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Mathematics and the Anthropocene equation: Comment on Gaffney O and Steffen W (2017) The Anthropocene equation. *The Anthropocene Review* 4:53-61

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Abstract
A recent paper in *The Anthropocene Review* introduces an 'Anthropocene equation', which is assumed to model the Anthropocene. While welcoming mathematical treatment as such, we criticize the specific approach for being sloppy, wrong and empty. While the use of mathematics in the criticized paper suggests a high level of scientific rigor, it actually weakens the message rather than strengthens it.

Keywords
Anthropocene equation, mathematics, rhetorics

A recent paper by Gaffney and Steffen (2017) in this journal introduces 'the Anthropocene equation', written as

$$\frac{dE}{dt} = f(H)$$
$$A, G, I \to 0$$

where E represents the Earth System, H is human activity, A astronomical forcing, G geophysical forcing, and I the internal dynamics of the Earth System. The term t we assume to represent time. The symbol f we consider to be a function with a list of arguments, as in $f(H)$ above, or as in $f(A,G)$, $f(A,G,I)$ and $f(A,G,I,H)$ in earlier places.

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We agree with the paper's overall message that the influence of human activity (H) has become more important than it has ever been, in certain aspects even exceeding the influence of the other forces (astronomical, geological and internal). Our concern is the way the authors formulate their statement. Gaffney and Steffen (G&S from here) basically repeat arguments and facts by others, citing a large number of sources from Milanković to the present. Their prime contribution is the formulation of the Anthropocene equation, which is indeed the title of their paper as well as the central term in their abstract. This Anthropocene equation as cited above is, in our view, a meaningless element. So, while the paper reviews a large number of valid points, its final synthesis in an Anthropocene equation discredits the paper.

In this commentary, we first review the correctness and meaning of the Anthropocene equation. Towards the end, we will broaden the discussion to the role of mathematics as a rhetorical tool in Anthropocenic research.

To see what is wrong with the Anthropocene equation, we study G&S's equations (1) and (2) which describe the Earth's fate during the first billion years of history:

$$\frac{dE}{dt} = f\left(A, G\right) \quad \text{or} \quad \frac{dE}{dt} = f\left(A, G, I\right)$$

Our first problem with these two equations is that all variables are basically undefined. E represents the 'Earth System', but that is a system, not a measurable variable. We can (and do) study the Earth System in many ways, in terms of temperature, species richness, concentrations of O_2, entropy, etc. But the Earth System itself is not a quantity, and can therefore never enter a formula.

Let us take this for granted and read for E a typical quantifiable and measurable variable of the Earth System, such as sea level or oxygen concentration. Then the next question is what are A (astronomical forcing) and G (geophysical forcing). The very words 'forcing' suggest a force-type of variable. Indeed, the gravitational pull by the sun, the centrifugal forces from a rotary movement, and influx of cosmic radiation and extraterrestrial objects are forces that have left many birthmarks on the Earth. Likewise, the geophysical forces by weathering, oceanic currents, and tectonic and seismic activity have had a huge influence on our planet. Merging all these forces into one abstract term A and G requires a more precise definition, specifying measurable variables and dimensions. Do we indeed mean forces in the strict physical sense, so measured in newton? If not, what else do we mean?

Next comes the problem that the left-hand side of the equation specifies not E but rather $\frac{dE}{dt}$. This gives the Anthropocene the flavour of a differential equation, which may be right because differential equations are the heart of dynamic systems, and the Earth System is a dynamic system. But this dynamics assumes of course that A , G and I (or at least some of them) are functions of t :

$$\frac{dE}{dt} = f\left(A(t), G(t)\right) \quad \text{or} \quad \frac{dE}{dt} = f\left(A(t), G(t), I(t)\right)$$

We postponed discussing the term I, which is supposed to represent the 'internal dynamics' of the Earth System. We guess this 'internal dynamics' is a term that basically represents the forces that result from the state of the Earth System itself. Presence of life forms, temperature, oxygen levels, etc. all influence the force on the Earth System E . So, we should consider

$$I(t) = I\left(E(t)\right)$$

With that change, $f\left(A(t),G(t),I\left(E(t)\right)\right)$ can also be written as $f\left(A(t),G(t),E(t)\right)$, obviating the need for the spurious variable I. The differential equation structure now becomes more outspoken:

$$\frac{dE}{dt}=f\left(A(t),G(t),E(t)\right)$$

G&S next set out to separate human activity from the rest of the Earth System, by adding an additional term H as an argument to f. Again, human activity is further undefined in terms of how it is quantified or measured, but we will suppose it to represent humanity's force on the Earth System, in newton. The final step is taken by arguing that H has increased since the industrial era and in particular since 1950. In fact, the 'astronomical and geophysical forcings in the Holocene … approximate to zero compared with the impact of current human pressures on the rate of change of the Earth System … We also note from the rates of change described above that I now is also significantly less than H' (Gaffney and Steffen, 2017: 3). Reading 'pressure' once more as 'force', and interpreting 'significantly' not in the statistical meaning but instead as a synonym to 'much', this statement signifies that the ratio of A, G and I to H tends to 0. Mathematically, we would write this for the comparison with A as

$$\frac{A}{H}\to 0 \quad \text{or} \quad \frac{A}{H}<<1 \text{or}\, A<<H$$

In any case, it does not imply that $A\to 0$, as is written as a subscript in equation (4).

