{-1/4} e^{-i\omega t + \vec{k}\vec{x}} \phi_{\pm}. \quad (1.88)$$

The Dirac equations then become

$$\sqrt{\frac{g_{ii}}{g_{rr}}} (\partial_r \mp m\sqrt{g_{rr}}) \phi_{\pm} = i\sqrt{-\frac{g_{ii}}{g_{tt}}} \left(\omega + eQ \left(1 - \frac{1}{r}\right)\right) \phi_{\pm} - ik_i \sigma^i \phi_{\mp}. \quad (1.89)$$

We are interested in the fermionic Green's function. As shown in [32, 53], in order to derive the Green's function, instead of evaluating the classical action at the saddle point, one just can analyze the near boundary asymptotics of the Dirac bulk wave function:

$$\phi_+ = A(\omega, \vec{k}) r^m + B(\omega, \vec{k}) r^{-m-1} + \dots, \quad r \rightarrow \infty \quad (1.90)$$

$$\phi_- = C(\omega, \vec{k}) r^{m-1} + D(\omega, \vec{k}) r^{-m} + \dots. \quad (1.91)$$

Then

$$G_R = -iD(\omega, \vec{k}) A^{-1}(\omega, \vec{k}). \quad (1.92)$$

As described in [31, 32], imposing infalling boundary conditions at the horizon we can evaluate this function. What has been found is that the theory has a number of remarkable features that distinguish it from the conventional Fermi liquid:

- At zero temperature, the spectral function of the theory has a Fermi surface pole, but the corresponding dispersion relation is not of the linear Landau type  $\omega \sim v_F(k - k_F)$ , but rather

$$\omega \sim (k - k_F)^z, \quad (1.93)$$

where  $z$  can be any value dependent on the parameters of the model.

- As the temperature is raised, the peak is smoothed out. However unlike the Fermi liquid case, its width is not just quadratic in frequency/temperature, but depends non-trivially on frequency and momentum [31]:

$$\Gamma = \tan \gamma |\omega^*(k_F)|, \quad (1.94)$$

where  $\omega^*$  is the resonant frequency at the Fermi level where the peak is located, and  $\gamma$  is a numerical parameter.

- Another non-FL feature of the model is that, near the Fermi momentum  $k_F$ , it has a strong particle-hole asymmetry:

$$G_{ii}(\omega, \vec{k}) \neq G_{ii}(-\omega, \vec{k}). \quad (1.95)$$

- In the limit of small frequencies, the Green function exhibits logarithmic oscillations  $G \sim e^{i \log \omega}$ .
- The system can have a multitude of Fermi-surfaces.

It remains to be checked experimentally what properties of the holographic non-Fermi liquids can be observed in nature. However, it is already clear that the *AdS/CFT* provides a powerful tool for modeling fermionic systems at finite density beyond the quasi-particle paradigm, and even if this simplest model does not capture all possible physics, the holographic approach has the capacity for constructing more realistic setups [19, 54].

## 1.5 This thesis

In this thesis we apply the AdS/CFT correspondence to three problems belonging to different areas of theoretical physics.

Chapter 2 is dedicated to the holographic description of superconductivity. While most of the holographic setups describe this phenomenon on the level of the scalar order parameter, a realistic theory should take into account the strong pairing between microscopic fermionic degrees of freedom. Here we make a first step towards filling in the gap and study pairing-induced superconductivity in strongly coupled systems at finite density. The inroad is to study the pairing of quasi-particles. We have just described a holographic model of a non-Fermi liquid, but we can easily change it to a regular Fermi liquid by introducing an IR hard wall cut-off.

This removes gapless critical excitations and allows one to controllably address the dynamics of a single confined Fermi surface. Then, in the weakly coupled dual gravitational theory, the mechanism is that of conventional BCS theory. We study in detail the interplay between the scalar order parameter field and fermion pairing. It is very natural in holography to introduce independent bulk dynamics for the scalar field as well, which corresponds to a non-trivial RG flow of the order parameter. One can then demonstrate that the theory experiences a BCS/BEC crossover controlled by the relative scaling dimensions of the scalar and fermionic operators. A novel technical issue we encounter here is unexpected resonances in the canonical expectation value of the scalar operator at certain values of the scaling dimension, which indicate that in the presence of interacting fields in the bulk, the standard holographic dictionary requires modification.

In chapter 3 we analyze the holographic quark gluon plasma (QGP) formed at a very early stage right after the collision of heavy ions. The  $T - \mu$  phase diagram of the QGP is constructed by using a holographic dual model for the heavy ion collision. In this dual model, colliding ions are simulated by charged gravitational shock waves. In accordance with the suggestion in [55], the formation of the deconfined QGP phase is associated in dual terms with the creation of a black hole in the collision of shock waves, which can be detected in the appearance of a trapped surface. Hadronic matter and other confined states correspond to the absence of a trapped surface after the collision.

In addition, we estimate the multiplicity of the ion collision process, i.e. the number of hadrons that forms when the quark gluon plasma has frozen out, which in the dual language is proportional to the area of the trapped surface. We show that a non-zero chemical potential reduces the multiplicity. To plot the phase diagram we use two different dual models of colliding ions, the pointlike and the wall shock waves, and find qualitative agreement of the results.

Finally, in chapter 4 we address a more exotic issue, namely the dynamical evolution of a quantum field in a time machine. Three dimensional gravity in  $AdS$  has a very simple eternal time machine solution based on two conical defects moving around their center of mass on a circular orbit. Closed time-like curves in this spacetime extend all the way to the boundary of  $AdS_3$ , violating the causality of the boundary field theory. We apply  $AdS/CFT$  to obtain the dual interpretation of this spacetime. By use of the geodesic approximation, we address the “grandfather para-

dox” in the dual  $1 + 1$  dimensional field theory and calculate its two-point causal Green’s function. It has a non-trivial analytical structure both at negative and positive times, providing us with constructive intuition on how an interacting quantum field could behave once causality is broken. In particular a clear effect we can see is revivals of the field at certain time moments in the past, preceding the act of the Green’s function sourcing.

We conclude with a discussion of the obtained results and put them into a wider context of the holographic Glass Bead Game.





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