Extension of operators on pre-Riesz spaces

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Stellingen

behorende bij het proefschrift

Extension of Operators on Pre-Riesz Spaces

van

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1. On a pervasive pre-Riesz space X, a regular norm can be extended to a Riesz norm on a vector lattice cover of X. If X is, moreover, Archimedean, then an order continuous monotone seminorm on X yields an order continuous seminorm on a vector lattice cover of X. [Lemma 2.1.6 and Theorem 3.1.8]

2. Inverses of disjointness preserving bijections from a uniformly complete vector lattice to a pervasive pre-Riesz space with a monotone norm are disjointness preserving. More restrictive conditions involving order and norm convergences are required for both domain and range space to be pre-Riesz spaces to conclude the similar result. [Corollary 2.1.7 and Theorem 2.1.20]

3. A positive operator S between two pre-Riesz spaces which is dominated by a compact operator is compact, provided certain conditions for the domain and range space are satisfied. Under a different set of conditions the conclusion that S^3 is compact is obtained. [Theorem 3.3.7 and Theorem 3.3.25]

4. A disjointness preserving C_0-semigroup on an ordered Banach space with a semimonotone norm admits a local generator. [Theorem 4.3.2]

5. Every Archimedean directed partially ordered vector space has a vector lattice cover. The Riesz completion and the Dedekind completion of an Archimedean directed partially ordered vector

space are determined isomorphically. This is a very important fact for the study of partially ordered vector spaces.

6. The C_0-semigroup $T(t)_{t\geq 0}$ on a Banach space is contractive with respect to a continuous sublinear functional p if and only if the generator A of $T(t)_{t\geq 0}$ is p-dissipative. The positivity of $T(t)_{t\geq 0}$ can be characterized by the positive off-diagonal property of A on an ordered Banach space with an order unit, and the order unit is essential.

7. The spectrum of an order bounded and disjointness preserving operator on a Banach lattice has 1 as its only element if and only if this operator is an identity operator. In the case of an ordered Banach space, one direction of this conclusion is easy, but the other one is not clear so far.

8. If a positive compact operator T on a Banach lattice has a strictly positive spectral radius r, then r is an eigenvalue of T with a corresponding strictly positive eigenvector. This conclusion holds as well on ordered Banach spaces. (Krein-Rutman)

9. Finding a counterexample is always more difficult than finding a good example.

10. Sometimes, solving a problem is just like looking for a diamond. Once you have found it, all you need to do is pick it up.