



Universiteit
Leiden
The Netherlands

Strategy dynamics

Jacobs, F.J.A.

Citation

Jacobs, F. J. A. (2020, December 8). *Strategy dynamics*. Retrieved from <https://hdl.handle.net/1887/138513>

Version: Publisher's Version

License: [Licence agreement concerning inclusion of doctoral thesis in the Institutional Repository of the University of Leiden](#)

Downloaded from: <https://hdl.handle.net/1887/138513>

Note: To cite this publication please use the final published version (if applicable).

Cover Page



Universiteit Leiden



The handle <http://hdl.handle.net/1887/138513> holds various files of this Leiden University dissertation.

Author: Jacobs, F.J.A.

Title: Strategy dynamics

Issue date: 2020-12-08

Strategy Dynamics

Proefschrift

ter verkrijging van
de graad van Doctor aan de Universiteit Leiden,
op gezag van Rector Magnificus prof. mr. C.J.J.M. Stolker,
volgens besluit van het College voor Promoties
te verdedigen op dinsdag 8 december 2020
klokke 16.15 uur

door

Franciscus Johannes Aloysius Jacobs

geboren te Asten
in 1960

Promotor: Prof. dr. R.M.H. Merks

Co-promotor: Prof. em. dr. J.A.J. Metz

Promotiecommissie: Prof. dr. O. Diekmann (Universiteit Utrecht)

Dr. S. Galam (SciencesPo, Frankrijk)

Dr. E. Kisdi (Helsinki University, Finland)

Prof. dr. A. Doelman

Prof. dr. P. Stevenhagen

CONTENTS

List of Figures	vi
-----------------	----

List of Tables	viii
----------------	------

1 INTRODUCTION	3
1.1 The notion of strategy	4
1.2 Opinion dynamics	6
1.3 Adaptive dynamics	7
1.4 Overview	9

I Opinion Dynamics 13

2 THE ROLE OF INFLEXIBLE MINORITIES IN THE BREAKING OF DEMOCRATIC OPINION DYNAMICS	15
2.1 Introduction	15
2.2 Group size 3	17
2.3 Conclusions	28
3 TWO-OPINIONS-DYNAMICS GENERATED BY INFLEXIBLES AND NON-CONTRARIAN AND CONTRARIAN FLOATERS	29
3.1 Introduction	30
3.2 Group size 1	34
3.3 Group size 2	37
3.4 Group size 3	39
3.4.1 The fully symmetric case: $\alpha_A = \alpha_B$ and $\gamma_A = \gamma_B$	39
3.4.2 The general case	42
3.5 Conclusions	52
3.6 Appendices	57
3.6.1 Table 1: group size $L = 1$	58
3.6.2 Table 2: group size $L = 2$	59
3.6.3 Table 3: group size $L = 3$	60
3.6.4 $L = 3$: analysis for the fully symmetric case $\alpha_A = \alpha_B$ and $\gamma_A = \gamma_B$	62
3.6.5 $L = 3$: analysis of the general case	62

II	Adaptive Dynamics	65
4	ON THE CONCEPT OF ATTRACTOR FOR COMMUNITY-DYNAMICAL PROCESSES: THE CASE OF UNSTRUCTURED POPULATIONS	67
4.1	Introduction	67
4.2	Chaining, Chain Attractors and Basin of Chain Attraction	68
4.3	Extinction Preserving Chain Attractors for Immigration-Free Communities	73
4.4	Four Examples	78
4.5	Discussion	81
5	ADAPTIVE DYNAMICS FOR LOTKA-VOLTERRA COMMUNITY DYNAMICS	85
5.1	Introduction	86
5.2	An introduction to the mathematical framework	90
5.2.1	Preliminaries on trait spaces and Lotka-Volterra community dynamics	90
5.2.2	Trait-dependent ODE community-dynamical systems	92
5.2.3	Properties of the maps $\mathbb{T}^k \rightarrow LV_k(\mathbb{T}^k)$	96
5.2.4	c -Attractors and invasion fitness: from community dynamics towards adaptive dynamics	99
5.2.5	Mono-, di- and trimorphisms: first steps towards a generalisation	114
5.3	Adaptive dynamics: the mathematical framework	136
5.3.1	A generalisation of the invasion function and its consequences	137
5.3.2	A closer look at the mathematical conditions for coexistence	146
5.3.3	Permanence	149
5.4	Discussion	151
5.5	Appendices	155
5.5.1	Proof of Lemma 7	156
5.5.2	Proof of Lemma 8	164
5.5.3	Proof of Lemma 9	168
5.5.4	Proof of Lemma 10	173
5.5.5	Proof of Lemma 11	175
5.5.6	Proof of Lemma 12	180
5.5.7	Proof of Lemma 13	184
III	Discussion	189
6	DISCUSSION	191
6.1	Opinion dynamics	191
6.2	Adaptive dynamics	192
	BIBLIOGRAPHY	195

