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Maximum entropy models for financial systems

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Maximum Entropy Models for Financial Systems

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The cover shows a visualisation of the international trade network for the year 2011. The different colours represent densely connected clusters of countries detected by a community detection algorithm. The figure was created with VOSviewer (<http://www.vosviewer.com/>).

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To Liam, Daan and Tomer

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Introduction

Following the 2008 crisis, there has been a soaring interest in using ideas from different disciplines to make sense of economic and financial markets. The near-collapse of the financial system could not be explained, even more so predicted, by the traditional economic models. Ignoring properties like the complex network of interactions and non-linear relations in the system, these economic models are based on very restrictive assumptions. Confronting this gap, in recent years an alternative view has been developed using network theory and tools from statistical physics. A notable example of this shift of perspective is the move from traditional measures of “risk” of individual financial entities to new measures of “systemic risk,” defined as the risk of collapse of an entire system. Nevertheless, the pursuit of physicists to characterize financial and economic systems with empirical laws and “simple” global models started already two decades before, giving rise to the controversial field of ‘Econophysics.’ The interaction between economists and physicists, although immersed in frictions and resistance, led to some fruitful results and attracted major interest by central banks, regulators, and policy makers. Nonetheless, despite the various “real world” applications and implications, this field has to problems of a great social relevance, it requires the toolkit of theoretical physics and, in particular, statistical physics. This stimulating scientific context is partially reflected in the environment wherein the research described in this Ph.D. has been conducted. This study combined theoretical work and data analysis at the Lorentz Institute for Theoretical Physics, alongside important interactions with practitioners in finance (Duyfken Trading Knowledge) and bank supervisors (The Dutch National Bank).

Employing concepts from physics or mathematics in the field of economics is by no means a new phenomenon. In fact, Leiden University provides some remarkable examples of scientists with a background in physics, who left a significant impact on the field of economics. The most famous one is Jan Tinbergen, the first recipient of the Nobel Memorial Prize in Economics in 1969, who obtained his Ph.D. in physics at Leiden University under the supervision of Paul Ehrenfest in 1929. The title of his thesis was "Minimum Problems in Physics and Economics". In the early sixties, Tinbergen proposed the so-called Gravity Model of International Trade, presumably inspired by his physics training. The model

predicts the bilateral trade flows between two countries by a formula similar to Newton's law of gravitation, and is still being used by economists. As we explain later, part of this thesis focuses on extensions of the Gravity Model. Another great example is Tjalling Koopmans, which was a student of Jan Tinbergen in 1933. In 1936, Koopmans graduated with a Ph.D. from the faculty of mathematical and physical sciences at Leiden University, with a thesis entitled "Linear regression analysis of economic time series". Time series analysis is another core topic that we address in this thesis. Later in 1975, Koopmans was awarded the Nobel Memorial Prize in Economics (jointly with Leonid Kantorovich) for his contributions to the field of resource allocation, specifically the theory of the optimal use of resources. Looking back at these great scientists from a modern standpoint, they highlight the advantages of interdisciplinary research in tackling major challenges.

Coming back to the present, the research of complexity in economics has been steadily growing, gradually encompassing different scales: from 'microscopic' networks of financial assets, through 'mesoscopic' networks of firms, banks, and institutions, to 'macroscopic' networks of countries and economic sectors. In general, the dynamics of these complex financial systems is highly random and noisy. Nevertheless, they carry critical information. The 'universal' challenge, across the different scales, is the extraction of meaningful information regarding the state of the system from the observable (empirical) data. This problem is immensely complicated by the temporal heterogeneity, i.e. different dynamics in different time periods, and the structural heterogeneity, i.e. complex topology, in the systems. Most current models are much more homogeneous and cannot account for, or explain these complex properties. This takes us to the main research question of the thesis, where we want to introduce a new class of statistical models which enforce partial empirical information that accurately controls for the heterogeneity in the system. A very promising approach to this problem is the use of maximum-entropy ensembles. Maximum-entropy models can be used in different settings, and typically serve as a reference to identify non-random patterns or properties. The power of the maximum-entropy approach is that it applies to very different fields and systems, from neuroscience to social network analysis. In this work, we review various maximum-entropy models and their powerful applications to financial systems. As a by-product, we also apply our framework to brain data. This was possible due to a collaboration with Leiden University Medical Center (LUMC).

The thesis is divided into three independent chapters, where each chapter covers results from multiple scientific publications addressing a particular system or problem. For a better comprehensibility, in each chapter we start by introducing the theoretical models and framework, and later proceed to the different applications of our models to real-world systems. In Chapter 1 we aim at characterizing and quantifying the information encoded within the so-called binary projections (i.e. the signs of the increments) of financial time series. We introduce maximum-

entropy ensembles of binary matrices that represent projections of single and multiple binary time series, subject to a set of desired constraints defined as simple empirical observables. Our approach leads to a family of analytically solved null models that allow us to quantify the amount of information encoded in the chosen constraints, i.e. the selected observables of the binary projections of real-time series. Lastly, we show that our approach is able to reproduce and mathematically characterize certain empirical non-linear relationships between binary and non-binary properties of real time series.

In Chapter 2 we focus on economic networks, in particular, the International Trade Network (ITN). The network describes the exchange of capital, goods, and services between countries, and plays a significant role in the propagation of shocks. Modelling this complex system has been tackled by different disciplines, starting in the early sixties with the aforementioned Gravity Model. However, the empirical topology of the ITN is much more heterogeneous than the one predicted by the Gravity Model. The complete characterization of the ITN via a simple, yet accurate, model is still an open problem. We propose two different GDP-driven models which reconcile the different approaches of macroeconomics and network theory. Specifically, one model is a maximum-entropy generalization of the popular Gravity Model that embeds the latter in a realistic network topology. Thus, it represents significant improvement with respect to current models.

In Chapter 3 we discuss the identification of functional structure from correlation matrices measured from empirical time series driven by a common non-stationary trend. We discuss a recent community detection method and generalize it using a complete maximum-entropy framework that builds on the results from the previous chapter 1. In this setting, we introduce a null model serving as a random benchmark for the identification of non-random patterns in the correlation matrix. We apply the method to various real-world financial markets, examining the emergent functional structure generated by financial time series and their binary projections. We show that the simple binary representation can replicate to a large degree the complex structure which is induced by the full weighted time series. Next, we show that our method also has a great potential in a biological setting, specifically in an empirical detection of functional brain organization. In collaboration with LUMC, we apply the method to the biological clock of mice (suprachiasmatic nucleus). This is a very small brain region that can be represented as a complex network of oscillating neurons. While other methods failed, our method consistently revealed a core-periphery structure associated with two populations of neurons, a result that has been cross-checked with independent analysis.

Finally, we end this thesis with some concluding remarks, reviewing the key findings presented in this work.
