On transport properties of Majorana fermions in superconductors: free & interacting

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Summary

Majorana fermions in superconductors are the subgap quasiparticle excitations that are their own antiparticles. They are an equal weight quantum superposition of a positively charged electron and a negatively charged hole, that makes them charge-neutral but not an eigenstate of charge. Although the expectation value of charge is therefore zero, there are fluctuations of charge. This would enable a purely electrical detection of Majorana states, outdoing low-temperature thermal measurements. The fluctuations of the charge carried by Majorana fermions in superconductors are studied in the first two chapters of this thesis.

Majorana fermions occur on the boundary of so-called topological superconductors, either the one-dimensional edge of a two-dimensional system or the two-dimensional surface of a three-dimensional system. In chapter 2 we show that the charge fluctuations (electrical shot noise) are proportional to the thermal conductance, with a coefficient made up only from fundamental constants and the temperature. This result stems from the combination of time reversal and electron-hole symmetries.

The electrical shot noise (the second moment of the charge fluctuations) generated by the Majorana fermion in a single edge channel is quantised in units of $\frac{1}{2} \times e^2/h$ per eV voltage bias. In chapter 3 we extend the quantization to all even moments of the charge fluctuations. At low temperatures the full probability distribution of the charge fluctuations is trinomial: a Majorana fermion transfers either $-e$, 0 or $+e$ charge, with the corresponding probabilities 1/4, 1/2 and 1/4. This result is drastically different from the usual binomial statistics of electrons.

In chapter 4 we proceed from topological superconductors to graphene, which is a single carbon layer. Although graphene is not superconducting, we demonstrate that there exists an analogy with Andreev reflection in a topological superconductor. In a superlattice of graphene on a suitable substrate an anti-unitary symmetry occurs. The latter one is mathemati-

cally equivalent to electron-hole symmetry in a superconductor. The "valley index" K, K′ in graphene then plays the role of the electron-hole degree of freedom. Thus, the electron approaching perpendicularly to the interface between pristine graphene and a graphene superlattice is reflected in the opposite valley, just as Andreev reflection scatters an electron into a hole and vice versa.

The properties of the systems mentioned above can be captured by non-interacting theories at low energy scale. In the second part of the thesis, we deal with the effects of strong interactions between the particles. In particular we focus on the Sachdev-Ye-Kitaev (SYK) model that describes a metal with strong infinite-range interactions (mean strength J) between N fermions (Dirac/Majorana). The model is solvable in the large N limit, where a conformal symmetry emerges in the infrared. The transport observables resulting from the SYK model are considered in the three following chapters.

In chapter 5 we predict that the tunneling spectroscopy of the complex-valued SYK model shows a duality between low and high voltage: The differential conductance G depends on the applied voltage eV only through the dimensionless parameter $\xi = eVJ/\Gamma^2$ and obeys the duality relation $G(\xi) = G(\pi/\xi)$, where Γ characterizes coupling to the probe. This property of the current-voltage characteristic reveals the underlying conformal symmetry of the SYK model.

In chapter 6 we show that in a system that contains at least two discrete states that are coupled to a quantum continuum, one can see the isolated zero point in the spectral function of one of the states at the frequency of the other state. This phenomenon makes it possible to test the continuous behavour of the spectral function of the SYK model. An electron propagating through a lattice of the SYK nodes is scattered by those in a way that produces a characteristic "holographic" spectral function with a line of zeros.

In chapter 7 we point out that a quantum dot becomes superconducting close to Majorana fermions described by the SYK model. Despite the induced superconducting order parameter, which is an odd function of frequency, there is no excitation gap. The occurrence of "gapless superconductivity" can be measured spectroscopically.

In the final chapter of this thesis, we consider a system with an infinitely ranged interaction without disorder, in contrast with the SYK model. The interaction arises from the gauge-invariant coupling of N

low-capacitance Josephson junctions to a photon in a microwave cavity. The effective theory of the system turns out to be an extended Dicke Hamiltonian of N spins-1/2 with all-to-all repulsive interaction. We propose a rotating Holstein-Primakoff representation for the total spin of N particles, which enables us to uncover the first-order phase transition in the large N limit.