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Higgs dynamics in the early universe

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Chapter 1

Introduction

Cosmology is the study of the universe as a whole. It aims to understand the evolution from a state of very large energy density, 13.8 billion years ago, to the universe that we live in today, that is filled with radiation, dust and clusters of galaxies and the mysterious dark matter and dark energy. Cosmological structures and observations involve very large length scales. Particle physics, on the contrary, is about the very small (subatomic) scales. The Standard Model (SM) of particle physics describes the elementary particles and their interactions. This thesis describes research at the interface of particle physics and cosmology.

Even though the length scales of cosmological observations today are very large, the data contain traces of particle physics processes in the early universe. These processes happened at energy scales much larger than the scales accessible at particle accelerators on earth. Observations of the early universe are thus a unique probe of particle physics. Properties of elementary particles guide us in unravelling the history of the universe whilst cosmological observations also constrain models of Beyond the Standard Model (BSM) physics. We list some examples of this interplay.

- The particles that dominate the energy density of the universe determine its rate of expansion. Very early in the history of the universe, the energy density was dominated by relativistic particles. This epoch is referred to as radiation-dominated. At a later time, the main constituent became massive, non-relativistic particles. In the matter-dominated epoch the rate of expansion of the universe is different from the expansion rate in the radiation-dominated epoch.
- Measurements of e.g. rotation curves of galaxies [7–9], gravitational lensing [10] and oscillations in the power spectrum of the Cosmic Microwave Background [11] indicate that there is a large amount of ‘dark matter’ (DM) in the universe. This massive contribution to the energy budget

of the universe does not interact through the electromagnetic interaction and is therefore not visible. The observations above can be explained by DM with a particle nature, which is an indication of BSM physics. There are also searches for alternative descriptions of gravity [12, 13], but these alternatives have so far not managed to explain all observational indications for DM.

- The value of the neutrino masses affects the evolution of cosmological perturbations and structure formation. The sum of neutrino masses can be constrained by observations of the Cosmic Microwave Background and Large Scale Structure [14].

There are many more examples of the interplay between particle physics and cosmology but in this thesis we will focus on two: reheating after inflation and electroweak baryogenesis. Before diving into these two topics, we will give a brief introduction into the Standard Model of particle physics and cosmology.

1.1 The Standard Model of particle physics

It would be an impossible task to introduce the Standard Model of particle physics in its full glory in a short introductory section. We will therefore only focus on the aspects of the SM that are relevant for this work. More extensive introductions than the one presented here, can be found in Refs. [15–17].

The formulation of the SM was a formidable task that took several decades. A very important first step was the theory of Quantum Electrodynamics. This theory was formulated and proved to be renormalizable in the '40s, with major contributions from Bethe, Tomonaga, Schwinger and Feynman [18–24]. The full electroweak sector was written down by Glashow, Weinberg and Salam [25–27] and shown to be renormalizable by 't Hooft and Veltman [28].

Of all the particles of the SM, the Higgs particle plays the main role in this thesis. Since a Higgs field on its own does not lead to very interesting particle physics, we will first present the full particle content of the SM in section 1.1.1. We will then introduce the electroweak Lagrangian and its gauge invariance in section 1.1.2. In section 1.1.3 we show how the SM particles obtain their masses through spontaneous symmetry breaking.

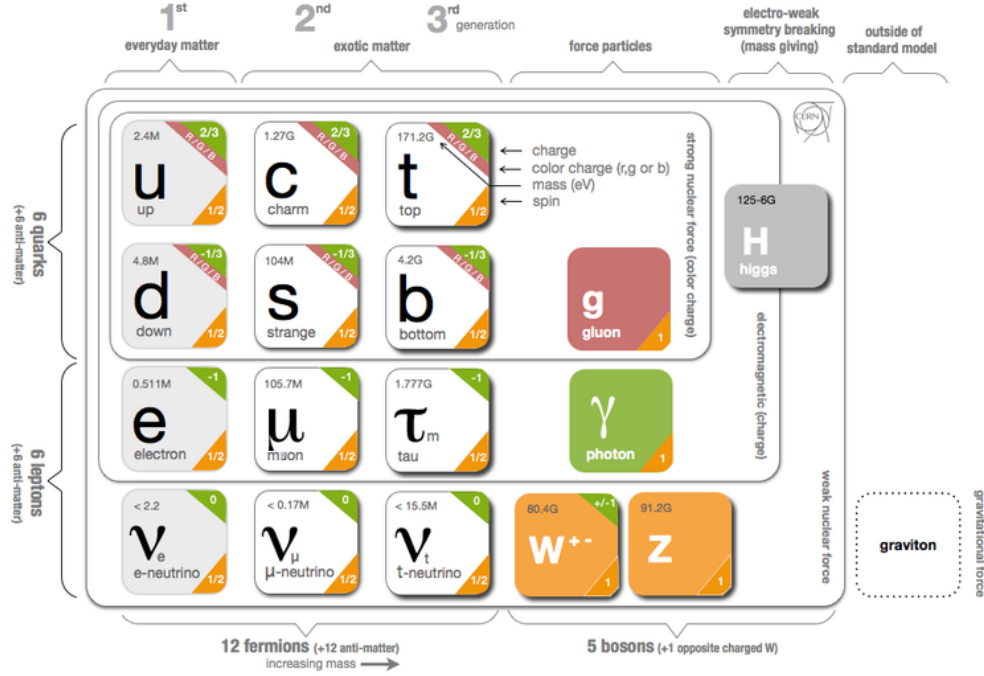


FIGURE 1.1: Particle content of the Standard Model (credit: CERN). Fermions are shown on the left, with quarks in the top two rows and leptons in the bottom two rows. On the right-hand side the bosons are shown.

1.1.1 Particle content

Figure 1.1 shows the particle content of the SM. The matter sector, consisting of fermions with spin $1/2$, is shown on the left. There are three generations of quarks and three generations of leptons. Quarks participate in all three fundamental interactions described by the SM: the strong interaction, the weak interaction and the electromagnetic interaction. Leptons are not charged under the strong interaction, and therefore only interact via weak and electromagnetic interactions. The neutrinos have zero electromagnetic charge and consequently only interact through the weak interaction.

In the SM Lagrangian, fermions are represented by four-component spinors. An important property of these spinors is ‘chirality’ or ‘handedness’. A right-handed spinor ψ_R is an eigenvector of the chirality matrix γ^5 with eigenvalue $+1$. A left-handed spinor ψ_L has eigenvalue -1 ,

$$\gamma^5 \psi_R = +\psi_R, \quad \gamma^5 \psi_L = -\psi_L. \quad (1.1)$$

We will see below that the properties of right- and left-handed particles are not identical.