SAMENVATTING	203
DANKWOORD	209
CURRICULUM VITAE	211
PUBLICATIONS	213

LIST OF FIGURES

Figure 1.1	A plot of the subsequent phenotypic trait compositions of community-dynamical attractors present on the evolutionary timescale	10
Figure 2.1	The bare model with only floaters	16
Figure 2.2	One side inflexibles at low density	17
Figure 2.3	Equal presence of inflexibles on both sides	18
Figure 2.4	Unequal densities of inflexibles	19
Figure 2.5	One sided inflexibles fixed points as a function of their density x	21
Figure 2.6	p_{t+1} as a function of p_t for one sided inflexibles	22
Figure 2.7	Comparison of update series	23
Figure 2.8	Two side symmetric inflexibles fixed points as a function of their density b	24
Figure 2.9	p_{t+1} as a function of p_t for two sided symmetric inflexibles	24
Figure 2.10	Two side asymmetric inflexibles	25
Figure 2.11	Evolution of an initial A support in time	26
Figure 2.12	The dynamics map	27
Figure 3.1	Examples for group size $L = 1$	37
Figure 3.2	Overview of the opinion dynamics for group size $L = 1$	38
Figure 3.3	Graphs of $f_{3;0.1,0.1;\gamma,\gamma}$	40
Figure 3.4	Critical curves for the fully symmetrical case	41
Figure 3.5	Equilibrium densities $\hat{p}_{3;\alpha_A,\alpha_B;\gamma_A,1-\gamma_A}$	43
Figure 3.6	Sign plots of the discriminant, for small densities of inflexibles for both opinions	46
Figure 3.7	Examples of opinion dynamics for small densities of inflexibles and small fractions of contrarians for both opinions	48
Figure 3.8	Examples of opinion dynamics for small densities for inflexibles for both opinions, and with a positive discriminant $D(\alpha_A, \alpha_B; \gamma_A, \gamma_B)$	49
Figure 3.9	Examples of opinion dynamics for small inflexible densities and large fractions of contrarians for both opinions	50
Figure 3.10	Examples of small inflexible densities and large fractions of contrarians that allow for a collapse to equilibrium	51
Figure 3.11	Sign plots of the discriminants $D(0.1, 0.4; \gamma_A, \gamma_B)$ and $D(0.5, 0.3; \gamma_A, \gamma_B)$	52
Figure 3.12	Examples of opinion dynamics for small densities of inflexibles for the A opinion and intermediate densities for the B inflexibles	53

Figure 3.13	Examples of opinion dynamics for intermediate densities of inflexibles for both opinions	54
Figure 4.1	An illustration of an ε -pseudo-orbit	69
Figure 4.2	A degenerate dynamical system	79
Figure 4.3	The simplest perturbation of the dynamical system from Example 1	80
Figure 4.4	The May-Leonrad dynamical system	80
Figure 4.5	An example of a dynamical system with an open ep-chain recurrent set	81
Figure 5.1	An example of the sets $\mathbb{A}_1$ and $\mathbb{A}_2$	110
Figure 5.2	An example of a PIP	115
Figure 5.3	The TEP resulting from Fig. 5.2	118
Figure 5.4	Classification of PIPs and TEPs	128
Figure 5.5	Example of a PIP and TEP in the absence of a monomorphic singularity	129
Figure 5.6	Example of a PIP and TEP for a zero set of s_1 with self-intersection	129
Figure 5.7	The 1- and 2-isocline obtained from the intersection of the zero set of z_2 with $\mathbb{A}_2$ embedded in $\triangle_{1,3}^3$ and $\triangle_{2,3}^3$	132

LIST OF TABLES

Table 1	Densities for the A opinion after application of the local majority rule and the switch by contrarians for all possible individual cases ($L = 1$)	58
Table 2	Densities for the A opinion after application of the local majority rule and the switch by contrarians for all possible compositions of groups of size $L = 2$	59
Table 3	Densities for the A opinion after application of the local majority rule and the switch by contrarians for all possible compositions of groups of size $L = 3$	60