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Extension of operators on pre-Riesz spaces

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Extension of Operators on Pre-Riesz Spaces

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To my beloved parents.

Preface

The theory of Riesz spaces, as a branch of functional analysis, has been developed extensively in the last decades. It was first considered by F. Riesz, L. Kantorovic, and H. Freudenthal in the middle nineteen thirties, and subsequently studied by scholars from the Soviet Union, Japan and the United States. Then the theory has grown rapidly in Dutch and German schools since the middle of the nineteen seventies, and it has also attracted scholars from the United Kingdom and Spain. Later on, more and more researchers have joined in the developments, for instance from China, South Africa, Slovenia and so on.

Most vector spaces admit a partial order in a pointwise manner or in another way. In a nutshell, Riesz spaces (vector lattices) are the partially ordered vector spaces in which any pair of elements has a least upper bound with respect to the order (lattice operation). Even though this is a very concise fact, the study of Riesz spaces in conjunction with Banach's theory of normed vector spaces has progressed slowly and steadily from the mid-1930s to 1960s. The last 30 years of the previous century was a fruitful age. The theory of Riesz space has systematically been established in *Riesz Spaces I* by W. A. J. Luxemburg and A. C. Zaanen [40]. H. H. Schaefer [48] has bridged the gap between the theory of positive operators in Banach lattices and the mainstream of operator theory in Banach spaces in *Banach Lattices and Positive Operators*. The book *Positive Operators* by C. D. Aliprantis and O. Burkinshaw [6], and the book *Banach Lattices* by P. Meyer-Nieberg [42] were remarkable next steps. There are too many monographs to list them one by one at here, we refer the reader to the previously mentioned four books for more history of Riesz spaces. In the area of dynamic systems, the theory of Riesz spaces and positive operators has been applied as well. We only mention two books which are related to this thesis. One is *One-parameter Semigroups* [16] by Dutch scholars, and the other one is *One-parameter Semigroups of Positive Operators* [45] by German scholars.

Compared with the high level of accomplishment in Riesz space theory, there are only few results for general partially ordered vector spaces, due to the lack of lattice

structure. Nonetheless, some scholars, e.g. C. D. Aliprantis and E. Langford [7] in 1984, M. van Haandel [54] in 1993, have developed a theory of the vector lattice covers of partially ordered vector spaces in a categorical approach. Based on the theory in the latter approach, O. van Gaans and A. Kalauch have extended some basic results from Riesz spaces, viz. the concepts of ideals, bands and disjointness etc. to a new setting of pre-Riesz spaces. This thesis proceeds their works. It extends the study of operators on pre-Riesz spaces.

This thesis includes five main chapters. The first chapter introduces some basic terminologies in ordered vector spaces and vector lattices. The second chapter investigates disjointness preserving operators on partially ordered vector spaces, in particular pre-Riesz spaces. The third chapter is concerned with then extension of the theory of compact operators, in particular the positive domination property in pre-Riesz spaces. The fourth chapter studies disjointness preserving C_0 -semigroups in partially ordered vector spaces with respect to a suitable norm. In the last chapter, the dissipativity and positive off-diagonal property of operators on ordered vector spaces are considered.

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