The gluon, photon and W- and Z-bosons are spin 1 particles. They are the gauge bosons of the strong, electromagnetic and weak interaction respectively. The gauge bosons are represented by

vector fields in the SM Lagrangian. The gluon and photon are massless, but the mediators of the weak interaction are massive.

The particle on the very right of figure 1.1 is the Higgs boson. It has spin 0 and is therefore represented by a scalar field in the SM Lagrangian.

1.1.2 Gauge symmetry

The SM is a gauge theory with gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ (see Ref. [29] for an introduction into gauge theories in particle physics). $U(1)$ is an Abelian gauge group, but $SU(3)$ and $SU(2)$ are non-Abelian, which means that the elements of the group do not commute.

The spin 1 gauge fields correspond to the generators of the different gauge groups. The group $SU(3)_C$ is associated to the strong force. It has 8 generators, corresponding to 8 gluons. The weak and electromagnetic interaction are unified in the $SU(2)_L \times U(1)_Y$ electroweak (EW) interaction. We denote the three gauge bosons associated to $SU(2)_L$ by A_μ^i and the gauge boson corresponding to $U(1)_Y$ (hypercharge) by B_μ . We will see below that the photon and W- and Z-bosons are linear combinations of the fields A_μ^i and B_μ . The subscript L in $SU(2)_L$ indicates that the $SU(2)_L$ -part only interacts with left-handed fermions. The left-handed fermions form $SU(2)_L$ -doublets

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad (1.2)$$

where Q_L and L_L denote quark and lepton doublets respectively. u_L (d_L) is a left-handed up- (down-)type quark, and ν_L (e_L) is a left-handed neutrino- (electron-)type lepton. The corresponding right-handed fields

$$u_R, \quad d_R, \quad e_R, \quad (1.3)$$

are singlets under $SU(2)_L$. The SM does not include right-handed neutrino fields.

To see how the gauge symmetry works, we will write down the Lagrangian for the electroweak sector. For simplicity, we only consider a single generation of quarks and leptons.

$$\begin{aligned} \mathcal{L}_{EW} = & -\bar{Q}_L i \not{D} Q_L - \bar{u}_R i \not{D} u_R - \bar{d}_R i \not{D} d_R - \bar{L}_L i \not{D} L_L - \bar{e}_R i \not{D} e_R - (D_\mu \Phi)^\dagger D^\mu \Phi \\ & - \frac{1}{4} (A_{\mu\nu}^i A^{i\mu\nu} + B_{\mu\nu} B^{\mu\nu}) \\ & - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 - \left(\bar{Q}_L Y_u \tilde{\Phi} u_R + \bar{Q}_L Y_d \Phi d_R + \bar{L}_L Y_e \Phi e_R + \text{h.c.} \right). \end{aligned} \quad (1.4)$$

The first line is the kinetic term for the fermions and the Higgs field Φ . As usual, the ‘barred’ spinor $\bar{\psi}$ is defined as $\bar{\psi} = \psi^\dagger \gamma^0$. The derivatives D_μ are covariant derivatives instead of ordinary partial derivatives to make the Lagrangian gauge invariant. We have used the notation $\not{D} = \gamma^\mu D_\mu$, with

$$D_\mu = \partial_\mu - ig A_\mu^i \frac{\tau^i}{2} - ig' \frac{Y}{2} B_\mu, \quad (1.5)$$

where g and g' are the coupling constants of $SU(2)_L$ and $U(1)_Y$ respectively and the $\tau^i/2$ are the generators of $SU(2)$, with the τ^i the Pauli matrices. A summation over the repeated index i is implied. Y denotes the hypercharge of the field that the covariant derivative is acting on.

The second line of the Lagrangian (1.4) contains the kinetic terms for the gauge fields. The third line describes the Higgs potential and the interactions between the Higgs and the fermions. $\tilde{\Phi}$ is defined as $\tilde{\Phi}^a = \epsilon^{ab} \Phi^{b*}$, where ϵ^{ab} is the antisymmetric tensor in two dimensions ($\epsilon^{12} = +1$).

Let’s see what happens to the terms in the Lagrangian (1.4) under a $SU(2)_L \times U(1)_Y$ gauge transformation. The transformations are described by 3 + 1 spacetime-dependent parameters $\{\vartheta^1(x), \vartheta^2(x), \vartheta^3(x), \xi(x)\}$. The unitary representations of the transformations are

$$U(\vec{\vartheta}) = \exp \left[\frac{i\vartheta^i \tau^i}{2} \right], \quad U(\xi) = \exp \left[\frac{i\xi Y}{2} \right], \quad (1.6)$$

or, combined

$$U(\vec{\vartheta}, \xi) = \exp \left[\frac{i\vartheta^i \tau^i}{2} + \frac{i\xi Y}{2} \right]. \quad (1.7)$$

Under a $SU(2)_L \times U(1)_Y$ -transformation, the left-handed quark and lepton doublets transform as

$$\begin{aligned} Q_L &\rightarrow Q'_L = U(\vec{\vartheta}, \xi) Q_L = \exp \left[\frac{i\vartheta^i \tau^i}{2} + \frac{i\xi}{6} \right] Q_L, \\ L_L &\rightarrow L'_L = U(\vec{\vartheta}, \xi) L_L = \exp \left[\frac{i\vartheta^i \tau^i}{2} - \frac{i\xi}{2} \right] L_L, \end{aligned} \quad (1.8)$$

where we used that quark doublets have hypercharge $Y = 1/3$ and lepton doublets $Y = -1$. Since the right-handed fields are singlets under $SU(2)_L$, they simply transform as

$$u_R \rightarrow u'_R = \exp \left[\frac{2}{3} i\xi \right] u_R, \quad d_R \rightarrow d'_R = \exp \left[-\frac{1}{3} i\xi \right] d_R, \quad e_R \rightarrow e'_R = \exp [-i\xi] e_R, \quad (1.9)$$

where we used that the hypercharges of u_R , d_R and e_R are $4/3$, $-2/3$ and -2 respectively. The Higgs field, which has hypercharge $+1$, transforms as

$$\Phi \rightarrow \Phi' = \exp \left[\frac{i\vartheta^i \cdot \tau^i}{2} + \frac{i\xi}{2} \right] \Phi. \quad (1.10)$$

Finally, we write down the transformation of the gauge fields themselves:

$$\begin{aligned} A_\mu^i \frac{\tau^i}{2} &\rightarrow A_\mu'^i \frac{\tau^i}{2} = U(\vec{\vartheta}) \left[A_\mu^i \frac{\tau^i}{2} - \frac{i}{g} \partial_\mu \right] U^{-1}(\vec{\vartheta}), \\ B_\mu \frac{Y}{2} &\rightarrow B_\mu' \frac{Y}{2} = U(\xi) \left[B_\mu \frac{Y}{2} - \frac{i}{g'} \partial_\mu \right] U^{-1}(\xi). \end{aligned} \quad (1.11)$$

Plugging in the transformation of the gauge fields into the covariant derivatives, we find that $D_\mu \rightarrow D'_\mu = U(\vartheta, \xi) D_\mu U^{-1}(\vartheta, \xi)$ and this implies that the kinetic terms are indeed invariant under gauge transformations.

