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## **Methods to simulate fermions on quantum computers with hardware limitations**

Steu dtner, M.

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# **Methods to simulate fermions on quantum computers with hardware limitations**

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