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## Strategy dynamics

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# **Part III**

## **Discussion**



## DISCUSSION

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### 6.1 OPINION DYNAMICS

In the first chapter on opinion dynamics we have studied the role of inflexibles and floaters on the support for two opinions under repeated application of the local majority rule in groups of size 3. In the second chapter on opinion dynamics we refined this study by allowing for contrarian and non-contrarian floaters. We derived several scenarios for the development of opinion support in time. We shall now point to topics for further research in opinion dynamics. All these directions for research in opinion dynamics deal with adding more structure to opinion-dynamical models.

A first and straightforward way to add more realism to opinion-dynamical models is by allowing groups of varying sizes in which individuals encounter other opinions. This can be done by means of probability distributions on group sizes. Also an increase in the number of opinions will be useful in connecting with real opinion-dynamical processes.

More realism can also be achieved by by incorporation social structures. These can be of a spatial, economic and/or educational kind. Spatial structure enables to study the spreading of opinions in both rural and urban areas. In addition it allows to model a possibly higher abundance of opinions in urban areas compared to rural areas. Adding economic structure takes into account that support of opinion may depend on income, whereas educational structure deals with the support of opinion in relation to educational level. A further realistic extension of opinion models is to incorporate the role of media in the spreading of opinions, as a global influence on opinion bearers in contrast to the local influences which individuals encounter in the groups they repeatedly form.

Apart from inflexibles and (contrarian and non-contrarian) floaters it may be useful to include a class of neutrals composed of individuals who do not yet have an opinion, but may develop one by repeated encounters with opinion bearers. A further extension is the incorporation of a delay effect for floaters, which expresses that a floater builds up the inclination to change its opinion to that of the majority instead of making this change at first encounter with a majority. Another extension of opinion dynamics models is to allow for the change of inflexible into floater or vice versa. E.g., contrarian behaviour may express the adolescent attitude to strive for individualisation, which may transform into inflexible behaviour when one grows older. Fuzzy set theory can be applied to express doubt in supporting an opinion.

Political parties in general represent a number of political opinions, and part of these opinions may be characteristic for several political parties. It may therefore be insightful to model the dynamics of support for vectors of opinions instead of single opinions. An individual then is characterised by the vector of opinions he or she supports, and the opinions for which the individual is an inflexible determines the political party that one associates with. For the opinions for which an individual is not an inflexible, non-contrarian or contrarian behaviour can be assumed.

With chapters on opinion dynamics as well as adaptive dynamics in one thesis it is tempting to reflect on mutual applications. One immediate notion that comes to mind is to model with adaptive-dynamical means a phenomenon that is well-known in Dutch politics: the raise and development of a new alternative opinion or political splinterparty, and to determine under which conditions such an opinion or party becomes part of the establishment, remains present in a marginal way or disappears altogether.

## 6.2 ADAPTIVE DYNAMICS

Many works has already been done and published on the theory and applications of adaptive dynamics (see the site [60] for an overview). However, an extensive study of adaptive dynamics based on Lotka-Volterra community dynamics had not yet been performed, and Chapter 5 on adaptive dynamics in this thesis provides an opening in this direction. As is shown in this chapter, the approach allows for an explicit expression for the invasion fitness function, which turned out to be useful in stating various conclusions of adaptive dynamics. We mention several open problems of adaptive dynamics that follow from this chapter.

A main problem is the determination of the zero sets of the invasion fitness functions  $s_k(\tau_1, \dots, \tau_k; \nu)$  for arbitrary  $k \geq 1$ . The intersections of these zero sets with the diagonal hyperplanes  $\Delta_{i,k+1}^{k+1}$  determine the position and shape of the isoclines. Also, these zero sets determine the local shapes of the sets  $\mathbb{A}_{k+1}$  attached to the isoclines, and thus the shape of the space into which an evolutionary trajectory proceeds in an evolutionary branching process. In particular it can be deduced from these shapes when a trajectory is close enough to the boundary of an  $\mathbb{A}_k$  for a sufficiently large step to cause evolutionary pruning.

After evolutionary branching from  $\mathbb{A}_1$  into  $\mathbb{A}_2$  the trajectory the evolutionary path follows can be deduced by embedding  $\mathbb{A}_2$  into the two hyperplanes  $\Delta_{1,3}^3$  and  $\Delta_{2,3}^3$  and then analyse how the invasion fitness function  $s_2(\tau_1, \tau_2; \nu)$  behaves on the two embeddings of  $\mathbb{A}_2$ . A change in the first trait of a dimorphic community  $(\tau_1, \tau_2)$  is determined by the sign pattern of  $s_2$  on  $\Delta_{1,3}^3$ , and similarly a trait substitution in the second trait is determined by the pattern on  $\Delta_{2,3}^3$ . The combination of these changes composes the evolutionary path the community follows in  $\mathbb{T}^2$ . To avoid making these combinations of informations coming from two different local

patterns it would be useful to have one space available in which conclusions on how the evolutionary path for a community  $\tau_1, \tau_2$  proceeds can be concluded from the local properties of a suitable function at  $(\tau_1, \tau_2)$ . Similar questions on combining information from diagonal hyperplanes to conclude how an evolutionary path proceeds, and overcoming this by means of a suitable representation of course also hold for evolutionary paths in any  $\mathbb{A}_k, k \geq 2$ .

A next question deals with the relation between the function  $s_1$  and the evolutionary trajectories that it allows. What properties must  $s_1$  or, more precisely, the per capita initial growth function  $r$  and the interaction function  $a$  of a Lotka-Volterra model, satisfy to enable an evolutionary trajectory with an arbitrary number of branches, or with a specified number of branches?

A forthcoming paper will discuss bifurcation theory for adaptive dynamics based on Lotka-Volterra community dynamics. This bifurcation theory studies in particular if a IESS can change into an evolutionary branching point due to changes in the values of community-dynamical parameters that appear in the interaction function  $a$  and the per capita initial growth function  $r$  as they appear in the Lotka-Volterra differential equations in the previous chapter. The bifurcation theory gives insight in the way the set  $\mathbb{A}_2$  is attached to the 1- and 2-isoclines when one moves along these isoclines, i.e., about how the local shape of  $\mathbb{A}_2$  deforms along these isoclines.