Gauge invariance in the second line of the Lagrangian (1.4) is obtained for the following field strength tensors

$$\begin{aligned} A_{\mu\nu}^i &= \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + g\epsilon^{ijk} A_\mu^j A_\nu^k, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \quad (1.12)$$

with ϵ^{ijk} the structure constant of $SU(2)$. Since the $B_{\mu\nu} B^{\mu\nu}$ contains only terms that are quadratic in B_μ , this field does not have self-interactions. The last term in $A_{\mu\nu}^i$ *does* allow for self-interactions, which is a typical property of non-Abelian gauge groups.

1.1.3 Spontaneous symmetry breaking

It is known from experiments that the W- and Z-bosons, which mediate the weak interaction, are massive. This raises a problem for the theory as it was just described. For concreteness, consider the Abelian hypercharge field (the same problem arises for the non-Abelian gauge fields). The gauge transformation of eq. (1.11) can be written as:

$$B_\mu \rightarrow B'_\mu - \frac{1}{g'} \partial_\mu \xi. \quad (1.13)$$

If we added a mass term

$$\mathcal{L}_m = -\frac{1}{2} m^2 B_\mu B^\mu, \quad (1.14)$$

this would clearly break gauge invariance.

The fermions have a related problem: the left-handed particles are part of $SU(2)_L$ -doublets, but the right-handed particles are singlets under $SU(2)_L$. A fermion mass term would mix the left- and right-handed components, and would therefore violate $SU(2)_L$ -gauge invariance.

A consistent solution of these problems is provided by spontaneous symmetry breaking using the Higgs mechanism¹ which generates masses for the Z- and W-bosons and fermions. This is achieved by introducing a doublet of scalar fields parameterized as

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \theta^1 + i\theta^2 \\ \phi + i\theta \end{pmatrix}. \quad (1.15)$$

It will be useful to split the component ϕ into a classical background value φ and an excitation h (the Higgs boson), $\phi(x, t) = \varphi + h(x, t)$ ².

The Higgs field couples to the electroweak gauge bosons through its kinetic term and to the fermions through the Yukawa interactions. The potential can be read off from eq. (1.4)

$$V(\Phi, \Phi^\dagger) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad (1.16)$$

where λ is assumed to be positive, such that the potential is bounded from below³ and μ^2 is negative. The shape of this potential is often described as a Mexican hat. The potential is minimized for

$$\Phi^\dagger \Phi = \frac{v_0^2}{2}, \quad \text{with } v_0 = \sqrt{-\frac{\mu^2}{\lambda}}, \quad (1.17)$$

which defines the vacuum state. v_0 is referred to as the ‘vacuum expectation value’ (vev). Combining eq. (1.17) with the requirement that electromagnetic gauge symmetry is unbroken, we obtain the expectation value of the Higgs field:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_0 \end{pmatrix}, \quad (1.18)$$

¹The terms ‘Higgs field’ and ‘Higgs mechanism’ do not do justice to all scientists that introduced these concepts. The often-used names ‘Higgs field’, ‘Higgs boson’ and ‘Higgs mechanism’ refer to Peter Higgs, who introduced this mechanism in Ref. [30]. He also pointed out the existence of a neutral spin 0 boson, that would eventually be detected at the LHC [31, 32]. The same mechanism that gives mass to the fermions and gauge bosons was introduced by Robert Brout and François Englert in Ref. [33], who however did not mention the existence of the corresponding boson. It could thus be argued that the name ‘Higgs boson’ is appropriate, but that the corresponding field and mechanism should be called ‘BEH-field’ and ‘BEH-mechanism’. In order to avoid confusion, we will stick to the names that are most commonly used in literature, and thus refer to the ‘Higgs field’ and ‘Higgs mechanism’.

²In cosmology, $\varphi(t)$ is time-dependent.

³The measured values of the Higgs and top quark mass indicate that λ could run negative at large energy scales $\gtrsim 10^{11}$ GeV. We study this possibility in chapter 3. If this is the case, the potential should be bounded from below by higher-dimensional operators.

where v_0 is the value of φ . The vev spontaneously breaks the $SU(2)_L \times U(1)_Y$ -symmetry into a residual $U(1)_Q$ -symmetry. The spontaneous symmetry breaking generates mass terms for the gauge bosons. To see this, we write Φ in unitary gauge:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_0 + h \end{pmatrix}, \quad (1.19)$$

and plug this into the Higgs Lagrangian $\mathcal{L}_{\text{Higgs}} = -(D_\mu \Phi)^\dagger D^\mu \Phi - V(\Phi, \Phi^\dagger)$:

$$\mathcal{L}_{\text{Higgs}} = -\frac{1}{2} \partial_\mu h \partial^\mu h - \frac{g^2}{4} (v_0 + h)^2 W_\mu^\dagger W^{\mu\dagger} - \frac{g^2}{8 \cos^2 \theta_W} (v_0 + h)^2 Z_\mu Z^\mu - \frac{\lambda}{4} (h^2 + 2v_0 h)^2. \quad (1.20)$$

We can read off the Higgs boson mass

$$m_h = \sqrt{-2\mu^2}, \quad (1.21)$$

where we used eq. (1.17). The W- and Z-bosons are linear combinations of the A_μ^i and B^μ fields defined above

$$W_\mu = \frac{A_\mu^1 - iA_\mu^2}{\sqrt{2}}, \quad Z_\mu = \cos \theta_W A_\mu^3 - \sin \theta_W B_\mu, \quad (1.22)$$

where θ_W is the Weinberg angle [26]. The masses of the gauge bosons are

$$m_W = \frac{gv_0}{2}, \quad m_Z = \frac{gv_0}{2 \cos \theta_W}. \quad (1.23)$$

Spontaneous symmetry breaking has thus resulted in mass terms for the gauge bosons without breaking gauge symmetry at the level of the Lagrangian. The photon, which is the combination of A_μ^3 and B_μ orthogonal to Z_μ , does not couple to the Higgs boson. It remains massless and mediates the electromagnetic force; it is associated with the residual $U(1)_Q$ -symmetry, with Q standing for electric charge. Q is related to Y, hypercharge, and I_3 , weak isospin, by

$$Q = I_3 + \frac{Y}{2}. \quad (1.24)$$

The value of I_3 is zero for $SU(2)_L$ -singlets and $+1/2$ ($-1/2$) for the upper (lower) components of $SU(2)_L$ doublets.

