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Hydrodynamics and the quantum butterfly effect in black holes and large N quantum field theories

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Hydrodynamics and the quantum butterfly effect in black holes and large N quantum field theories

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Cover: the front cover represents Poseidon, the Greek god who rules the sea and the water, as represented on a Greek vase (ca. 480 - 460 BCE) by the Aegisthus Painter. The vase is now part of the collection of the Yale University Art Gallery, New Haven, CT, USA. In front of Poseidon, a representation of a black hole, the most chaotic object in nature, is displayed, surrounded by Hawking radiation. The relation between quantum chaos and hydrodynamics is the main focus of this thesis. On the back cover, one sees a photo of a wood-inlay work by Giovan Francesco Capoferri, dated 1524, in the Basilica di Santa Maria Maggiore in Bergamo, Italy, with the title: *Magnum chaos*. The black hole on the front of the cover is a modification of this work.

The original pictures of the wood-inlay and the vase are in the public domain and can be respectively found at: https://upload.wikimedia.org/wikipedia/commons/7/79/Lotto_Capoferri_Magnum_Chaos.jpg
and

<https://artgallery.yale.edu/collections/objects/25657>.

To my family

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