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Applications of topology to Weyl semimetals and quantum computing

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Applications of topology to Weyl semimetals and quantum computing

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Cover: Quantum circuits for interleaved surface code X and Z stabilizer
measurements on a Möbius strip, forcing their intersection.

To my family; Mum, Dad, Tess, and to Louise.

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