The Higgs field also gives masses to the fermions, for which gauge symmetry also forbids an explicit mass term in the Lagrangian. The Higgs field couples to fermions via so-called ‘Yukawa’ interactions, as written in the last line of eq. (1.4). After spontaneous symmetry breaking the mass of a fermion

f is

$$m_f = \frac{y_f v_0}{\sqrt{2}}. \quad (1.25)$$

Since the neutrinos do not have Yukawa interactions, they are massless in the SM. However, the observation of neutrino oscillations indicates that they must be massive. This mass is an indication of BSM physics.

In the upcoming chapters we will often encounter the situation where the Higgs field does not sit at its zero-temperature vev. Instead, its background value $\varphi(t)$, will be dynamical. In the adiabatic limit, the masses of the fermions and gauge bosons are obtained by replacing v_0 with φ in eqs. (1.23) and (1.25).

1.2 Cosmology

We will now give a very brief overview of what is currently known about the evolution of the universe. The estimated age of the universe is 13.8×10^9 years [34], but we will mostly focus on the first 380,000 years. The processes that are described in this thesis, reheating and baryogenesis, both take place before the universe was even a second old. We will nevertheless describe the further evolution of the universe, because observational constraints on the very early universe can be derived from processes that happened later.

The formation of light elements during Big Bang Nucleosynthesis (BBN) is the first process that can be probed by observations. Strictly speaking, any description of the universe before BBN is speculation. However, data from the Cosmic Microwave Background (CMB) and BBN and properties of elementary particles strongly restrict the history of the universe. In section 1.2.2 we will introduce the widely accepted Hot Big Bang (HBB) model, which gives accurate predictions for BBN and CMB observables. The inflationary phase, which is the topic of section 1.2.4, was introduced to solve some shortcomings of the HBB model that are explained in section 1.2.3. We will start with an important pillar of cosmology: the cosmological principle. Extensive introductions into the phenomenology of the early universe can be found for example in Refs. [35–38].

1.2.1 Cosmological principle

Modern cosmology is built on the so-called cosmological principle, which states that, on large scales, the universe is homogeneous and isotropic. The cosmological principle is not just some philosophical

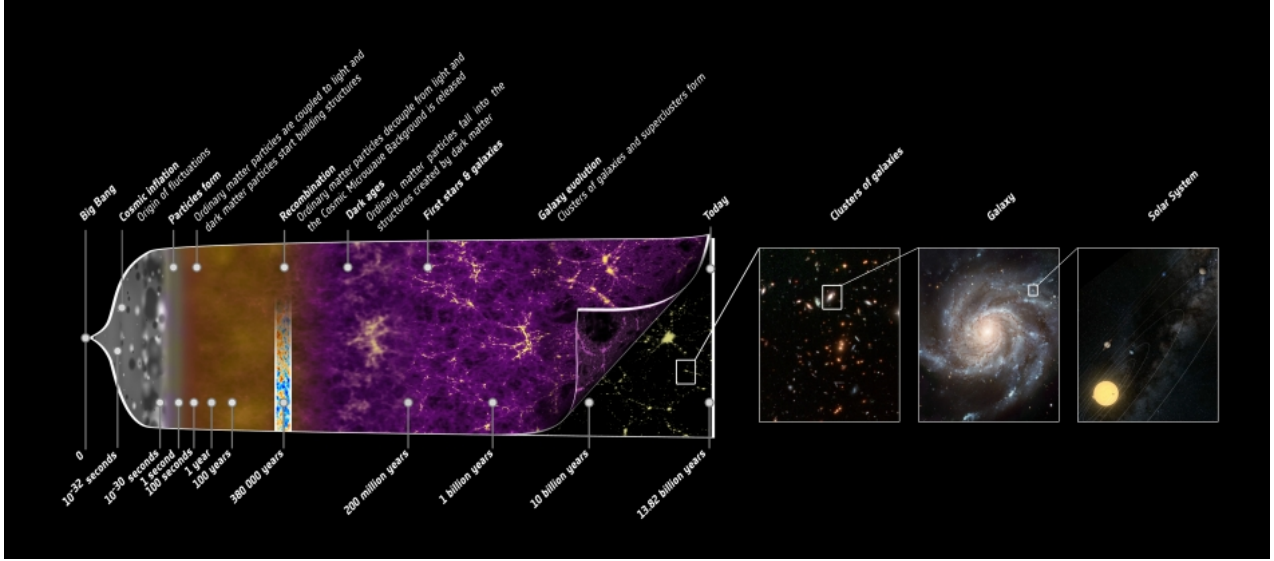


FIGURE 1.2: Sketch of the evolution of the universe (credit: NASA).

idea, but it is also confirmed by observations [39–41]. A homogeneous and isotropic universe is described by the Friedman-Lemaître-Robertson-Walker [42–45] solution of the Einstein equations [46], with line element in spherical coordinates:

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \quad (1.26)$$

where t is cosmic time, r, θ, ϕ spherical polar coordinates and $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$. k is the curvature parameter, it has dimensions of length^{-2} . A positive k corresponds to a closed universe, $k = 0$ to a spatially flat universe and negative k to an open universe. $a(t)$ is the time-dependent scale factor. The metric can also be written in conformal form

$$ds^2 = a^2(\tau) \left[-d\tau^2 + \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \quad (1.27)$$

which defines conformal time as $d\tau = dt/a(t)$. The coordinates r, θ and ϕ are called comoving coordinates: particles that are at rest in these coordinates, will remain at rest. If we take two comoving particles, with comoving distance d_{com} , the *physical* distance d_{phys} between these two particles is given by

$$d_{\text{phys}}(t) = a(t) d_{\text{com}}. \quad (1.28)$$

Since d_{com} remains constant, the physical distance between the two particles increases if the universe is expanding ($\dot{a}(t) > 0$):

$$\dot{d}_{\text{phys}}(t) = \frac{\dot{a}(t)}{a(t)} d_{\text{phys}}(t) \equiv H(t) d_{\text{phys}}(t). \quad (1.29)$$

This relation is known as the Hubble-Lemaître law [43, 47] and $H(t)$ as the Hubble parameter. The value of the Hubble parameter today is denoted as H_0 . If we let d_{phys} denote the physical distance between two galaxies instead of two particles, we see that galaxies move away from each other with increasing velocity, which has also been confirmed by observations [47].

