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## Lattice models for Josephson junctions and graphene superlattices

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### Citation

Ostroukh, V. (2018, June 27). *Lattice models for Josephson junctions and graphene superlattices*. *Casimir PhD Series*. Retrieved from <https://hdl.handle.net/1887/63217>

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**Author:** Ostroukh, V.

**Title:** Lattice models for Josephson junctions and graphene superlattices

**Issue Date:** 2018-06-27

# Lattice models for Josephson junctions and graphene superlattices

PROEFSCHRIFT

TER VERKRIJGING VAN  
DE GRAAD VAN DOCTOR AAN DE UNIVERSITEIT LEIDEN,  
OP GEZAG VAN RECTOR MAGNIFICUS PROF. MR. C.J.J.M. STOLKER,  
VOLGENS BESLUIT VAN HET COLLEGE VOOR PROMOTIES  
TE VERDEDIGEN OP WOENSDAG 27 JUNI 2018  
KLOKKE 15.00 UUR

DOOR

Viacheslav Ostroukh

GEBOREN TE LUGANSK, OEKRAÏNE IN 1991

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Casimir PhD Series Delft-Leiden 2018-14

ISBN 978-90-8593-346-5

An electronic version of this thesis can be found  
at <https://openaccess.leidenuniv.nl>

Cover: current distribution in a disordered graphene quantum dot (front),  
computed in a tight-binding model (back).

*To my parents.  
Моїм батькам.*



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