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Lines in the Sand

Behaviour of self-organised vegetation patterns in dryland ecosystems

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Laat ons een bloem en wat gras dat nog groen is
Laat ons een boom en het zicht op de zee
Vergeet voor één keer hoeveel geld een miljoen is
De wereld die moet nog een eeuwigheid mee
— Louis Neefs, Laat ons een bloem

Leave them a flower, some grass and a hedgerow
A hill and a valley, a view to the sea
These things are not yours to destroy as you want to
A gift given once for eternity
— Wally Whyton, Leave them a flower

Table of Contents

1 Introduction **9**

2 Multistability of model and real dryland ecosystems through spatial self-organization **21**

2.1 Introduction 21
2.2 Theory 23
2.2.1 Model description 23
2.2.2 Theoretical outcomes 23
2.2.3 Testable predictions 26
2.3 Data acquisition & processing 26
2.4 Results 27
2.5 Discussion 29

Appendices

2.A Dimensional extended-Klausmeier 31
2.B Description of study sites 31
2.C Data sets 32
2.C.1 Topographical data 32
2.C.2 Biomass measurements 32
2.C.3 Optical data 33
2.D Data processing 33
2.D.1 Spectral analysis, direction of anisotropy and wavelength . . . 33
2.D.2 Pattern classification 34
2.D.3 Cross-spectral analysis and migration speed 35
2.D.4 Assessment of uncertainty in calculations of slope gradient and aspect from topographical data 35
2.D.5 Assessment of uncertainty in estimation of pattern frequency from optical imagery 36

3 The dynamics of disappearing pulses in a singularly perturbed reaction-diffusion system with parameters that vary in time and space **41**

3.1 Introduction 41
3.1.1 Size assumptions 48
3.2 PDE to ODE reduction 50
3.2.1 The inner regions 51
3.2.2 The outer regions 53
3.2.3 Pulse location ODE under assumption (A3) 54
3.2.4 Pulse location ODE under assumption (A3’) 57
3.3 Linear Stability 59

3.3.1 Stability of homoclinic pulses on $\mathbb{R}$ . . . 60
3.3.2 Stability of N-pulse solutions . . . 66
3.4 Numerical Simulations . . . 70
3.4.1 Flat terrains . . . 71
3.4.2 The effect of sloped terrains . . . 77
3.5 Discussion and Outlook . . . 84
Appendices
3.A The movement of water on a varying terrain . . . 87
3.B Fixed Points of the pulse-location ODE - Proofs . . . 88
3.B.1 Properties of $R_\pm$ . . . 89
3.B.2 Unbounded domains . . . 90
3.B.3 Bounded domains with periodic boundaries . . . 90
3.B.4 Bounded domains with Neumann boundary conditions . . . 93

4 Pulse solutions for an extended Klausmeier model with spatially varying coefficients **95**
4.1 Introduction . . . 95
4.1.1 Assumptions . . . 100
4.2 Analysis of stationary pulse solutions . . . 101
4.2.1 Stationary pulse solutions for $f = 0$ and $g = 0$ . . . 102
4.2.2 Stationary pulse solutions for varying f and g . . . 105
4.2.3 Some basic results from the theory of exponential dichotomies . 110
4.2.4 Dynamics on $\mathcal{M}$ (Proof of Proposition 4.2.9) . . . 112
4.2.5 Existence results . . . 115
4.2.6 Proof of existence results . . . 118
4.3 Linear stability analysis . . . 118
4.3.1 Qualitative description of the point spectrum location (Proof of Theorem 4.3.2) . . . 120
4.3.2 Small eigenvalue close to $\lambda = 0$ (Proof of Theorem 4.3.5) . . . 130
4.4 The effect of the small eigenvalue: movement of pulse . . . 136
4.4.1 Formal derivation of pulse location ODE . . . 137
4.4.2 Stability of fixed points of pulse location ODE (4.4.1) . . . 138
4.4.3 Small eigenvalue in case of small spatially varying coefficients . 139
4.4.4 Examples of stationary single-pulse solutions . . . 141
4.4.5 Stationary multi-pulse solutions . . . 145
4.5 Discussion . . . 148

5 Stable planar vegetation stripe patterns on sloped terrain in dryland ecosystems **151**
5.1 Introduction . . . 151
5.2 Slow-fast analysis of traveling wave equation . . . 157
5.2.1 Critical manifolds . . . 157
5.2.2 Layer fronts . . . 160
5.2.3 Slow flow . . . 163
5.2.4 Singular orbits . . . 163
5.2.5 Main existence results . . . 168
5.3 Persistence of solutions for $0 < \varepsilon \ll 1$ . . . 169
5.3.1 Transversality along singular orbits . . . 169
5.3.2 Proof of existence results . . . 171
5.4 Stability . . . 172

5.4.1 Essential spectrum . . . 173
5.4.2 Point spectrum . . . 175
5.4.3 Formal computation of small eigenvalues . . . 175
5.4.4 Main stability results . . . 178
5.5 Rigorous proof for stability theorems . . . 181
5.5.1 Exponential dichotomies/trichotomies . . . 181
5.5.2 Reduction to region of bounded $|\ell|$ . . . 182
5.5.3 The region $(\tilde{\lambda}(\ell), \ell) \in R_3(M) \times [-L_M, L_M]$ . . . 184
5.5.4 Setup for the region $(\tilde{\lambda}(\ell), \ell) \in R_1(\delta) \cup R_2(\delta, M) \times [-L_M, L_M]$ 184
5.5.5 The region $(\tilde{\lambda}(\ell), \ell) \in R_1(\delta) \times [-L_M, L_M]$ . . . 187
5.5.6 The region $(\tilde{\lambda}(\ell), \ell) \in R_2(\delta, M) \times [-L_M, L_M]$ . . . 195
5.5.7 Proof of Theorem 5.4.3 . . . 196
5.6 Defects and curved vegetation pattern solutions . . . 197
5.7 Numerics . . . 198
5.7.1 Continuation of traveling stripes and gaps . . . 199
5.7.2 Direct simulations . . . 200
5.8 Discussion . . . 200
Appendices
5.A Stability of steady states . . . 206
5.B Absence of point spectrum in $R_2(\delta, M)$ . . . 206

References **209**

Dutch Summary **221**

Curriculum Vitae **223**