The evolution of the scale factor $a(t)$ depends on the energy content of the universe. We model the matter and energy in the universe as a perfect fluid with energy density ρ and pressure P . It will be convenient to define the equation of state parameter w

$$w \equiv \frac{P}{\rho}. \quad (1.30)$$

The equations of state for matter, radiation and vacuum energy (cosmological constant) are shown in table 1.1. Plugging the energy-momentum tensor of the perfect fluid into the Einstein equation, we obtain the two Friedmann equations

$$H^2 = \frac{\rho}{3m_{\text{pl}}^2} - \frac{k}{a^2}, \quad (1.31)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{2m_{\text{pl}}^2} \left(\frac{\rho}{3} + P \right). \quad (1.32)$$

From energy-momentum conservation, $\nabla_\nu T^{\mu\nu}$, we obtain a continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (1.33)$$

which can be used instead of the second Friedmann equation. The continuity equation gives us a relation between the energy density and the scale factor

$$\rho \propto a^{-3(1+w)}, \quad (1.34)$$

which depends on the equation of state w . In reality, the universe consists of components with different equations of state and the total energy density is the sum of these components. The dominant contribution to the energy density then determines the dependence on the scale factor. Solving eq. (1.31) with $k = 0$ ⁴, for the dominant constituent gives the time-dependence of the scale factor

$$a(t) \propto \begin{cases} t^{2/3(1+w)} & w \neq -1, \\ \exp Ht & w = -1. \end{cases} \quad (1.35)$$

⁴Observations show that the energy density associated to curvature is small: $\Omega_k \equiv -\frac{k}{H^2 a^2} = 0.001 \pm 0.002$ [34]. Setting $k = 0$ is thus a good approximation.

	w	$a(t)$	$\rho(a)$
Matter	0	$t^{2/3}$	a^{-3}
Radiation	$\frac{1}{3}$	$t^{1/2}$	a^{-4}
Vacuum energy	-1	$\exp Ht$	a^0

TABLE 1.1: Equation of state, time-dependence of the scale factor and scale-factor-dependence of the energy density for matter, radiation and vacuum energy.

Table 1.1 shows the dependence of the energy density on the scale factor and the time-dependence of the scale factor for matter, radiation and vacuum energy.

The very early universe has a temperature that is much larger than the masses of the SM particles⁵. As a consequence, all particles are relativistic and radiation is thus the main contribution to the energy density. The scale factor evolves as $a \propto t^{1/2}$. As table 1.1 shows, the energy density in radiation decreases faster with the expansion of the universe than the energy density in matter. When the universe is approximately 60,000 years old, at a temperature of $T = 0.75$ eV, the energy density in matter becomes equal to the energy density in radiation. The matter-dominated expansion $a \propto t^{2/3}$ persists until the energy density in matter has diluted to ρ_Λ , the energy density of dark energy. This happens when the universe is about 9×10^9 years old, at $T = 0.33$ meV. Since then, the universe has been in a phase of accelerated expansion.

1.2.2 Standard Hot Big Bang Cosmology

The original Hot Big Bang (HBB) model [48, 49] describes the evolution of a universe that started in a very hot and dense state. Even though the model was later complemented with a period of inflation that preceded Hot Big Bang evolution, this should not affect the predictions of HBB cosmology. We will thus start our description of the evolution of the universe in the HBB model and find out why the inflationary phase was introduced along the way.

We will focus on processes involving SM particles, but we stress that HBB evolution does not apply to SM physics only. There are strong indications (e.g. DM and neutrino oscillations) that the SM is not complete. Any BSM degrees of freedom can be incorporated into the HBB model, as long as the BSM physics does not spoil the successful predictions of HBB. The BBN and CMB data are thus useful tools to constrain BSM physics.

⁵Since we have set $k_B = c = 1$, we can express temperature and mass in units of energy. A temperature of 1 eV corresponds to $T = 1.16 \times 10^4$ K.

1.2.2.1 Plasma of relativistic particles

The HBB evolution starts at some high temperature T at which the Higgs potential is dominated by finite-temperature contributions, such that it only has a minimum at $\varphi = 0$ (more about this in section 5.2). All SM particles are thus massless and relativistic^{6,7}. The universe is radiation-dominated, with energy density

$$\rho = \frac{\pi^2}{30} g^*(T) T^4, \quad (1.36)$$

where g^* is the total number of relativistic degrees of freedom. The SM particles can exchange momentum and energy efficiently, since interaction rates Γ are large with respect to the Hubble parameter H . If $\Gamma \gg H$ holds for a certain reaction

$$A + B \leftrightarrow C + D, \quad (1.37)$$

the particles are said to be in chemical equilibrium, which sets a relation between the chemical potentials

$$\mu_A + \mu_B = \mu_C + \mu_D. \quad (1.38)$$

In the early universe, particles that have a large number density typically have very small chemical potentials that can be neglected [37, 38]. A relativistic particle i that is in chemical equilibrium has the following number density

$$n_i = \frac{\zeta(3)}{\pi^2} g_i T^3 \begin{cases} 1 & \text{bosons,} \\ \frac{3}{4} & \text{fermions,} \end{cases} \quad (1.39)$$

where g_i is the number of internal degrees of freedom of species i .

1.2.2.2 Electroweak phase transition and annihilation

Combining eq. (1.36) with the scale-factor dependence of the energy density in a radiation-dominated universe, we see that the temperature T decreases as $T \propto a(t)^{-1}$. When the temperature reaches $T \approx 100 \text{ GeV}$ the finite-temperature contributions to the Higgs potential become small enough such that the potential obtains the Mexican-hat shape described in section 1.1.3. The vev transitions

⁶The SM particles would also be relativistic if the Higgs field would have a nonzero vev, as long as $T > m$.

⁷Technically, the temperature at which HBB evolution starts is only constrained by Big Bang Nucleosynthesis (see section 1.2.2.3) to be above a few MeV. For this relatively low temperature, the Higgs is already in its broken phase and the SM particles are massive. However, most inflationary models end in a thermal state with a temperature many orders of magnitude above $\mathcal{O}(1 \text{ MeV})$. In our description we will assume that this is the case.

from $\varphi = 0$ to $\varphi \neq 0$ and the SM fermions and W- and Z-bosons become massive. This is referred to as the Electroweak Phase Transition (EWPT).

Once the particles become massive, the top quark becomes non-relativistic, since $T \lesssim m_t$. Its number density then gets Boltzmann-suppressed due to annihilations of top quarks and anti-top quarks. The equilibrium number density of a nonrelativistic particle i is given by

$$n_i = g \left(\frac{m_i T}{2\pi} \right)^{3/2} \exp \left[\frac{\mu_i - m_i}{T} \right]. \quad (1.40)$$

Soon after top-antitop annihilation, the Higgs boson and the W- and Z-bosons become non-relativistic and annihilate as well, followed by the bottom and charm quarks and the tau lepton. The temperature lowers further to $T \approx 150$ MeV, the temperature of the QCD phase transition [50]. Below this scale, quarks and gluons are confined to baryons (consisting of three quarks) and mesons (consisting of a quark-antiquark pair). The baryons and mesons become immediately nonrelativistic during the phase transition and thus get Boltzmann-suppressed, except for the light pions. At a slightly lower temperature, the pions and muons annihilate as well. At a temperature of a few MeV all SM particles are still in chemical equilibrium and most of them are Boltzmann suppressed. If this equilibrium would persist forever, all massive particles would eventually annihilate and the universe would be filled with photons only. Fortunately, chemical equilibrium is disturbed when the weak interaction freezes out and light nuclei are formed in Big Bang Nucleosynthesis.

