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## On transport properties of Majorana fermions in superconductors: free & interacting

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# On transport properties of Majorana fermions in superconductors: free & interacting

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Front cover: Spectral function of the SYK impurity lattice with the lines of zeros. Chiral Majorana mode propagating along the edge of a two-dimensional topological superconductor, with a graphene fragment in the lower left corner. Back cover: Tunneling spectroscopy of a black hole chip (SYK quantum dot), on the background of a graph of the low-high voltage duality derived in this thesis. The author's name is tilted representing the "rotating" Holstein-Primakoff approach proposed in this thesis.

*To my parents.  
Моим родителям.*



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