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Extension of operators on pre-Riesz spaces

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Summary

This thesis mainly extends the theory of positive operators on Riesz spaces to a setting of pre-Riesz spaces. It includes five chapters. The first chapter is on preliminaries, and the rest of the four chapters present main results. These results can be roughly divided into two parts: the second and third chapter extend work on disjointness preserving operators and compact operators to pre-Riesz spaces, and the last two chapters concern the theory of C_0 -semigroups in ordered Banach spaces.

Our work is based on the theory which was established by M. van Haandel in 1993, which yields that every directed Archimedean partially ordered vector space (pre-Riesz space) owns a vector lattice cover, that is, it can be embedded order densely into a Riesz space. We establish that for a pervasive pre-Riesz space X with monotone norm, there exists a Riesz norm on the Riesz completion X^ρ . Moreover, we show that if for every nonzero positive element x in X^ρ , there exists an increasing sequence which is less equal than x and norm converges to x , then every positive linear operator $T : X \rightarrow Y$ which is disjointness preserving and injective can be extended to $\widehat{T} : X^\rho \rightarrow Y^\delta$ where \widehat{T} is disjointness preserving and injective, and where X, Y are pervasive pre-Riesz spaces with monotone norms, X^ρ is the Riesz completion of X and Y^δ is the Dedekind completion of Y . In addition, we explore that for an operator $T : X \rightarrow Y$ with X a pervasive Archimedean pre-Riesz space with the Riesz decomposition property and Y a Dedekind complete vector lattice, there exists an extension of T to all of X^ρ which preserves order boundedness and disjointness. In a pervasive Archimedean pre-Riesz space X with the Riesz decomposition property, for a positive element in X^ρ , we construct a net in X which is order convergent to it. Using this result, we extend order continuous norms on pre-Riesz spaces. Furthermore, we show that the compact domination property of positive operators from X to Y , where X and Y are Banach lattices, holds as well in pervasive Archimedean pre-Riesz spaces with the Riesz decomposition property and regular norms, provided $(X^\delta)'$ and Y having order continuous norms.

In the second part of the results, we establish that a disjointness preserving C_0 -semigroup $T(t)_{t \geq 0}$ on an ordered Banach space with a semimonotone norm always admits a local generator A . We show that the reverse direction of this implication holds if A and A^{-1} are local. We also consider the contractivity and positivity of semigroups $T(t)_{t \geq 0}$ on ordered Banach spaces, which are related to the dissipativity of A with respect to a suitable sublinear functional.

We provide many examples to support our results, where one typical example of a pre-Riesz space is $C^1[0, 1]$.