1.2.2.3 Big Bang Nucleosynthesis

Big Bang Nucleosynthesis [49, 51] describes the formation of light elements in the early universe. Recent reviews of BBN can be found in Refs. [52–55].

At a temperature of a few MeV neutrons n and protons p are non-relativistic and their interactions with electrons and neutrinos

$$p + e \leftrightarrow n + \nu_e, \quad (1.41)$$

(and the interactions that are related by crossing symmetry) are in equilibrium. As the chemical potentials of electrons and neutrinos are negligible at this stage, chemical equilibrium implies $\mu_n = \mu_p$ and the neutron-to-proton ratio is given by

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p} \right)^{3/2} \exp \left[-\frac{m_n - m_p}{T} \right]. \quad (1.42)$$

If equilibrium persisted forever, the number density of neutrons would drop to zero. This does not happen, since the weak interaction becomes too slow to compete with the expansion of the universe around $T \approx 1$ MeV (step 1 in figure 1.3). The neutron-to-proton ratio then remains constant up to neutron decay:

$$\frac{n_n}{n_p} = \frac{n_n}{n_p} \Big|_{\text{freeze out}} \exp[-t/\tau], \quad (1.43)$$

with $\tau = 886.7 \pm 0.8$ s the neutron lifetime and $\frac{n_n}{n_p} \Big|_{\text{freeze out}}$ the value of the neutron-to-proton ratio at the instant when the weak interactions freeze out. Figure 1.3 clearly shows that the neutron abundance (light blue line) decreases much more slowly than it would in the equilibrium situation (black line). But again, if nothing would change, all neutrons would decay into protons. Fortunately, the temperature has now dropped below the nuclear binding energy ($B_D = 2.2$ MeV) of deuteron D (an isotope of hydrogen consisting of a proton and a neutron), that is formed via the reaction

$$n + p \rightarrow D + \gamma. \quad (1.44)$$

Deuteron formation actually only becomes efficient at $T \lesssim 100$ keV, because the number density of photons is much larger than the number densities of protons and neutrons. Even though the *average* photon energy at $T \lesssim 2.2$ MeV is too small to disassociate a deuterium nucleus, there are enough photons in the energetic tail of the spectrum to block production of significant amounts of deuteron before $T \lesssim 100$ keV. As soon as deuteron becomes available, it combines with protons to form ^3He and subsequently ^3He and D form ^4He (formation of ^4He can also proceed through combination of tritium and deuteron). Since the nuclear binding energy of ^4He is larger than the nuclear binding energy of deuteron and tritium, the abundance of ^4He is energetically favored. As a consequence, virtually all available neutrons combine into ^4He , which then makes up $\sim 25\%$ of the baryonic mass. Hydrogen nuclei make up $\sim 75\%$ and there are still some small traces of other light elements (D, ^3He and ^7Li). BBN completes at a temperature of ~ 10 keV.

The predictions of light element abundances from BBN match observations very well [53]. This is one of the main triumphs of the HBB model. The success of BBN tells us that, at a temperature of a few MeV, the universe needs to be radiation-dominated with SM particles in chemical equilibrium. This constrains BSM physics. Furthermore, the value of the helium-to-hydrogen ratio depends on the neutron-to-photon ratio at neutron freeze-out. The latter depends on the temperature at freeze-out and thus on the number of relativistic degrees of freedom, which puts another constraint on new physics. See Ref. [56] for other examples of BSM constraints from BBN.

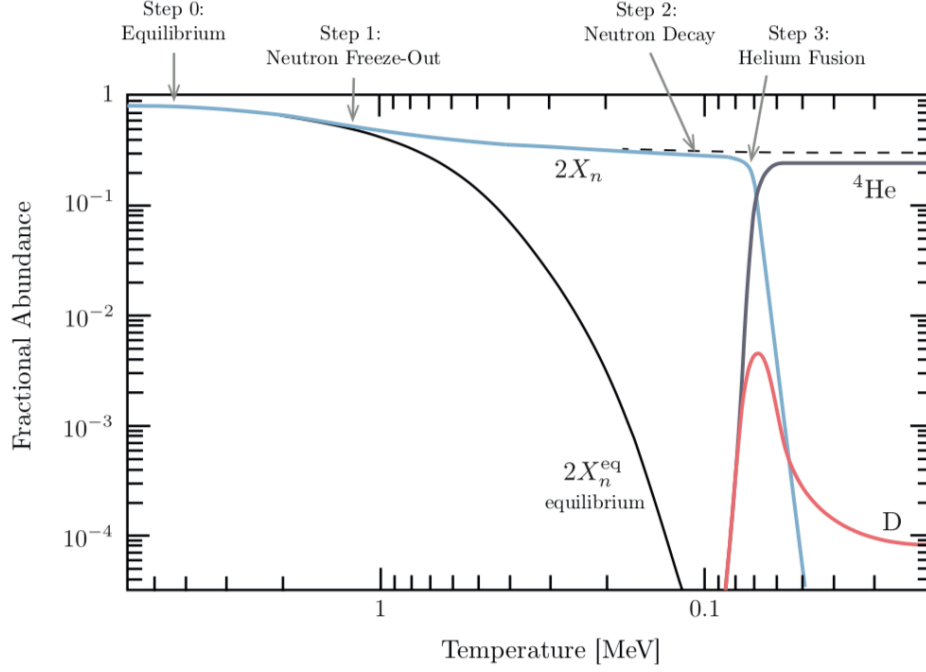


FIGURE 1.3: Numerical results for the evolution of neutron, deuteron and helium abundance. Figure copied from Ref. [38].

1.2.2.4 Recombination and photon decoupling

At a temperature of $T \approx 10$ keV, BBN has completed and the universe is filled with photons, relativistic electrons and positrons and baryons, mostly in the form of H and ${}^4\text{He}$ nuclei. When the temperature drops below the electron mass, electrons and positrons become non-relativistic and their density becomes Boltzmann-suppressed. Despite Boltzmann suppression, Thomson scattering between photons and electrons

$$e + \gamma \leftrightarrow e + \gamma, \quad (1.45)$$

is in equilibrium, such that the universe is still opaque. Neutral hydrogen and helium do not yet form, as the temperature is above the atomic binding energies. As the temperature drops, neutral helium forms first, as it has the larger atomic binding energy. The atomic binding energy of hydrogen is 13.6 eV, but as in the case of deuteron above, the formation of neutral hydrogen becomes efficient at a lower temperature $T \approx 0.3$ eV, because of the relatively large number density of photons. The formation of neutral atoms from free electrons and nuclei is called ‘recombination’.