Equation (4) is anyhow worth a closer look. It says

$$\frac{dE}{dt}=f(H)$$

but with a subscript added: '$A,G,I\to 0$'. While many types of subscripts have a clear meaning in mathematics (such as $\lim_{A\to 0}\cdots$ or $\sum_{A}\cdots$), the notation

$$\underset{A,G,I\to 0}{f(H)}$$

does not exist in mathematics and does not have a meaning as long as the authors do not define it. If the authors mean that A (etc.) is close to 0, there is still no reason to take it out of the argument list of the function f. After all, the function $f(H,A)=(12-H)(5-A)$ is an example of a function where $A\to 0$ doesn't mean that A isn't doing anything. If the authors instead mean that $A<<H$, A is still an indispensable argument of the function.

What they mean to say is, we think, that in the present state (we shall call it $t=0$, the 'now' moment), the variables A, G, I and H have particular values, say $A(0)$, etc, and that the present rate of change of E is higher than ever because of a higher value of H. So, in the long distant past (say, at $t=-\infty$) we have

$$\left.\frac{dE}{dt}\right|_{t=-\infty}=f\left(A(-\infty),G(-\infty),I(-\infty),H(-\infty)\right)$$

where the vertical line with subscript $t=-\infty$ means 'evaluated at $t=-\infty$', and at the present moment (so at $t=0$) it is

$$\left.\frac{dE}{dt}\right|_{t=0} = f\left(A(0), G(0), I(0), H(0)\right)$$

where the consensus claim is that

$$\left.\frac{dE}{dt}\right|_{t=-\infty} << \left.\frac{dE}{dt}\right|_{t=0}$$

mainly because $H(-\infty) << H(0)$, while $A(-\infty) \approx A(0)$ and similar approximate identities for G and I.
With this in mind, it follows that

$$\left.\frac{\partial f}{\partial H}\right|_{t=0} >> 0$$

which is just a difficult way of expressing that the present increase of human activity induces an historically large rate of change of the Earth System.

This conclusion and some of the numbers reported ('Over the past 45 years … the rate of the temperature rise is about … 170 times the Holocene baseline rate.') have made it to the headlines of many news sites (see https://sage.altmetric.com/details/16295889/news for a surprisingly large number of news items, blogs and tweets). However, conclusion and numbers were not found by applying the Anthropocene equation. For instance, the number 170 is just taken from the cited report by NOAA (2016): 'the global annual temperature has increased … at an average rate of 0.17°C … per decade'.

So, the mathematics is not doing anything. Why, then, is it in the paper? In general, we think mathematics can serve three different roles in a scientific paper.

The first role is that of providing rigor to an argument. Josiah Willard Gibbs coined the phrase that 'Mathematics is a language' (Samuelson, 1967), a phrase that should be understood as signifying that mathematics is a vehicle for arguing concisely and precisely. It allows one to manipulate long sequences of deductions in a transparent and reproducible way. But it only works if it is used in a precise context in which all concepts and variables are clearly defined. When scientists use the language of mathematics for ill-defined variables (such as E and H above) and employ undefined notations (such as the subscript $A, G, I \to 0$ underneath a function $f(H)$), the language of mathematics adds no advantage over the conventional language of words.

A second role is that of providing a 'heuristic' equation, an equation that is not meant to provide a precise and provable relationship, but rather reflects an approximate pattern. An example is the *IPAT*-equation (Holdren and Ehrlich, 1974) that expresses an approximate tendency that reflects the dependency of environmental impact (*I*) on population size (*P*), affluence (*A*) and technological level (*T*). While its authors admit that application of this equation should be handled with care, it is an equation that can be and has been tested in practice. This is possible because the equation, although heuristic, is defined in operational terms. For instance, it defines T to represent technology in a specific operational way, namely 'damage per unit of consumption' (Holdren and Ehrlich, 1974), which can be made more concrete in specific cases. Did G&S have such a heuristic equation in mind? We do not know, it is not made clear in their paper. By contrast, their paper puts the focus on the equation as such.

A third reason reason for using mathematics is that it can strengthen the rhetorical qualities: it is supposed to increase the status of a paper from a 'merely qualitative' paper into a 'scientifically rigorous' paper. G&S do not use mathematics to derive new conclusions. So, why do they use mathematics at all, instead of providing a verbal synthesis of the literature? We conjecture that they think that mathematics strengthens their point, rather than that it weakens it. Mathematics has the status of a reputed body of knowledge which conveys eternal truths. Pythagoras' theorem is a clear example: which other field of science can boast to have delivered insights that survived for 2500 years?

Our critique of the G&S paper is not confined to the Anthropocene equation. Their figure 2 is another case of meaningless mathematics. While the orbits shown are clearly inspired by the Lorenz attractor, it is not even clear what is on the axes. And equations (5) and (6) are completely built up of undefined symbols. T, for instance, is called 'Technosphere', but the technosphere is not a number but another system, which as such can never enter the list of arguments of a function f. As mentioned, Holdren and Ehrlich (1974) do something similar, but they give an operational meaning to their symbols. This is absent in the G&S paper.

Precisely because the rules of mathematics are unambiguous, its introduction in a primarily verbal paper introduces a vulnerability, as debunking the argument becomes simpler as we have shown above. Even when the overall message is correct, a bad argument, in this case sloppy mathematics, will undermine the validity of the message.

We invite all scientists in the field of sustainability research to use mathematics whenever appropriate and whenever good. Let us collectively retry to do the mathematics of the Anthropocene.

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