During recombination, the number density of free electrons drops dramatically. At $T \approx 0.27$ eV the number density of free electrons has become so small that the interaction rate for Thomson scattering drops below the expansion rate H . When this interaction freezes out, photons are no longer coupled to the primordial plasma. This implies that the universe becomes transparent to

photons and they start to stream freely. These photons, that last interacted with the primordial plasma when the universe was about 380,000 years old, are observed today as the Cosmic Microwave Background (CMB) radiation.

The existence of background radiation as a consequence of HBB evolution was first predicted by Alpher and Herman in 1948 [57]. It was discovered by Penzias and Wilson in 1964 [58] and has become a very important probe of the early universe. Over the past decades, it has been measured with ever increasing accuracy [34, 59, 60]. The background radiation has a blackbody spectrum with a very uniform temperature $T_{\text{CMB}} = 0.23 \text{ meV}$ (or $T_{\text{CMB}} = 2.7 \text{ K}$). There are, however, small temperature fluctuations, that form the seeds of stars and galaxies that start to form later.

Even though the detection of the CMB corroborates the HBB model, the fact that it is so homogeneous also indicates that the HBB model is not complete, as we will see in section 1.2.3.

1.2.2.5 Intermezzo: asymmetry between baryons and antibaryons

In our description of BBN and recombination we tacitly assumed that our universe has a baryon asymmetry: it contains more baryons (matter) than antibaryons (antimatter). As a result, before the QCD phase transition, the number density of quarks is slightly larger than the number density of antiquarks. After the phase transition, the quarks and antiquarks combine into baryons and a slightly smaller amount of antibaryons. As the (anti)baryons become immediately non-relativistic, baryons and antibaryons start to annihilate. The slight excess of baryons prevents these annihilations to complete and some nonzero baryon number remains. These leftover baryons, which have a number density that is much smaller than the number density of photons, eventually end up in neutral hydrogen and helium.

BBN and CMB observations are two independent methods to determine the value of the baryon asymmetry [61]. The value of the baryon asymmetry determines T_{BBN} , the temperature at which the production of light elements starts. The final abundances depend on the initial neutron-to-proton ratio, which is determined by T_{BBN} . To determine the abundances of light elements for a given value of the baryon asymmetry one needs to solve a coupled set of Boltzmann equations. By comparing the predicted abundances to the observed values, the correct value of the baryon asymmetry can be determined. Alternatively, the value of the baryon asymmetry can be obtained from the angular power spectrum of the CMB [62].

The origin of the baryon asymmetry is unknown. Since the SM can not explain the value of the observed asymmetry, this is a strong hint for BSM physics. Part II of this thesis focuses on electroweak baryogenesis, a mechanism in which the baryon asymmetry is generated during the electroweak phase transition. For an introduction into the generation of the baryon asymmetry, with a focus on electroweak baryogenesis, see chapter 5.

1.2.2.6 Structure formation

The small temperature fluctuations observed in the CMB correspond to slightly overdense and underdense regions. Due to gravity, matter collapses onto the overdense regions, which then become even denser. The overdense regions form the seeds of stars and galaxies. The first stars formed when the universe was approximately 700 million years old [63]. An overview of galaxy formation can e.g. be found in Ref. [64].

1.2.3 Shortcomings of the HBB model

The predictions of the abundances of light elements were already a success of the Hot Big Bang model, but especially the confirmation of the existence of a CMB lead to the acceptance of the HBB model as the correct description of the history of the universe (during the ‘50s there was a fierce debate between supporters of the HBB model and supporters of the Steady State Theory [65, 66]).

The HBB seems a very good model for the description of the early universe, but it has two main issues: the horizon problem and the flatness problem.

1.2.3.1 Horizon problem

Let’s ask the following question: how far could a light ray, emitted at t_i , the time of the beginning of Hot Big Bang evolution, have travelled at the time of photon decoupling, t_{CMB} ? At t_{CMB} , this is the largest distance over which information could have been exchanged within the lifetime of the universe. This distance is called the ‘particle horizon’, R_p .

We take a light ray moving in the radial direction: $\theta = \phi = 0$. Light rays travel along null geodesics, so $dr = \pm \frac{1}{a(t)} dt$. The physical distance that the light ray could have travelled is thus

$$R_p = a(t_{\text{CMB}}) \int_{t_i}^{t_{\text{CMB}}} \frac{1}{a(t)} dt = a(t_{\text{CMB}}) \int_{\ln a_i}^{\ln a_{\text{CMB}}} \frac{d(\ln a)}{aH} \approx \frac{2}{1+3w} (H(t_{\text{CMB}}))^{-1}. \quad (1.46)$$

We assumed the same equation of state throughout (this is not completely correct, as the universe becomes matter-dominated before recombination, but this is a small effect and it does not change the main point) and in the last step neglected the contribution from the lower bound on the integral. We can do this if $1 + 3w > 0$, which holds for all matter sources that we have so far encountered in HBB evolution. The length scale $(aH)^{-1}$ is called the ‘comoving Hubble radius’ (or comoving Hubble horizon). For standard HBB cosmology it is approximately equal to the (comoving) particle radius, but this is not true in general.

The horizon problem is the following [67]: comparing any two points on the CMB map, the differences in the temperature are as small as $\frac{\delta T}{T} = 10^{-5}$, suggesting that all regions had been in causal contact at the time of recombination. However, using eq. (1.46) to compute the particle horizon at that time, one finds that patches larger than 1 squared degree on the CMB map were not causally connected at the time of recombination! How did the temperature of the universe become so homogeneous?

1.2.3.2 Flatness problem

The first Friedmann equation (1.31) contains a contribution from the spatial curvature k . So far, we did not pay much attention to this contribution, since observations show that it is small: $\Omega_k \equiv -\frac{k}{a^2 H^2} = 0.001 \pm 0.002$ [34].

For matter satisfying $1 + 3w > 0$, Ω_k *increases* as the universe expands. To get a small value of Ω_k today, the value of Ω_k in the early universe had to be extremely small. The value of Ω_k needs to be tuned by $\sim \mathcal{O}(10^{-60})$ (the exact amount of tuning depends on the initial temperature) [68]. The requirement of such a large amount of fine-tuning to explain the small value of the spatial curvature is called the flatness problem.

The horizon problem and the flatness problem both concern the initial conditions of the HBB evolution. In principle, the spatial curvature and fluctuations in the temperature could be tuned to match the values that are observed today. The enormous amount of fine-tuning would be a very unattractive feature of the HBB model. It turns out that a period of inflation prior to the HBB evolution can solve the flatness and horizon problems, without further modifications to the HBB model.

1.2.4 Inflation

Both the horizon and flatness problem could be solved if, before the standard HBB evolution starts, there is a phase in which the comoving Hubble horizon $(aH)^{-1}$ decreases. For such a phase, the *lower* bound in the integral of eq. (1.46) dominates and the particle horizon becomes much larger than the value from HBB. During this phase the value of Ω_k *decreases* such that the small value of Ω_k does no longer require fine-tuning. We call this phase *inflation* [69–71]. Readers that want to read more about inflation than this very brief introduction can for example consult Refs. [36, 38, 72, 73].

A phase of decreasing comoving Hubble horizon is equivalent to a phase of accelerated expansion

$$\frac{d}{dt}(aH)^{-1} < 0 \iff \ddot{a} > 0. \quad (1.47)$$

This also implies that the fractional change of H per e-fold N (an increase of the scale factor by a factor e) is small

$$\frac{d}{dt}(aH)^{-1} < 0 \iff \epsilon < 1, \quad \text{where } \epsilon \equiv -\frac{\dot{H}}{H^2} = -\frac{d \ln H}{dN}. \quad (1.48)$$

Assuming that the energy scale of the universe at the end of inflation is $\sim 10^{15}$ GeV, it can be shown that inflation solves the hierarchy and flatness problems if the scale factor grows by at least a factor $\sim 10^{28}$ between the beginning I and end E of inflation ⁸

$$\frac{a_I}{a_E} \gtrsim 10^{28} \rightarrow N = \ln \frac{a_I}{a_E} \gtrsim 64. \quad (1.49)$$

Note that this is only a lower bound on the duration of inflation.

We have seen above that matter with an equation of state satisfying $1 + 3w > 0$ will not lead to a decreasing comoving Hubble horizon. Instead we need some source of energy density with negative pressure

$$w = \frac{P}{\rho} < -\frac{1}{3}. \quad (1.50)$$

In eq. (1.35) and table 1.1 we have already seen an example of matter satisfying the condition $w < -1/3$: vacuum energy has an equation of state $w = -1$, leading to $a \propto \exp Ht$, a deSitter universe. The inflationary phase should however not be exactly deSitter, because inflation has to

⁸If inflation happens at a much lower scale, for example the electroweak scale, the required amount of e-folds of expansion is smaller.

end and allow for HBB evolution. Fortunately, there are other ways of to generate an inflationary phase. We will focus on the one that is most widely used.

1.2.4.1 Scalar field inflation

A phase of accelerated expansion occurs when the energy density is dominated by a homogeneous scalar field $\chi(t)$ moving in a flat potential $V(\chi)$. The energy density and pressure of the inflaton field χ are

$$\rho = \frac{1}{2}\dot{\chi}^2 + V(\chi), \quad P = \frac{1}{2}\dot{\chi}^2 - V(\chi), \quad (1.51)$$

and the equation of motion

$$\ddot{\chi} + 3H\dot{\chi} + V_{,\chi} = 0, \quad (1.52)$$

with the subscript $_{,\chi}$ denoting a derivative with respect to the χ -field. By plugging the energy density and pressure of the inflaton into the Friedmann equations, we find that $\dot{H} = -\dot{\chi}^2/(2m_{\text{pl}})^2$. Substituting into eq. (1.48) gives

$$\epsilon = \frac{\frac{1}{2}\dot{\chi}^2}{m_{\text{pl}}^2 H^2}. \quad (1.53)$$

Remembering that $m_{\text{pl}}^2 H^2 = \rho/3$, we find that the requirement $\epsilon < 1$ is satisfied if the kinetic energy is smaller than 1/3 of the full energy density, or

$$\dot{\chi}^2 < V(\chi), \quad (1.54)$$

and this indeed leads to an equation of state $w < -1/3$.

Inflation ends when ϵ becomes larger than 1. In many models, and also the ones that we study in chapters 2, 3 and 4 the inflaton then starts to oscillate at the bottom of its potential. Since inflation was introduced to solve the problems of the initial conditions of the HBB model, but not to replace it, we should connect the end state of inflation to the beginning of HBB evolution. The energy density of the inflaton field has to be transferred to the particles of the SM that then thermalize. This process is called reheating and is the subject of the first part of this thesis. It will be introduced more extensively in chapter 2.

1.2.4.2 Fluctuations and CMB constraints

Besides solving the horizon and flatness problems, inflation can also explain the small fluctuations

that are observed in the CMB and that eventually formed stars and galaxies. The temperature fluctuations correspond to fluctuations in the energy density that are caused by quantum fluctuations in the inflaton field [74].

The fluctuations in the CMB can be described by their power spectrum. The measured amplitude of the fluctuations and the scale dependence of the power spectrum can be used to constrain models of inflation [72, 73, 75]. Furthermore, the non-detection of tensor modes puts an upper bound on the value of the Hubble parameter during inflation [76, 77].

It is a remarkable fact that even the simplest models, where inflation is caused by a single scalar field, can correctly predict the power spectrum of fluctuations as observed in the CMB [77]. There are many different models for scalar field inflation [78]. In chapter 3 we will not stick to a specific model, but in chapter 4 we will study the case where the Higgs is the inflaton.

1.3 Outline of this thesis

In this thesis we focus on two stages of the early universe. The topic of Part I is reheating: the phase connecting inflation to HBB evolution. In chapter 2 we show why studying the reheating phase is interesting and sketch how the energy of the inflaton field is transferred to SM particles. Chapter 3 addresses a problem that arises from the running of the Higgs self-coupling λ , which becomes negative at energy scales above 10^{11} GeV. Efficient production of Higgs modes during preheating would cause the Higgs field to end up in an energetically favorable vacuum at $\varphi \gg 246$ GeV, which is in contradiction with observations. In chapter 4 we study reheating after inflation caused by the Higgs field. We focus on production of Higgs modes and gauge modes, as these are most efficiently produced.

The topic of Part II is electroweak baryogenesis. Chapter 5 sums up the necessary conditions for baryogenesis. We then show how the baryon asymmetry can be obtained during the electroweak phase transition and give an overview of the computation of the baryon asymmetry. The central question of chapter 6 is whether electroweak baryogenesis can be studied in the framework of the Standard Model Effective Field Theory. We focus on the contribution from top quarks to the baryon asymmetry, but find that this does not result in a value of the baryon asymmetry that is consistent with observations. In chapter 7 we will then study the importance of leptons for generating the baryon asymmetry.